CS230 : Computer Graphics

Lecture 11: Curves and Surfaces (cont.)

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Acknowledgments

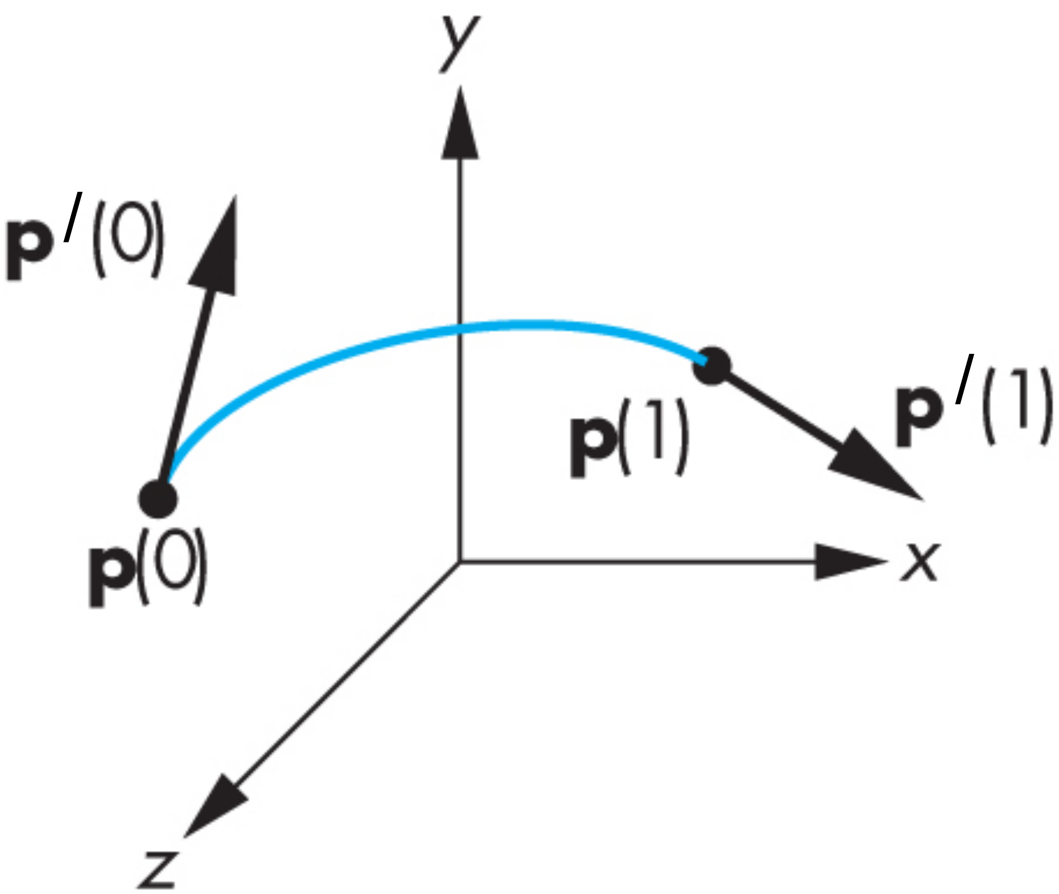
- Sources: Figures from Angel and Shreiner 6th edition, unless otherwise noted
- Some slides courtesy of V. Zordan

