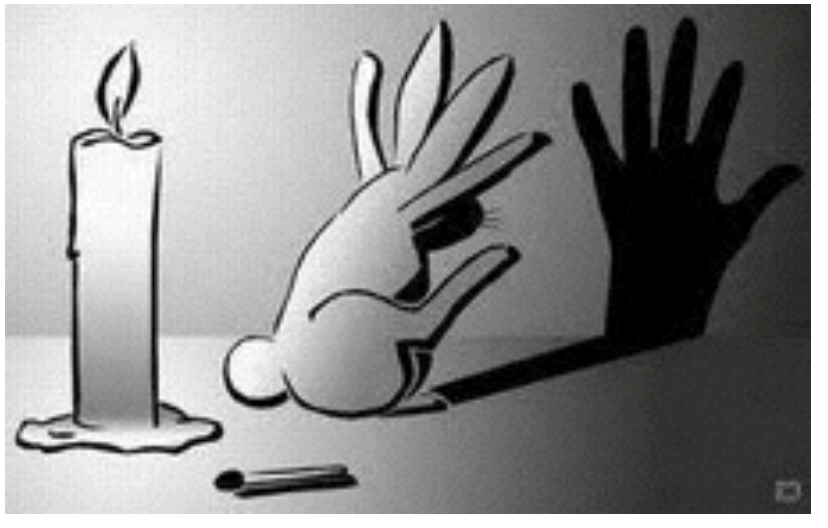
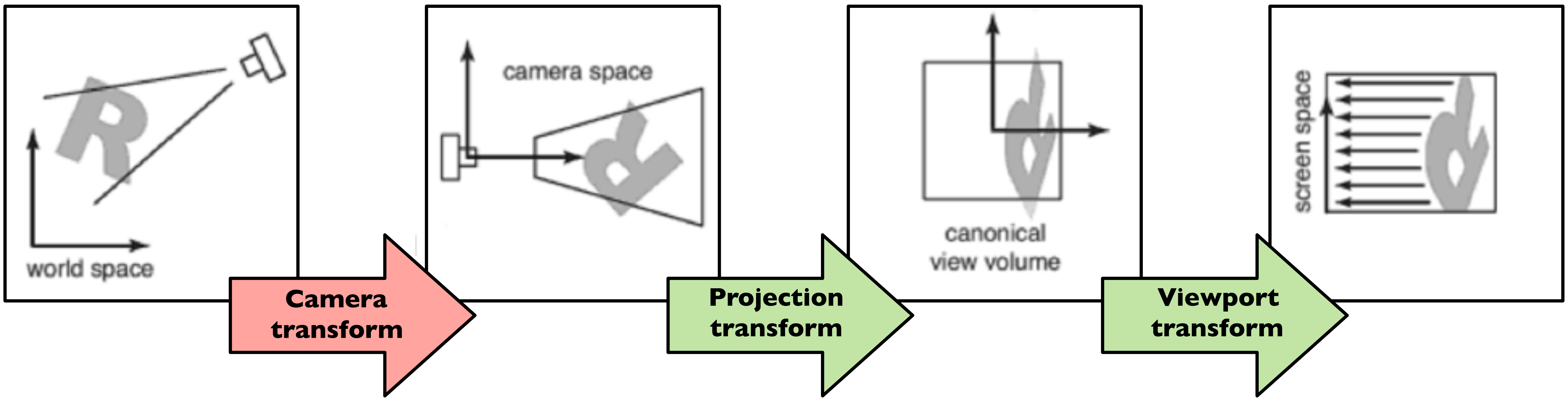
Viewing Transformations



Camera Transform



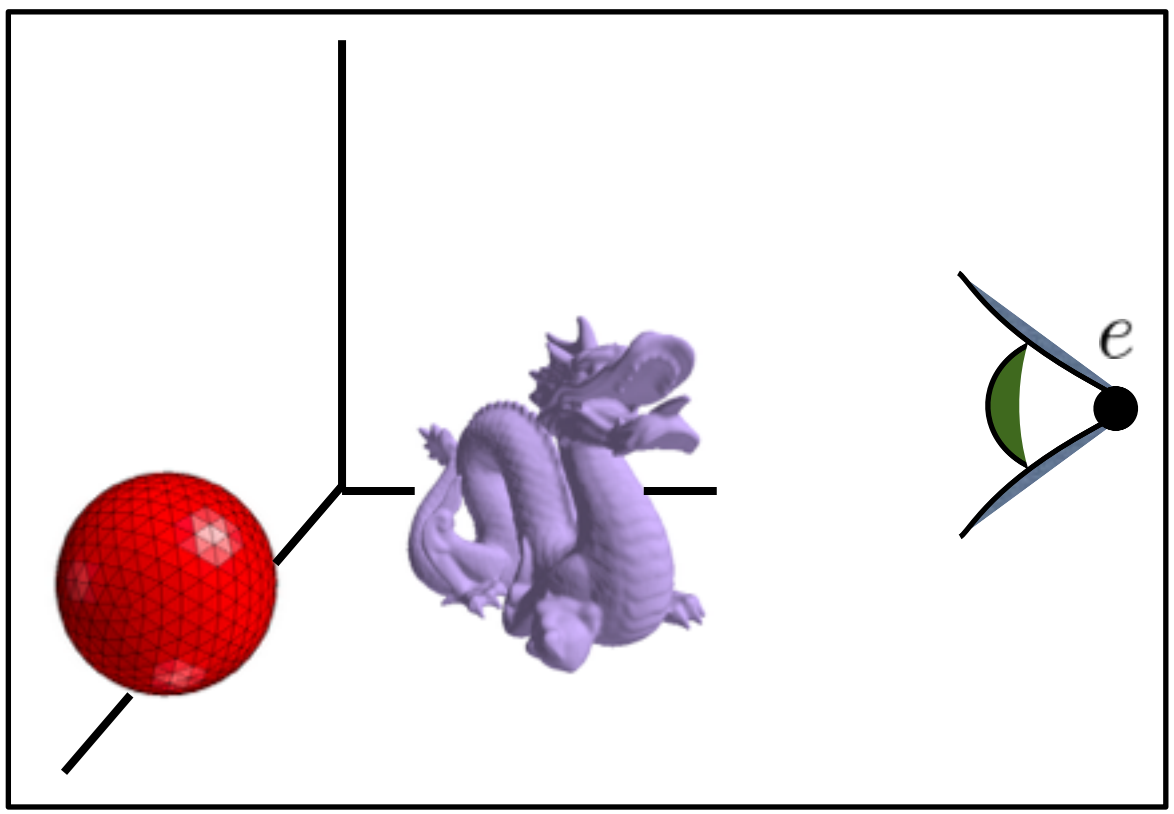
Camera Transform

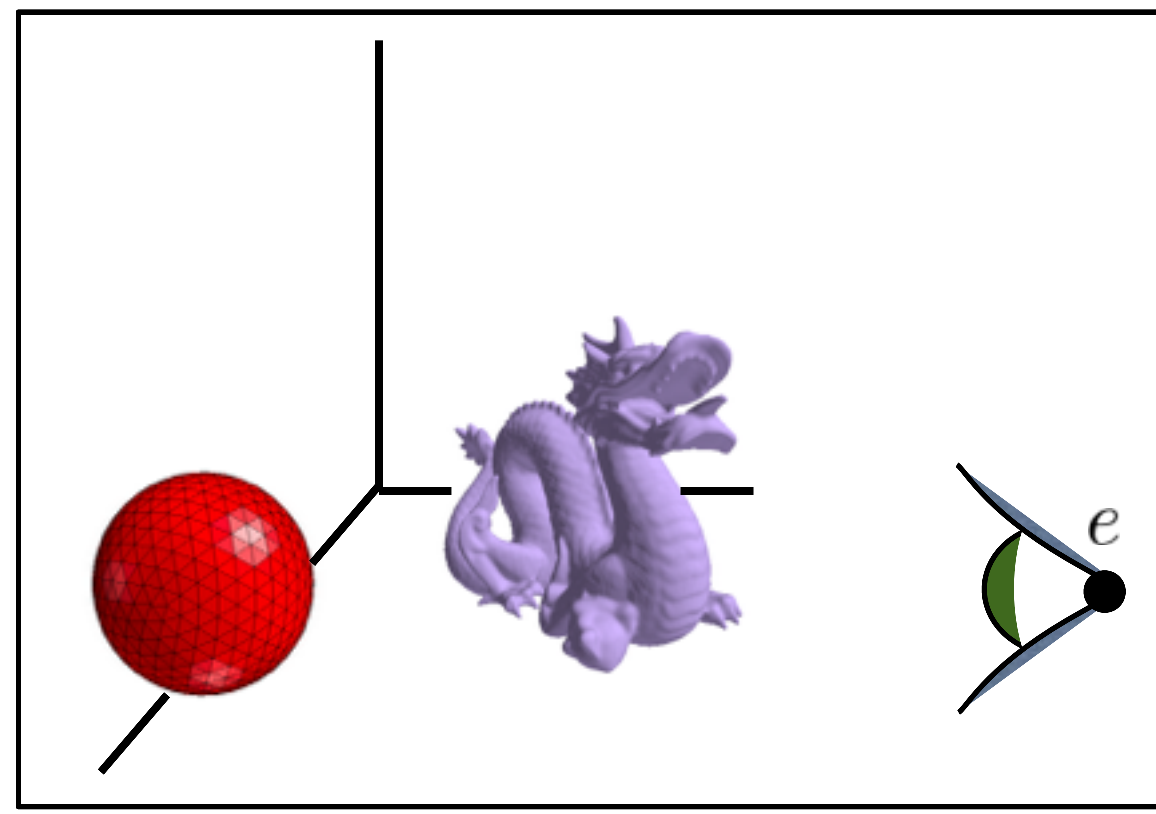
How do we specify the camera configuration?

