For office use only	Team Control Number	For office use only
T1 ________________	**51170**	F1 ________________
T2 ________________		F2 ________________
T3 ________________	Problem Chosen	F3 ________________
T4 ________________	**E**	F4 ________________

2016

MCM/ICM

Summary Sheet

Water scarcity has been a major issue that limits the development of human society, for the decreasing resources could hardly meet the increasing demands.

As for the requirement of first three tasks, we choose the supply/demand ratio to estimate the water scarcity situation. To obtain the water supply and demand, we build two models respectively, considering the dynamic social and natural factors that affect water scarcity.

To construct our water demand model, we first list 6 factors that could affect demands and eliminate the less important factors by performing ***self-organizing data mining***. Then by using ***non-linear regression*** we depict how selected key factors changes as time varies. So at a given time, those factors can be calculated through regression function and water demand can be obtained subsequently. Water supply model employ the same logic. Water supply is composed of two major parts: surface water and ground water, which show obvious correlation with time. So by treating them as ***time series*** and depicting their functions with variable as time, we can easily predict the supply at given time.

Then we adapt our models to Shandong Province in China, which is marked as heavily exploited region (Smakhtin, Revenga and Doll,2004) to carry further analysis. First, we validate our model by comparing the predicted supply and demand with the real data of Shandong Province from 2000-2014. Then we predict the water demand and supply for the next 15 years. As the result shows, Shandong is under severe water scarcity and the increasing water demand mainly comes from domestic consumption and agriculture, which makes sense because Shandong is loaded with large population and always plays a major role in the primary industry.

As for the task 4 and 5, we proposed the intervention plan transferring water from Jiangsu Province to Shandong. According to our analysis, the plan would greatly ease the scarcity in Shandong by increasing the water supply in short-term and benefit the water resources storage in long-term. However, the water transferred would result in the decrease of total water resources in Jiangsu Province, which would reduce the demand and supply at the same time. Finally, we performed stability analysis to the intervention plan, illustrating the possible scarcity situation Shandong might face when Jiangsu is in dry year.

A Model Saving the Thirsty Planet

Contents

1 Symbol Description

D	The water demand
$W_S(t)$	Total surface water at time *t*
ρ_e	The evaporation rate
$P(t)$	The precipitation at time *t*
$C(t)$	The water consumption at time *t*
$W_g(t)$	Total volume of ground water
$E(t)$	The extracted volume of ground water at time *t*
ρ_p	The utilizing efficient of total amount of water resources available
$S(t)$	The amount of water supply at time *t*
R	The water scarce degree
R_1	Index of population growth fits as linear fitting
R_2	Index of population growth fits as square polynomial fitting
R_3	Index of population growth fits as cubic polynomial fitting
P_1	Primary Industry Index fits as linear fitting
P_2	Primary Industry Index fits as square polynomial fitting
P_3	Primary Industry Index fits as cubic polynomial fitting
T_1	Tertiary Industry Index fits as linear fitting
T_2	Tertiary Industry Index fits as square polynomial fitting
T_3	Tertiary Industry Index fits as cubic polynomial fitting
$Tuple_{i,j,k}$	The combination of three different factors
$\triangle$ D	Increased water demand in Shandong after water transferred
β	Coefficients reflexes the different factors' ability to affect water demand

2 Basic Assumptions

Assumption 1. The water demands of certain region is affected by six factors: population, primary industry, secondary industry, tertiary industry growth and irrigated area and total water resource. The ability of these factors effects water demand varies in different areas, but we assume the ability doesn't change in certain area as time changes.
Assumption 2. The precipitation is random.

3 Supply/Demand Model

3.1 Overview

Many indices and methodologies are built to evaluate water scarcity. (Brown, 2011) But considering the fact that the essence of this issue is the relationship between supply and demand. Here we choose the ratio of supply/demand as the index to evaluate the ability of region to provide clean water. Since there are many equally important facets to water use and supply, quantification the effects of those dynamic factors to water scarcity would be the major difficulty to analysis, which is also the focus of our modeling process.

3.2 Water Demand Model

Human consumption for water can be divided into two categories: domestic purpose and GDP growth, which are affected by six factors: population, primary industry, secondary industry, tertiary industry growth and irrigated area and total water resource according to our assumption. Apart from which, environmental water requirements (EWR) (Smakhin, et al,2005) should also be considered, as it is required for ecosystem maintain and tapping into EWR would be regarded as overexploit.

The basic idea of our water demand model is: we want to build a function describing the change of water demand with in certain region as time goes by. Since the EWR are usually constant within a region, we only need to consider correlation between the water demands with the six factors that affect human consumption in certain area. Here is the demand function:

$$\mathrm{D} = \alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6$$

D is the water demand, while $x_1(t)$ $x_2(t)$ $x_3(t)$ $x_4(t)$ $x_5(t)$ $x_6(t)$ each stands for population, primary industry index, secondary industry index, tertiary industry index, irrigated area, total water resource. α, $\beta_i(i = 1,2 \dots 5)$ are the coefficients, that reflexes the different factors' ability to affect the water demand. Because every region has its own characteristic, the coefficients are totally different in different area. For example, in a developing country where industry is less developed, the agriculture might be the major consumer of water. So β_2 and β_5, the coefficients of primary industry index and irrigated would be our prior concern while β_3 corresponding to secondary industry index would be less important and neglected.

First we standardize the data by calculating its relative growth, in order to exclude the effect causing by different dimension There is no need to standardize total water resource, because its absolute value has critical influence on the demand of water.

$$x_i(t) = \frac{x_i(t)}{x_i(t-1)} \times 100$$
$$\mathrm{i} = 1,2 \dots 5$$

Then we employ the method of the self-organizing data mining into the construction of our multi-driving forces model. We divide data into training set and testing set. We use data in training set to set up linear regression, then use data in testing set to make competition between coefficients and eliminate some of them that make no sense.