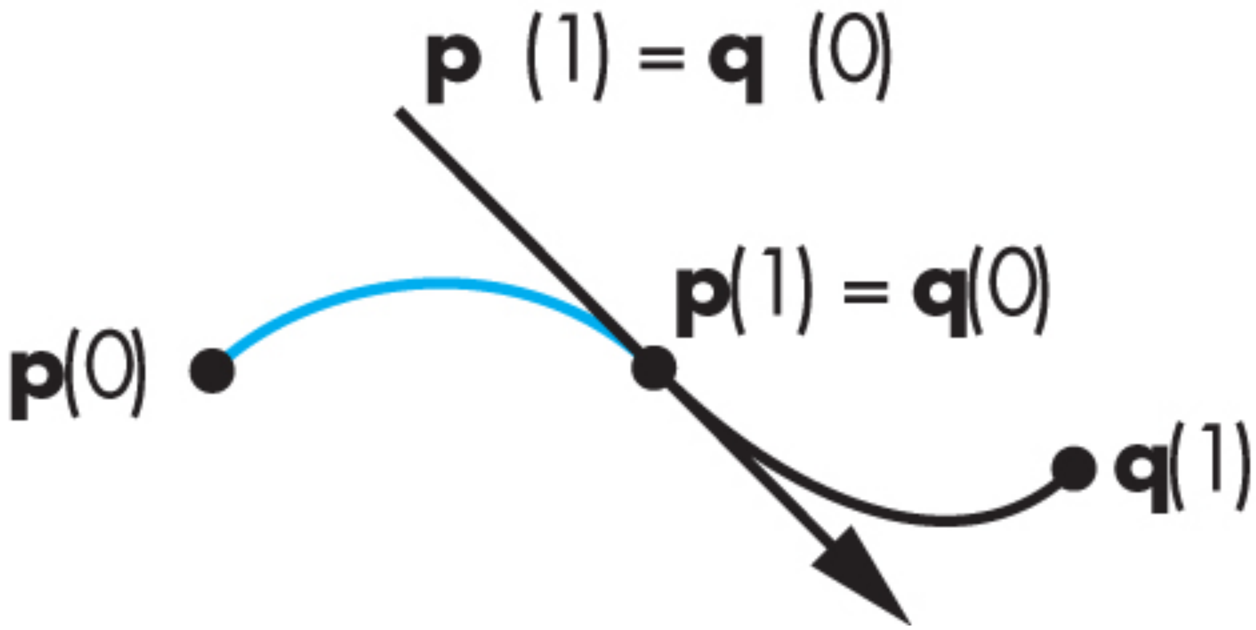
Hermite curves

Hermite Curves

Interpolate endpoints
and match derivatives

this gives
C^1 continuity

