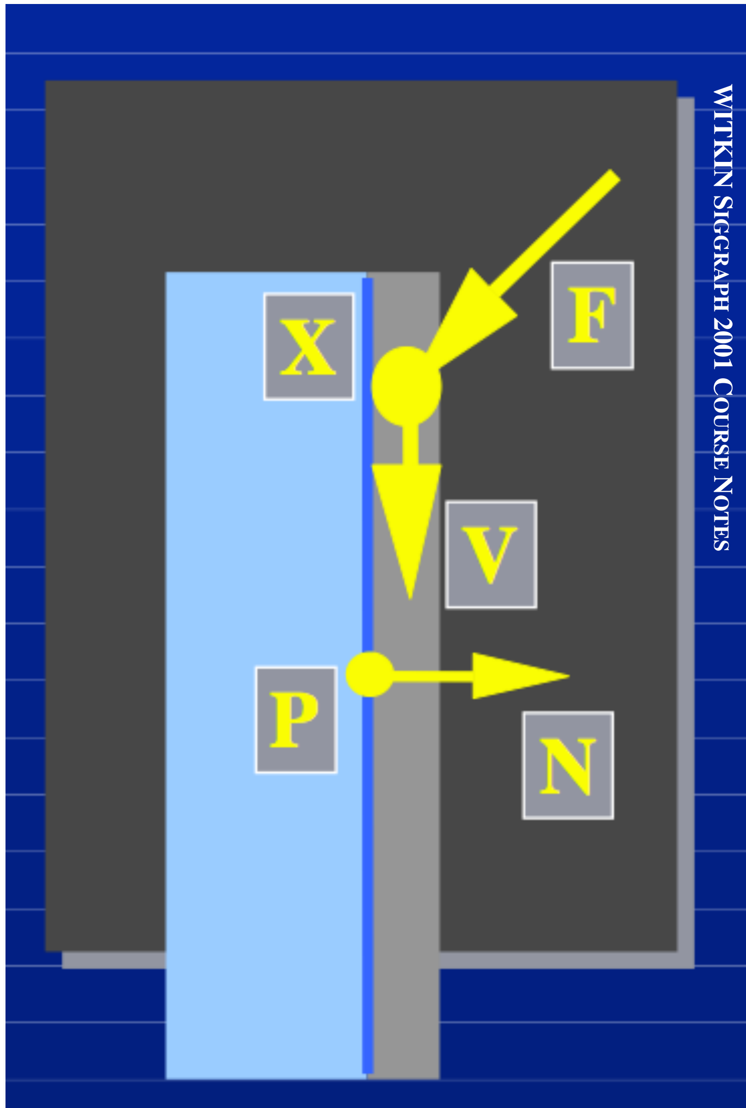
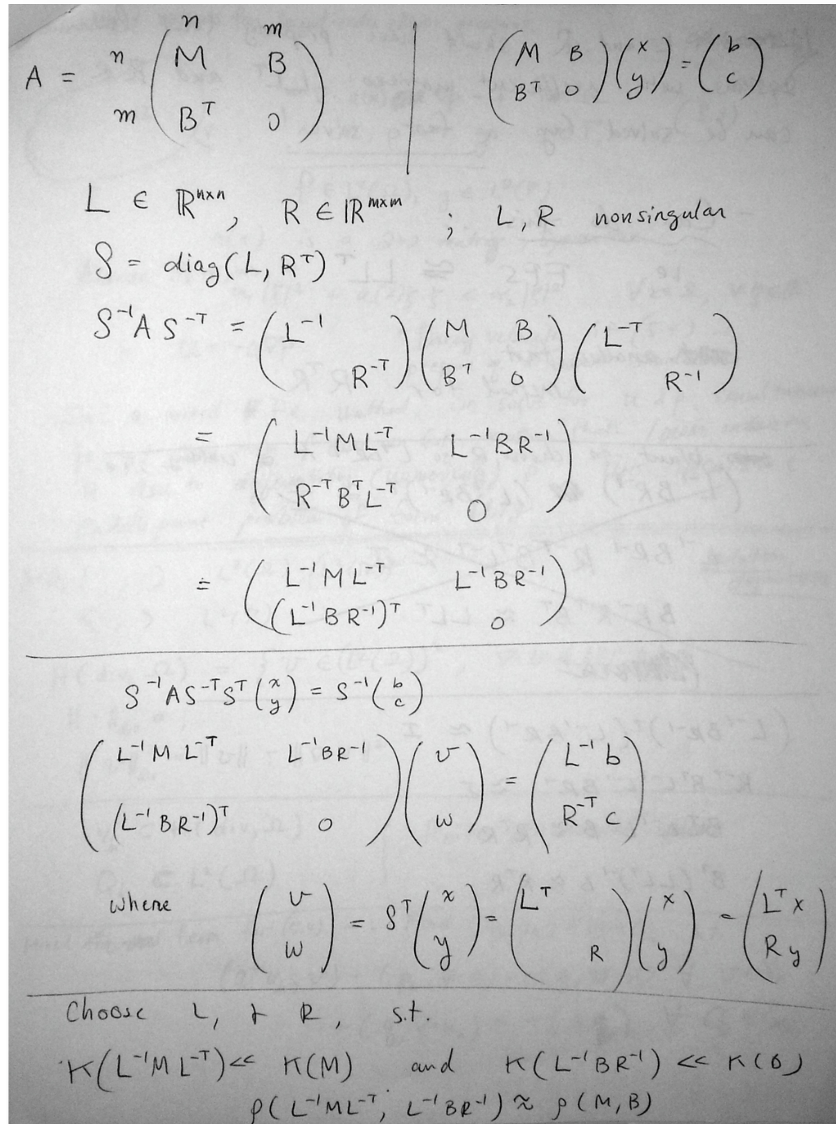
CS260
Physics-Based Simulation for Computer Graphics

Physics-Based Simulation



Physics


$$A = \begin{array}{c c} & \begin{array}{cc} n & m \end{array} \\ \begin{array}{c} n \\ m \end{array} & \begin{pmatrix} M & B \\ B^T & 0 \end{pmatrix} \end{array} \qquad \begin{pmatrix} M & B \\ B^T & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} b \\ c \end{pmatrix}$$

$L \in \mathbb{R}^{n\times n}$, $R \in \mathbb{R}^{m\times m}$; L, R nonsingular

$S = \mathrm{diag}(L, R^T)$

$$S^{-1} A S^{-T} = \begin{pmatrix} L^{-1} & \\ & R^{-T} \end{pmatrix}\begin{pmatrix} M & B \\ B^T & 0 \end{pmatrix}\begin{pmatrix} L^{-T} & \\ & R^{-1} \end{pmatrix}$$

$$= \begin{pmatrix} L^{-1} M L^{-T} & L^{-1} B R^{-1} \\ R^{-T} B^T L^{-T} & 0 \end{pmatrix}$$

$$= \begin{pmatrix} L^{-1} M L^{-T} & L^{-1} B R^{-1} \\ (L^{-1} B R^{-1})^T & 0 \end{pmatrix}$$

$$S^{-1} A S^{-T} S^T \begin{pmatrix} x \\ y \end{pmatrix} = S^{-1}\begin{pmatrix} b \\ c \end{pmatrix}$$

$$\begin{pmatrix} L^{-1} M L^{-T} & L^{-1} B R^{-1} \\ (L^{-1} B R^{-1})^T & 0 \end{pmatrix}\begin{pmatrix} v \\ w \end{pmatrix} = \begin{pmatrix} L^{-1} b \\ R^{-T} c \end{pmatrix}$$