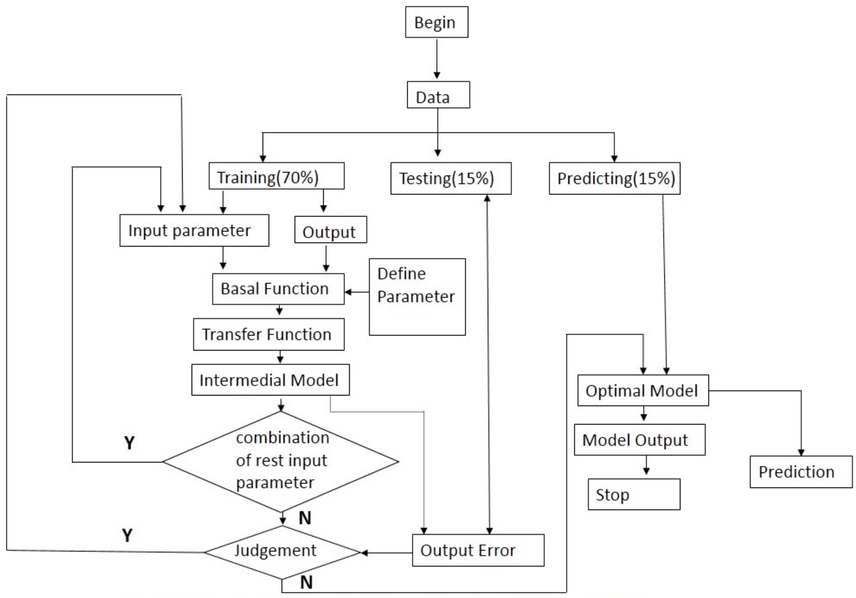

Figure 1, Flow diagram of Self-organizing data mining algorithm

Thus we will get all the α and β_i in linear regression function. These coefficients can express the difference in all kinds of areas. As a result, given the specific data of six factors that determine the water demand, we can get every single year's predicted demand value.

3.3 Water Supply Model

Water supply is closely related to two aspects: natural factors and human factors. First let's start with natural factors. The freshwater can be divided into two separately parts, surface water and ground water.

Surface water consists of lakes and rivers and rainfalls. These are dynamic and associated with time, since wet year or mean year or dry year will change the volume randomly. Then we have function of surface water,

$$W_S(t) = (1-\rho_e)W_S(t-1) + (1-\rho_{p1})P(t) - C(t-1)$$

$W_S(t)$ denotes total surface water at time *t*, while $\rho_{p1}, \rho_e, P(t),\ C(t),$ respectively stands for exploitable rate of precipitation, evaporation rate, precipitation at time *t* and water consumption at time *t*.

Ground water is often hard to replenish, because the volume of infiltration can be very small. So the amount of ground water largely depends on the amount stored last year.

$$W_g(t) = \rho_1 P(t) + (1-\rho_{p2})W_g(t-1) - E(t-1)$$

$W_g(t)$ denotes total volume of ground water, while $\rho_1,\ \rho_{p2},\ E(t-1)$ respectively stand for infiltration rate, exploitable rate of ground water at time *t-1* and extracted volume of ground water at time *t-1*.

After discussion of nature factors now let's focus on human factors. To keep sustainable development, we mustn't run out of all the water resources at one time, thus proper policy must be made to restrict the use of water. The following efficient ρ_p denotes the affect of policy towards the usage of total water resources $W(t)$ at time *t*.
So we get the supply model,

$$W(t) = W_s(t) + W_g(t)$$
$$\mathrm{S(t)} = \rho_p W(t)$$

$S(t)$ denotes the amount of water supply at time *t* and ρ_p represents the utilizing efficient of total amount of water resources available at time *t*.
According to this model, if a year is a wet year, surface water will increase because of the increase of precipitation and storage. What's more, total ground water will also increase and has a positive effect on next year's total ground water. On the other hand, if a year is a dry year, total surface water and total ground water will both decline due to the decrease of rainfall. Next year's total ground water will decrease, too.

After building both demand and supply model, we can finally derive our evaluation standard from both two model.

R= Supply / Demand

Water Scarce Degree	R
Dry Region	R<1
Mean Region	R=1
Wet Region	R>1

4 Analysis of Water Scarcity Situation in Shandong Province

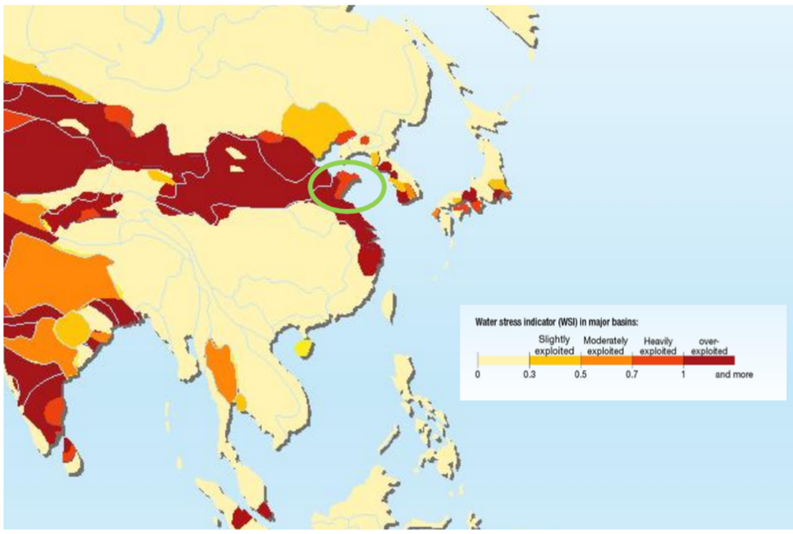

Figure : The water scarcity situation of Shandong Province in China (highlighted with green circle) (adapted from Smakhtin, Revenga and Doll,2004)

As for Task 2 we choose Shandong province in eastern China to analysis its water scarcity situation. The shortage of water resources is the basic situation in Shandong province, is the key constraint that restricts both economic and social development. Usually, Shandong province is regarded as the key region in domestic agriculture production. So the problem has enough influence to affect China domestic production.