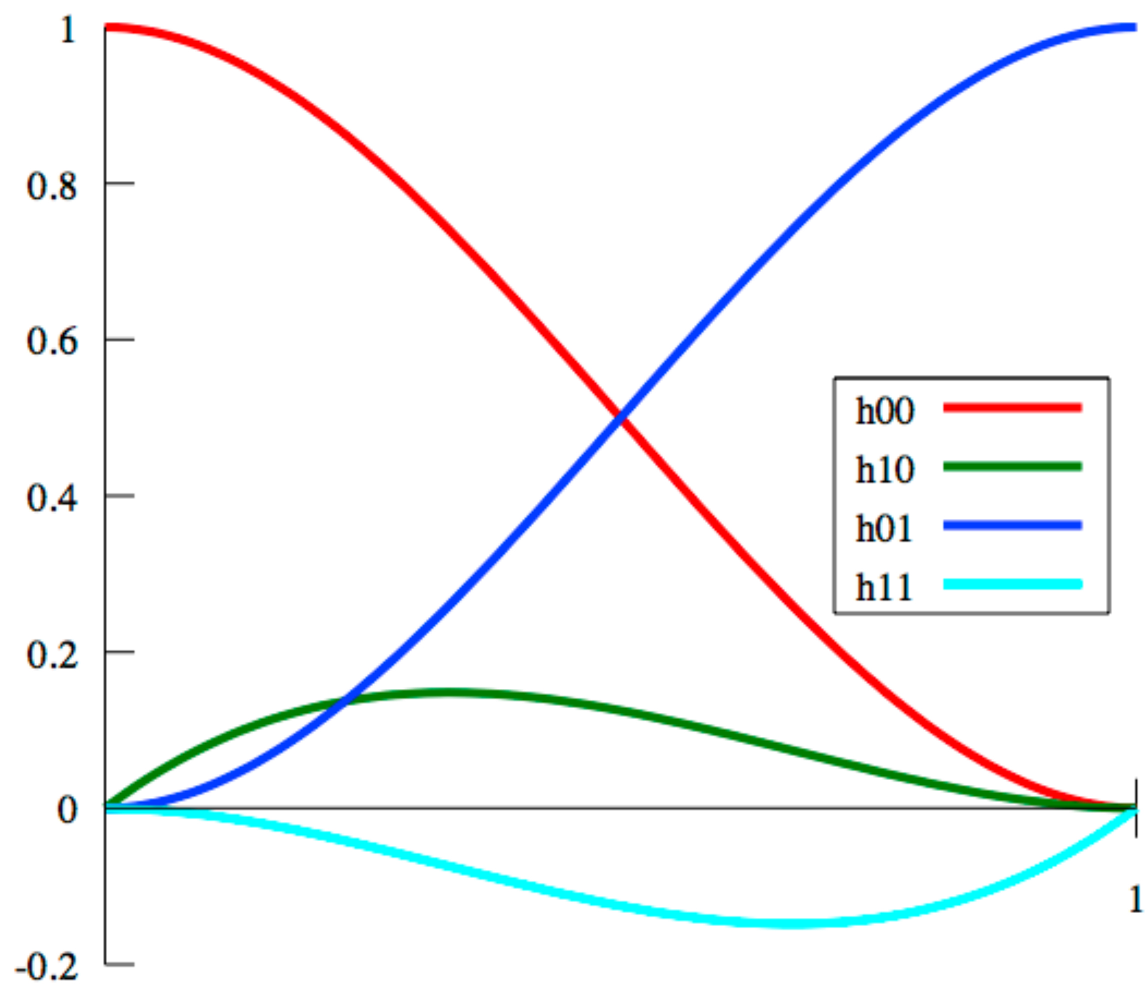




Hermite Curves Basis

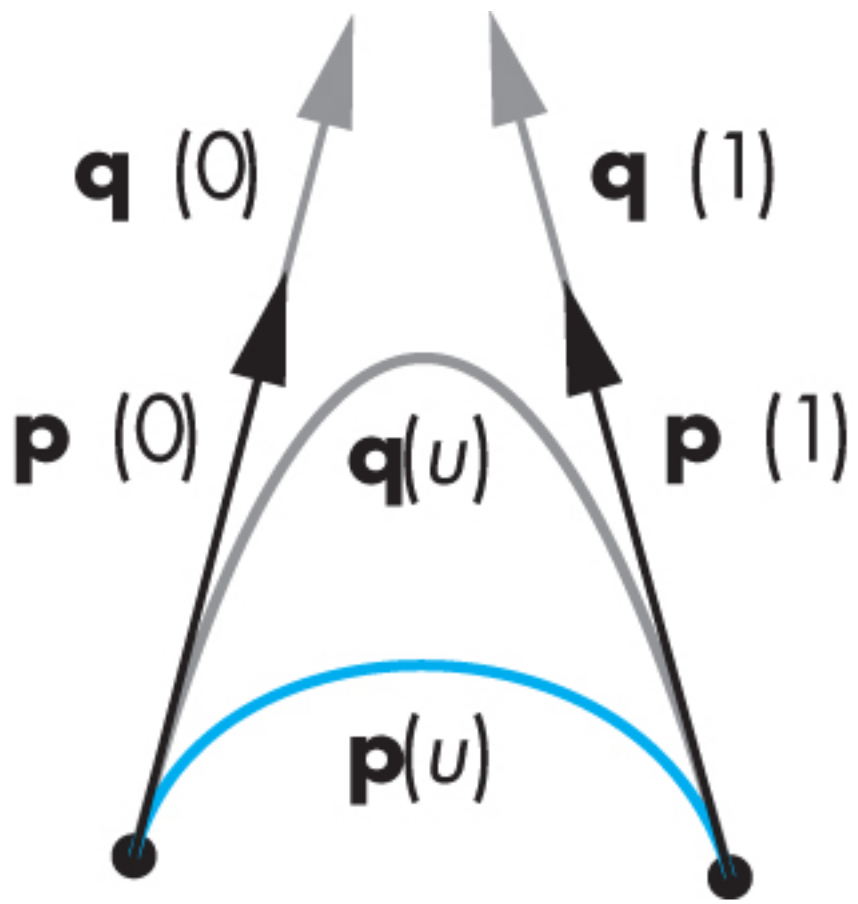
$$p(u) = h_{00}(u)p(0) + h_{01}(u)p(1) + h_{10}(u)p'(0) + h_{11}(u)p'(1)$$

