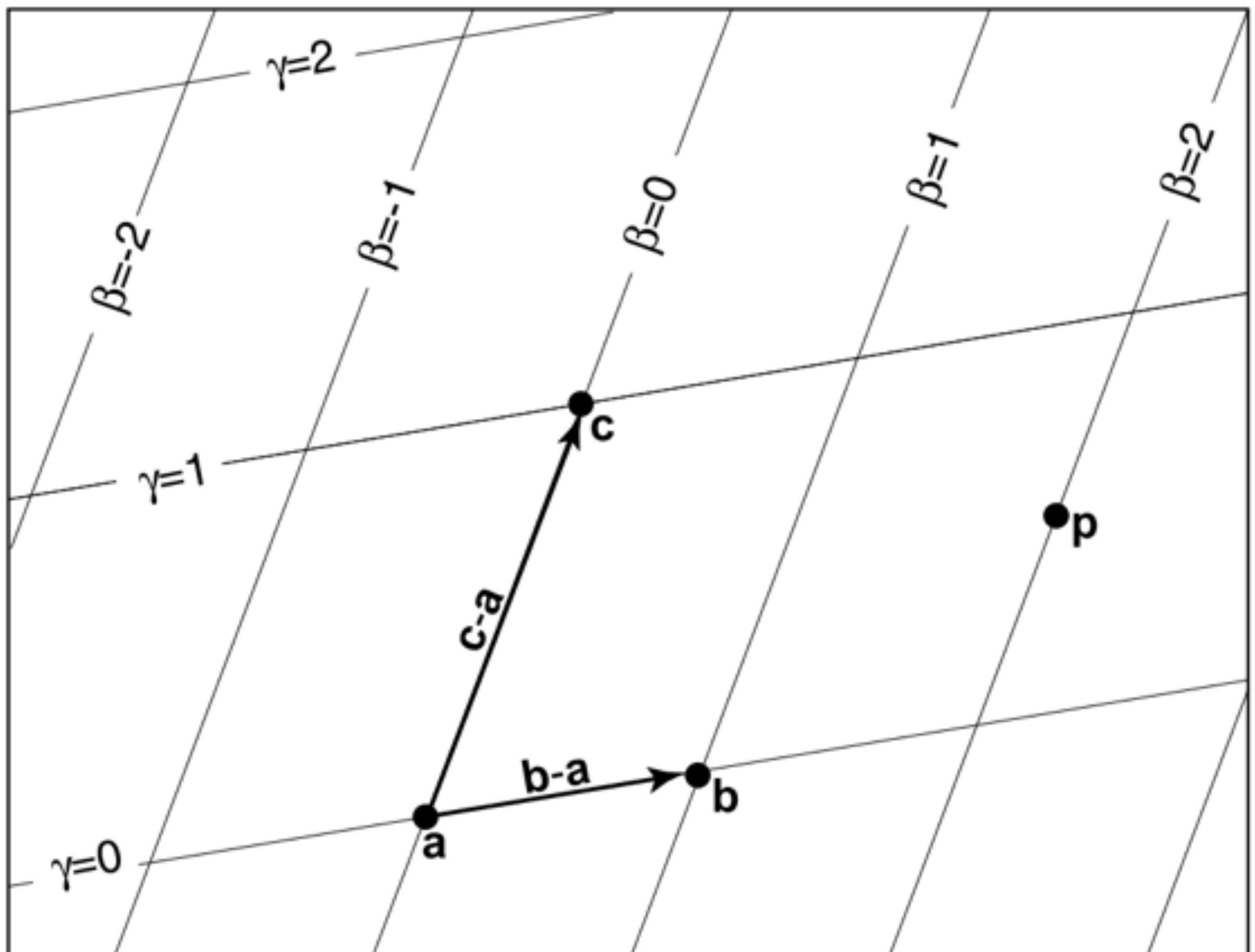
Triangles

barycentric coordinates

$$\mathbf{p} = \mathbf{a} + \beta(\mathbf{b} - \mathbf{a}) + \gamma(\mathbf{c} - \mathbf{a})$$



barycentric coordinates

$$\mathbf{p} = \mathbf{a} + \beta(\mathbf{b} - \mathbf{a}) + \gamma(\mathbf{c} - \mathbf{a})$$

$$\mathbf{p} = (1 - \beta - \gamma)\mathbf{a} + \beta\mathbf{b} + \gamma\mathbf{c}$$

$$\alpha \equiv 1 - \beta - \gamma$$

$$\mathbf{p}(\alpha, \beta, \gamma) = \alpha\mathbf{a} + \beta\mathbf{b} + \gamma\mathbf{c}$$

$$\alpha + \beta + \gamma = 1$$

barycentric coordinates

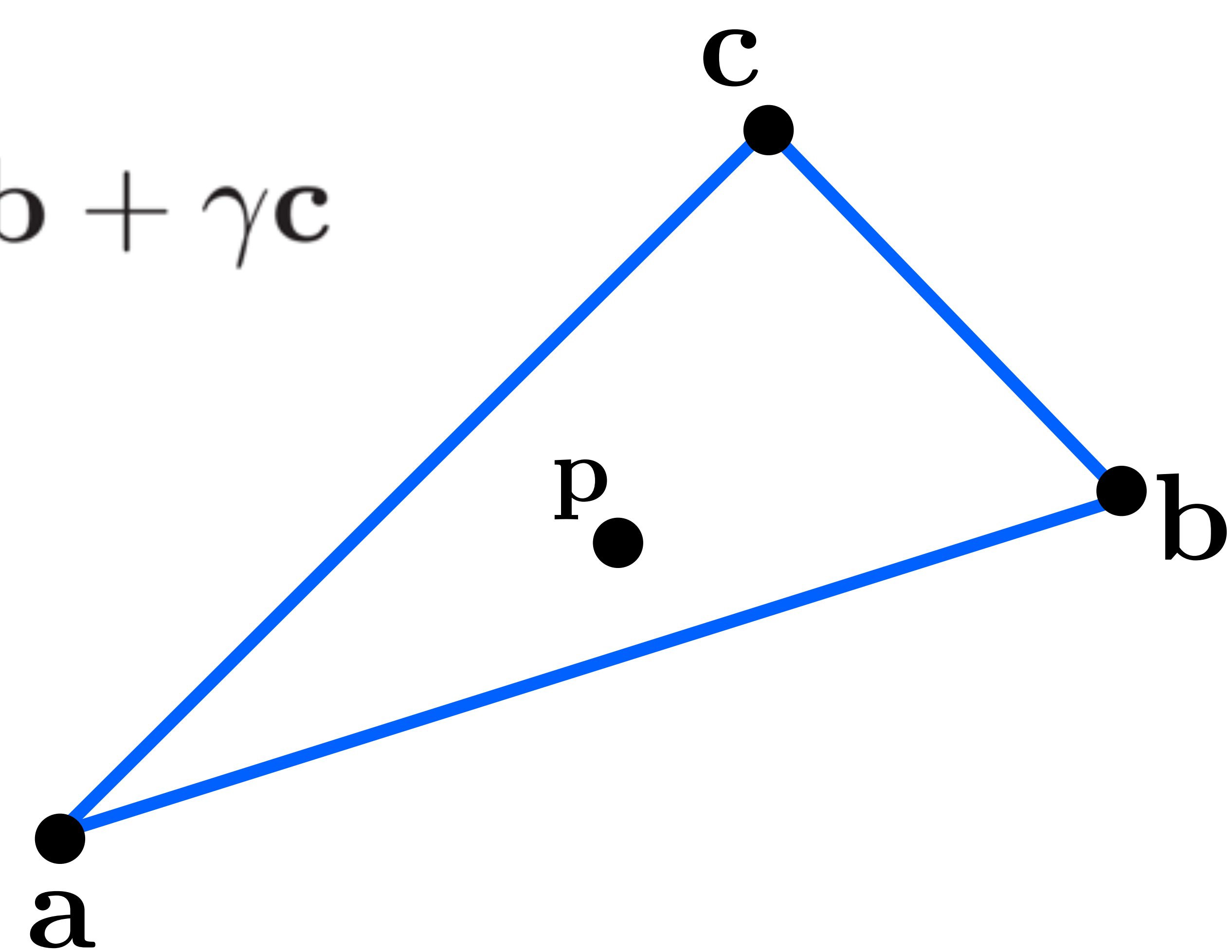
If **p** inside the triangle,

$$\mathbf{p}(\alpha, \beta, \gamma) = \alpha\mathbf{a} + \beta\mathbf{b} + \gamma\mathbf{c}$$