4.1 Water demand in Shandong Province

Water demand and associated factors from 2000 to 2014

Year	Population	Primary Industry Index	Secondary Industry Index	Tertiary Industry Index	Irrigated Area	Total Water Resource	Water Demand
2014	100.5754	103.8	109.2	108.9	103.6566	148.44	214.52
2013	100.4956	103.6	110.5	109.5	93.49401	291.7	217.94
2012	100.4981	104.7	110.5	109.8	101.4283	274.08	221.79
2011	100.5111	104	111.7	111.3	100.6373	374.01	224.05
2010	101.246	103.6	112.8	113.5	101.1922	309.12	222.74
2009	100.5628	104.2	113.9	111.2	100.8119	284.95	219.99
2008	100.5338	105.1	112	113.9	100.428	328.71	219.89
2007	100.6231	104	115.8	114.6	100.3865	387.11	219.55
2006	100.6596	105.2	116.6	114.5	100.5887	199.78	225.83
2005	100.7407	104.8	117.4	114.4	100.4856	415.86	211.03
2004	100.6027	106.9	119.3	112.3	100.1264	349.46	214.88
2003	100.4735	105.6	116.8	111.4	99.23605	489.69	219.34
2002	100.4535	102.5	115	110.9	99.2006	98.14	252.39
2001	100.4779	104.2	111	111.2	100.233	238.81	251.61

First we evaluate the water demand of Shandong Province, using the data from National Bureau of Statistics and Shandong statistical yearbook. By adapting our water demand model, we successfully calculate three coefficients for the three key factors and eliminate other three less important factors by performing non-linear regression.

Coefficient	α	β_1	β_2	β_3	β_4	β_5	β_6
Value	412.4543	1.2282	1.8094	0	-4.4639	0	0

Putting the value of six factors back to the water demand function, we got the predicted curve. By comparing the prediction value with actual water demand, we can draw the conclusion that the fitting result is very close to the actual value. So the coefficients make sense.

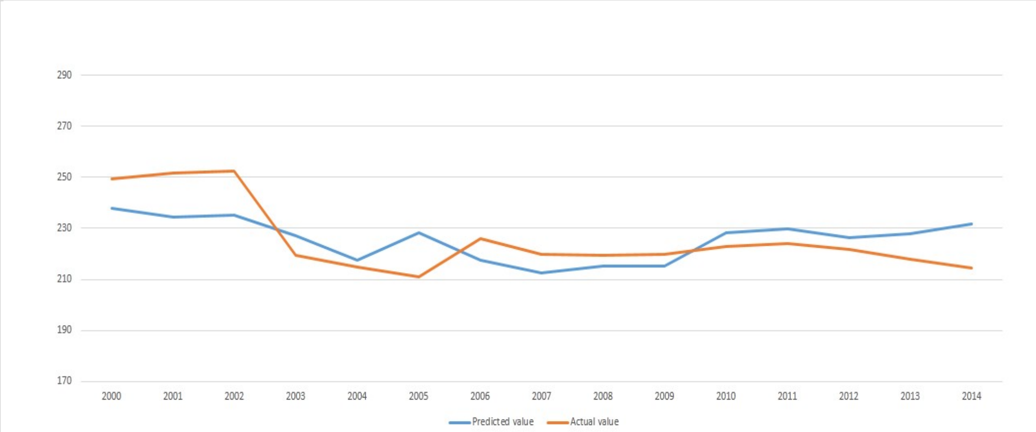

Figure 3 : The predicted water demand(blue) compared with actual demand(red) of Shandong Province

Shandong is a populous province with about 100 million population. Large population base leads to a gigantic increase in population when population growth rate is high. So the first reason is population issue. Agriculture accounts for a significant proportion of the economy by the rise of tertiary-industry, land use benefit numerical value will come up to a high point, which results in a process of economic transformation development. So primary industry which centers on agriculture increase the use of water, while tertiary industry decreases the demand of water by transferring economy growth way.

4.2 Prediction of Water Demand

Based on the coefficients calculated we get the function describing the water demand in Shandong Province,

$$\mathrm{D} = 412.4543 + 1.2282x_1 + 1.8094x_2 - 4.4639x_4$$

As we can see, water demand in Shandong province largely depends on three factors which are population growth, primary Industry and tertiary Industry. By performing non-linear regression we can depict the relationship between those three factors and time. We can get the water demand in certain year when we put the predicted factor back to equation(n).