Camera Transform

How do we specify the camera configuration?

eye position

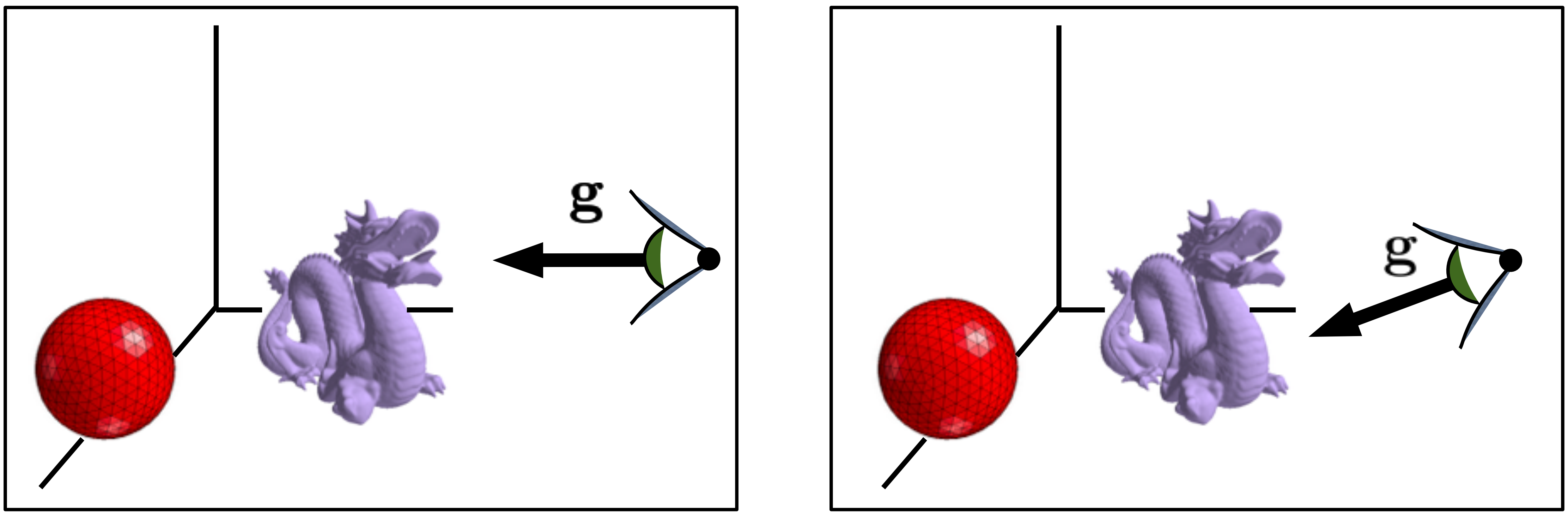




Camera Transform

How do we specify the camera configuration?

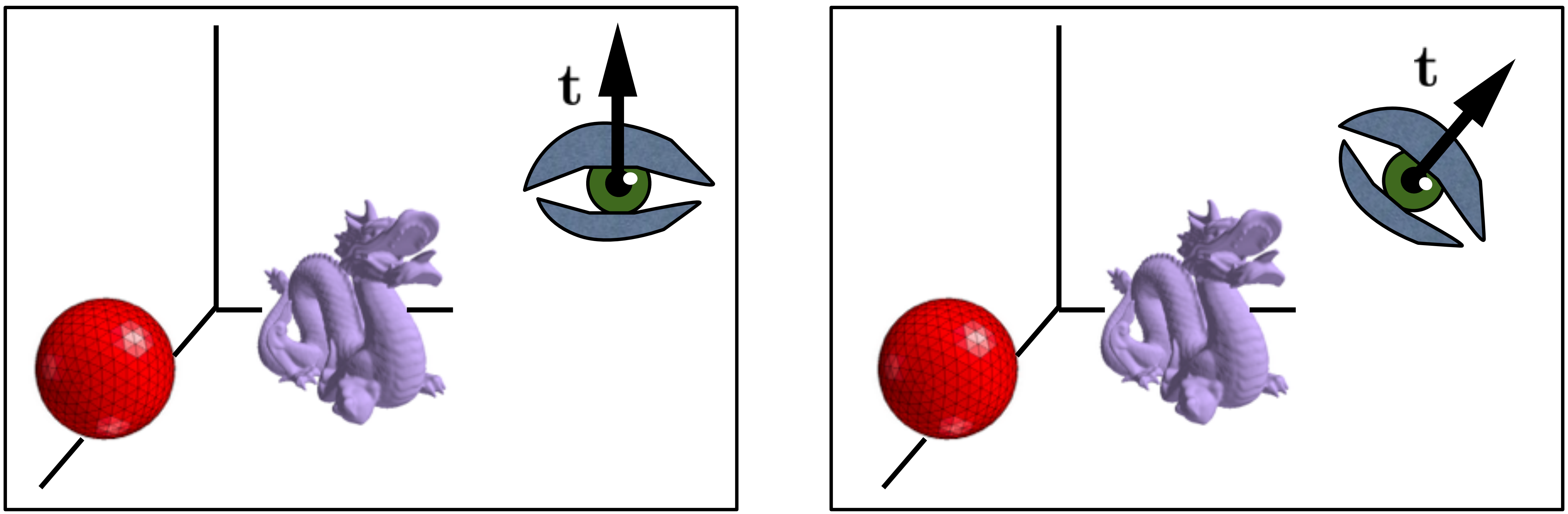
gaze direction



Camera Transform

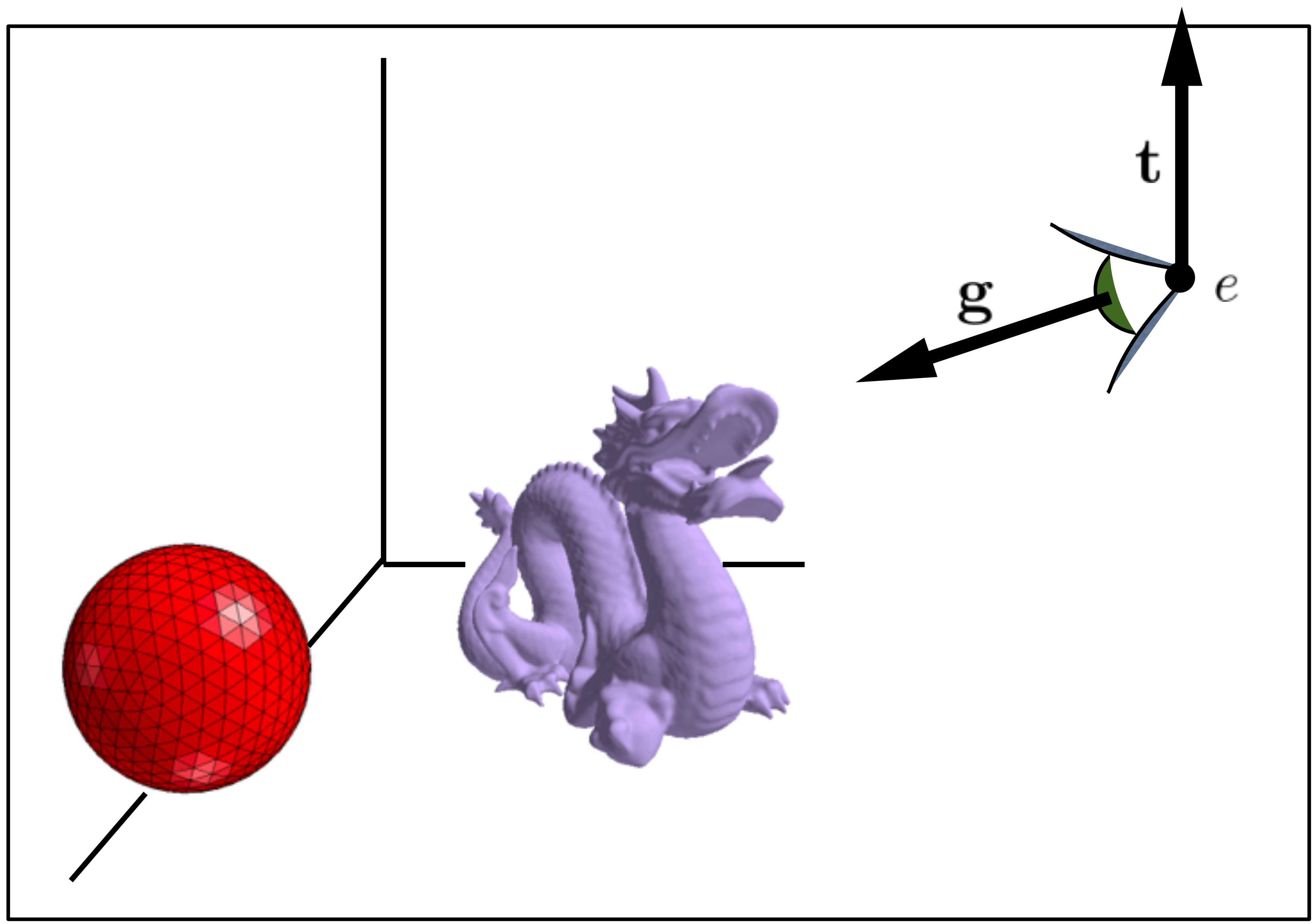
How do we specify the camera configuration?

up vector

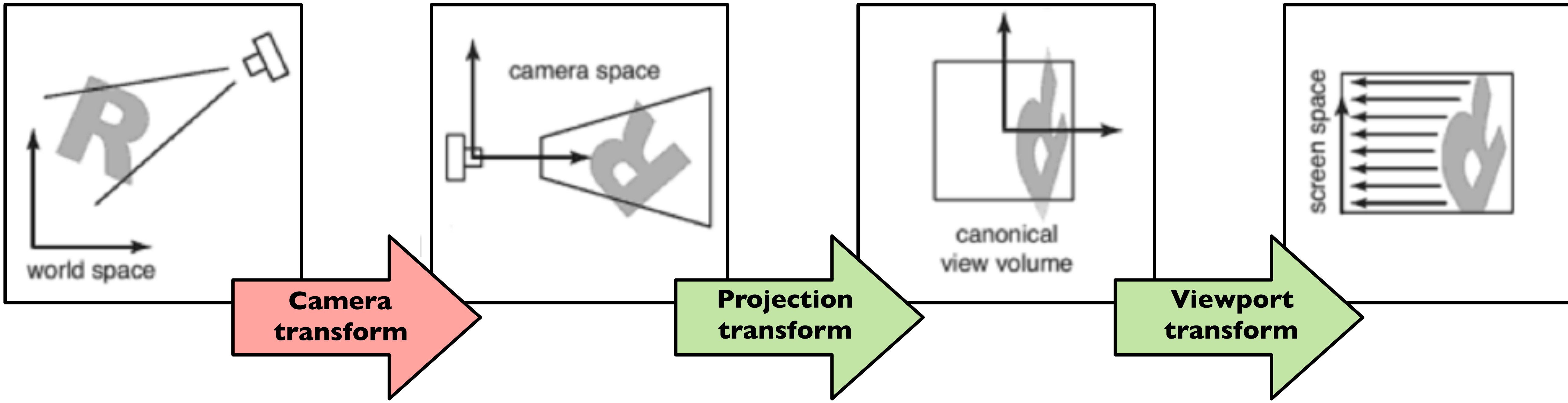


Camera Transform

How do we specify the camera configuration?