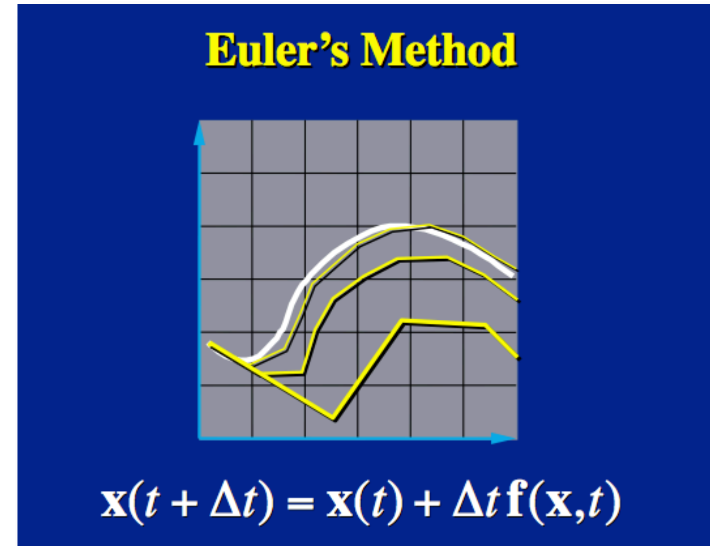
where $\begin{pmatrix} v \\ w \end{pmatrix} = S^T \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} L^T & \\ & R \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} L^T x \\ R y \end{pmatrix}$

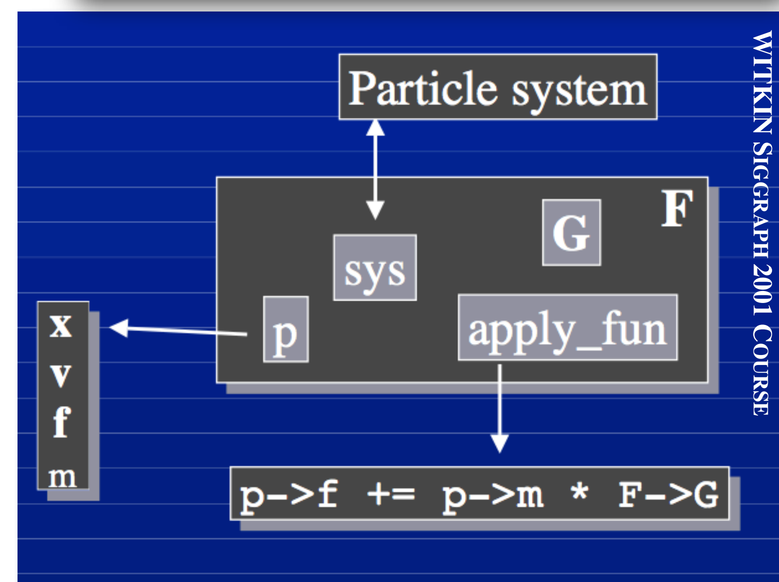
Choose L, + R, s.t.

$\kappa(L^{-1} M L^{-T}) \ll \kappa(M)$ and $\kappa(L^{-1} B R^{-1}) \ll \kappa(B)$

$\rho(L^{-1} M L^{-T}, L^{-1} B R^{-1}) \approx \rho(M, B)$

Mathematics





Numerical Methods and Algorithms

About the Instructor

- Name: Tamar Shinar
- Homepage: http://www.cs.ucr.edu/~shinar
- Office: WCH 419
- Office hours: Tuesdays, 2-3 pm, and by appointment
- email: shinar@cs.ucr.edu

About the Students

- Please introduce yourself:
  - Name
  - Year
  - Research area
  - Interest in taking this class

Class Structure

- course homepage:

  http://www.cs.ucr.edu/~shinar/courses/cs260
- Instructor lectures introducing basic principles
- Student presentations of literature

Grading

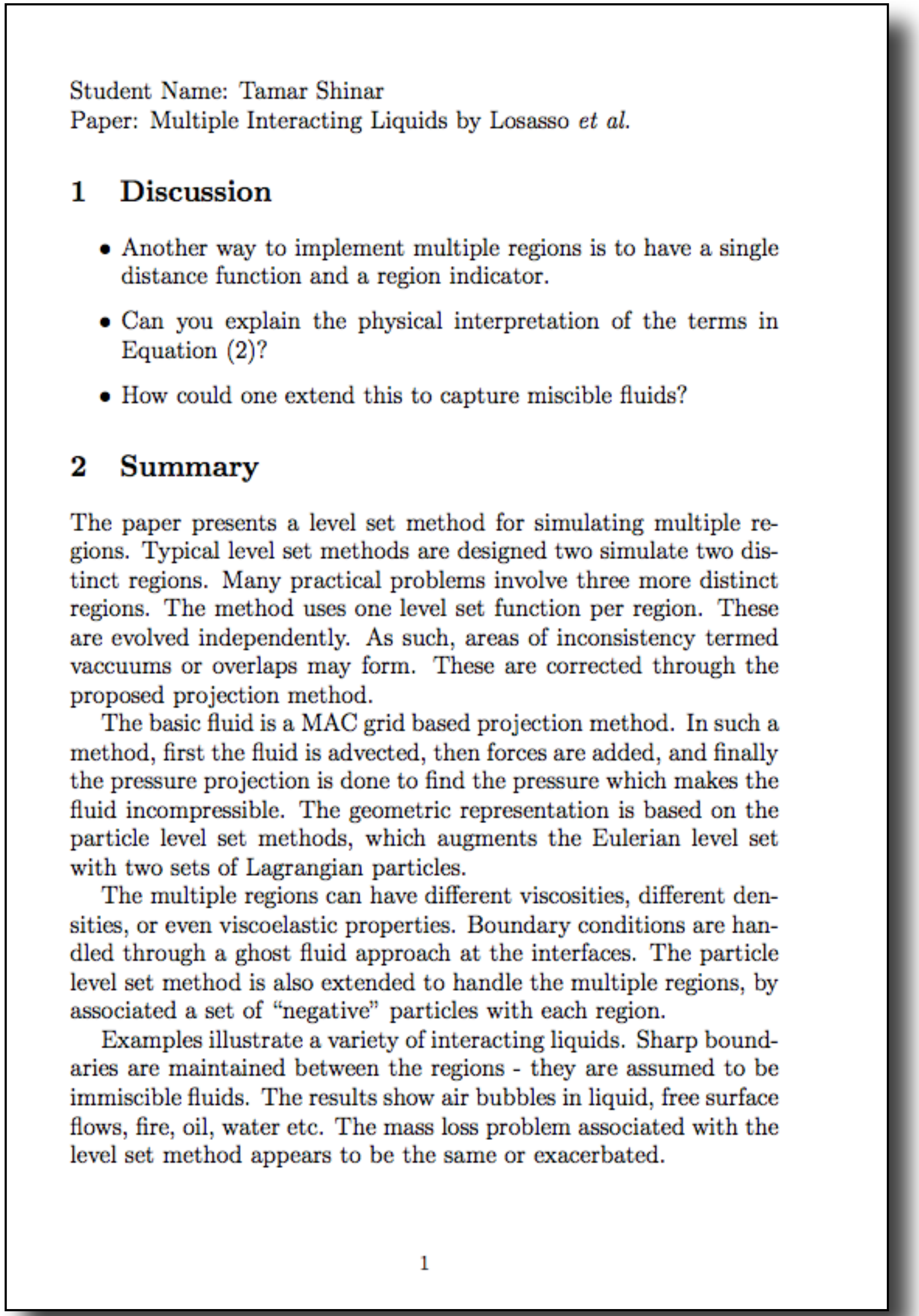
- Two paper presentations (80%)
- 1 page summary/paper of day's paper(s) and participation in class discussions (20%)

