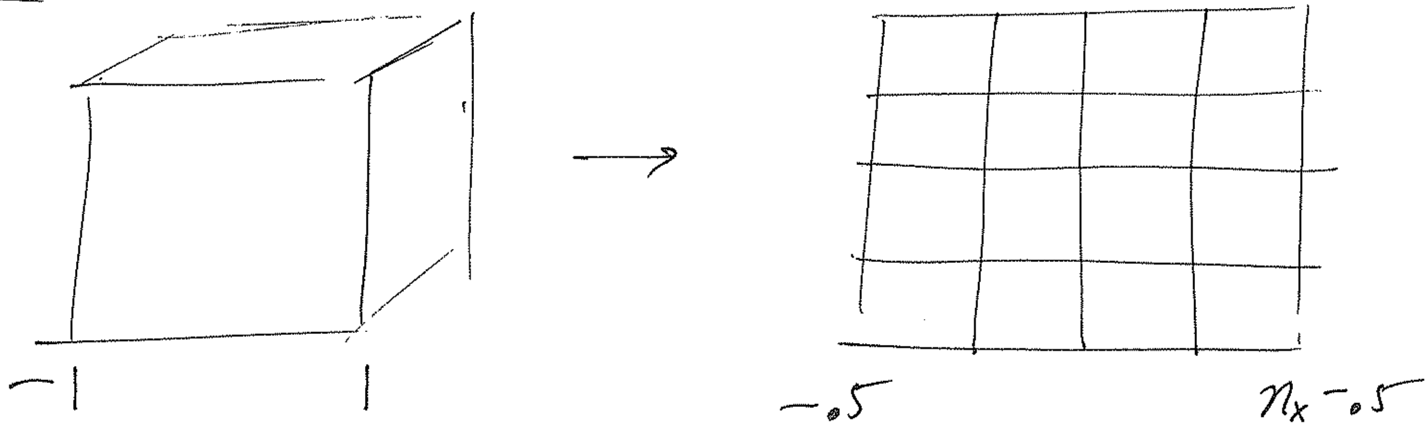
1 Viewport transform — LECTURE 4



X Recall basic transforms: ① translation ② ~~rotation~~ ③ scaling

viewing transform consists of translation & scaling

Consider x-coordinate:

Map interval $[-1, 1] \rightarrow [-.5, n_x - .5]$

① scale

$$[-1, 1] \longrightarrow [-.5, n_x - .5] \Rightarrow \boxed{s_x = \frac{n_x}{2}}$$

length = 2 $\qquad$ length = n_x

② translate

$$0 \longrightarrow \frac{n_x}{2} - .5 \Rightarrow \boxed{t_x = \frac{n_x - 1}{2}}$$

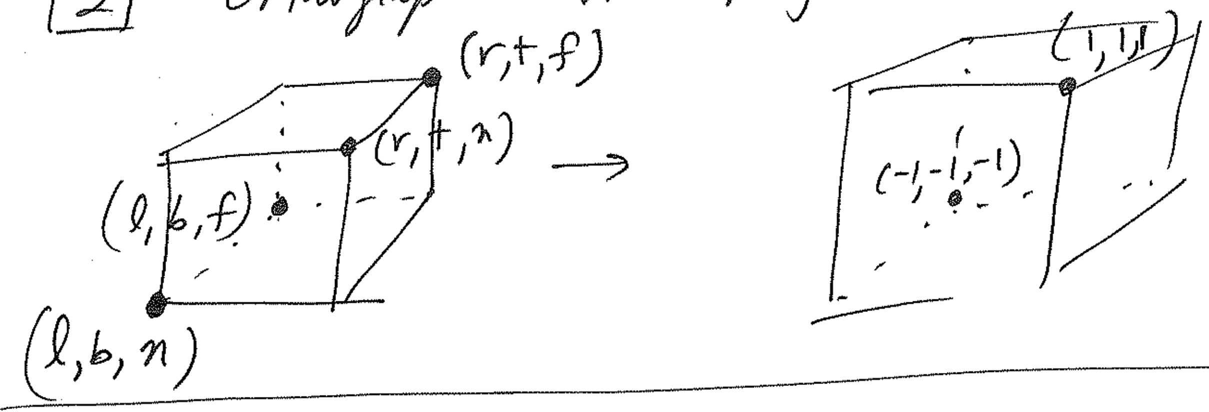
Y Same for y, with n_x replaced by n_y.