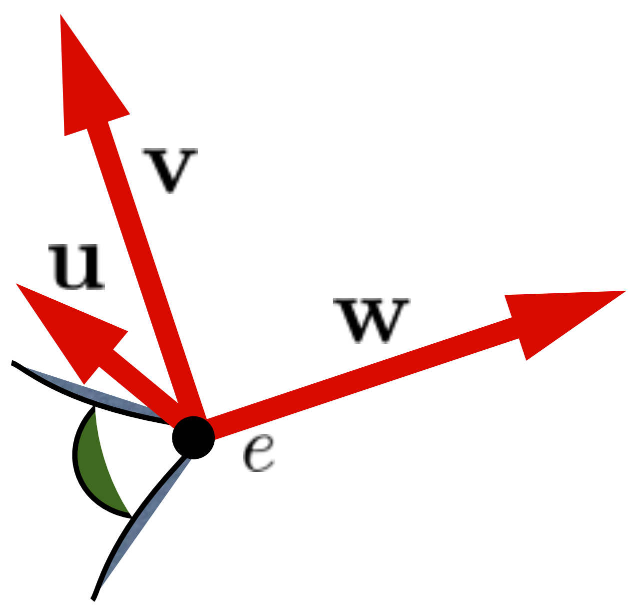
Camera Transform

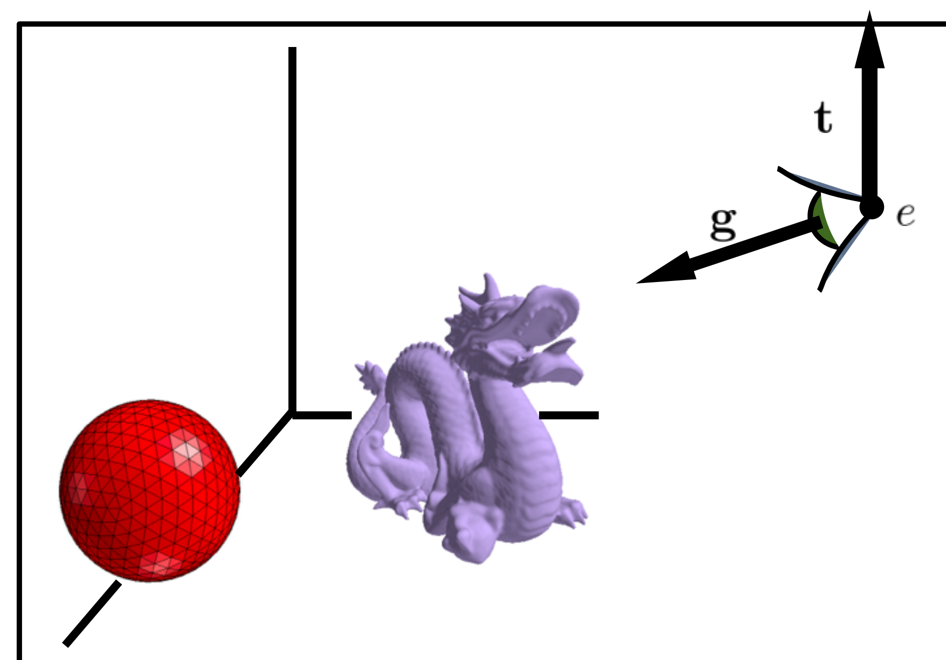


$$\mathbf{w} = -\frac{\mathbf{g}}{||\mathbf{g}||}$$

$$\mathbf{u} = \frac{\mathbf{t} \times \mathbf{w}}{||\mathbf{t} \times \mathbf{w}||}$$

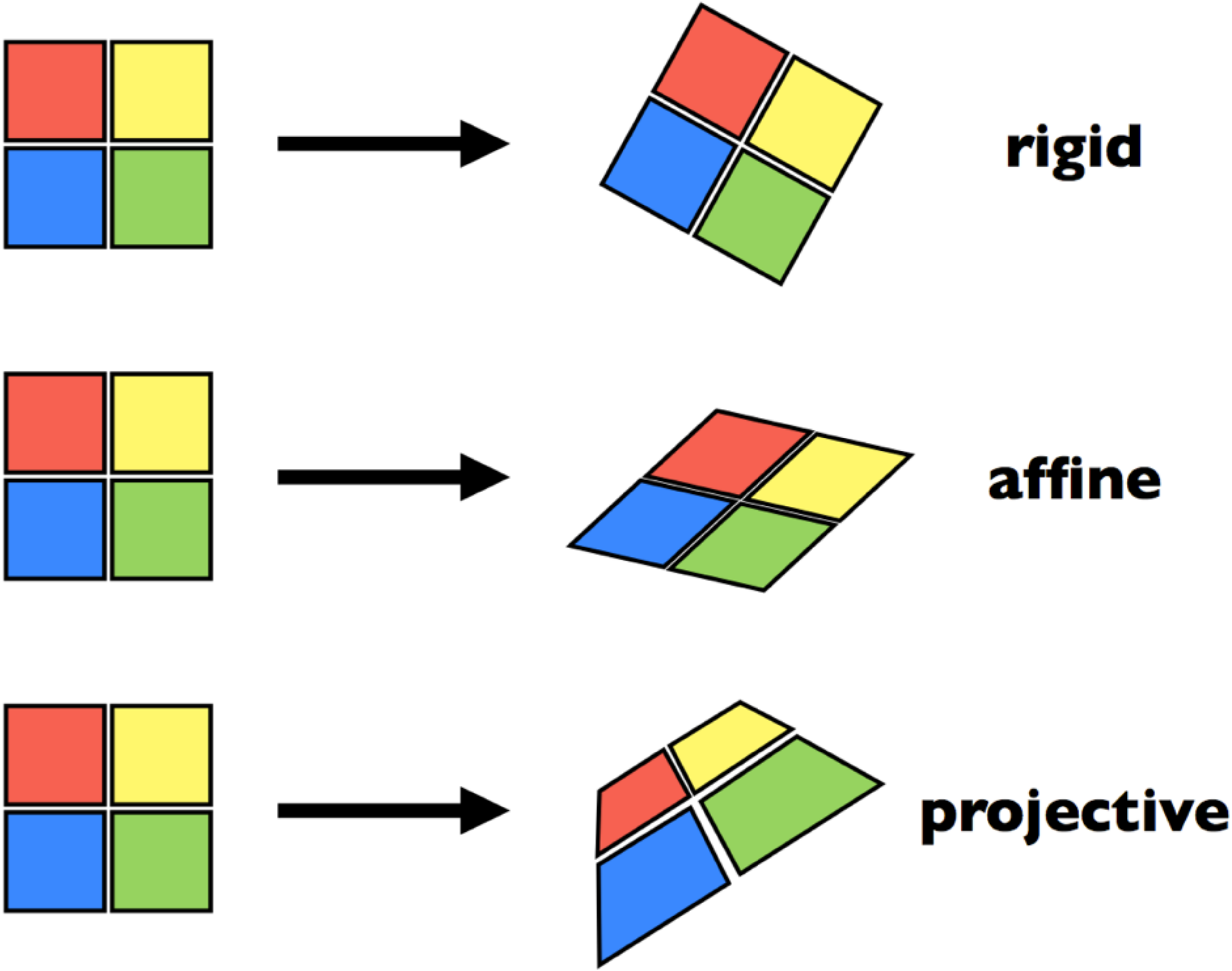
$$\mathbf{v} = \mathbf{w} \times \mathbf{u}$$



