CS 141 F20 Discussion Class

Week 1

$O, \Omega, \Theta, o, \omega$ notations

Functions	Real numbers
$f(n) = O(g(n))$	$a \leq b$
$f(n) = \Omega(g(n))$	$a \geq b$
$f(n) = \Theta(g(n))$	$a = b$
$f(n) = o(g(n))$	$a < b$
$f(n) = \omega(g(n))$	$a > b$

Asymptotical Bounds

$f(n) = \log^2 n$

$g(n) = \log\log n$	**$g(n) = \log^2 n$**	**$g(n) = n \log n$**
$\log^2 n \mathrel{!\in} O(\log\log n)$	$\log^2 n \in O(\log^2 n)$	$\log^2 n \in O(n \log n)$
$\log^2 n \mathrel{!\in} \Theta(\log\log n)$	$\log^2 n \in \Theta(\log^2 n)$	$\log^2 n \mathrel{!\in} \Theta(n \log n)$
$\log^2 n \in \Omega(\log\log n)$	$\log^2 n \in \Omega(\log^2 n)$	$\log^2 n \mathrel{!\in} \Omega(n \log n)$
$\log^2 n \mathrel{!\in} o(\log\log n)$	$\log^2 n \mathrel{!\in} o(\log^2 n)$	$\log^2 n \in o(n \log n)$
$\log^2 n \in \omega(\log\log n)$	$\log^2 n \mathrel{!\in} \omega(\log^2 n)$	$\log^2 n \mathrel{!\in} \omega(n \log n)$

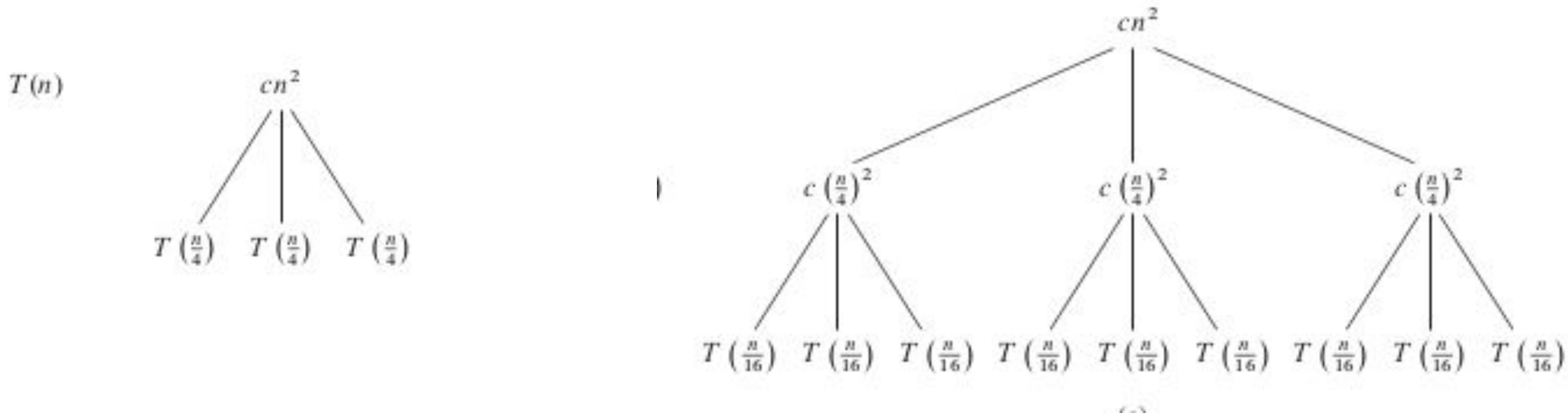
Asymptotical Bounds

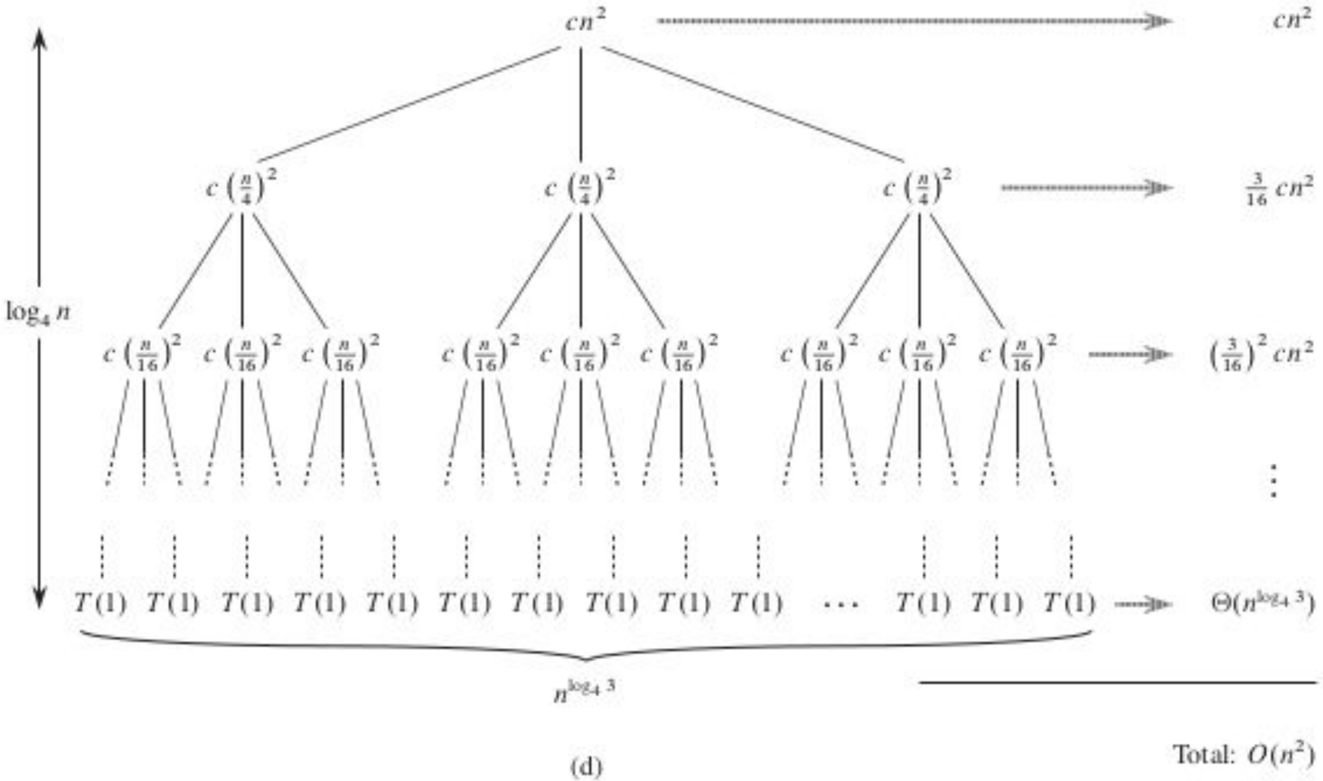
Indicate, for each pair of expressions (A, B) in the table below, whether A is **O**, **o**, **Ω**, **ω**, or **Θ**, of B. Assume that k >= 1, e > 0, and c > 1 are constants. Your answer should be in the form of the table with “yes” or “no” written in each box.