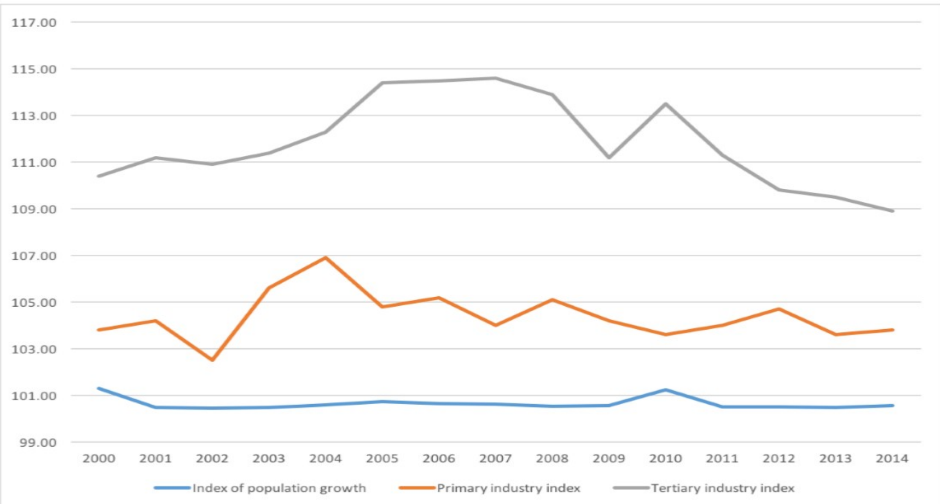

Figure 4 : The change of indices of population growth, primary industry, tertiary industry of Shandong Province

According to the characteristics of data, we decide to use polynomial fitting to determine each factor according to the change of time and predict these three factors' value. We place plots which is changed over time into linear fitting, square polynomial fitting and cubic polynomial fitting respectively.

Here is the R-Square table,

	Linear Fitting	Square Polynomial Fitting	Cubic Polynomial Fitting
Index of Population Growth	0.167	0.882	0.901
Primary Industry Index	0.119	0.891	0.911
Tertiary Industry Index	0.231	0.918	0.972

Then we permutate and combine three different fitting results of three factors to calculate the mean error in the forecast.

R_1 illustrates Index of population growth fits as linear fitting;
R_2 illustrates Index of population growth fits as square polynomial fitting;
R_3 illustrates Index of population growth fits as cubic polynomial fitting;
P_1 illustrates Primary Industry Index fits as linear fitting;
P_2 illustrates Primary Industry Index fits as square polynomial fitting;
P_3 illustrates Primary Industry Index fits as cubic polynomial fitting;
T_1 illustrates Tertiary Industry Index fits as linear fitting;
T_2 illustrates Tertiary Industry Index fits as square polynomial fitting;
T_3 illustrates Tertiary Industry Index fits as cubic polynomial fitting.

$Tuple_{i,j,k} = (R_i, P_j, T_k)$ $i, j, k \in \{1, 2, 3\}$ means combination of three different factors

Tuple	(R_1, P_1, T_1)	(R_1, P_1, T_2)	(R_1, P_1, T_3)	(R_1, P_2, T_1)	(R_1, P_2, T_3)	(R_1, P_3, T_1)	(R_1, P_3, T_2)
Mean error (%)	27.81	20.14	11.41	18.67	7.88	12.22	16.54
Tuple	(R_1, P_3, T_3)	(R_2, P_1, T_1)	(R_2, P_1, T_2)	(R_2, P_1, T_3)	(R_2, P_2, T_1)	(R_2, P_2, T_2)	(R_2, P_2, T_3)
Mean error (%)	17.86	16.98	10.01	6.12	6.91	5.48	2.92
Tuple	(R_2, P_3, T_1)	(R_2, P_3, T_2)	(R_2, P_3, T_3)	(R_3, P_1, T_1)	(R_3, P_1, T_2)	(R_3, P_1, T_3)	(R_3, P_2, T_1)
Mean error (%)	14.49	14.12	6.88	19.61	21.33	17.83	16.87
Tuple	(R_3, P_2, T_2)	(R_3, P_2, T_3)	(R_3, P_3, T_2)	(R_3, P_3, T_3)			
Mean error (%)	14.68	19.18	22.17	26.91			

From the above table, we can derive:

$$\min(Tuple_{i,j,k}) = (R_2, P_2, T_3)$$

So we use R_2, P_2, T_3 to fit,

$$x_1(t) = -0.0017t^2 - 0.0382\text{t} + 100.81$$
$$x_2(t) = -0.0253t^2 + 0.03685\text{t} + 103.54$$
$$x_4(t) = -0.0003t^3 - 0.0859t^2 + 1.5353t + 108.56$$

So at given time we can calculate factors above and put them back to the water demand function and get the predicted demand volume at that year.

Here is the predicted result,

Year	Index of population growth	Primary Industry Index	Tertiary Industry Index	Water Demand/10^8m^3
2015	100.63	103.5988	102.414	266.3387929
2016	100.6519	104.0432	102.666	266.0399723
2017	100.6732	104.5382	102.926	265.8011719
2018	100.6979	105.0838	103.194	265.6223919
2019	100.726	105.68	103.47	265.5036322
2020	100.7575	106.3268	103.754	265.4448928
2021	100.7924	107.0242	104.046	265.4461738
2022	100.8307	107.7722	104.346	265.507475
2023	100.8724	108.5708	104.654	265.6287966
2024	100.9175	109.42	104.97	265.8101385
2025	100.966	110.3198	105.294	266.0515007
2026	101.0179	111.2702	105.626	266.3528833
2027	101.0732	112.2712	105.966	266.7142861
2028	101.1319	113.3228	106.314	267.1357093
2029	101.194	114.425	106.67	267.6171528