$$0 < \alpha < 1$$

$$0 < \beta < 1$$

$$0 < \gamma < 1$$

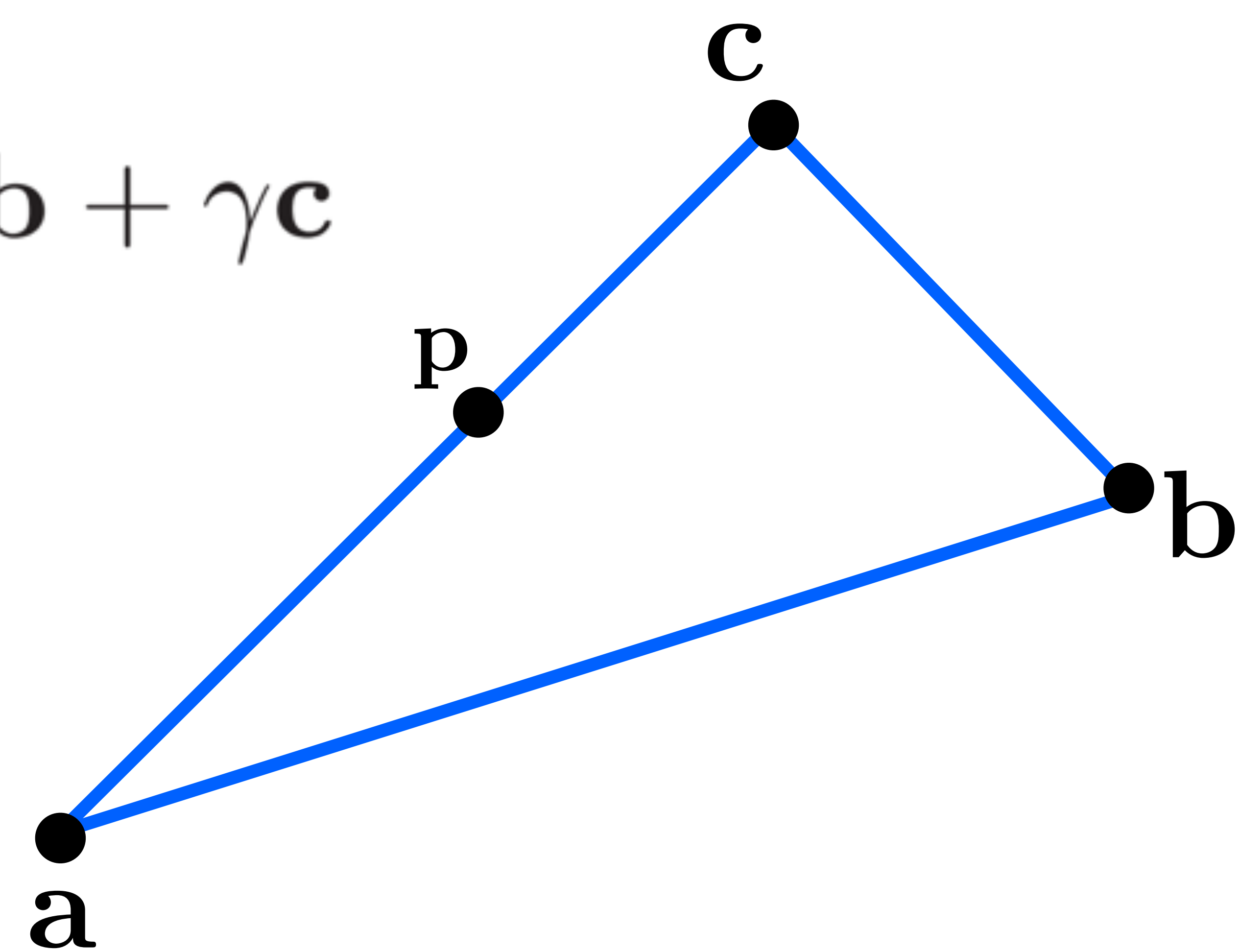


barycentric coordinates

If **p** on an edge, e.g.,

$$\mathbf{p}(\alpha, \beta, \gamma) = \alpha\mathbf{a} + \beta\mathbf{b} + \gamma\mathbf{c}$$