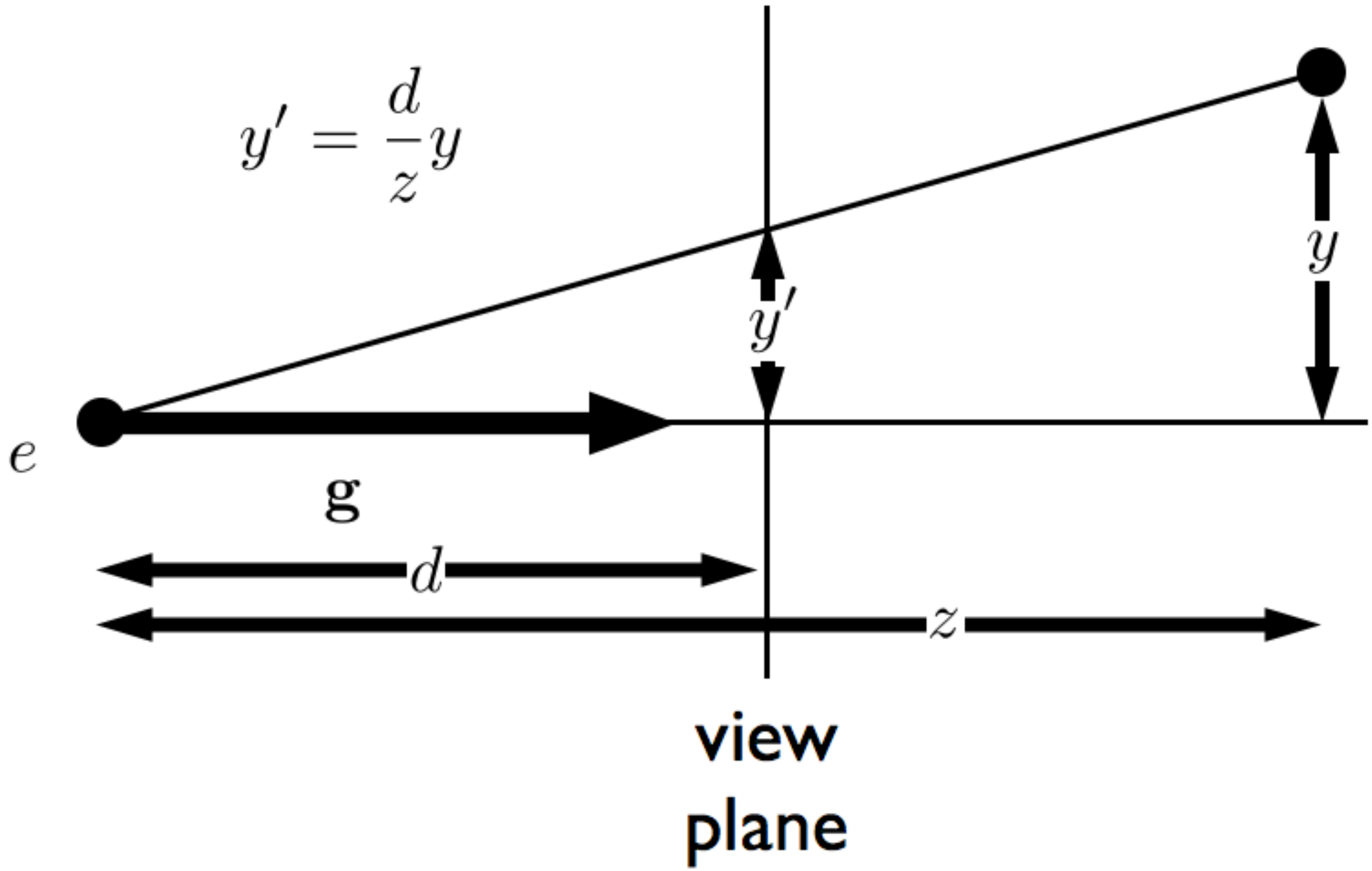


M_{cam} <whiteboard>

Perspective Viewing


rigid
affine
projective

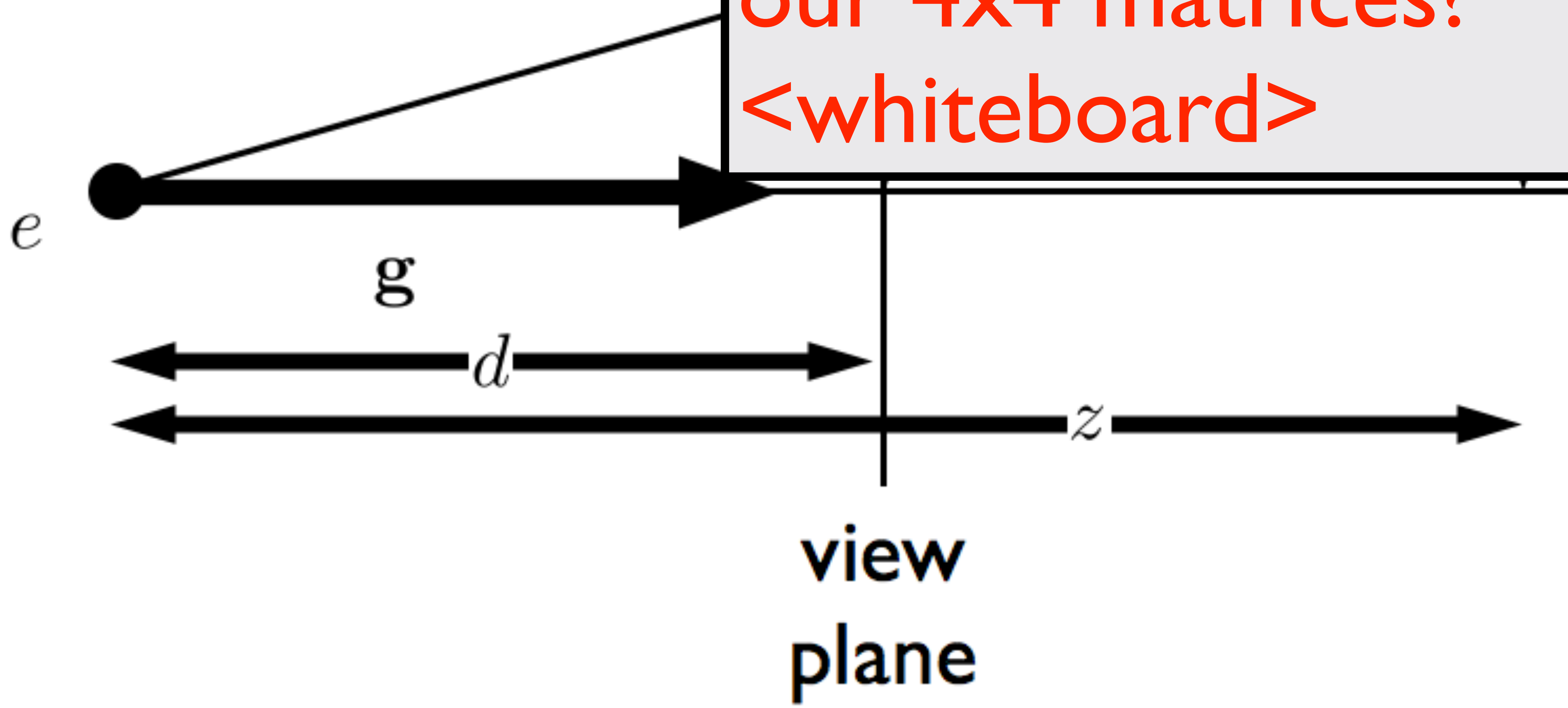
Projective Transformations



Projective Transformations

$$y' = \frac{d}{z} y$$

How can we represent this with our 4x4 matrices?
<whiteboard>

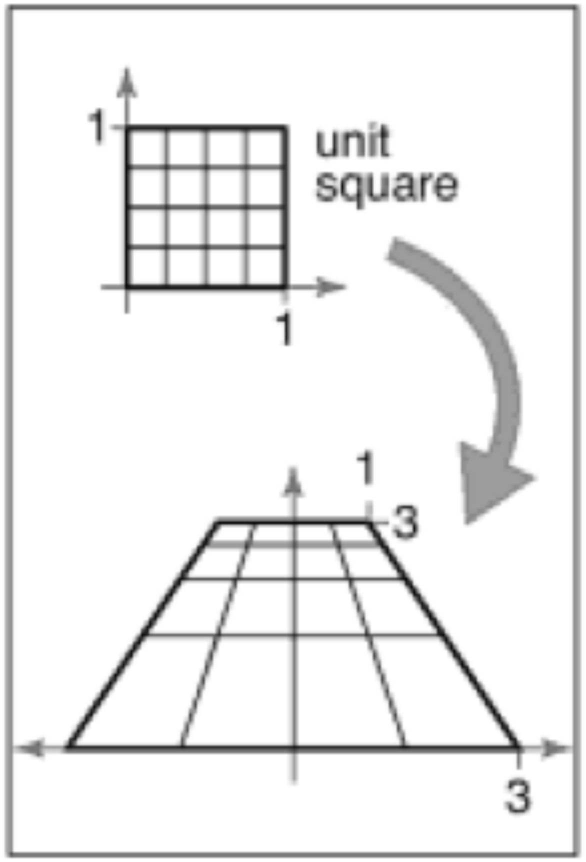


Projective Transformations

$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \\ w \end{pmatrix} \rightarrow \begin{matrix} x = \dfrac{\tilde{x}}{w} \\ y = \dfrac{\tilde{y}}{w} \\ z = \dfrac{\tilde{z}}{w} \end{matrix}$$

Example:

$$M = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 3 & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} \end{pmatrix}$$



[Shirley, Marschner]

<whiteboard>

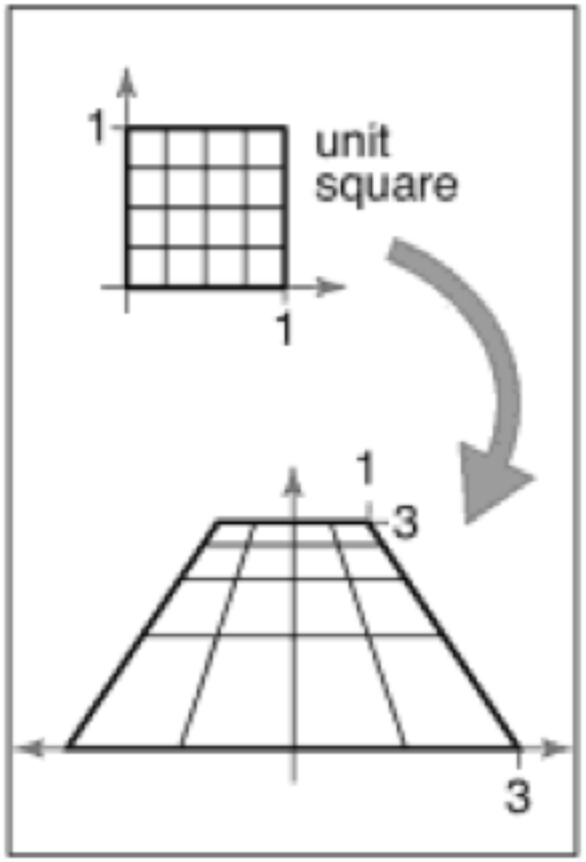
Projective Transformations

$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \\ w \end{pmatrix} \rightarrow \begin{array}{l} x = \dfrac{\tilde{x}}{w} \\ y = \dfrac{\tilde{y}}{w} \\ z = \dfrac{\tilde{z}}{w} \end{array}$$

We can now implement perspective projection!