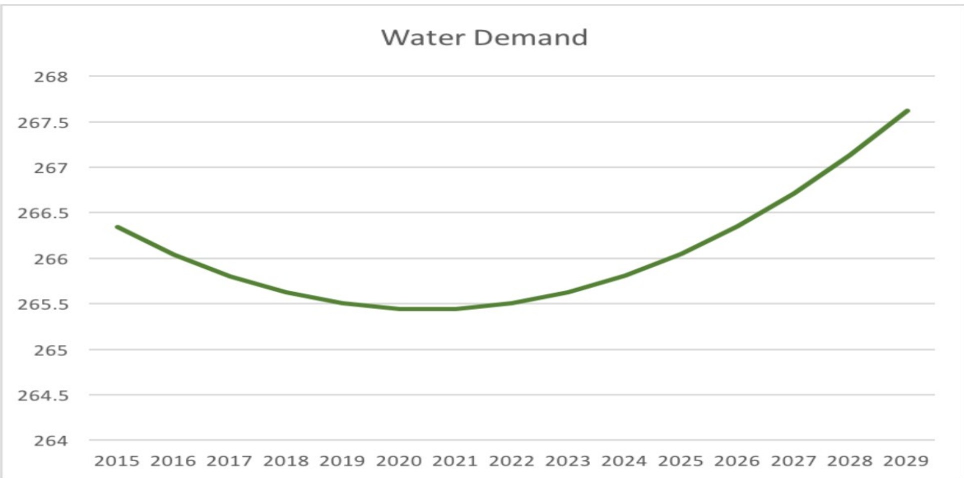

Figure 5 : Predicted water demand of Shandong Province from 2015-2029

4.3 Water Supply in Shandong Province

Water Supply in Shandong Province

Year	Water Supply (100 million/cu.m)	Surface Water (100 million/cu.m)	Ground Water (100 million/cu.m)
2000	246.21	114.40	131.81
2001	249.31	115.60	133.71
2002	250.63	117.66	132.96
2003	218.07	104.12	113.95
2004	213.68	106.28	107.40
2005	209.37	106.70	102.67
2006	223.67	119.77	103.90
2007	217.57	115.59	101.98
2008	216.74	115.51	101.23
2009	216.67	119.62	97.05
2010	218.46	127.15	91.31
2011	216.67	127.33	89.34
2012	215.38	126.12	89.26
2013	211.80	124.94	86.86

It is obvious to see that ground water supply decrease over time. Since water scarcity is a big problem, and government make effort to reduce the use of ground water, the lack of which will result in long-run economic decline. However, in order to keep the economy working and developing smoothly, water supply shouldn't decline suddenly. It can be seen that the government increase the use of surface water. So this is the trend that need to be predicted.

So we first establish the ground water trend.

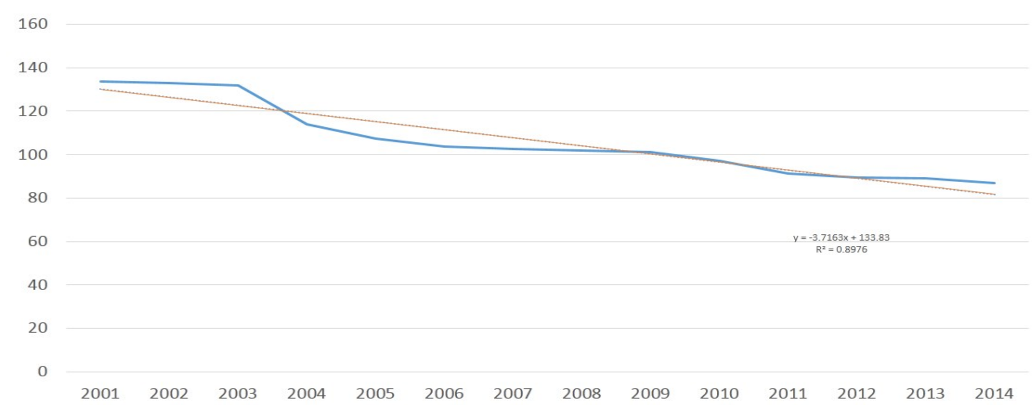

Figure 6: The changes of ground water supply in Shandong Province(blue denotes the actual data and red are predicted one)

It can be leant from the figure that the linear regression fit well. So we use this linear function to predict the value of ground water supply.

Next, we establish the relationship between surface water supply and time.

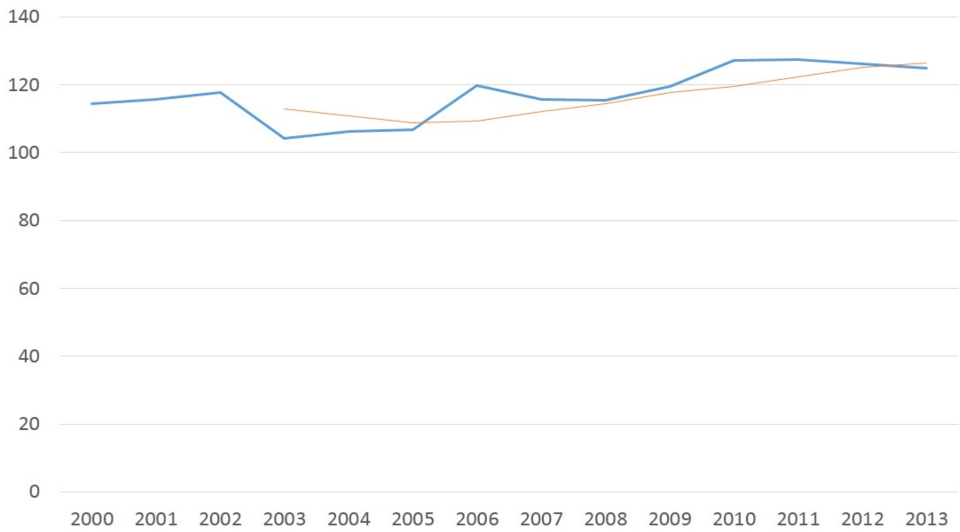

Figure 7: The changes of surface water supply in Shandong Province(blue denotes the actual data and red are predicted one)

Based on the figure, surface water supply moves generally upward. Its situation of growth is stepladder-like. When it reaches a certain level, it always spends time remaining there. After a time period, it finally increases. So we can use moving average method. Define the time period by 4 years. Then we can get the trend between time and surface water supply. It is clearly linear fitting relationship. As a result, we can predict the moving average value and resolve it to get the predicted surface water supply.