$$h_{00}(u) = 2u^3 - 3u^2 + 1$$
$$h_{01}(u) = -2u^3 + 3u^2$$
$$h_{10}(u) = u^3 - 2u^2 + u$$
$$h_{11}(u) = u^3 - u^2$$



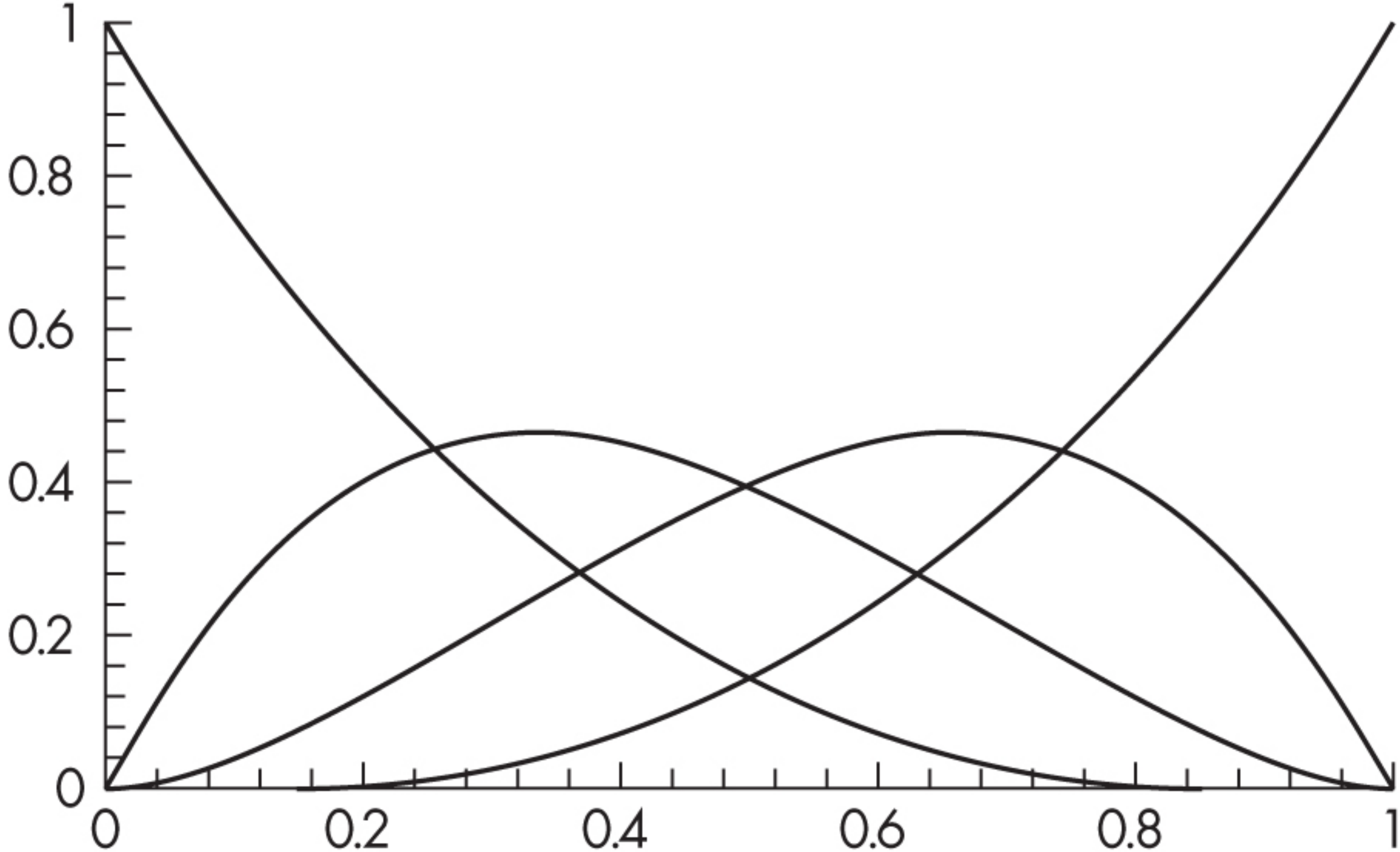
Wikimedia Commons

Geometric Continuity



Bezier curves