$$\beta = 0$$
$$\alpha + \gamma = 1$$

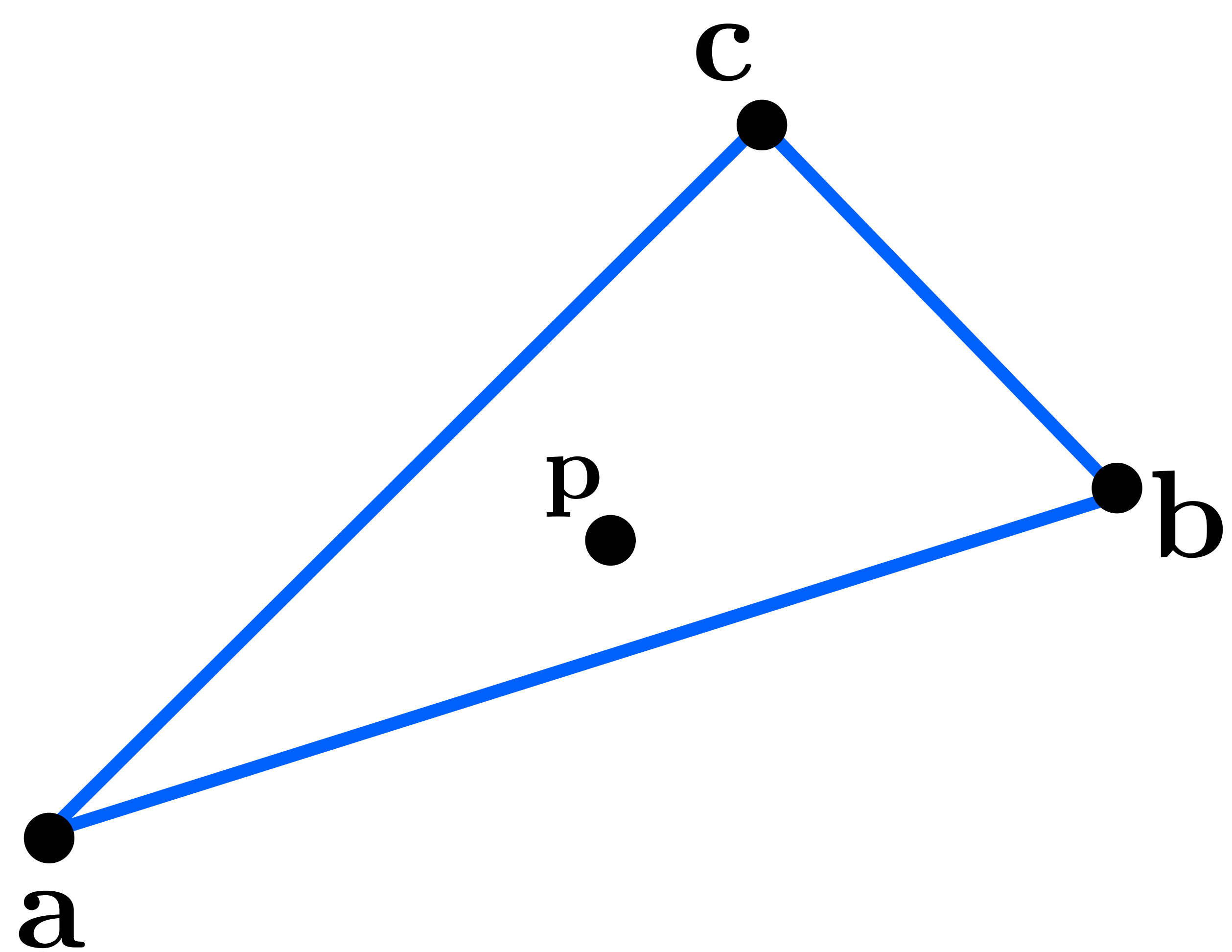


barycentric coordinates

$$\mathbf{p} = \alpha\mathbf{a} + \beta\mathbf{b} + \gamma\mathbf{c}$$

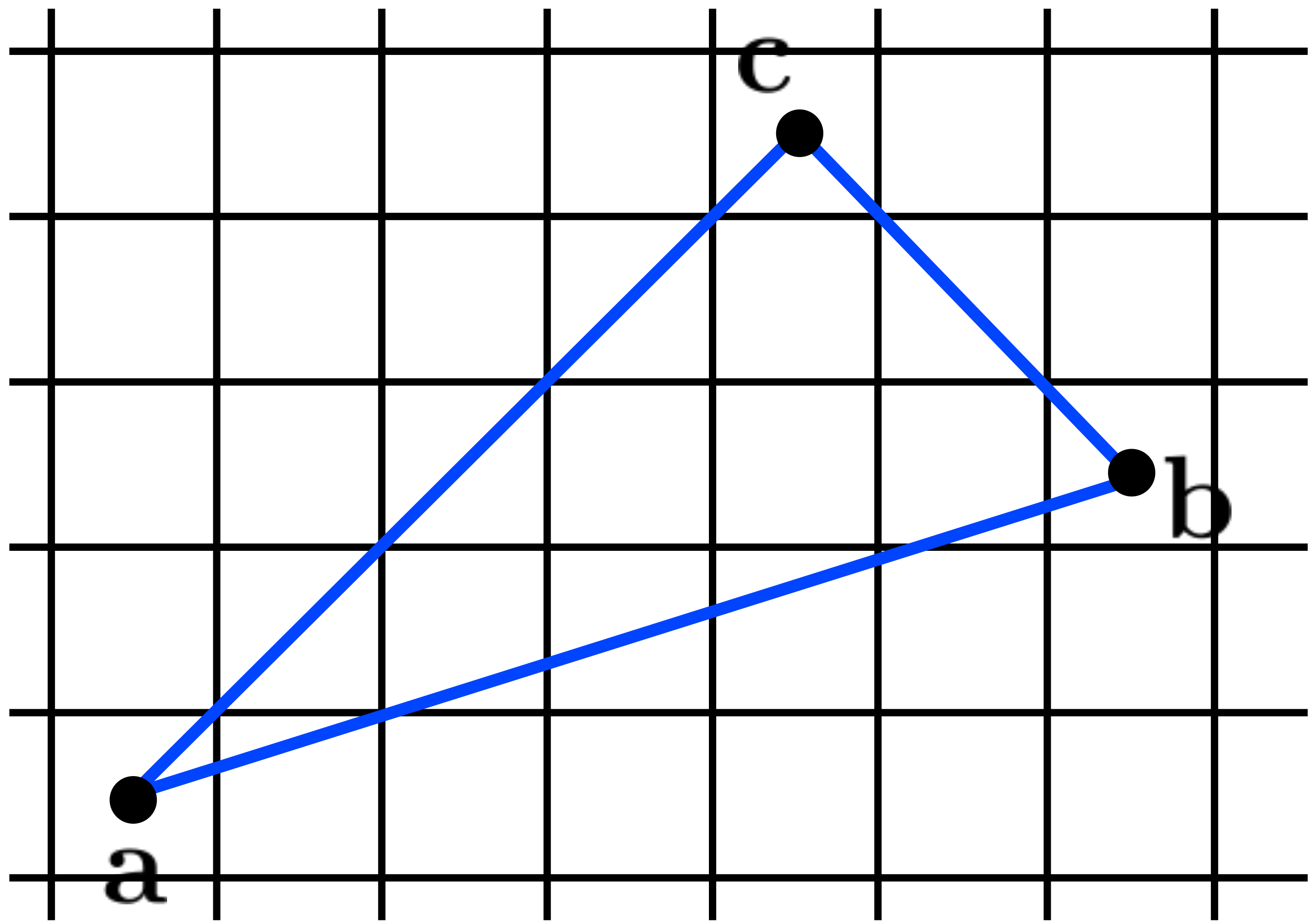
What are (α, β, γ) ?