Student Presentations

- 30-40 minutes + interactive discussion
- Introduction
- Problem statement
- Proposed solution
- Results
- Discussion (your assessment)

Paper Summary

- Typed, approximately one page
- First section: 3 bullets each with question/ comment for discussion
- Second section: A few paragraphs summarizing the paper (not the abstract)


Student Name: Tamar Shinar
Paper: Multiple Interacting Liquids by Losasso *et al.*

1 Discussion

- Another way to implement multiple regions is to have a single distance function and a region indicator.
- Can you explain the physical interpretation of the terms in Equation (2)?
- How could one extend this to capture miscible fluids?

2 Summary

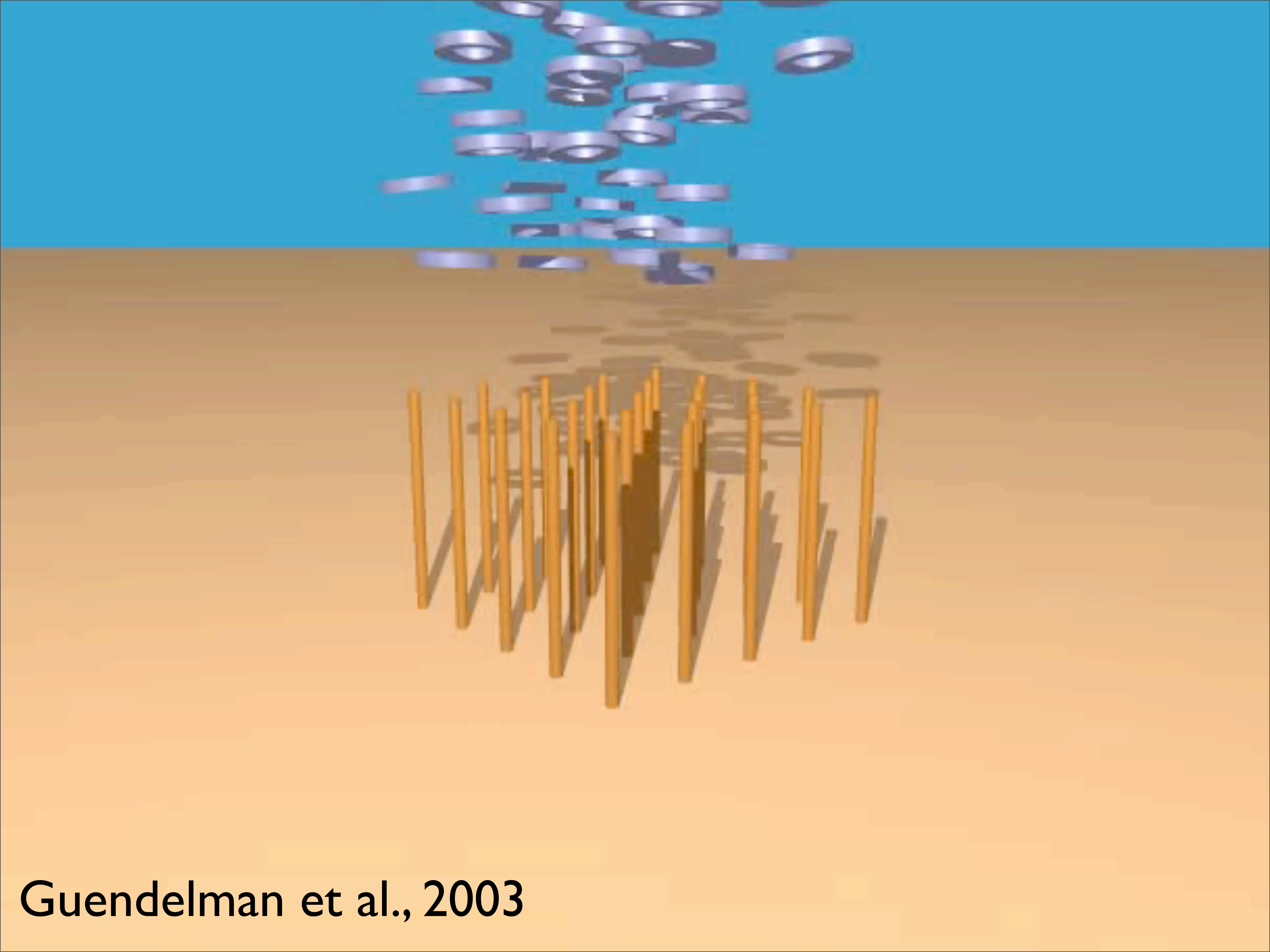
The paper presents a level set method for simulating multiple regions. Typical level set methods are designed two simulate two distinct regions. Many practical problems involve three more distinct regions. The method uses one level set function per region. These are evolved independently. As such, areas of inconsistency termed vaccuums or overlaps may form. These are corrected through the proposed projection method.

The basic fluid is a MAC grid based projection method. In such a method, first the fluid is advected, then forces are added, and finally the pressure projection is done to find the pressure which makes the fluid incompressible. The geometric representation is based on the particle level set methods, which augments the Eulerian level set with two sets of Lagrangian particles.

The multiple regions can have different viscosities, different densities, or even viscoelastic properties. Boundary conditions are handled through a ghost fluid approach at the interfaces. The particle level set method is also extended to handle the multiple regions, by associated a set of "negative" particles with each region.

Examples illustrate a variety of interacting liquids. Sharp boundaries are maintained between the regions - they are assumed to be immiscible fluids. The results show air bubbles in liquid, free surface flows, fire, oil, water etc. The mass loss problem associated with the level set method appears to be the same or exacerbated.