Bezier blending functions



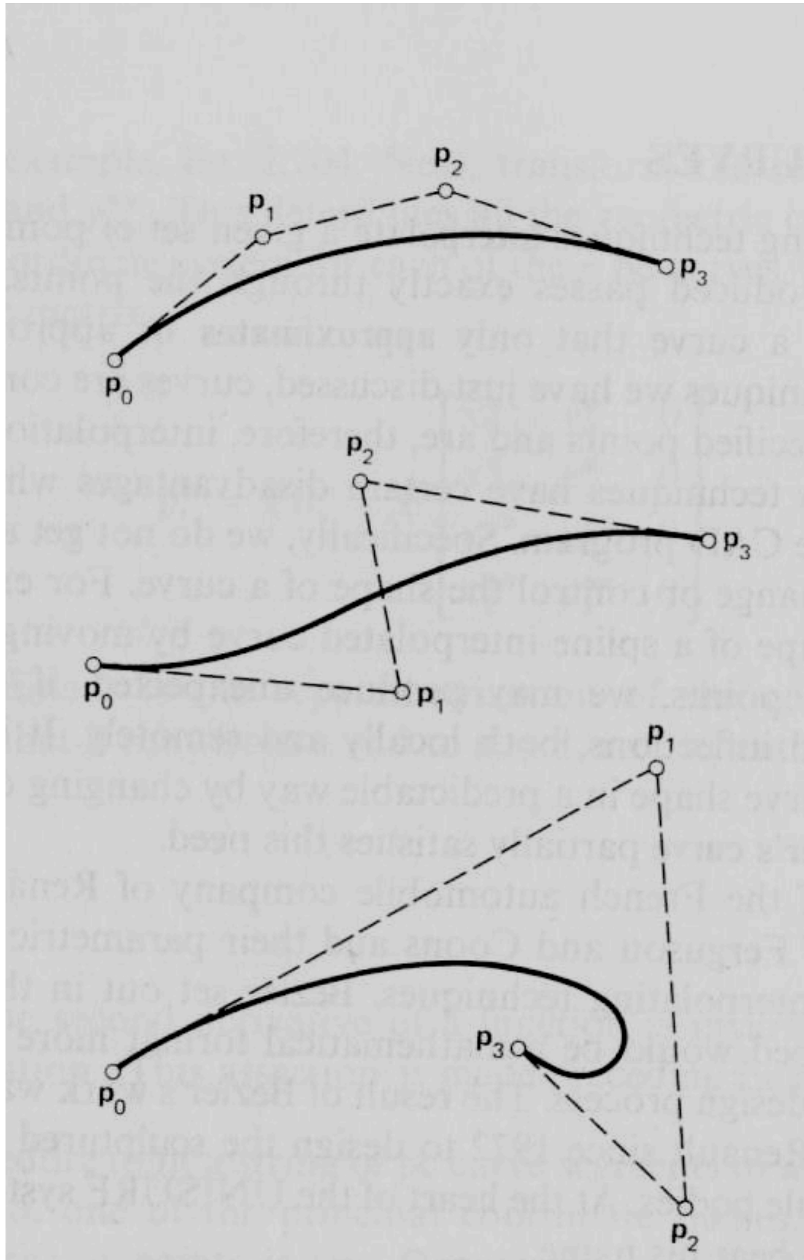
Bernstein Polynomials

- The blending functions are a special case of the Bernstein polynomials

$$b_{\mathrm{kd}}(u) = \frac{d!}{k!(d-k)!} u^k (1-u)^{d-k}$$