Finally, we can get total water supply curve.

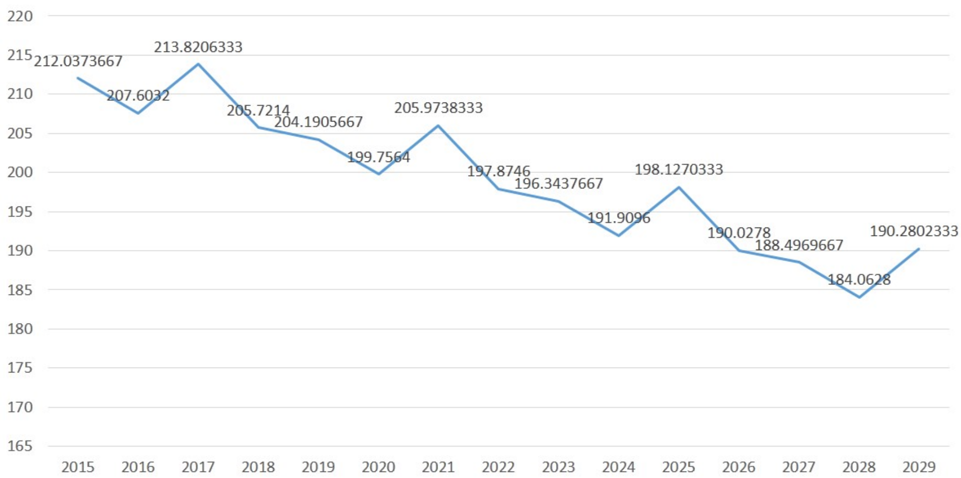

Figure 8: Predicted water supply in Shandong Province from 2015-2029

Because ground water supply decrease over time, while surface water supply's increment speed is too slow to catch up. Because surface water supply's increase is not uniform, when it steps up, there will be a sharp increase in surface water supply. But when it stays in a certain level, the total water supply just decline.

5 Intervention Plan Design

How to fill the gap? Chinese government has already come up with a plan called south-to-north water diversion, taking the advantage of the transportation pipeline. Jiangsu province is located in Yangtze River basin and is the start of south-to-north water diversion. In order to fill the gap in Shandong province, we figure out an intervention plan to transfer water in Yangtze River to Shandong. In this part we mainly evaluate the effect of this plan and discuss the effects for both provinces.

5.1 Improvement to Water Scarcity in Shandong Province

First we figure out the water transferring effect on Shandong province.
As for **water demand** of Shandong province, we have its related factors,

$$\mathrm{D} = \alpha + \beta_1 x_1 + \beta_2 x_2 - \beta_4 x_4$$

Population and Tertiary Industry will not be affected by transferring water, while agriculture depends a lot on water. So transferring water will lead to an increase in x_2, thus increase total water demand.

$$\vartriangle \mathrm{D} = \beta_2 \vartriangle x_2$$
$$\mathrm{D(t+1)} = \mathrm{D(t)} + \vartriangle D$$

The increase in demand depends on the region's ability to turn water into agriculture production. And it maintains a nature of diminishing marginal utility. So the region cannot just accept transferring water as much as possible. On the other hand, if cost per transferring water exceed the margin revenue in agriculture, there will be only certain amount of transferring water to meet the basic need of water. Since agriculture in Shandong province is fully loaded, the margin revenue is low.

As for **water supply** of Shandong province, our purpose of the intervention plan is to fill the gap. What's more, the cost of water transfer, including both engineering cost and transportation fee, is so large that the government cannot take transfer water into daily supply account. So the transferring water will not change the total water supply. This is short-run effect. There is also long-run effect which possibly lead to an increase in water supply and in turn reduce its own flow of transferring water.

$$\mathrm{S} = \rho_p W(t)$$

By using transferring water, the government need not to increase ρ_p to exploit more freshwater for demand. And if here comes large rainfall, the ground water or the surface water can be replenished.

5.2 Intervention Plan Effect on Jiangsu Province

Then, let's figure out the transferring plan's effect on Jiangsu Province.
Use our established total water demand model and the same algorithm, we can derive total water demand function based on the data table below.

$$D = -264.7636 + 19.2226x_2 - 10.8641x_3 + 0.1036x_6$$

Water demand and associated factors of Jiangsu Province

	Index of Population Growth	Primary Industry Index	Secondary Industry Index	Tertiary Industry Index	Irrigated Area	Total Water Resource	Water Demand
2000	100.4367408	103	110.9	111.5	99.97795352	492.4	556.2
2001	100.6386737	102.8	113.7	111.6	99.64230678	268.02	478.75
2002	100.7021334	99.9	117.2	112.4	98.84046484	619.1	421.5
2003	100.8715473	106	117.1	113.4	99.94897136	204.03	514.6
2004	100.864017	102.9	116	114.9	99.44386849	466.96	519.72
2005	100.8961518	105	116	115.5	100.5251894	404.4	546.38
2006	100.8751306	103.1	115.5	116.4	99.93303316	495.71	558.34
2007	100.5049851	104	113.2	113.4	99.52935348	378	558.32
2008	100.6183973	104.5	112.5	113.6	99.90987923	400.31	549.23
2009	100.7554417	104.9	113.1	113.3	100.1594269	383.5	552.19
2010	100.3812429	104	111.7	111.1	99.95235278	492.4	556.17
2011	100.2658564	104.6	111.1	109.7	102.928296	373.33	552.23
2012	100.239899	102.9	110	109.8	96.32416559	283.53	576.69
2013	100.2645169	103	108.2	110	102.7807792	399.34	591.29
2014	100.4367408	103	110.9	111.5	99.97795352	492.4	556.2

Use our established total water demand model and the same algorithm, we can derive total water demand function based on the data table below.

$$D = -264.7636 + 19.2226x_2 - 10.8641x_3 + 0.1036x_6$$

And we can use the same method to predict Primary Industry Index、Secondary Industry Index、Total Water Resource and Total Water Demand in 15 years.

