Ray tracing

LECTURE 9

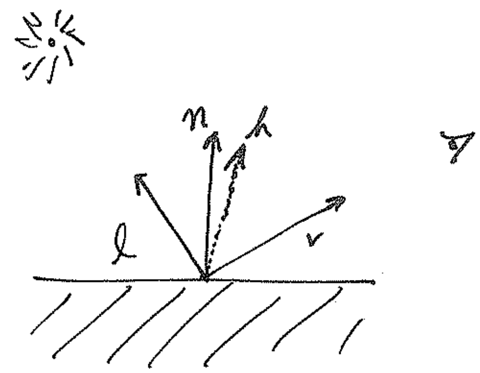
- Can build many nice effects on top of it.
  - much more difficult in rasterization framework.
  - e.g., shadows, reflections.

Recall Blinn-Phong ~~Sh.~~ Reflectance Model

$$L = k_a I_a + k_d I_d \max(0, \underset{\sim}{n} \cdot \underset{\sim}{l}) + k_s I_s \max(0, n \cdot h)^{\rho}$$

Multiple light sources

$$L = k_a I_a + \sum_{i=1}^{N} \left(k_d (I_d)_i \max(0, n \cdot l_i) + k_s (I_s)_i \max(0, n \cdot h_i)^{\rho} \right)$$



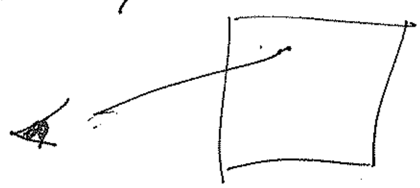
$$h = \frac{v + l}{|v + l|}$$

Types of Rays

- Eye/pixel rays
- Illumination/shadow rays.
- Reflection rays
- Transmission/transparency rays.

Eye rays

- automatically get a perspective view when trace from eye.

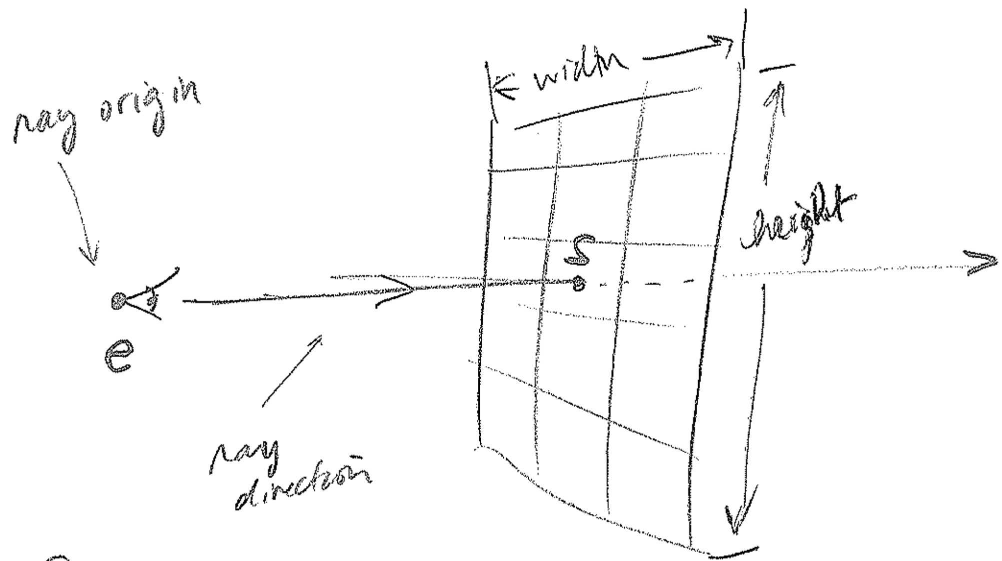


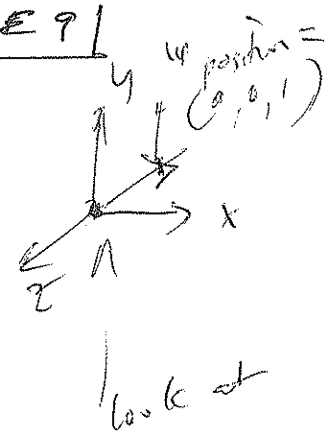
Q. How would you get an orthogonal view?