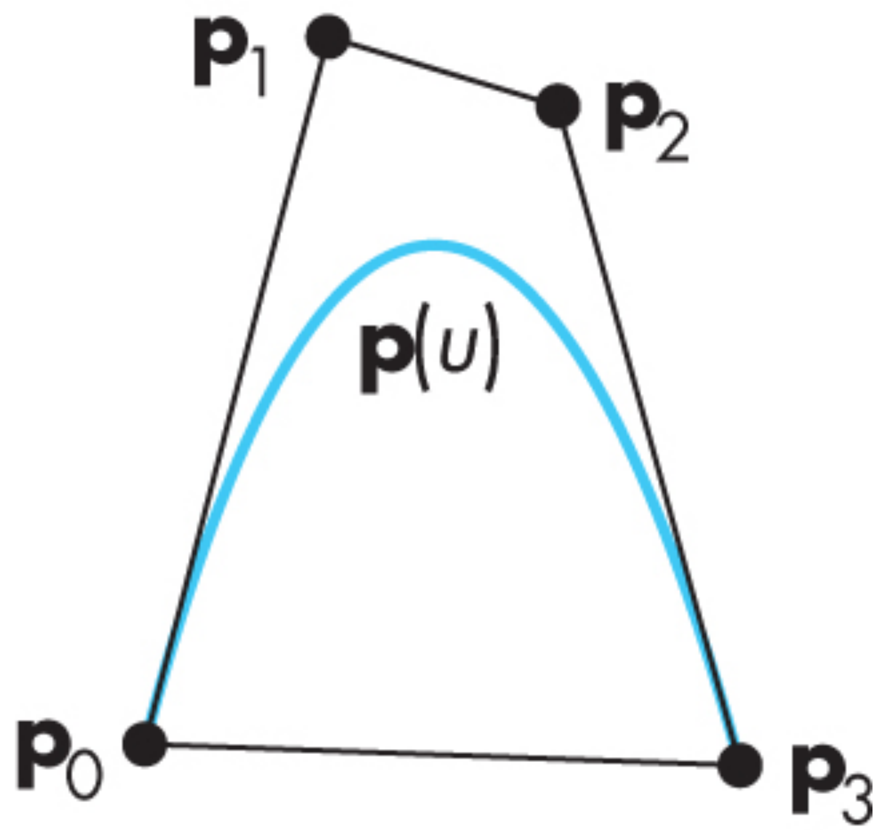
- These polynomials give the blending polynomials for any degree Bezier form
  - All zeros at 0 and 1
  - For any degree they all sum to 1
  - They are all between 0 and 1 inside (0,1)