A	B	O	o	**Ω**	**ω**	**Θ**
$\lg^k n$	n^e	Yes	Yes	No	No	No
n^e	c^n	Yes	Yes	No	No	No
sqrt(n)	$n^{\sin n}$	No	No	no	no	no
2^n	$2^{n/2}$	No	No	Yes	Yes	No
$n^{\lg c}$	$c^{\lg n}$	Yes	No	yes	No	Yes
$\lg(n!)$	$\lg(n^n)$	Yes	Yes	No	No	No

Solving Recurrence

$T(n) = 3T(n/4) + cn^2$




cn^2
$c\left(\frac{n}{4}\right)^2$
$c\left(\frac{n}{16}\right)^2$
$T(1)$
$\log_4 n$
$n^{\log_4 3}$
cn^2
$\frac{3}{16}cn^2$
$\left(\frac{3}{16}\right)^2 cn^2$
$\Theta(n^{\log_4 3})$
Total: $O(n^2)$

(d)

T(n) = T(n/2) + n^2

$$T(n) = T(n/2) + n^2$$

$$= \{T(n/4) + (n/2)^2\} + n^2$$

$$= T(n/2^2) + (1/4)n^2 + n^2$$

$$= \{T(n/2^3) + (n/4)^2\} + (1/4)n^2 + n^2$$

$$= T(n/2^3) + (1/4^2)n^2 + (1/4)n^2 + n^2$$

$$= T(n/2^i) + (1/4^{i-1})n^2 + (1/4^{i-2})n^2 + ... + (1/4^2)n^2 + (1/4)n^2 + n^2$$

$$= T(n/2^{log\ n}) + 1/4^{log\ n}n^2 + ... + (1/4^2)n^2 + 1/4n^2 + n^2$$

$$= T(1) + n^2 * \frac{1 - (1/4)^{log\ n}}{1 - 1/4}$$

$$= c + (4/3)n^2 * -(4/3)n^2(1/4)^{log\ n}$$

$$= c + (4/3)n^2 - (4/3)n^2 * \frac{1}{2^{2log\ n}} = c + (4/3)n^2 - (4/3)n^2 * \frac{1}{2^{log\ n^2}}$$

$$= c + (4/3)n^2 - (4/3)n^2 * \frac{1}{n^2} = c + (4/3)n^2 - (4/3)$$

$$\leq (4/3)n^2$$

$$= \Theta(n^2)$$

T(n) = 2T(n-1) + 1

$$T(n) = 2T(n-1) + 1$$
$$= 2\{2T(n-2) + 1\} + 1$$
$$= 2^2T(n-2) + 2 + 1$$
$$= 2^2\{2T(n-3) + 1\} + 2 + 1$$
$$= 2^3T(n-3) + 2^2 + 2 + 1$$
$$= 2^iT(n-i) + 2^{i-1} + 2^{i-2} + ... + 2^2 + 2 + 1$$

$$= 2^{n-1}T(1) + 2^{n-1} + ... + 2^2 + 2 + 1$$
$$= c2^{n-1} + 2^n - 1$$
$$\leq c2^n = \Theta(2^n)$$

Substitution Method

$T(n) = T(n/2) + n^2$

Substituting $T(n) \leq cn^2$

$$\begin{aligned} T(n) &\leq c(n/2)^2 + n^2 \\ &= (c/4 + 1)n^2 \\ &\leq cn^2 \end{aligned}$$

Substitution Method

$$T(n) = 2T(n-1) + 1$$

Substituting $T(n) \leq 2^n$

$$\begin{aligned} T(n) &= 2T(n-1) + 1 \\ &\leq 2(2^{n-1} + 1) + 1 = 2^n + 3 \\ &\leq 2^n \end{aligned}$$