Illumination/Shadow Rays.

LECTURE 9

Computing View Rays.





Ray equation

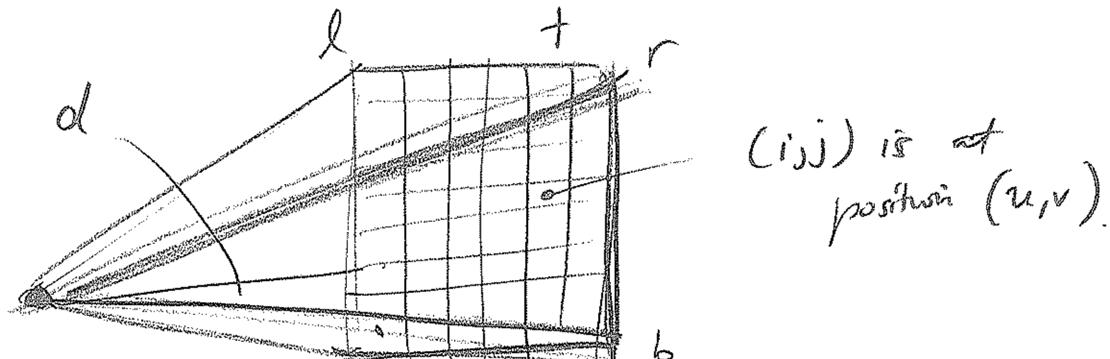
$$p(t) = \underset{\sim}{e} + t(\underset{\sim}{s} - \underset{\sim}{e})$$

$d = s - e$

Notes: $p(0) = \underset{\sim}{e}$

$p(1) = \underset{\sim}{s}$

$p(\alpha)$, $\alpha < 0$ behind the eye.



(i,j)

position $[0, n_x] \times [0, n_y] \Longrightarrow [l, r] \times [b, t]$

$u = l + \left(\frac{1}{2} + \frac{i}{n_x}\right)(r - l)$, $v = b + \frac{(j + \frac{1}{2})}{n_y}(t - b)$

Shadows

Ray_Color(ray) $\qquad$ ray = ray(e, d, 0, ∞)

if (object = closest_intersection (ray))

then

$p = e + \text{ray}.t\,d$ = ray.Point(ray.t_max)

color = $k_a I_a$

Shadow ray

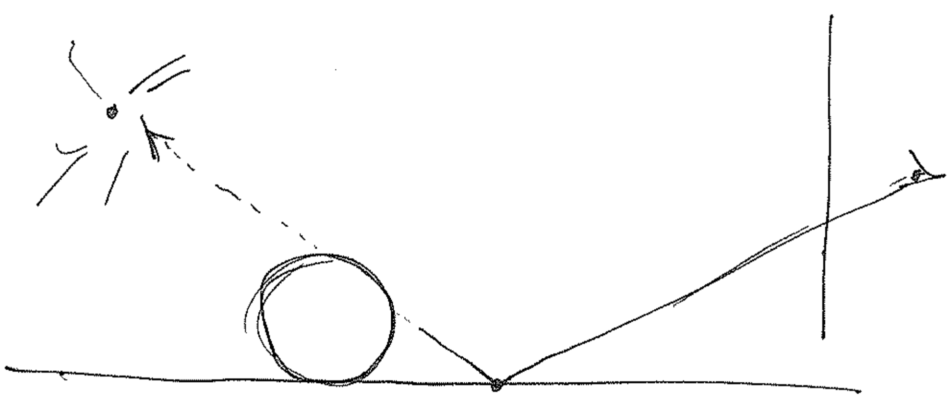
if (! closest_intersection (ray(p, l, ε, ∞)) then