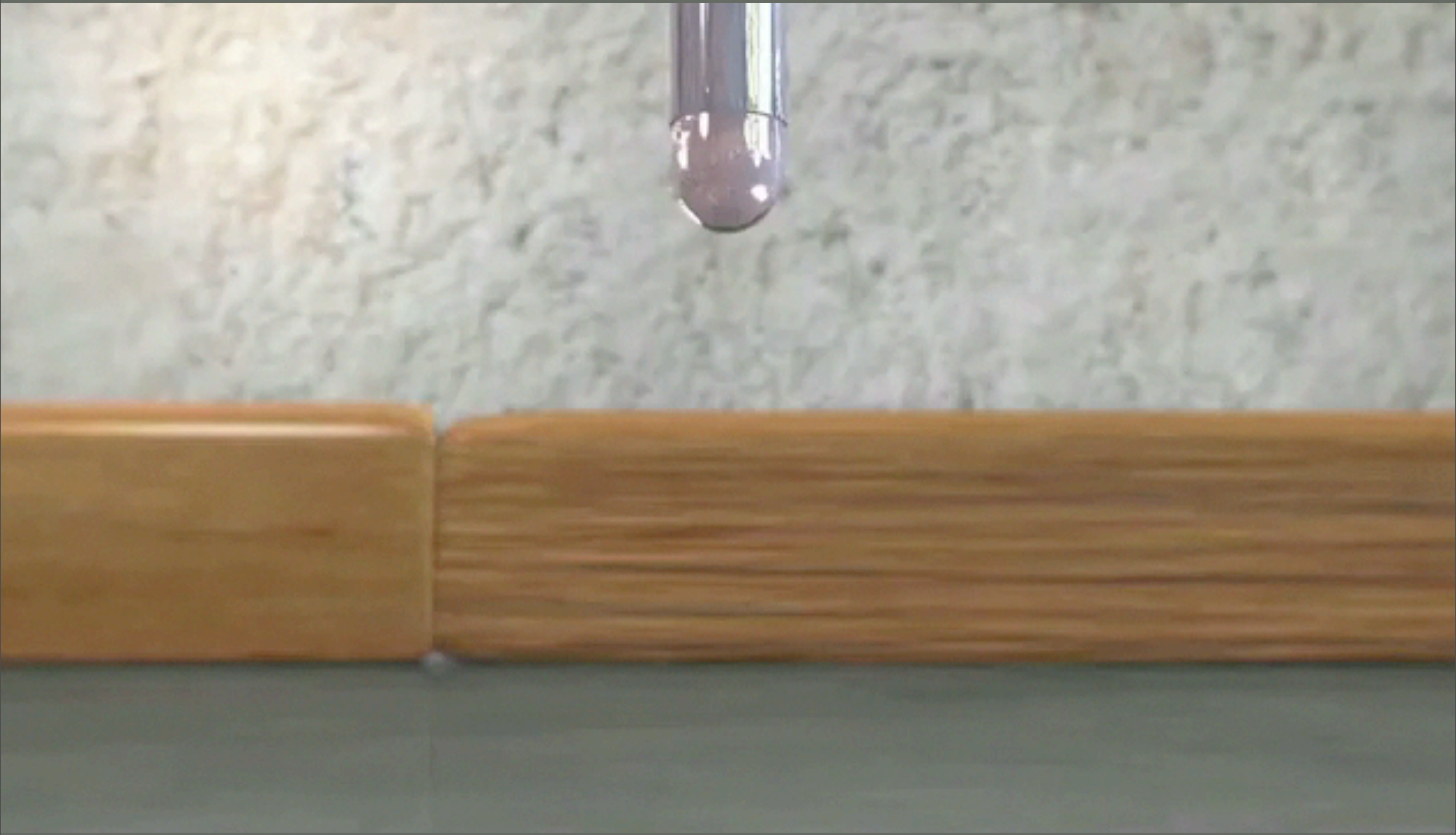
1



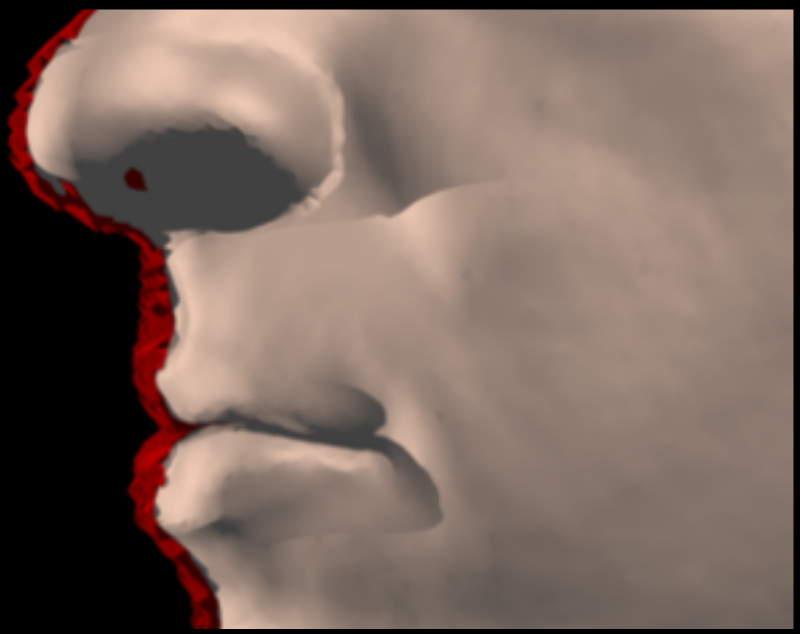
Guendelman et al., 2003

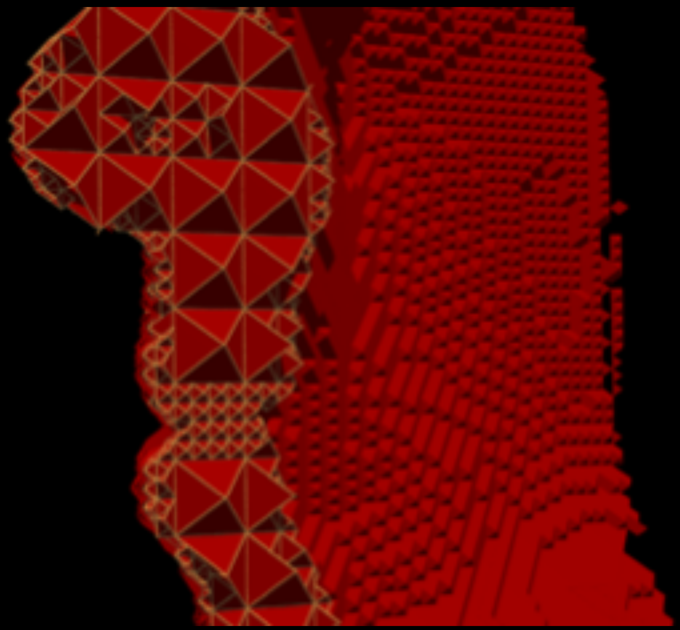
Selle et al. 2009

Hong et al. 2007

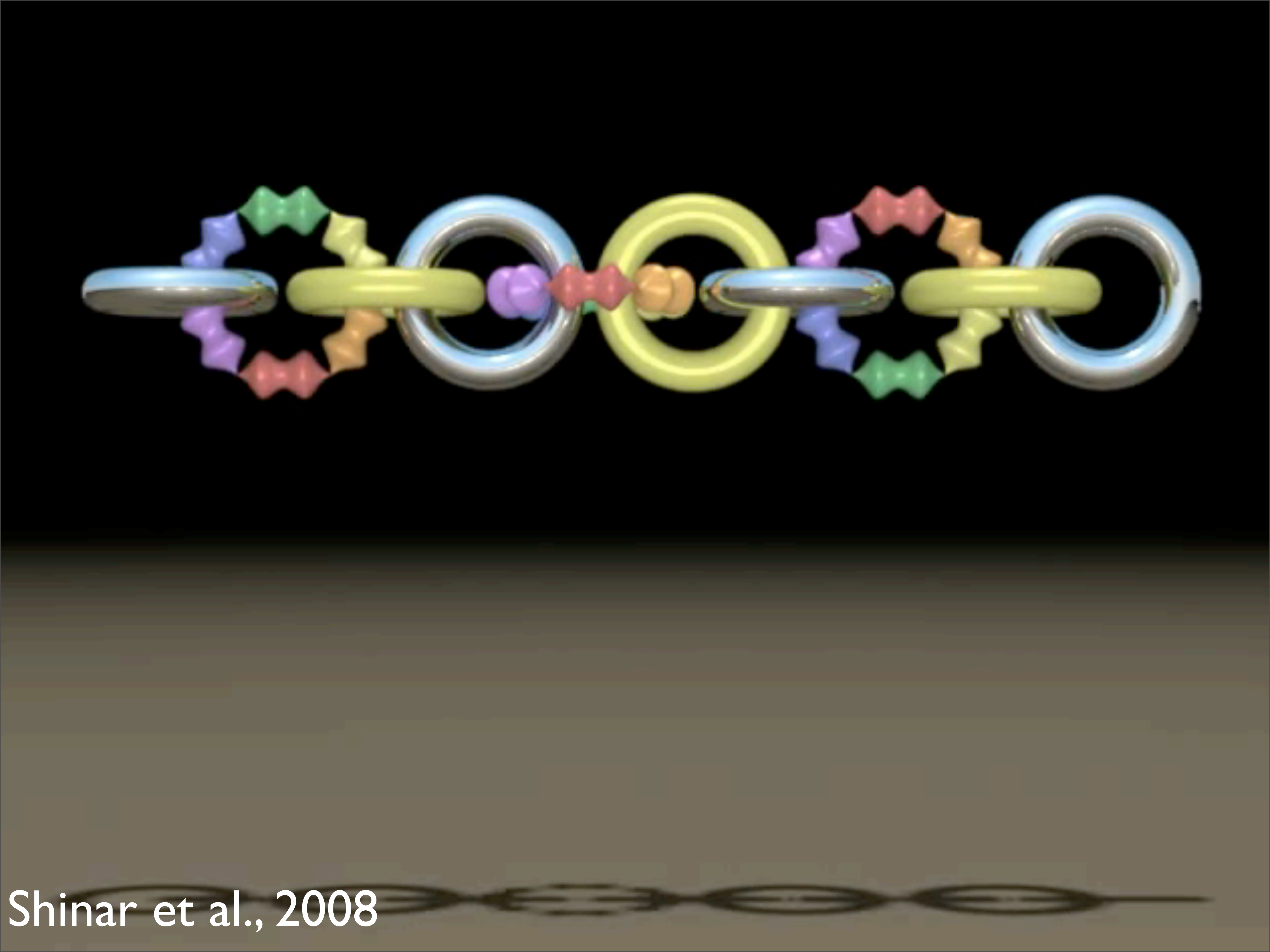


Clausen et al. 2013

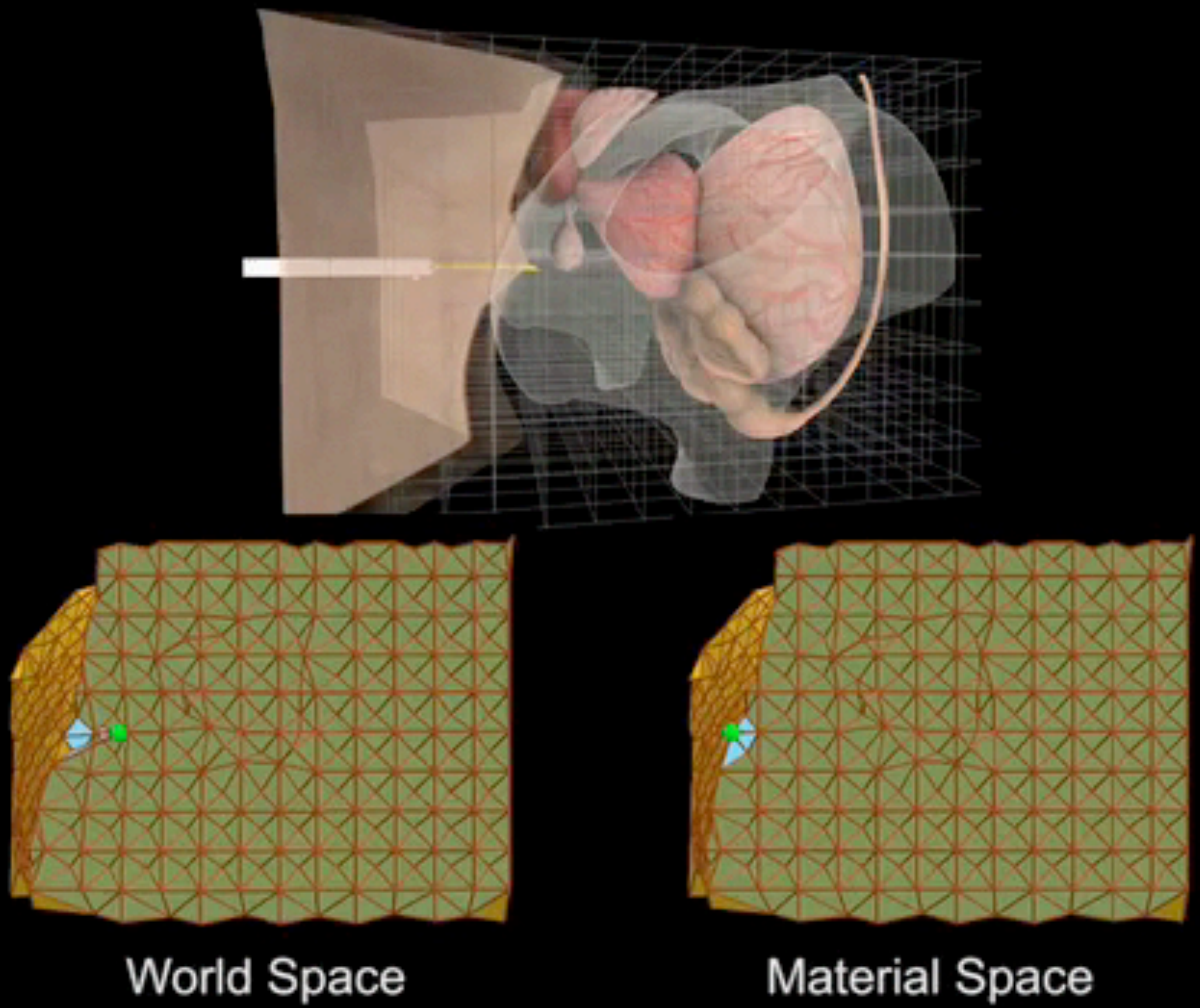




Sifakis et al., 2007



Shinar et al., 2008