<whiteboard>

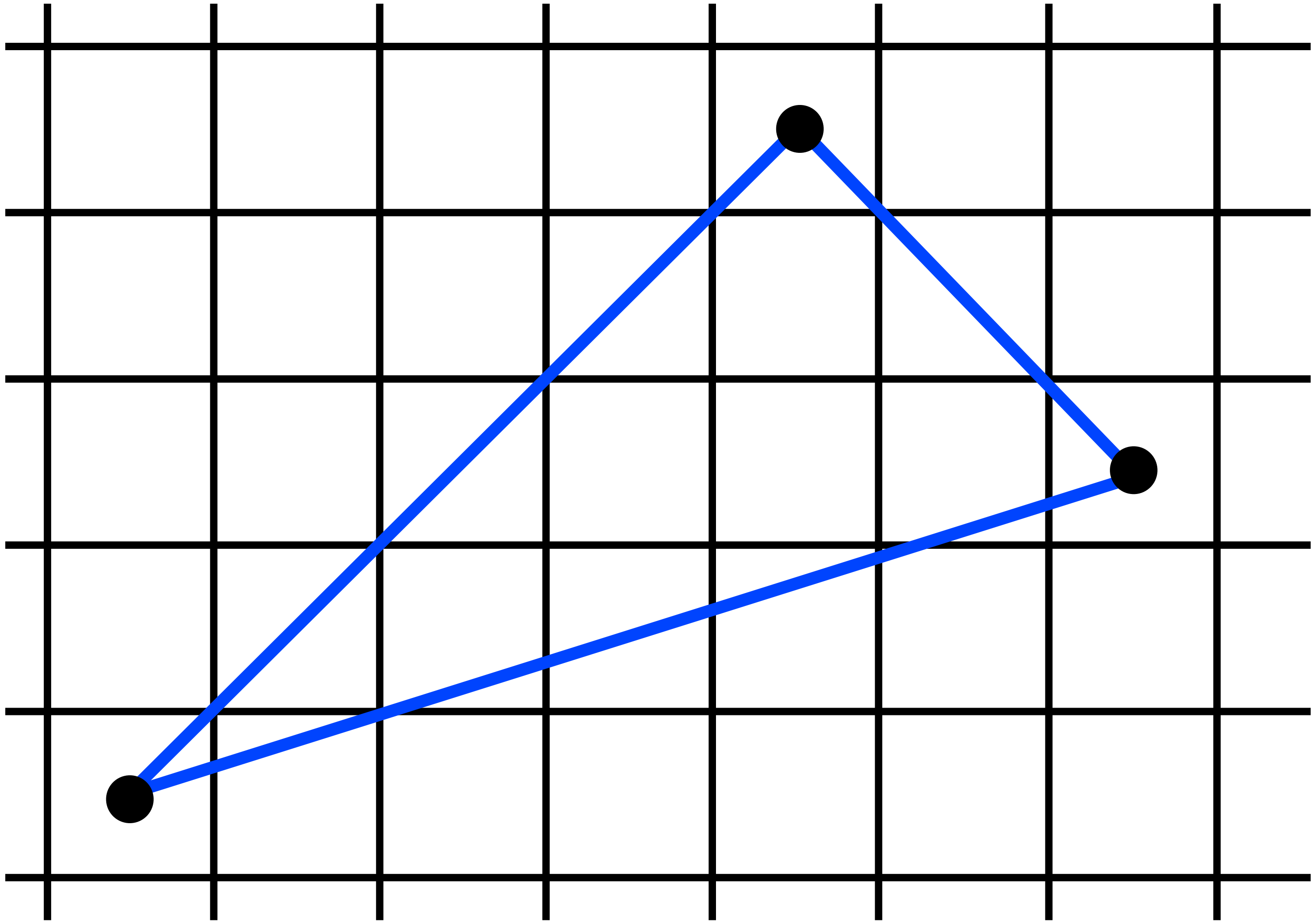


Triangle rasterization

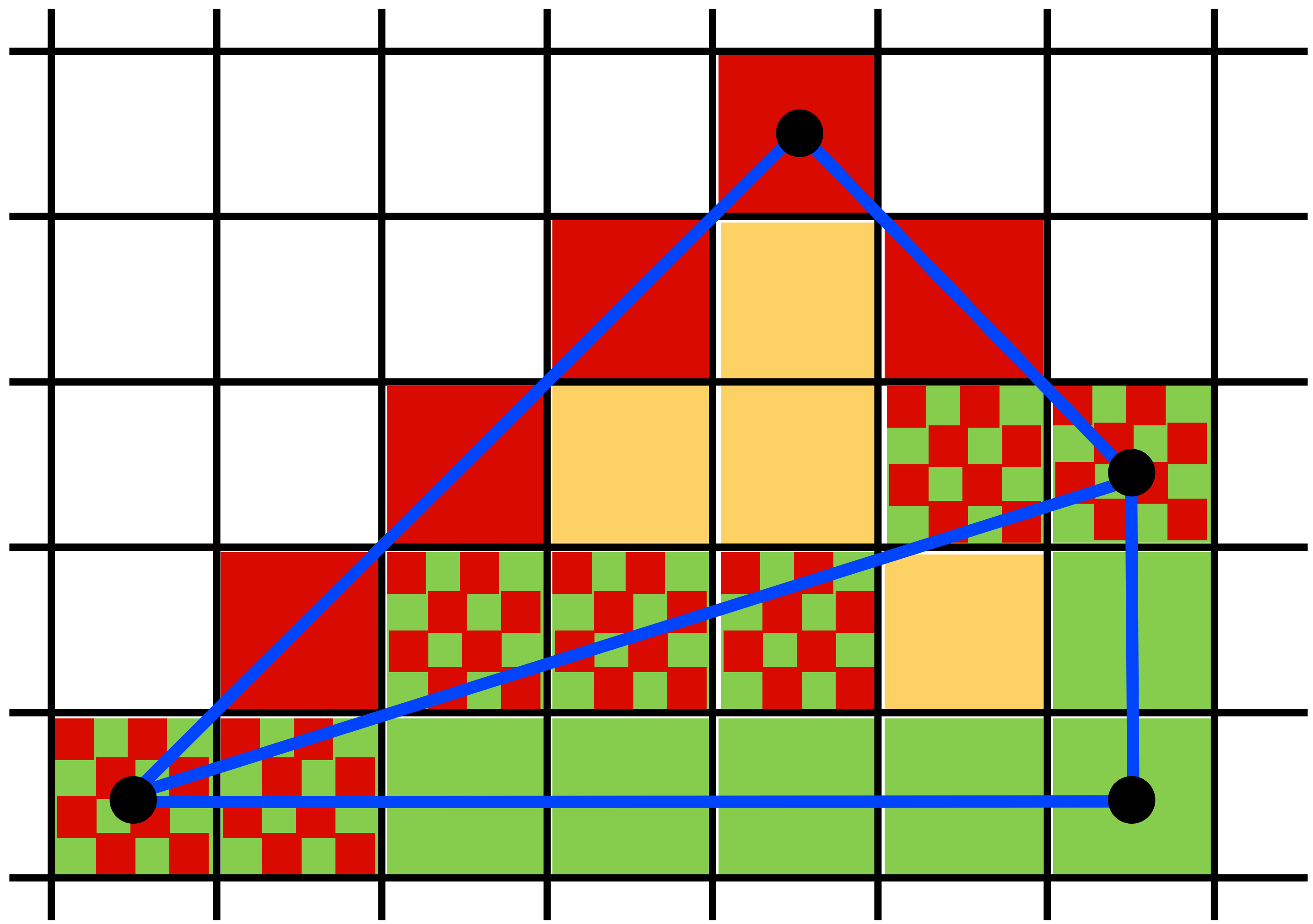
Which pixels should be used to approximate a triangle?



Triangle rasterization issues

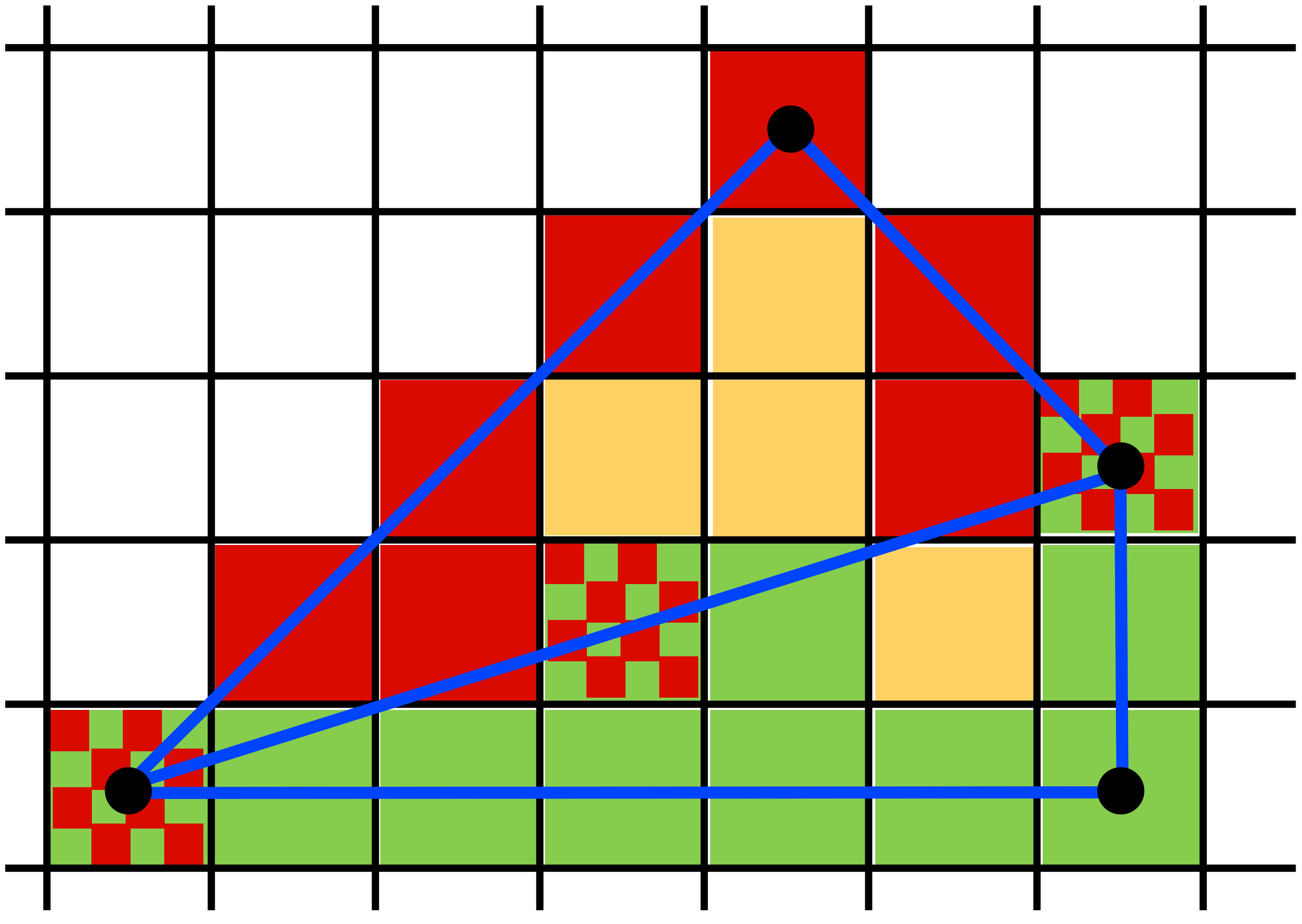


Which pixels should be used to approximate a triangle?