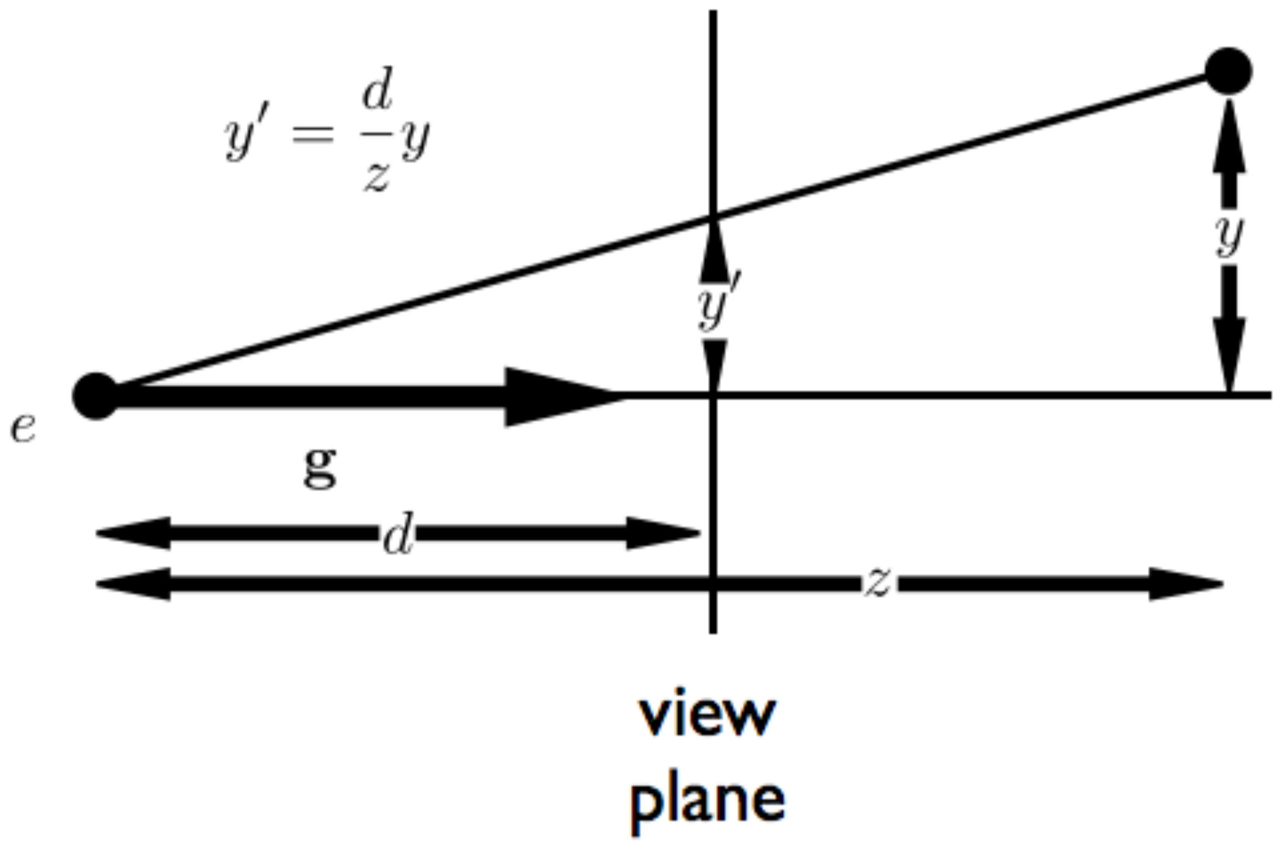
Example:

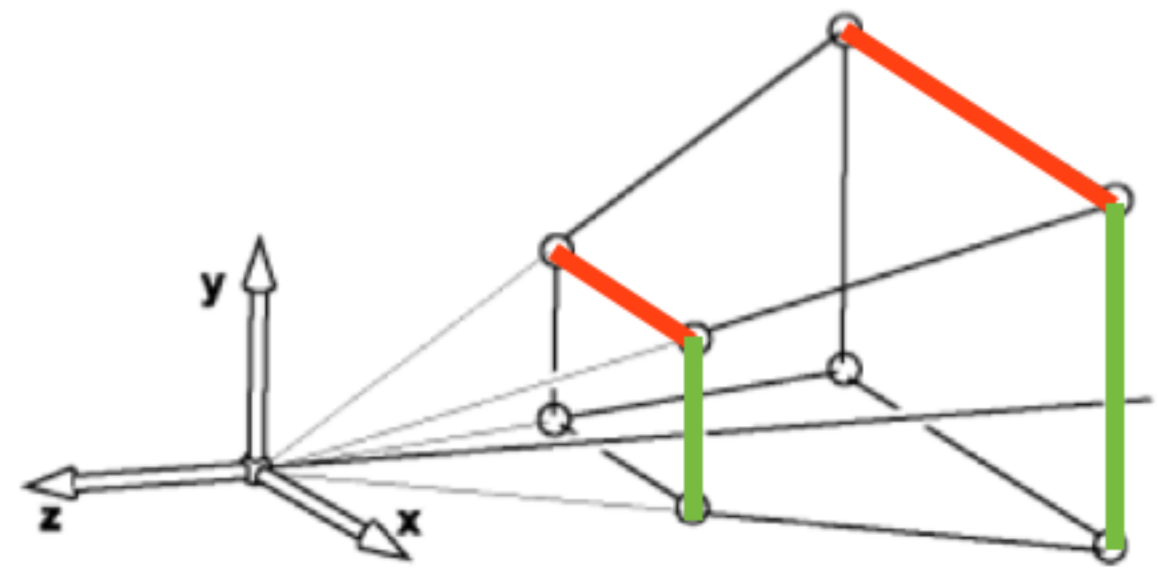
$$M = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 3 & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} \end{pmatrix}$$



[Shirley, Marschner]

Perspective Projection





both x and y get multiplied by d/z

[Shirley, Marschner]

Simple perspective projection

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \\ z/d \end{pmatrix} \Rightarrow \begin{cases} x' = \frac{d}{z} x \\ y' = \frac{d}{z} y \\ z' = \frac{d}{z} z = d \end{cases}$$

This achieves a simple perspective projection onto the view plane z = d

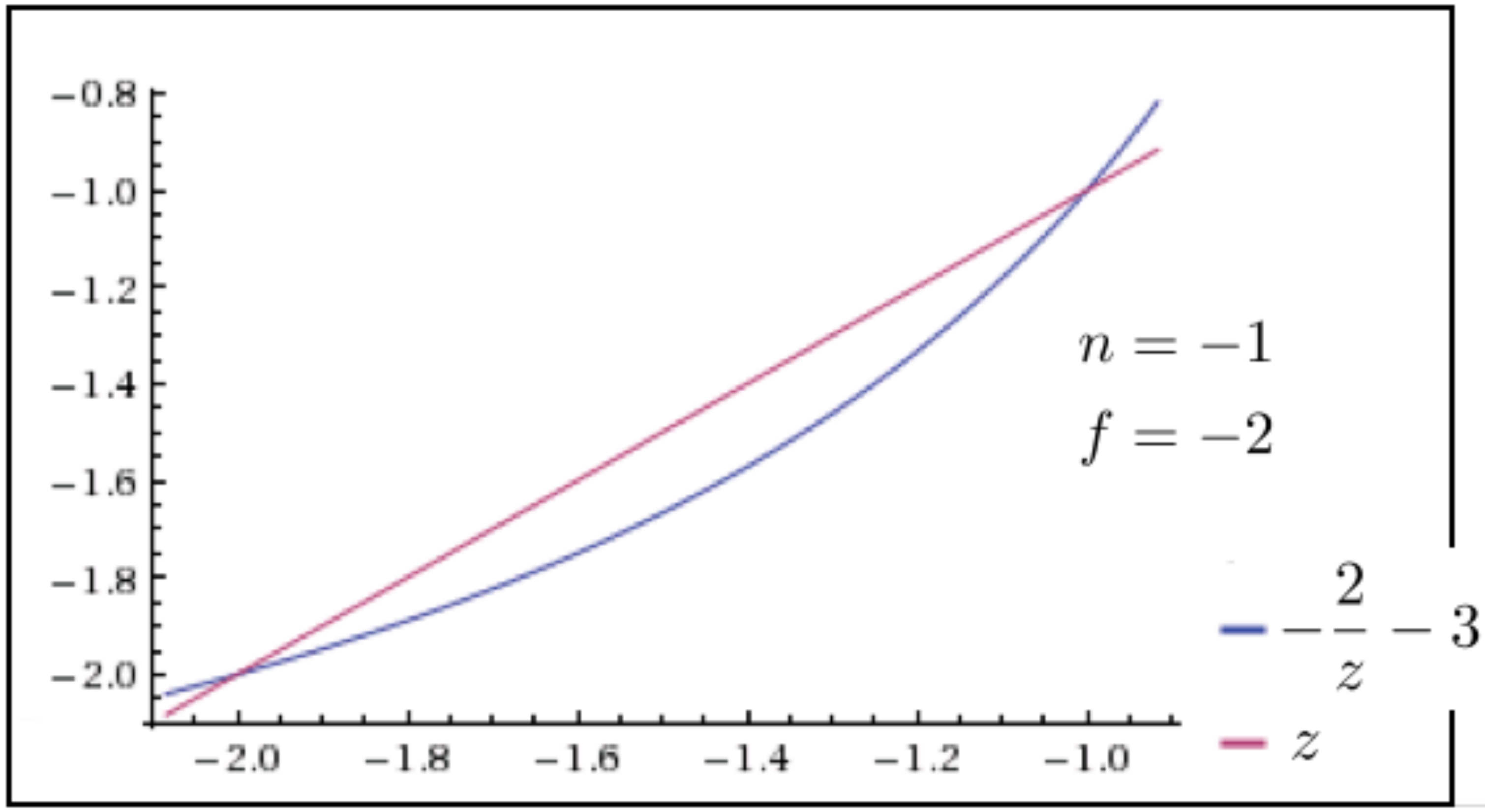
but we've lost all information about z!