Chentanez et al., 2009

Physics of Natural Phenomena

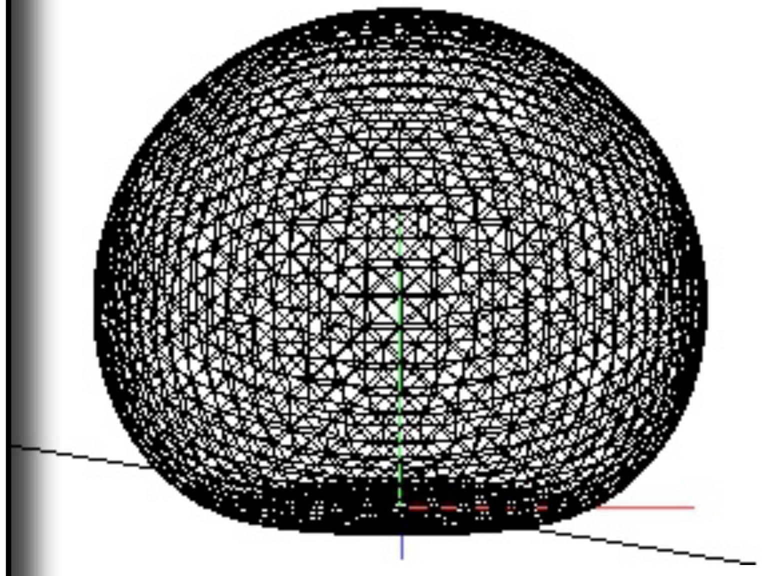
- **Newton's Second Law ($F = ma$)**

  *The acceleration **a** of a body is parallel and directly proportional to the net force **F** acting on the body, is in the direction of the net force, and is inversely proportional to the mass **m** of the body.*