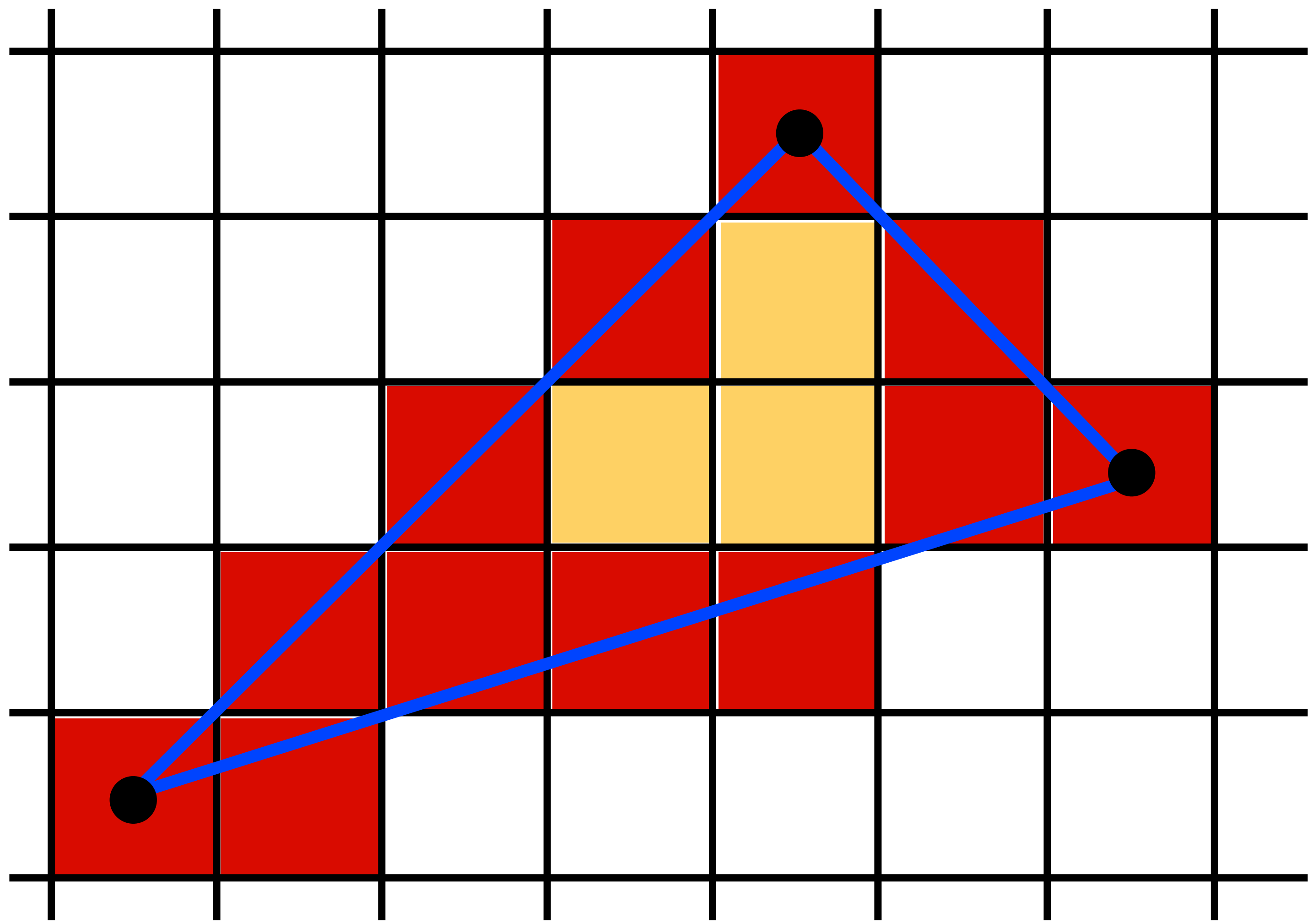
Who should fill in shared edge?

Which pixels should be used to approximate a triangle?



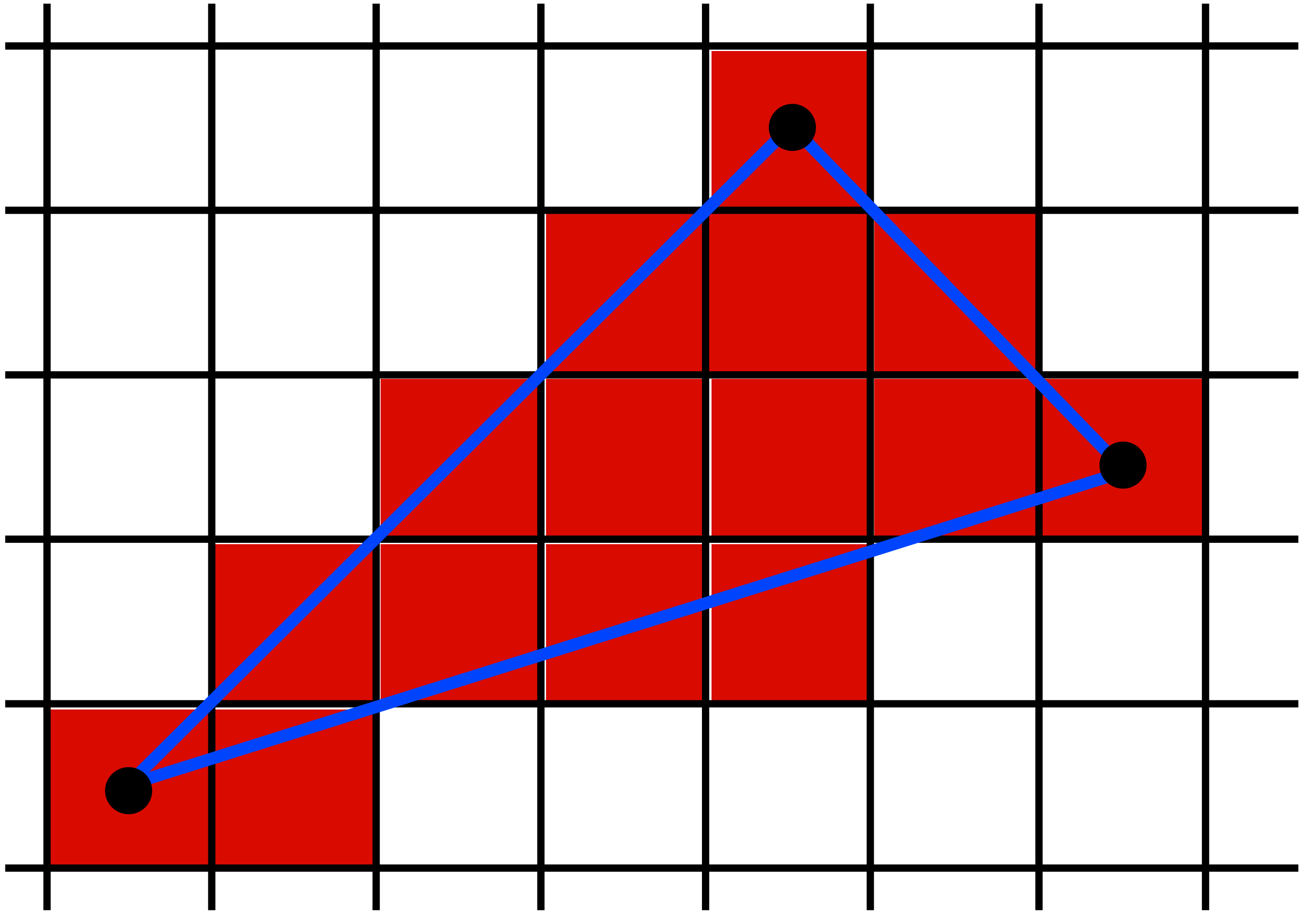
Who should fill in shared edge?