Bezier Curves



Bezier Curves

- curve lies in the convex hull of the data

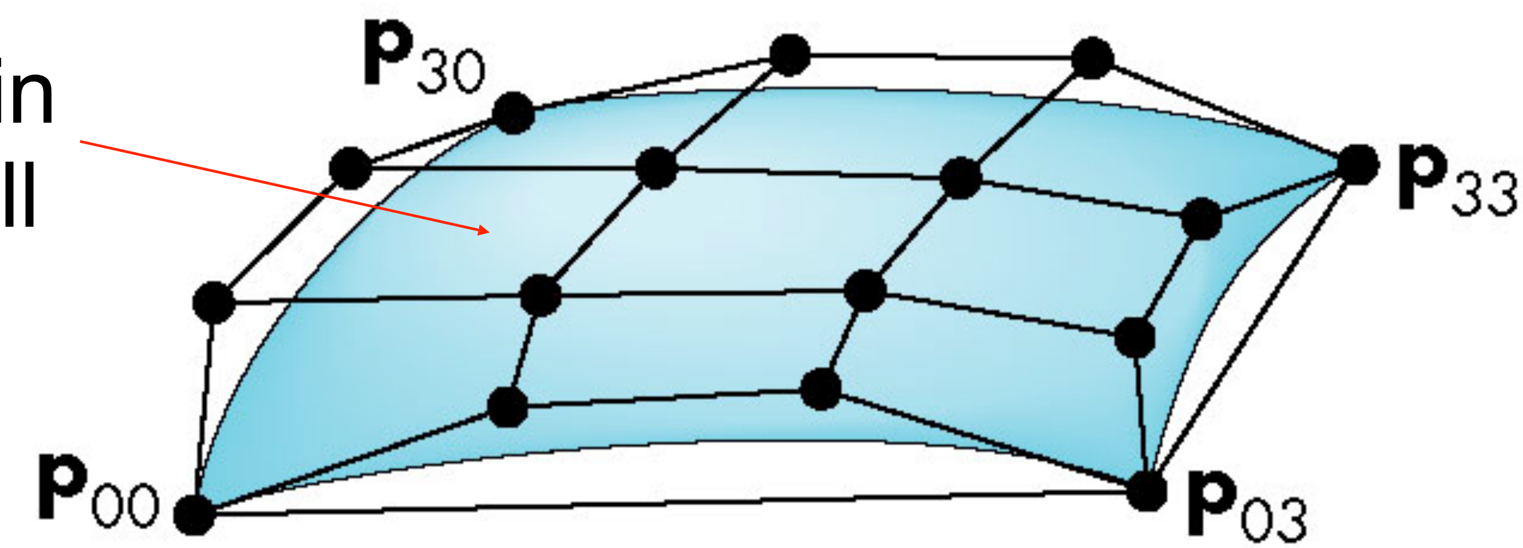


Bezier Patches

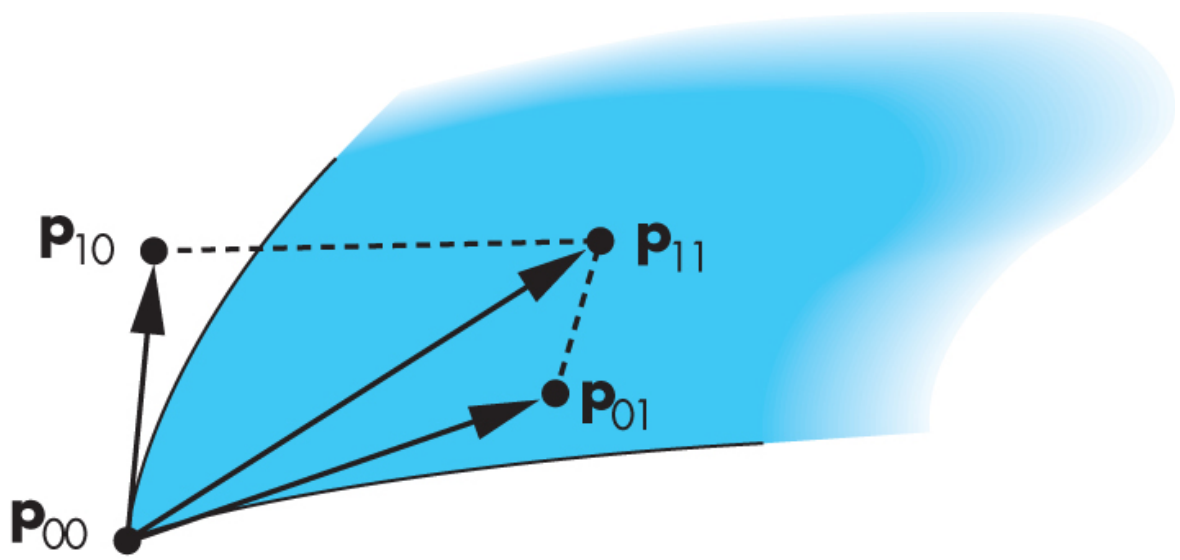
Using same data array $\mathbf{P}=[p_{ij}]$ as with interpolating form