- **Newton's Third Law** (Action/Reaction)

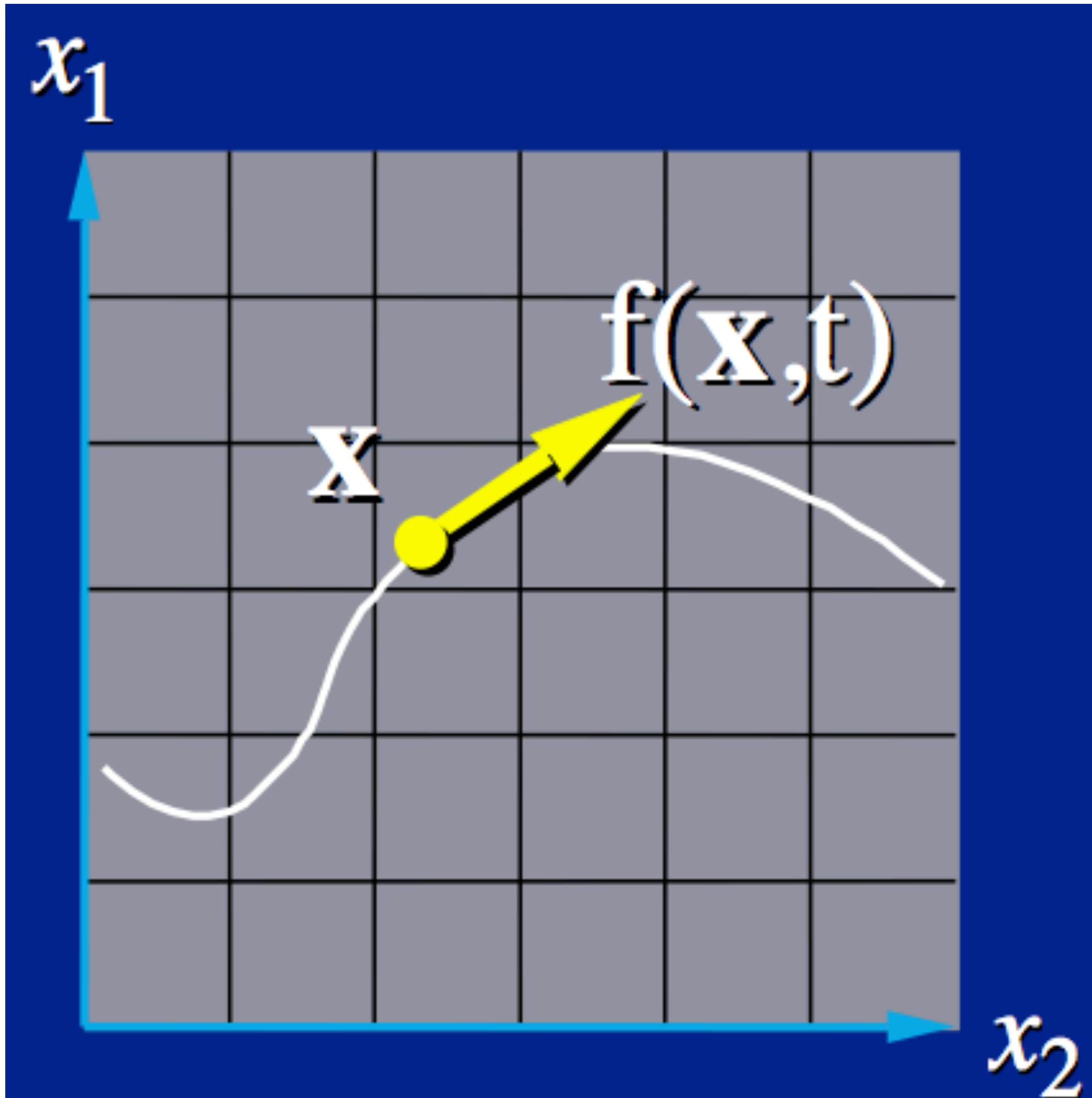
  When a body exerts a force $\mathbf{F_1}$ on a second body, the second body simultaneously exerts a force $\mathbf{F_2}=-\mathbf{F_1}$ on the first body. This means that $\mathbf{F_1}$ and $\mathbf{F_2}$ are equal in magnitude and opposite in direction.