Which pixels should be used to approximate a triangle?



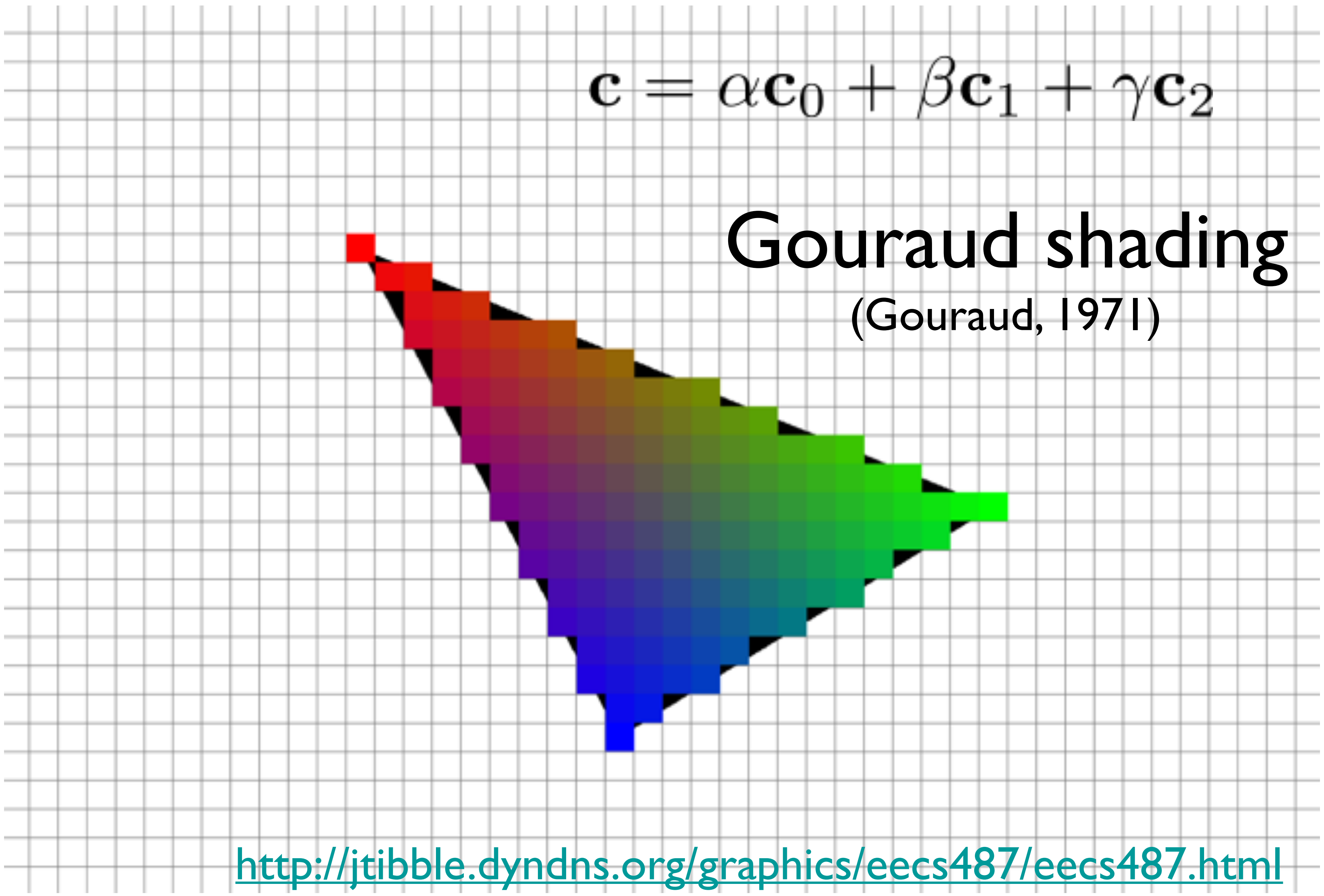
Use Midpoint Algorithm for edges and fill in?

Which pixels should be used to approximate a triangle?

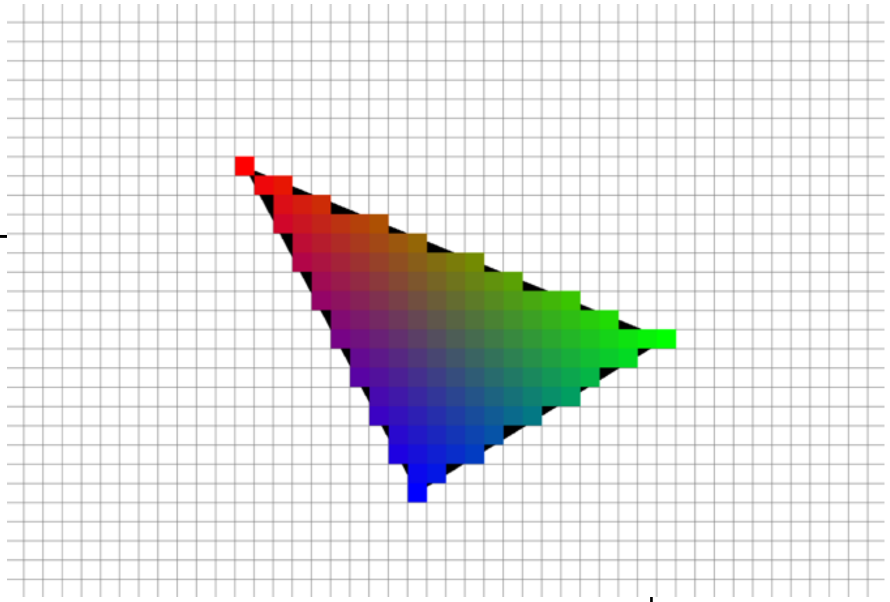


Use an approach based on barycentric coordinates

We can interpolate attributes using barycentric coordinates



Triangle rasterization algorithm



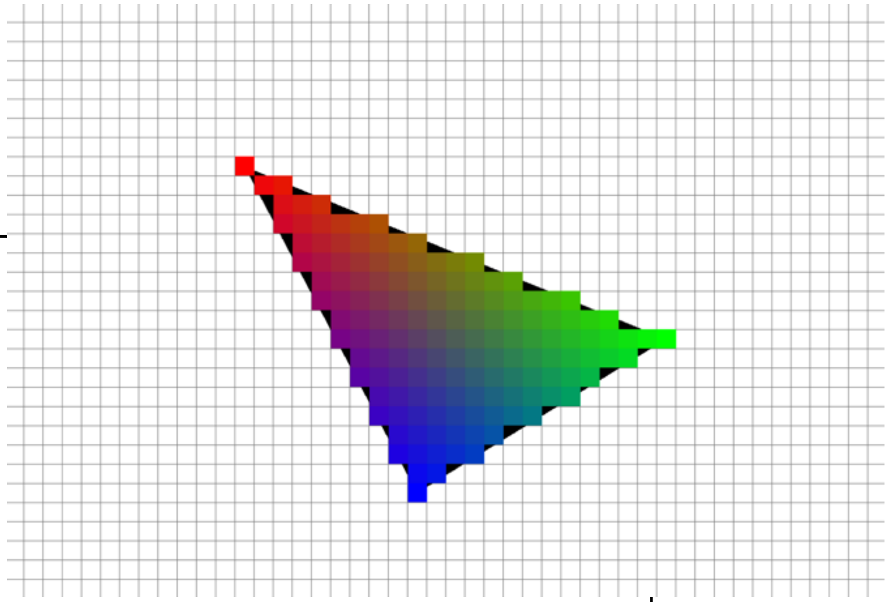
```
for all x do
    for all y do
        compute (α, β, γ) for (x,y)
        if (α ∈ [0,1] and β ∈ [0,1] and γ ∈ [0,1]) then
            c = αc_0 + βc_1 + γc_2
            drawpixel(x,y) with color c
```