	Primary Industry Index	Secondary Industry Index	Total Water Resource	Water Demand
2015	105.2580502	107.6235	385.8640979	604.4670311
2016	105.3225887	106.618	384.5500936	616.4928336
2017	105.3832133	105.9907	383.3157763	624.3395169
2018	105.4403718	104.3416	382.152031	643.2397884
2019	105.494439	102.9707	381.0512224	659.0623569
2020	105.5457323	101.378	380.0068909	677.2501352
2021	105.5945224	100.0635	379.0135232	692.3701504
2022	105.6410425	101.5272	378.0663756	677.2385906
2023	105.6854942	101.7691	377.1613378	675.3592685
2024	105.7280538	102.3892	376.294824	669.3350007
2025	105.7688758	103.5875	375.4636882	656.9933793
2026	105.8080965	104.164	374.6651545	651.3868287
2027	105.8458369	105.1187	373.8967614	641.6422925
2028	105.8822045	105.6516	373.1563162	636.4616711
2039	105.9172958	106.0627	372.4418569	632.5840633

The gap between supply and demands of Shandong Province in task 3 is shown on the figure below, which is also the basic volume that are required to be transferred from Jiangsu to Shandong.

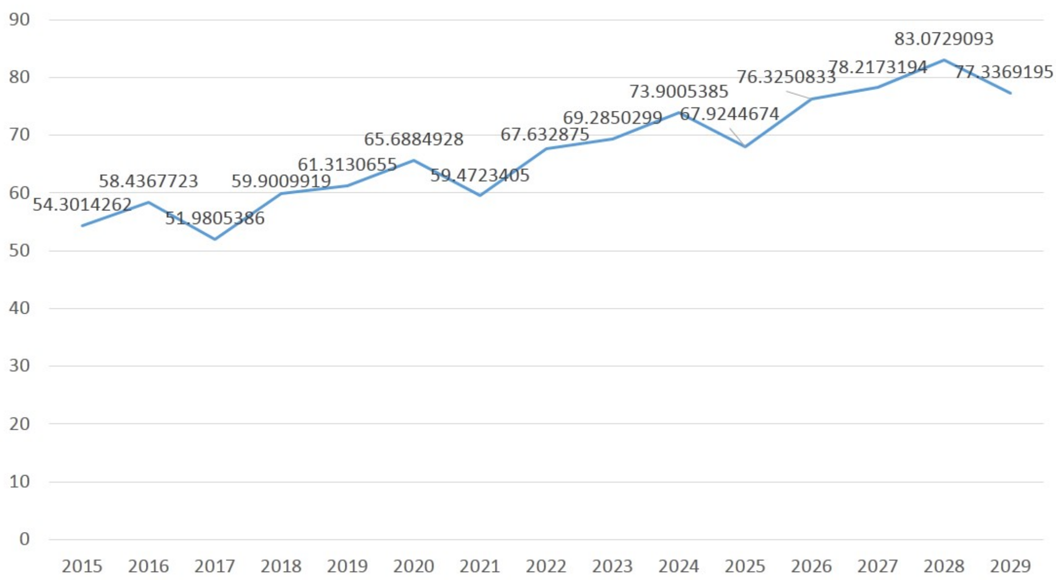

Figure 9: The gap between predicted supply and demand in Shandong Province from 2015-2029

The total water resource just decreases as much as the transferring water flow,

$$\beta_6 \triangle x_6 = \triangle D$$

So we can calculate total water demand after water transfer in 15 years.

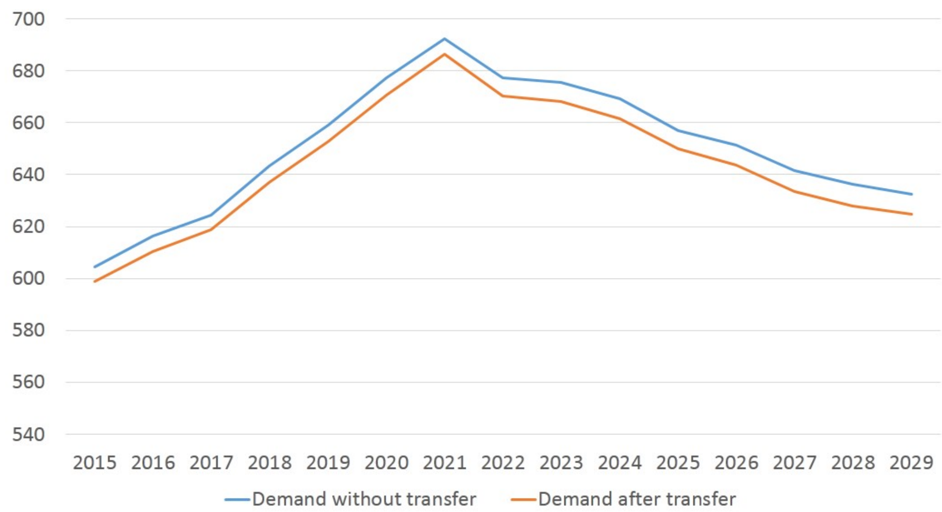

Figure 10 : The total water demand of Jiangsu Province with and without water transfer

Water transfer will decrease total water demand in Jiangsu province.

For supply, we collect data about surface water supply and ground water supply.