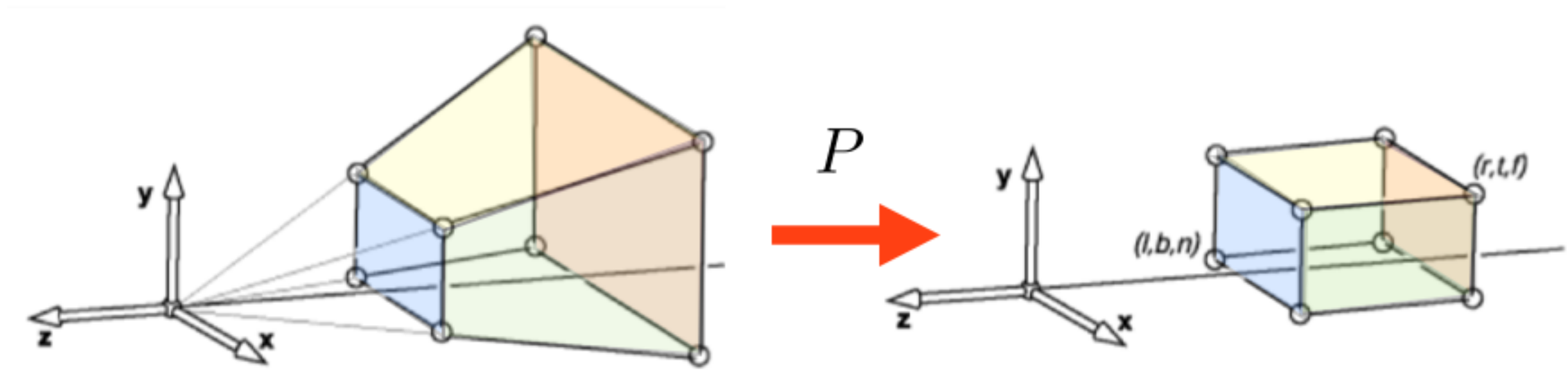
<whiteboard>

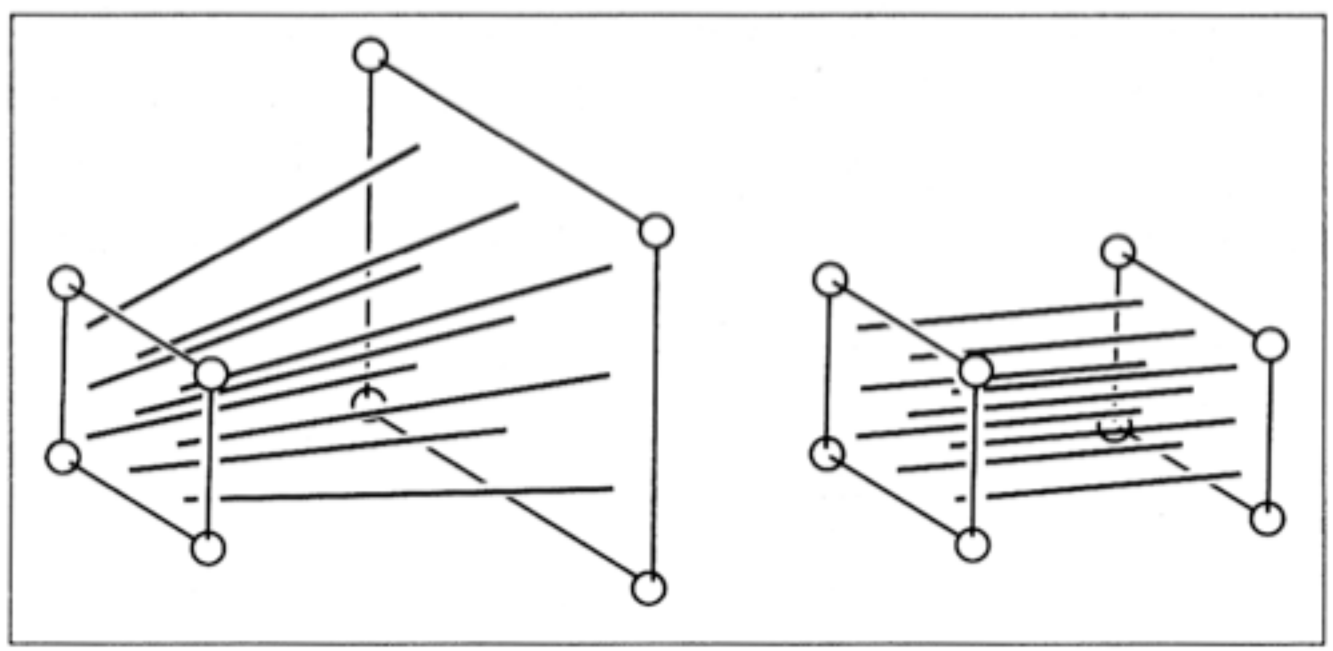
Perspective Projection

$$P = \begin{pmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad z' = (n+f) - \frac{nf}{z}$$

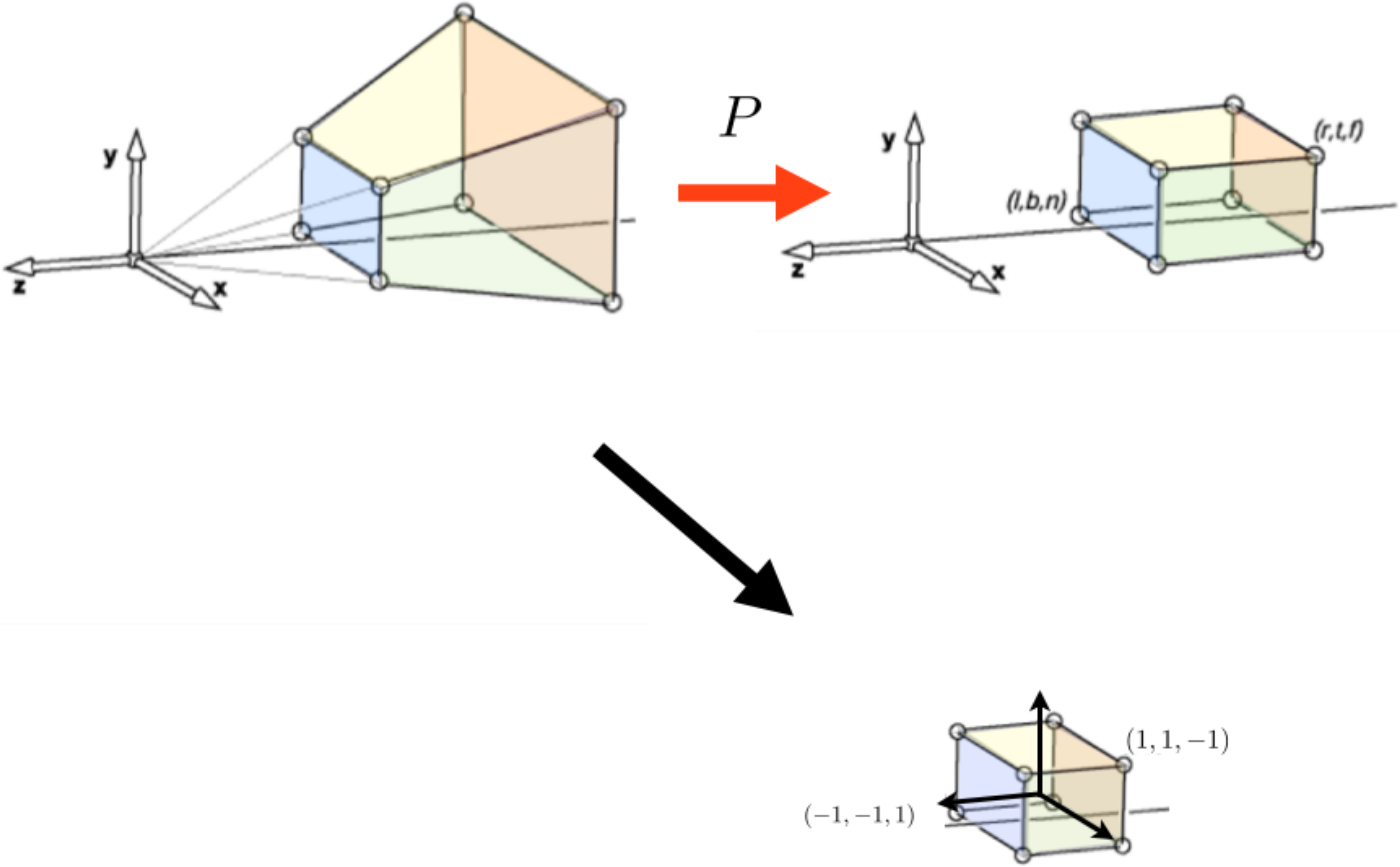
Example:



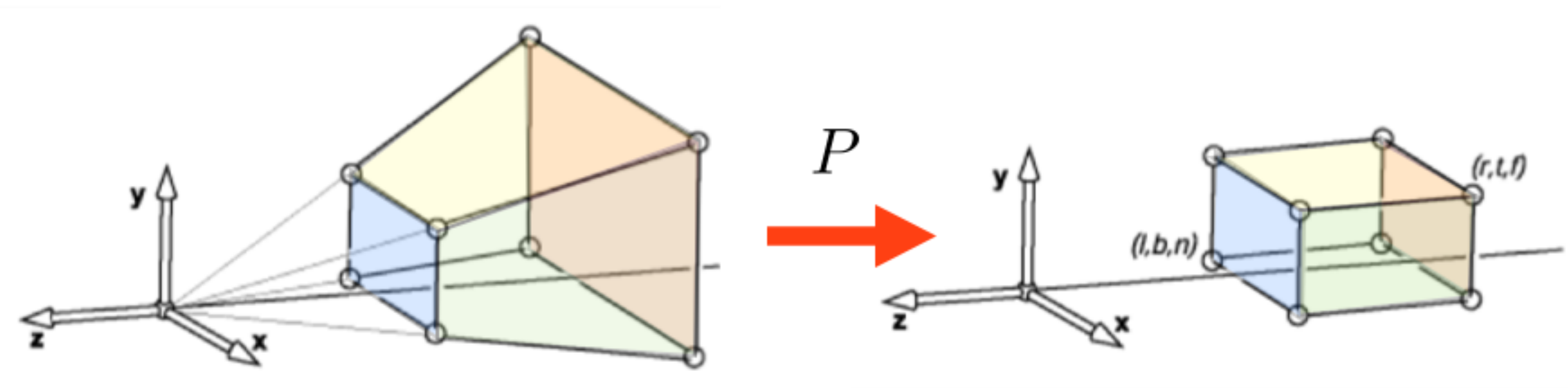


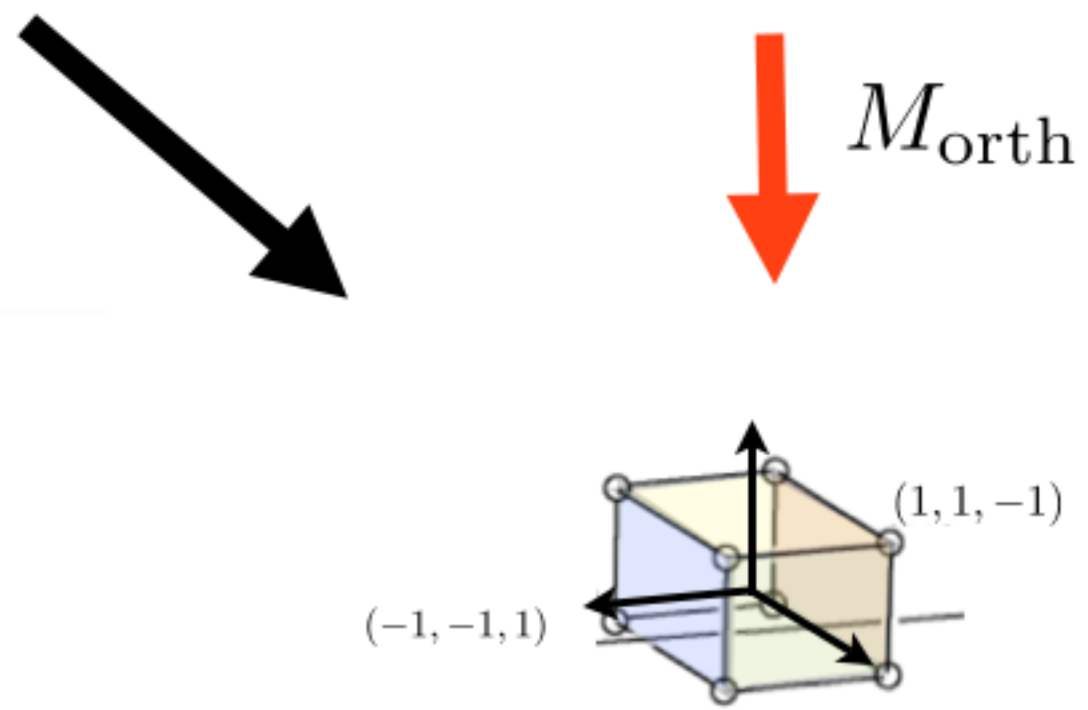


[Shirley, Marschner]



y
z
x
P
y
z
x
(l,b,n)
(r,t,f)
(1, 1, −1)
(−1, −1, 1)



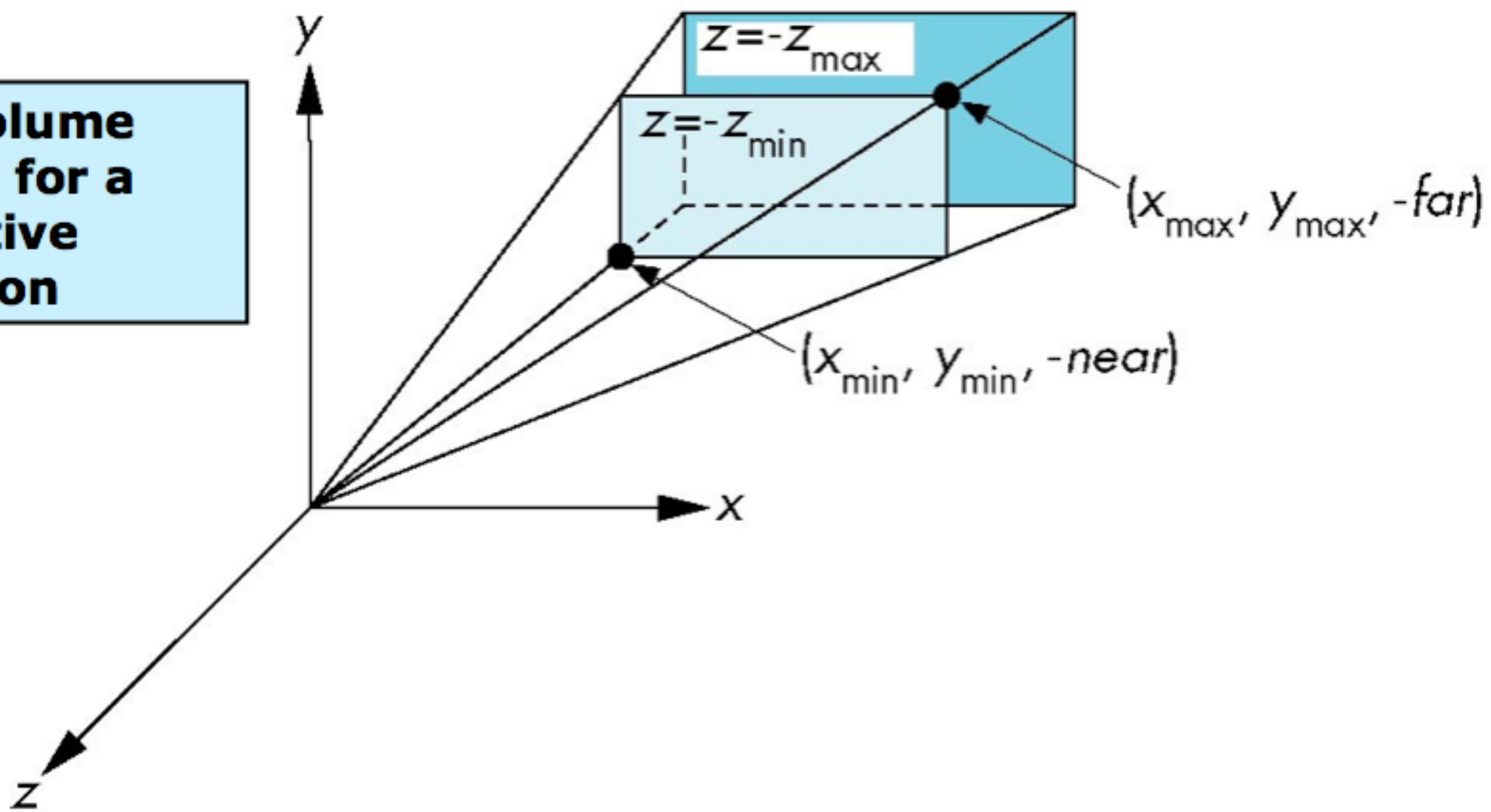


$$M_{\text{per}} = M_{\text{orth}} P$$

OpenGL Perspective Viewing

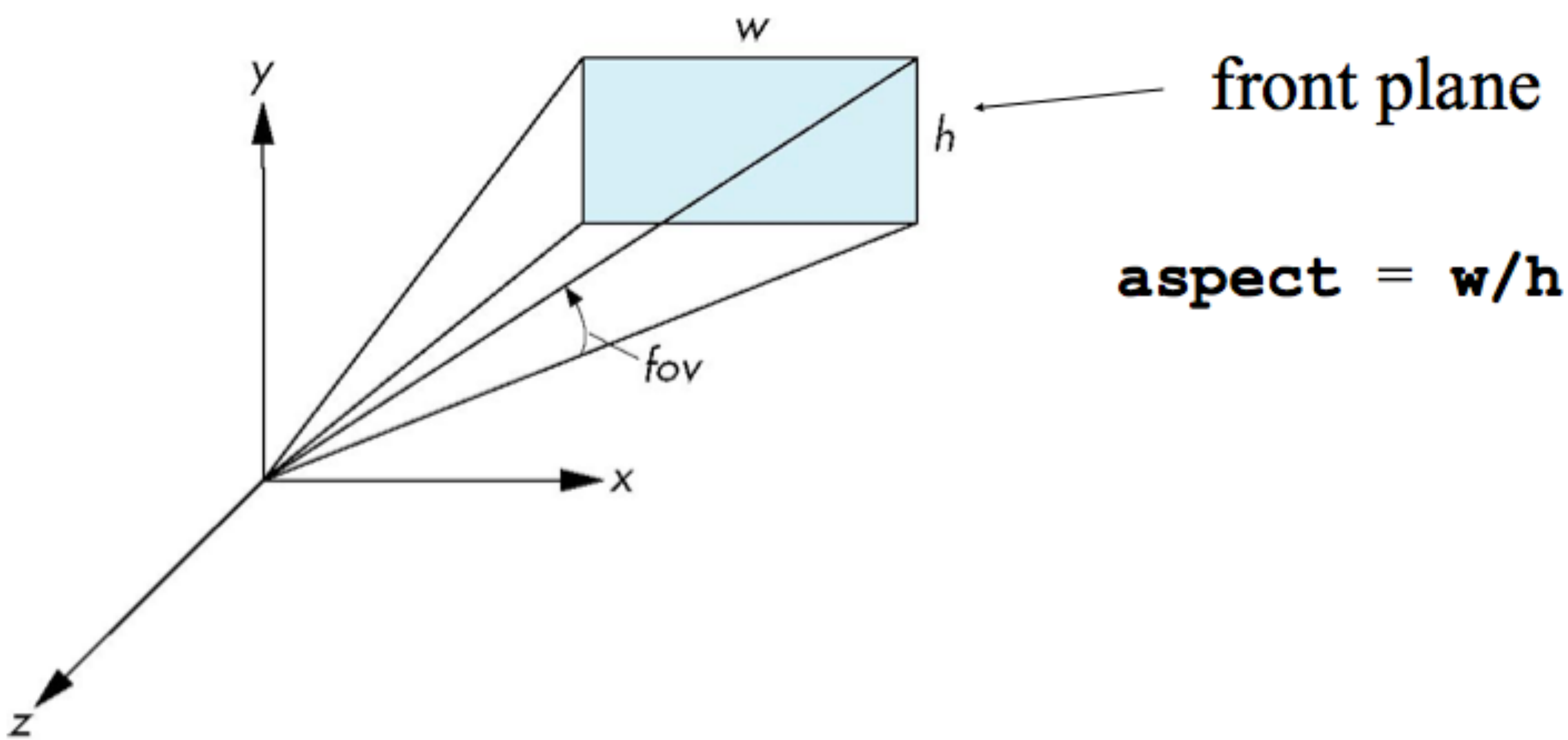
```
glFrustum(xmin,xmax,ymin,ymax,near,far)
```