[Wikipedia]



Math of Natural Phenomena

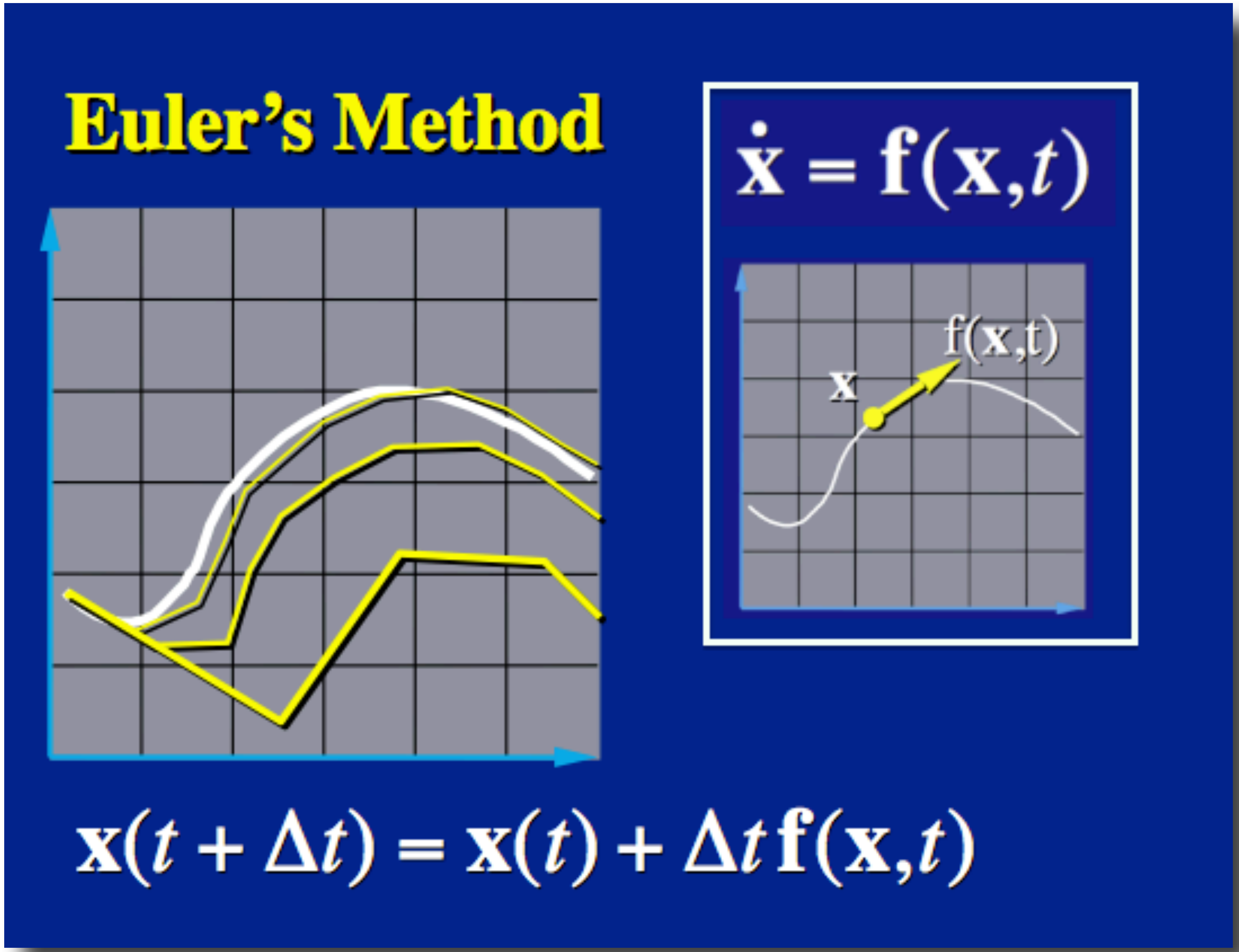
Ordinary Differential Equations



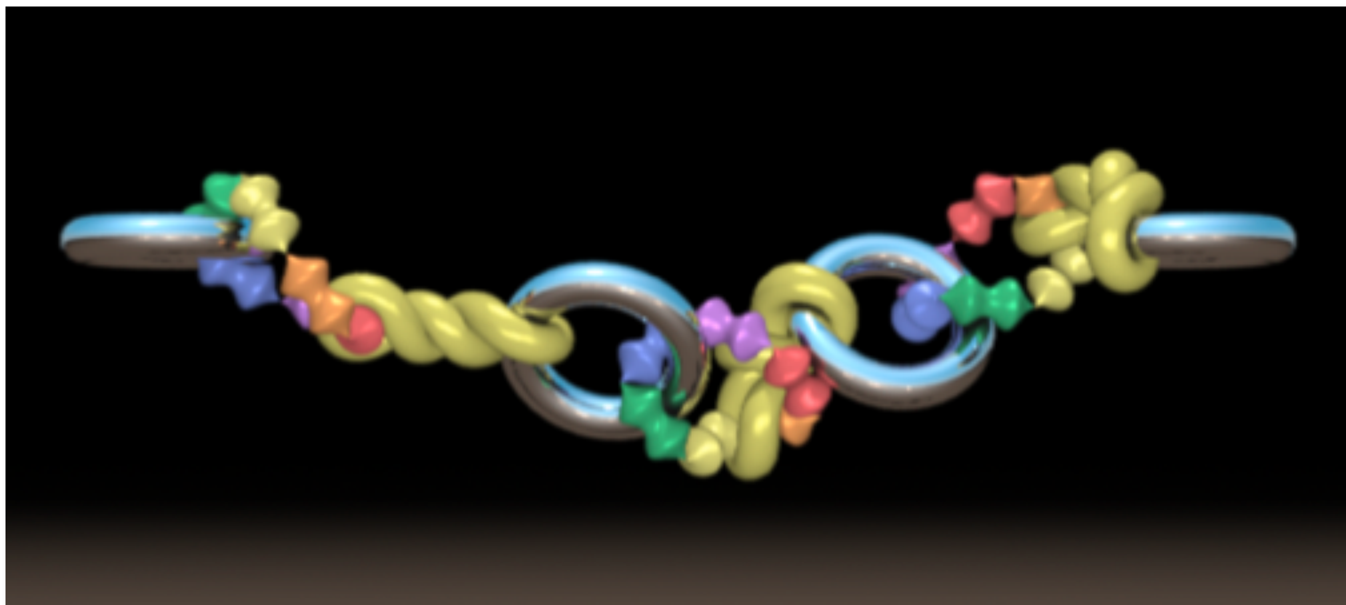
$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t)$$