$h = \ldots \left\| \frac{l + d}{\|l\| \; \|d\|} \right\|$

$c \mathrel{+}= k_d I_d \max(0, n \cdot l) + k_s I_s (n \cdot h)^p$

else

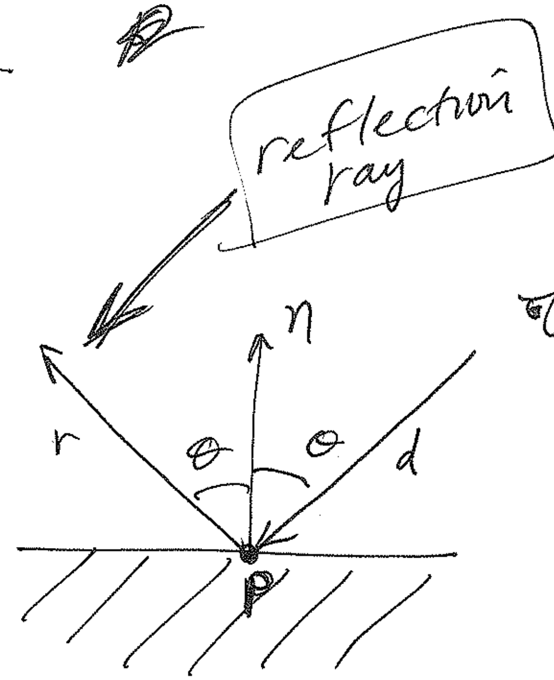
return background color.



point in shadow.

Ideal Specular Reflection

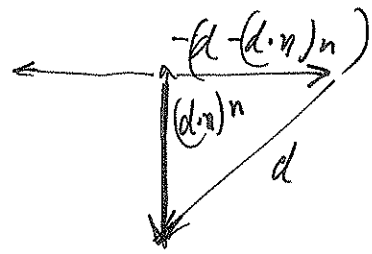
mirror reflection



$$r = d - 2(d \cdot n)n$$

Color $c = c + k_m$ Ray_Color(
$\underline{p} + s\underline{r}, \varepsilon, \infty)$

important to avoid intersecting w/ self



~~d + +~~

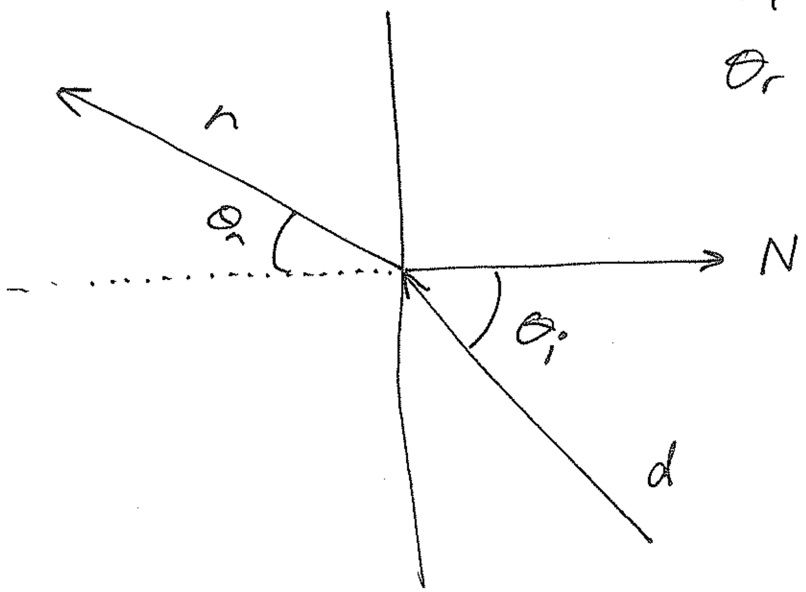
$$r = -d + 2(d - (d \cdot n)n)$$
$$= +d - 2(d \cdot n)n$$

- Add ~~t~~ a maximum recursion depth!

- don't generate a reflection if $k_m == 0$ (no reflection).

Transparency and Refraction

e.g. diamonds, glass, water



θ_i = angle of incidence
θ_r = " refraction

η_i, η_r = indices of refraction.

e.g.