$$(\alpha, \beta, \gamma)$$

$$(\alpha \in [0,1] \text{ and } \beta \in [0,1] \text{ and } \gamma \in [0,1])$$

$$\mathbf{c} = \alpha \mathbf{c}_0 + \beta \mathbf{c}_1 + \gamma \mathbf{c}_2$$

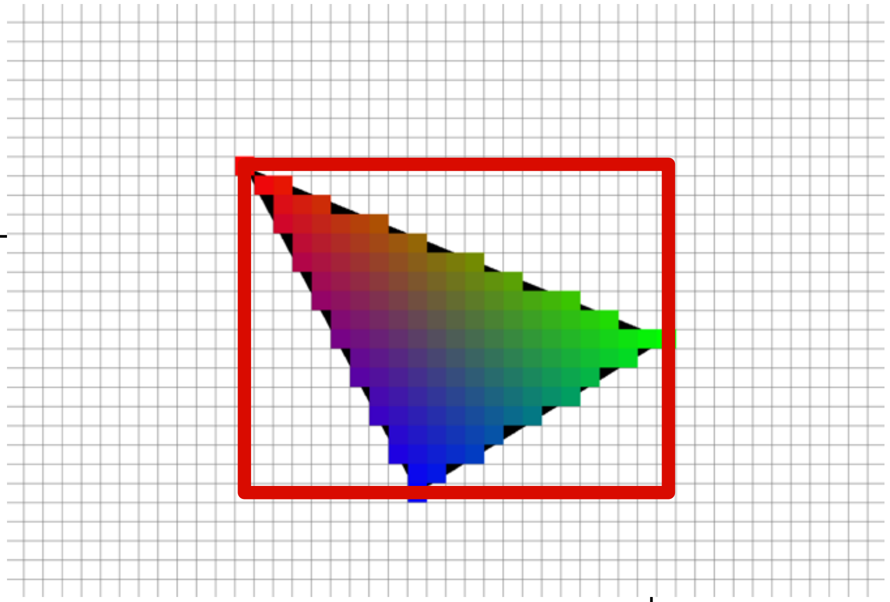
Triangle rasterization algorithm



for all x do
    for all y do
        compute (α, β, γ) for (x,y)
        if $(\alpha \in [0,1] \text{ and } \beta \in [0,1] \text{ and } \gamma \in [0,1])$ then
            $\mathbf{c} = \alpha\mathbf{c}_0 + \beta\mathbf{c}_1 + \gamma\mathbf{c}_2$
            drawpixel(x,y) with color c

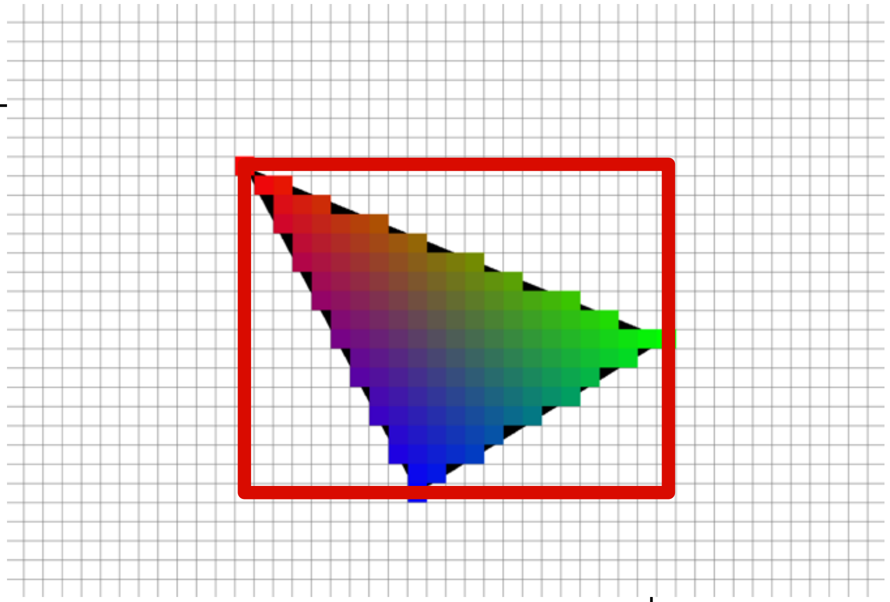
Triangle rasterization algorithm

use a bounding rectangle



for x in [x_min, x_max]
    for y in [y_min, y_max]
        compute (α, β, γ) for (x,y)
        if $(\alpha \in [0,1] \text{ and } \beta \in [0,1] \text{ and } \gamma \in [0,1])$ then
            $\mathbf{c} = \alpha \mathbf{c}_0 + \beta \mathbf{c}_1 + \gamma \mathbf{c}_2$
            drawpixel(x,y) with color c

Triangle rasterization algorithm



```
for x in [x_min, x_max]
    for y in [y_min, y_max]
```

$$\alpha = f_{bc}(x, y)/f_{bc}(x_a, y_a)$$

$$\beta = f_{ca}(x, y)/f_{ca}(x_b, y_b)$$

$$\gamma = f_{ab}(x, y)/f_{ab}(x_c, y_c)$$

if $(\alpha \in [0, 1] \text{ and } \beta \in [0, 1] \text{ and } \gamma \in [0, 1])$ then

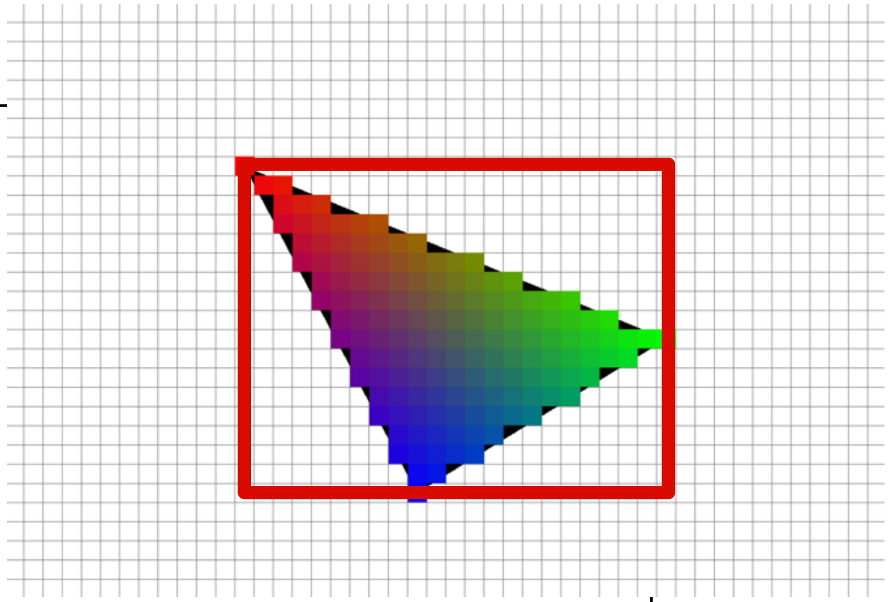
$$\mathbf{c} = \alpha\mathbf{c}_0 + \beta\mathbf{c}_1 + \gamma\mathbf{c}_2$$