	Surface Water Supply	Ground Water Supply
2014	574.7	9.69
2013	567.36	9.33
2012	542.41	9.83
2011	546.1	10.07
2010	543.52	8.67
2009	540.4	8.83
2008	548.65	9.67
2007	548.45	9.88
2006	535.66	10.72
2005	508.89	10.83
2004	515.1	10.48

The transferring water will reduce surface water supply. So the government can choose either to increase ground water supply or to increase ρ_p for surface water. Each of both choices will decrease water supply for next year. Since total water demand also decrease, it still has the opportunity to keep S/D ratio a good place.

5.3 Stability Analysis to Intervention Plan

Basically, Jiangsu province has extra water resources in every year and it can be used to help Shandong province releasing water scarce pressure. But there isn't always enough water to fill the gap every year, since Jiangsu might run into water shortage situation occasionally. This kind of situation should draw enough attention.

Consider the year *t*, when the transfer water cannot fill the gap.

(1) The government choose to increase ρ_{pi} to increase supply to meet the need.
Because the supply can guarantee the demand, the water consumption remains high level. And we regard the precipitation as its mean value.

$$\begin{cases} w_g(t+1) = \rho_1 E[P(t)] + (1-\rho_{p2})w_g(t) - C(t) \\ w_c(t+1) = (1-\rho_e)E[P(t)] + (1-\rho_{p1})w_c(t) - E(t) \end{cases}$$

$$\mathrm{S} = \rho_{p2} w_{g+} \rho_{p1} w_c$$

$$w_g(t+2) = \rho_1 E[P(t)] + \left(1-\rho_{p2}\right)\{\rho_1 E[P(t)] + (1-\rho_{p2})w_g(t) - C(t)\} - C(t+1)$$
$$w_c(t+2) = (1-\rho_e)E[P(t)] + \left(1-\rho_{p1}\right)\{(1-\rho_e)E[P(t)] + (1-\rho_{p1})w_c(t) - E(t)\} - E(t+1)$$

$$\vdots$$

$$w_g(t+n) = \sum_{i=0}^{n-1}(1-\rho_{p2})^i \rho_1 E[P(t)] + \left(1-\rho_{p2}\right)^n w_g(t) - \sum_{i=0}^{n-1}\left(1-\rho_{p2}\right)^{n-1-i} C(t+i)$$

$$w_c(t+n) = \sum_{i=0}^{n-1}\left(1-\rho_{p1}\right)^i (1-\rho_e)E\,[P(t)] + \left(1-\rho_{p1}\right)^n w_c(t) - \sum_{i=0}^{n-1}\left(1-\rho_{p1}\right)^{n-1-i} E(t+i)$$

Because $1-\rho_{pi}$ decreases and the use of freshwater increases, which results in a decrease in restore. When it is year *t+2*, the reduction will be conducted into this year. So the water resource will constantly decrease over time and is hard to recover.

(2) The government choose to ignore the exceeded demand and let demand reduce over time.

$$\mathrm{D} = \alpha + \beta_1 x_1 + \beta_2 x_2 - \beta_3 x_3$$

First, this method will reduce the use of domestic purpose. It may cause livelihood issue and make life more difficult for vulnerable groups, raising fears of social tension. Second, it will restrict the development of agriculture. Since Shandong province is an important agricultural base, some shifts in water may cause a wild scale of effects. Third, it may be a chance for transformation development. It will undoubtedly increase the ability to resist water scarce. Although it may cause social problems, people become easier to adopt and make economy more resistant.

6 Strengths and Weaknesses

6.1 Strengths

- **Intelligence:** Model is based on intelligent algorithm, has self-adaption. So it has good resilience to all kinds of environment.
- **Comprehensiveness:** Model considers both temporal and spatial dynamics.
- **Simplicity:** The coefficients in our model is easy to calculate and test.
- **Predictability:** Based on our model, we successfully forecast the value in 15 years.

6.2 Weaknesses

- **Data limitation:** The data obtained are spatial-located, resulted in the inability of our model to predict water scarcity in other region.

7 Reference

- Brown A, Matlock M D. A review of water scarcity indices and methodologies[J]. White paper, 2011, 106.
- Liu Dedi; Chen Xiaohong; Lou Zhanghua, Analysis on driving forces of water resources demand and its forecast. WATER RESOURCES AND HYDROPOWER ENGINEERING, 2010,41(3)
- IWMI. "Areas of physical and economic water scarcity." UNEP/GRID-Arendal Maps and Graphics Library,2008.
- Alcamo, Joseph, Thomas Henrichs, and Thomas Rosch. World Water in 2025: Global modeling and scenario analysis for the World Commission on Water for the 21st Century. Kassel World Water Series Report No. 2, Center for Environmental Systems Research, Germany: University of Kassel, 2000, 1-49.
- Falkenmark. "The massive water scarcity threatening Africa-why isn't it being addressed." Ambio 18, no.2 (1989): 112-118.
- Rijsberman, Frank R. "Water scarcity: Fact or Fiction?" Agricultural Water Management 80 (2006): 5-22.
- Raskin, P, P Gleick, P Kirshen, G Pontius, and K Strzepek. Waer Futures: Assessment of Long-range Patterns and Prospects. Stockholm, Sweden: Stockholm Environment Institute, 1997.
- Pfister, Stephan, Annette Koehler, and Stefanie Hellweg. "Assessing the Environmental Impacts of Freshwater Consumption in LCA." Environmental Science & Technology (American Chemical Society) 43 (2009): 4098-4104.
- [http://www.stats.gov.cn/] - National Bureau of Statistics of the People's Republic of China
- [http://www.sdwr.gov.cn/] - Water Resources Department of Shandong Province
- [http://xxgk.stats-sd.gov.cn/] - Shandong Statistical Information Net