$\eta_{air} \approx 1$
$\eta_{water} \approx 1.33$

Snell's Law:

$$\eta_i \sin\theta_i = \eta_r \sin\theta_r$$

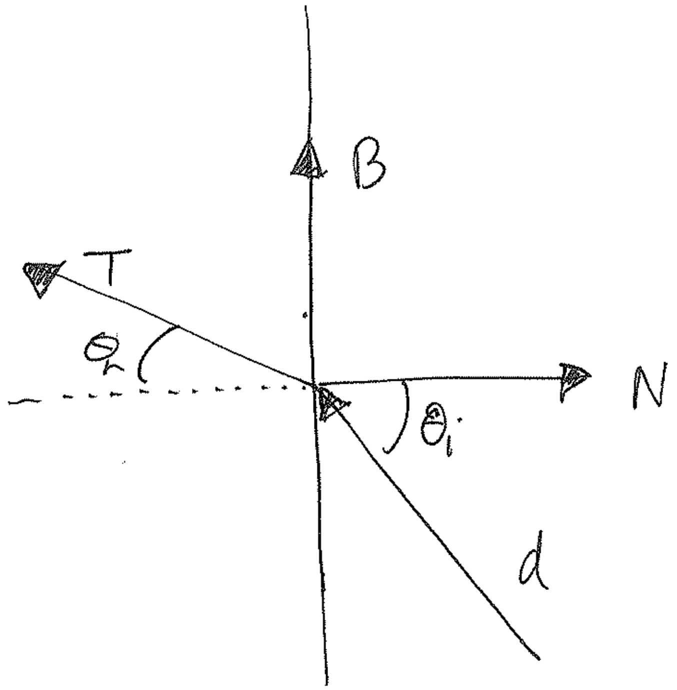
more convenient in cosines (dot product)

$$\eta_i^2 \sin^2\theta_i = \eta_r^2 \sin^2\theta_r$$

$$\eta_i^2 (1-\cos^2\theta_i) = \eta_r^2 (1-\cos^2\theta_r)$$

$$\Rightarrow \quad \cos^2\theta_r = \frac{\eta_r^2 - \eta_i^2(1-\cos^2\theta_i)}{\eta_r^2}$$

$$\Rightarrow \quad \cos\theta_r = \left[1 - \left(\frac{\eta_i}{\eta_r}\right)^2 (1-\cos^2\theta_i)\right]^{1/2}$$



① $\underset{\sim}{T} = -\cos\theta_r \underset{\sim}{N} + \sin\theta_r \underset{\sim}{B}$

② $\underset{\sim}{d} = -\cos\theta_i \underset{\sim}{N} + \sin\theta_i \underset{\sim}{B}$

② $\Rightarrow \quad \underset{\sim}{B} = \dfrac{1}{\sin\theta_i}\left(\underset{\sim}{d} + \cos\theta_i \underset{\sim}{N}\right)$

$\Rightarrow \quad T = -\cos\theta_r \underset{\sim}{N} + \dfrac{\sin\theta_r}{\sin\theta_i}\left(\underset{\sim}{d} + \cos\theta_i \underset{\sim}{N}\right)$

$\left(\eta_r \sin\theta_r = \eta_i \sin\theta_i\right) \Rightarrow \left(\dfrac{\sin\theta_r}{\sin\theta_i} = \dfrac{\eta_i}{\eta_r}\right)$

$\Rightarrow \quad T = -\cos\theta_r \underset{\sim}{N} + \dfrac{\eta_i}{\eta_r}\left(\underset{\sim}{d} + \cos\theta_i \underset{\sim}{N}\right)$

$$= \frac{\eta_i}{\eta_r}\left(\underset{\sim}{d} - (\underset{\sim}{N}\cdot\underset{\sim}{d})\underset{\sim}{N}\right) - \underbrace{\left(1 - \left(\frac{\eta_i}{\eta_r}\right)^2\left(1-\cos^2\theta_i\right)\right)}_{*}{}^{1/2} \underset{\sim}{N}$$

if ⊛ < 0, no refracted ray "total internal reflection"

Ray - Object Intersection

Find the first intersection with any object where $t > 0$.

$$f(p) = 0$$

implicit function describing surface.