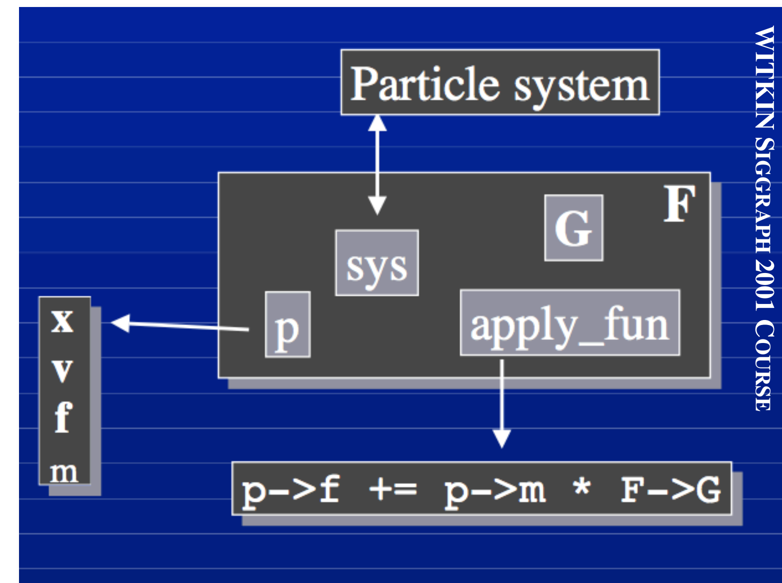
- $\mathbf{x}(t)$: a moving point.
- $\mathbf{f}(\mathbf{x}, t)$: x's velocity.

Numerical Solution of Diff. Eq.



Data Structures and Algorithms





I. Advance velocity $\mathbf{v}^n \longrightarrow \tilde{\mathbf{v}}^{n+\frac{1}{2}}$

II. Apply collisions $\mathbf{v}^n \longrightarrow \hat{\mathbf{v}}^n$, $\tilde{\mathbf{v}}^{n+\frac{1}{2}} \longrightarrow \hat{\mathbf{v}}^{n+\frac{1}{2}}$

III. Apply contact and constraint forces $\hat{\mathbf{v}}^{n+\frac{1}{2}} \longrightarrow \mathbf{v}^{n+\frac{1}{2}}$

IV. Advance positions $\mathbf{x}^n \longrightarrow \mathbf{x}^{n+1}$ using $\mathbf{v}^{n+\frac{1}{2}}$, $\hat{\mathbf{v}}^n \longrightarrow \bar{\mathbf{v}}^n$

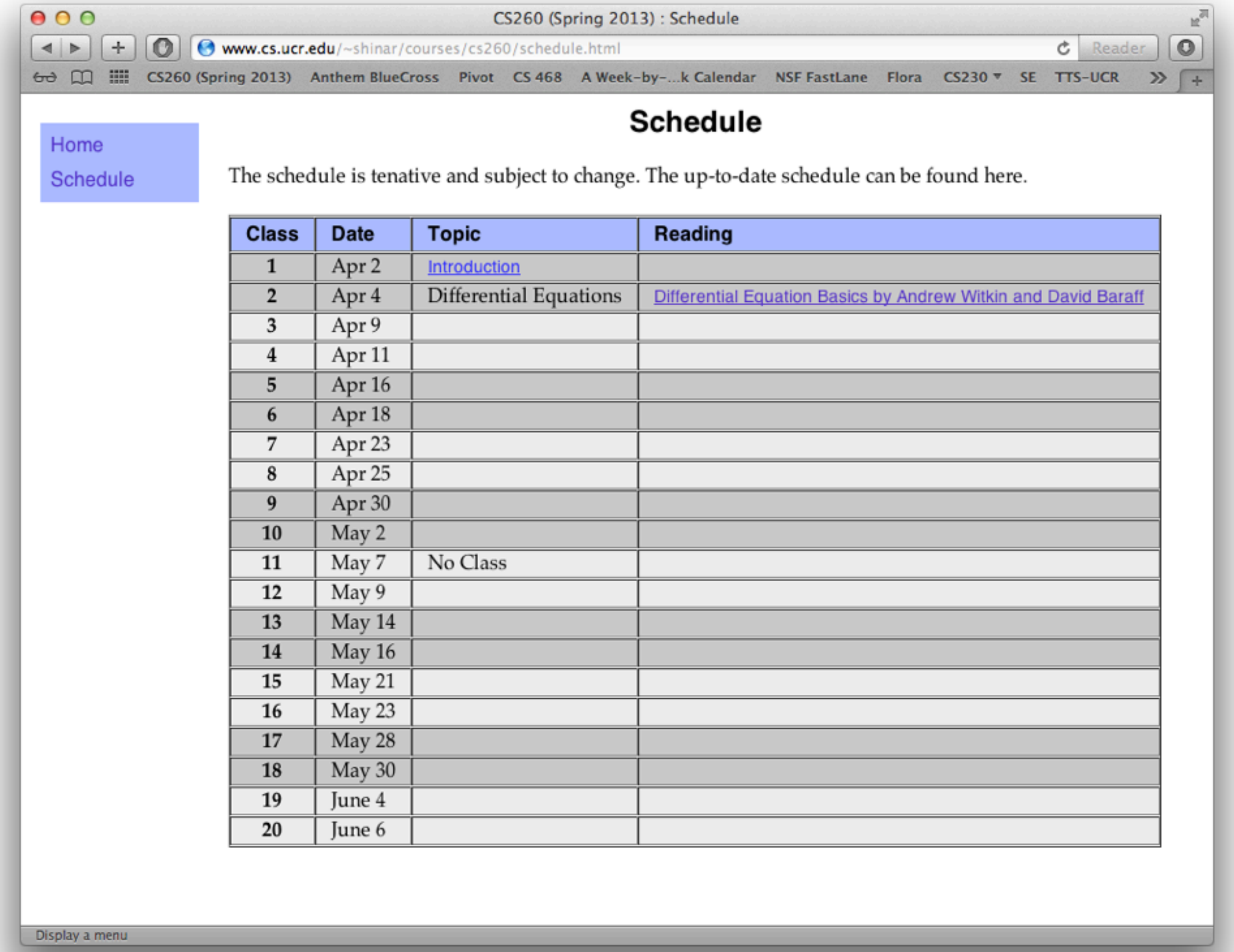
V. Advance velocity $\bar{\mathbf{v}}^n \longrightarrow \mathbf{v}^{n+1}$

Reading Assignment

Physically Based Modeling
Differential Equation Basics

Andrew Witkin and David Baraff
Pixar Animation Studios

(7 pages)



Schedule

The schedule is tenative and subject to change. The up-to-date schedule can be found here.

Class	Date	Topic	Reading
1	Apr 2	Introduction	
2	Apr 4	Differential Equations	Differential Equation Basics by Andrew Witkin and David Baraff
3	Apr 9		
4	Apr 11		
5	Apr 16		
6	Apr 18		
7	Apr 23		
8	Apr 25		
9	Apr 30		
10	May 2		
11	May 7	No Class	
12	May 9		
13	May 14		
14	May 16		
15	May 21		
16	May 23		
17	May 28		
18	May 30		
19	June 4		
20	June 6		