drawpixel(x,y) with color c

<whiteboard>

Triangle rasterization algorithm

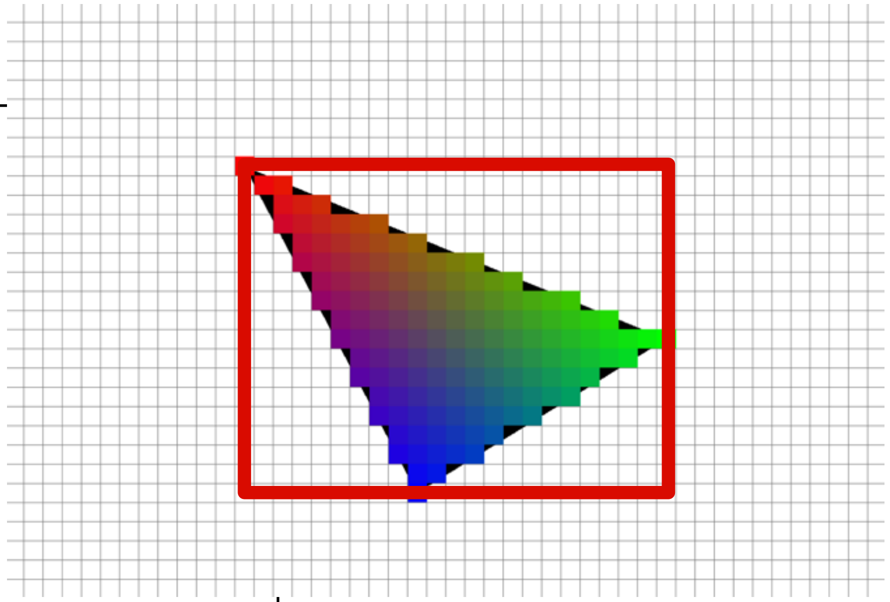
Optimizations?



for x in [x_min, x_max]
    for y in [y_min, y_max]
        $\alpha = f_{bc}(x, y)/f_{bc}(x_a, y_a)$
        $\beta = f_{ca}(x, y)/f_{ca}(x_b, y_b)$
        $\gamma = f_{ab}(x, y)/f_{ab}(x_c, y_c)$
        if $(\alpha \in [0,1] \text{ and } \beta \in [0,1] \text{ and } \gamma \in [0,1])$ then
            $\mathbf{c} = \alpha\mathbf{c}_0 + \beta\mathbf{c}_1 + \gamma\mathbf{c}_2$
            drawpixel(x,y) with color c

Triangle rasterization algorithm

Optimizations?

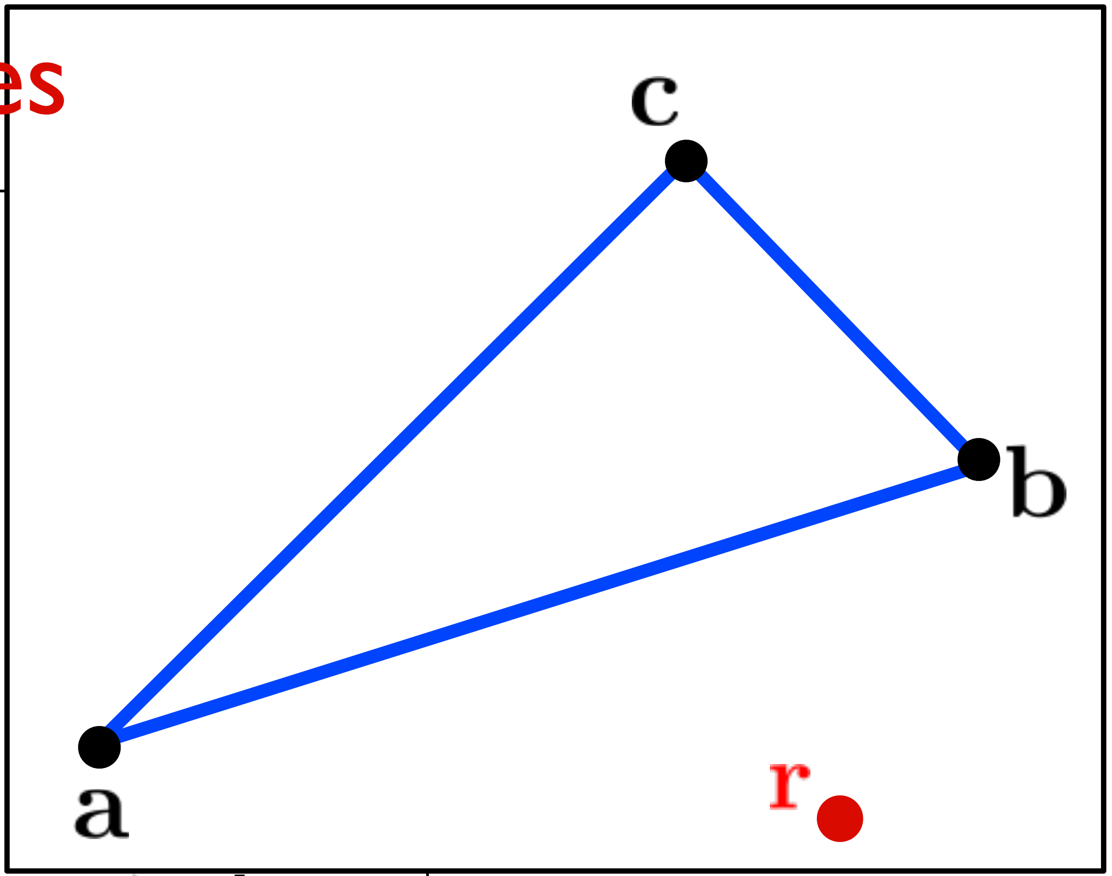


for x in [x_min, x_max]
    for y in [y_min, y_max]
        $\alpha = f_{bc}(x, y)/f_{bc}(x_a, y_a)$
        $\beta = f_{ca}(x, y)/f_{ca}(x_b, y_b)$
        $\gamma = f_{ab}(x, y)/f_{ab}(x_c, y_c)$
        if $(\alpha \geq 0 \text{ and } \beta \geq 0 \text{ and } \gamma \geq 0)$then
            $\mathbf{c} = \alpha\mathbf{c}_0 + \beta\mathbf{c}_1 + \gamma\mathbf{c}_2$
            drawpixel(x,y) with color c

make computation of bary. coords. incremental
color can also be computed incrementally
don't need to check upper bound

Triangle rasterization algorithm

dealing with shared triangle edges



for x in [x_min, x_max]

for y in [y_min, y_max]

$\alpha = f_{bc}(x,y)/f_{bc}(x_a,y_a)$

$\beta = f_{ac}(x,y)/f_{ac}(x_b,y_b)$

$\gamma = f_{ab}(x,y)/f_{ab}(x_c,y_c)$

if $(\alpha \geq 0 \text{ and } \beta \geq 0 \text{ and } \gamma \geq 0)$ then

if $(\alpha > 0 \text{ or } f_{bc}(\mathbf{a})f_{bc}(\mathbf{r}) > 0)$ and $(\beta > 0 \text{ or } f_{ca}(\mathbf{b})f_{ca}(\mathbf{r}) > 0)$ and $(\gamma > 0 \text{ or } f_{ab}(\mathbf{c})f_{ab}(\mathbf{r}) > 0)$ then

$\mathbf{c} = \alpha\mathbf{c}_0 + \beta\mathbf{c}_1 + \gamma\mathbf{c}_2$

drawpixel(x,y) with color c