Solve

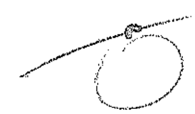
$$f(e + td) = 0.$$

E.g. sphere

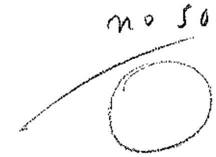
$$f(p) = (p-c)\cdot(p-c) = r^2$$

→ quadratic eq. in t.

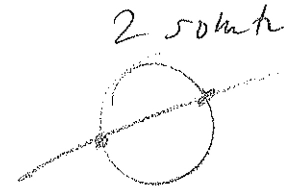
discriminant $= 0$
1 solution



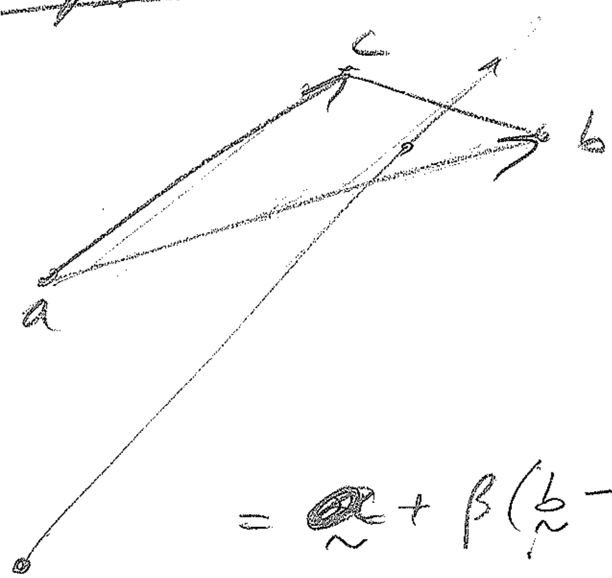
discriminant < 0
no solution



discriminant > 0
2 solutions



E.g. triangle



$$= \tilde{a} + \beta(\tilde{b} - \tilde{a}) + \gamma(\tilde{c} - \tilde{a})$$

$$\tilde{e} + t\tilde{d} = \alpha\tilde{a} + \beta\tilde{b} + \gamma\tilde{c}$$

$$\alpha = 1 - \beta - \gamma$$

$$x_e + t x_d = \alpha x_a + \beta x_b + \gamma x_c = x_a + \beta(x_b - x_a) + \gamma(x_c - x_a)$$

$$y_e + t y_d = \alpha y_a + \beta y_b + \gamma y_c = \text{''}$$

$$z_e + t z_d = \alpha z_a + \beta z_b + \gamma z_c = \text{''}$$

$$\cancel{\alpha + \beta}$$

$$x_e - x_a = \beta(x_b - x_a) + \gamma(x_c - x_a) - t x_d$$

$$y_e - y_a = \beta(y_b - y_a) + \gamma(y_c - y_a) - t y_d$$

$$z_e - z_a = \beta(z_b - z_a) + \gamma(z_c - z_a) - t z_d$$

$$\Rightarrow \begin{bmatrix} x_b - x_a & x_c - x_a & -x_d \\ y_b - y_a & y_c - y_a & -y_d \\ z_b - z_a & z_c - z_a & -z_d \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \\ t \end{bmatrix} = \begin{bmatrix} x_e - x_a \\ y_e - y_a \\ z_e - z_a \end{bmatrix}$$

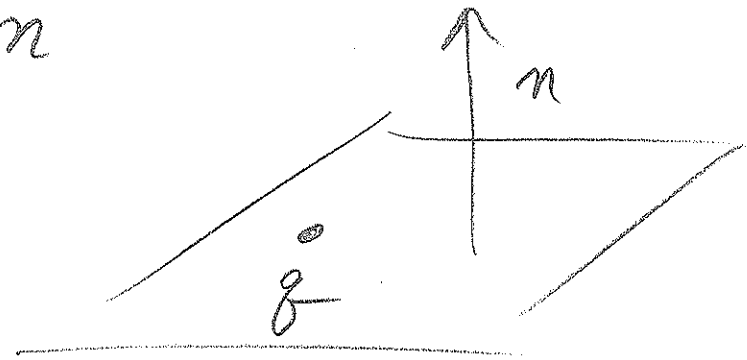
Cramer's Rule to invert 3x3 matrix & solve for β, γ, t.