Clipping volume (frustrum) for a perspective projection

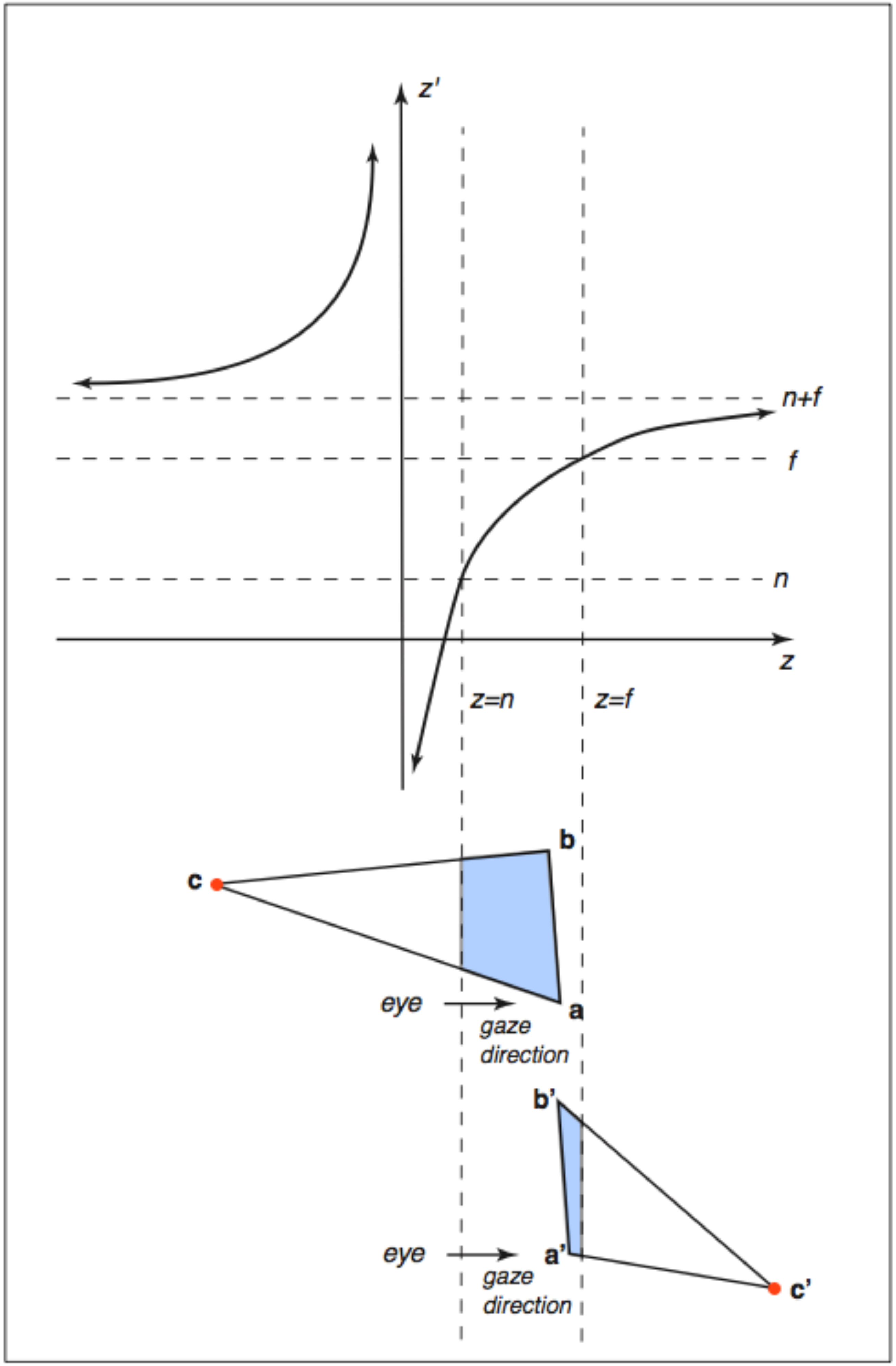


Using Field of View

With **glFrustum** it is often difficult to get the desired view **gluPerpective(fovy, aspect, near, far)** often provides a better interface

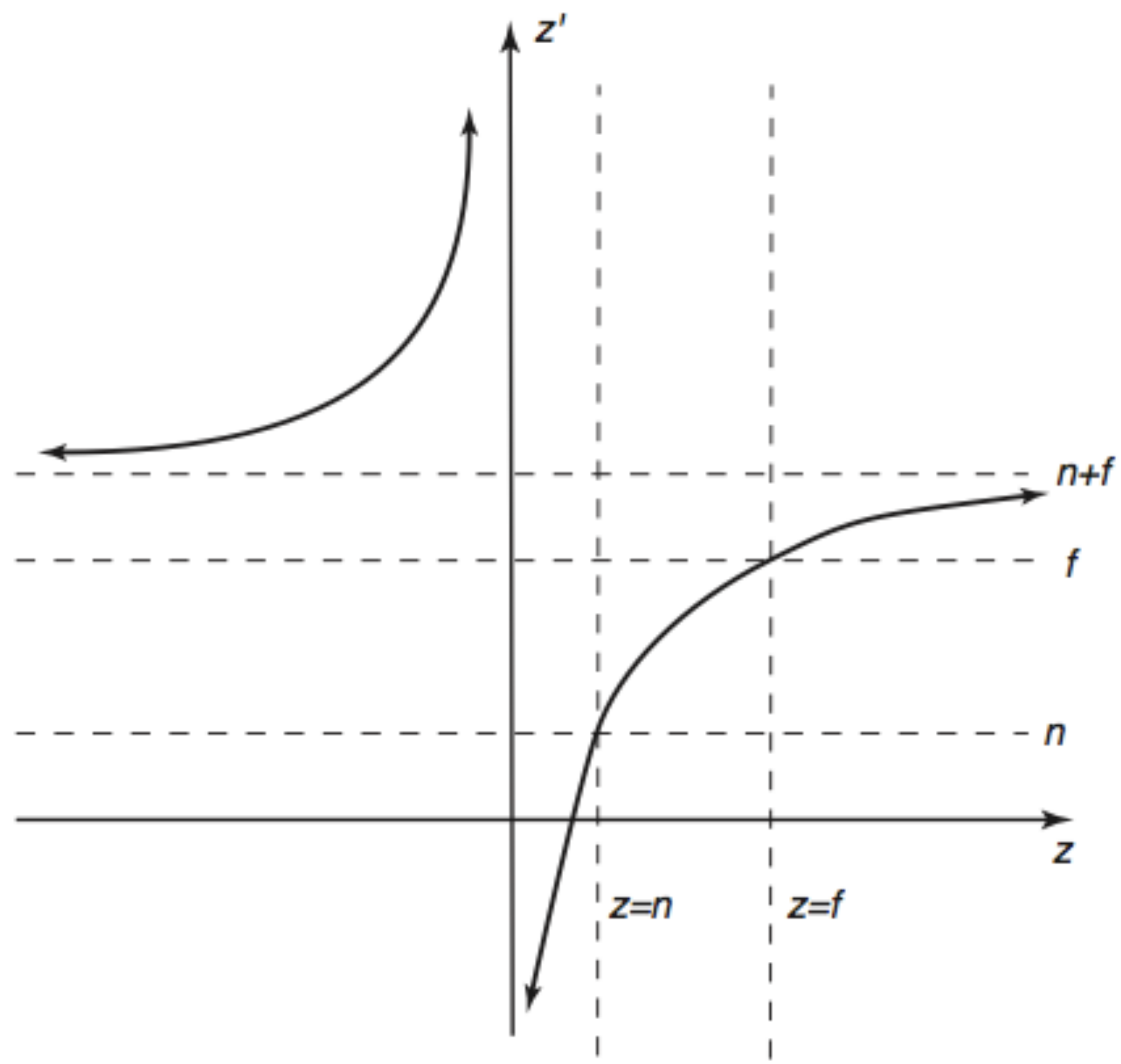


aspect = w/h



[Shirley, Marschner]

Clipping after the perspective transformation can cause problems

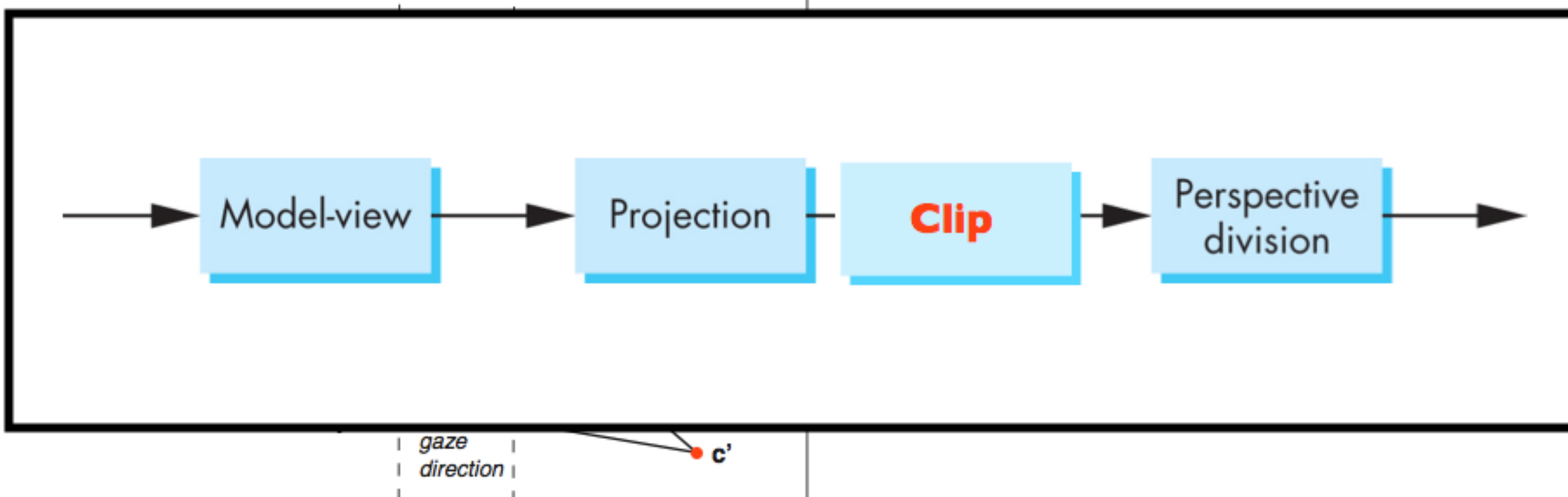


OpenGL clips **after** projection and **before** perspective division

$$-w \leq x \leq w$$

$$-w \leq y \leq w$$

$$-w \leq z \leq w$$



[Shirley, Marschner]