Z z is unchanged.

Result:

$$\begin{bmatrix} \frac{n_x}{2} & & & \frac{n_x - 1}{2} \\ & \frac{n_y}{2} & & \frac{n_y - 1}{2} \\ & & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = M_{vp}$$

[2] Orthographic View Projection



① first ~~...~~ translate center of viewing volume to $(0,0,0)$:

x: $\frac{l+r}{2} \to 0$

y: $\frac{b+t}{2} \to 0$

z: $\frac{f+n}{2} \to 0$

transform matrix

$$\begin{bmatrix} 1 & & & -\left(\frac{l+r}{2}\right) \\ & 1 & & -\left(\frac{b+t}{2}\right) \\ & & 1 & -\left(\frac{f+n}{2}\right) \\ & & & 1 \end{bmatrix}$$

②. Now scale the box edge lengths:

x: $r-l \to 2$

y: $t-b \to 2$

z: $n-f \to 2$

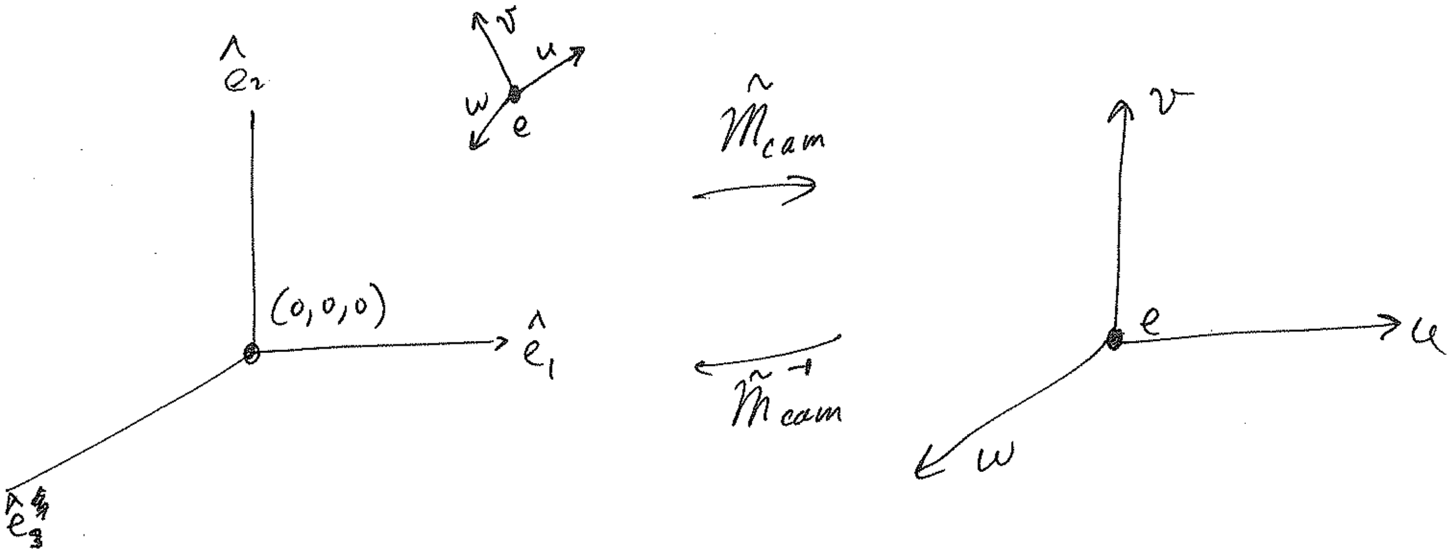
transform matrix

$$\begin{bmatrix} 2/(r-l) & & & \\ & 2/(t-b) & & \\ & & 2/(n-f) & \\ & & & 1 \end{bmatrix}$$