$\beta, \gamma > 0$, $\beta + \gamma < 1$

Matrix not invertible if ray parallel to triangle (no solutions).

ray, plane intersection

$$f(p) = (p - q) \cdot n$$



$$p(t) = e + (s-e)t$$

$$f(p(t)) = (e + (s-e)t - q) \cdot n = 0$$

$$\cancel{(t-t)e \quad t \quad q \cdot n}$$

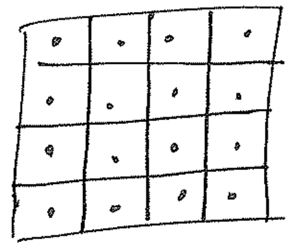
$$t\,(s-e) \cdot n = (q-e) \cdot n$$

$$t = \frac{(q-e) \cdot n}{(s-e) \cdot n}$$

Antialiasing

– basic idea: use local average color rather than single pixel color

regular sampling



single pixel

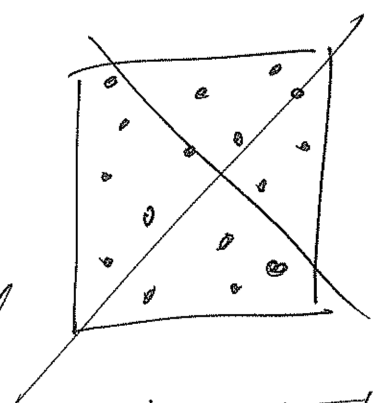
16 subsamples per pixel

→ some artifacts can still arise, e.g. Moiré patterns

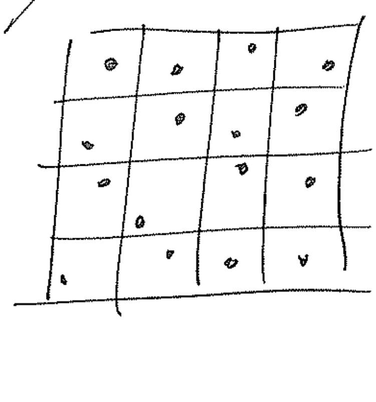
→ solution

random sampling

16 random subsamples per pixel



"jittering" or "stratified sampling"

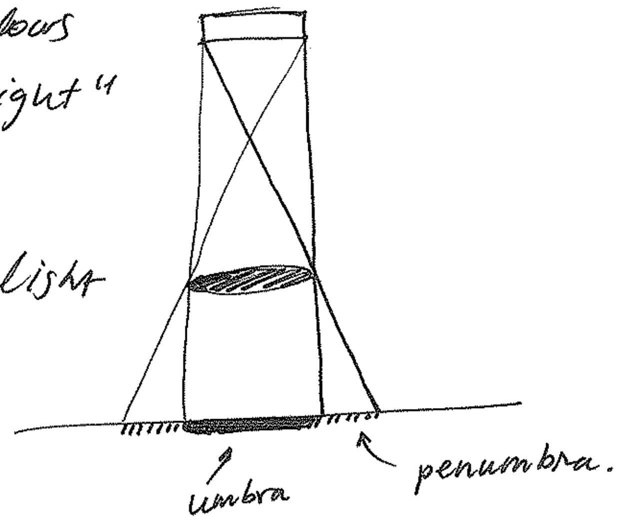


1 random sample/bin.

Soft Shadows

can achieve soft shadows by using an "area light"

randomly sample a point in the area light to use in the ray_color routine.



Depth of Field