Compose: ② ①

$$② \cdot ① = \begin{bmatrix} \frac{2}{r-l} & & & \frac{-(l+r)}{(r-l)} \\ & \frac{2}{t-b} & & -\frac{(b+t)}{(t-b)} \\ & & \frac{2}{n-f} & +\frac{(f+n)}{(n-f)} \\ & & & 1 \end{bmatrix} = M_{orth}$$

$\boxed{3}$ Camera transform



<u>We know</u> :

$$\left\{\begin{array}{l} \tilde{M}_{cam}\, u = \hat{e}_1 \\ \tilde{M}_{cam}\, v = \hat{e}_2 \\ \tilde{M}_{cam}\, w = \hat{e}_3 \end{array}\right. \Rightarrow \left\{\begin{array}{l} u = \tilde{M}_{cam}^{-1}\, \hat{e}_1 \\ v = \tilde{M}_{cam}^{-1}\, \hat{e}_2 \\ w = \tilde{M}_{cam}^{-1}\, \hat{e}_3 \end{array}\right.$$

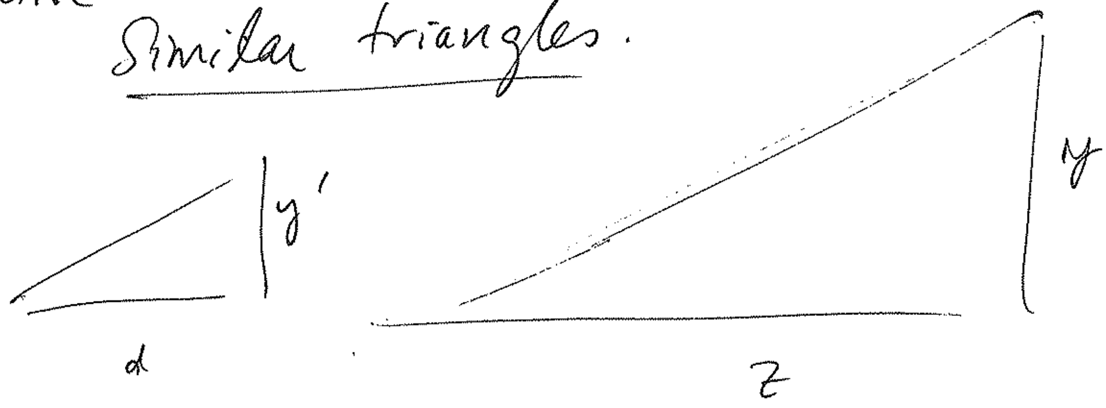
$$\tilde{M}_{cam}^{-1} = \begin{bmatrix} | & | & | \\ u & v & w \\ | & | & | \end{bmatrix} \cancel{\Rightarrow} \quad \cancel{M_{cam} = \begin{bmatrix} & \end{bmatrix}\begin{bmatrix} & \end{bmatrix}}$$

$$\Downarrow$$

$$M_{cam}^{-1} = \begin{bmatrix} u & v & w & e \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

[4] Projective Transformation

Similar triangles.

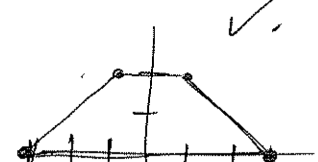


$$\frac{d}{z} = \frac{y'}{y} \Rightarrow \boxed{y' = \frac{d}{z}\, y}$$

$$\begin{pmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ e & f & g & h \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \\ w \end{pmatrix} \Rightarrow \begin{cases} x' = \tilde{x}/w \\ y' = \tilde{y}/w \\ z' = \tilde{z}/w \end{cases}$$

$$\begin{pmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2-1 \\ 0 \\ \frac{1}{3} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ \frac{1}{3} \end{pmatrix} \to \begin{cases} x = 3 \\ y = 0. \end{cases}$$

✓



$$\begin{pmatrix} & & \\ & '' & \\ & & \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \to \begin{cases} x = 1 \\ y = 2. \end{cases}$$

$$\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$$