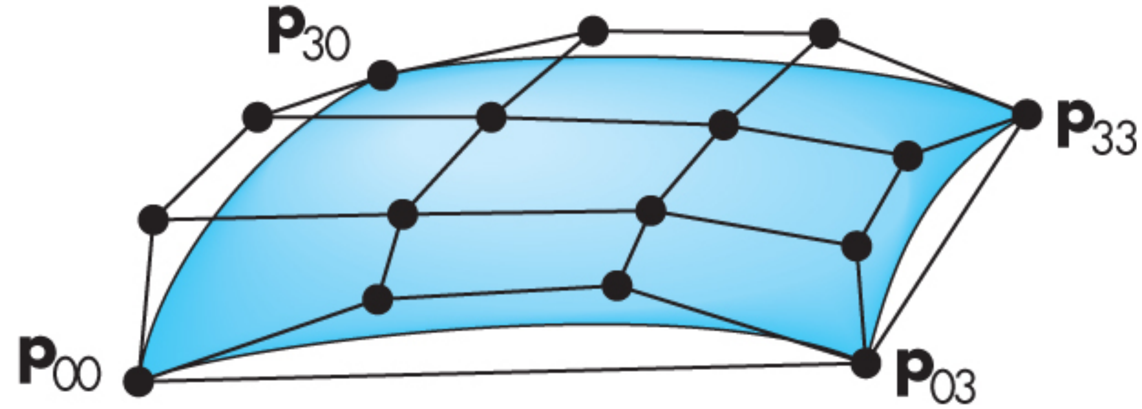
$$p(u,v) = \sum_{i=0}^{3}\sum_{j=0}^{3} b_i(u)\, b_j(v)\, p_{ij} = u^T \mathbf{M}_B \mathbf{P} \mathbf{M}_B^T v$$

Patch lies in convex hull



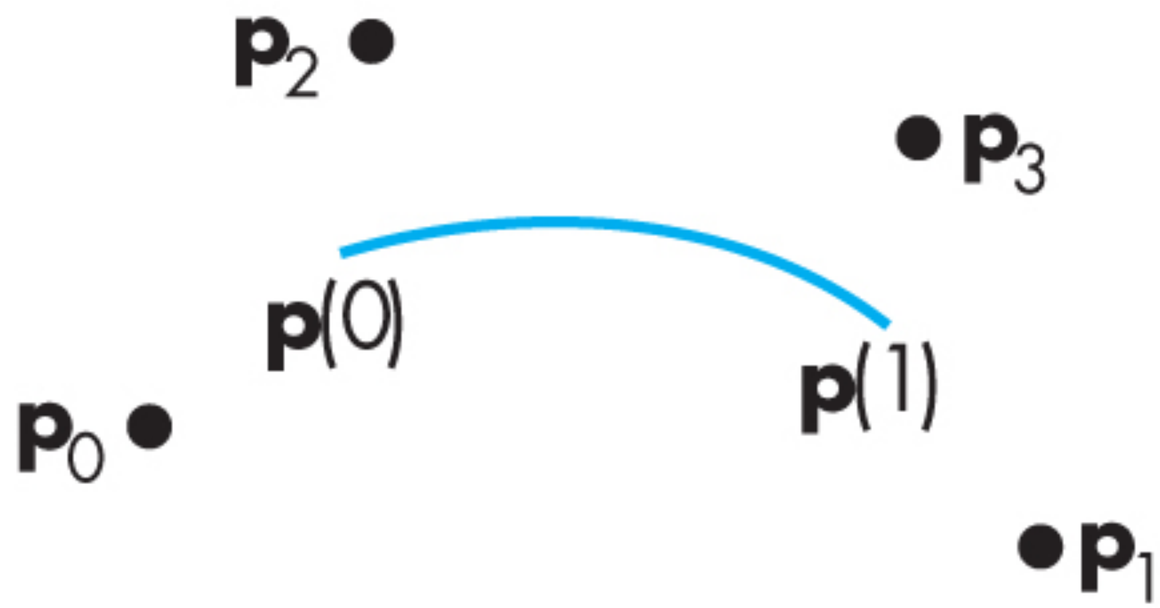
Bezier surface patches





Cubic B-Splines

Cubic B-Splines



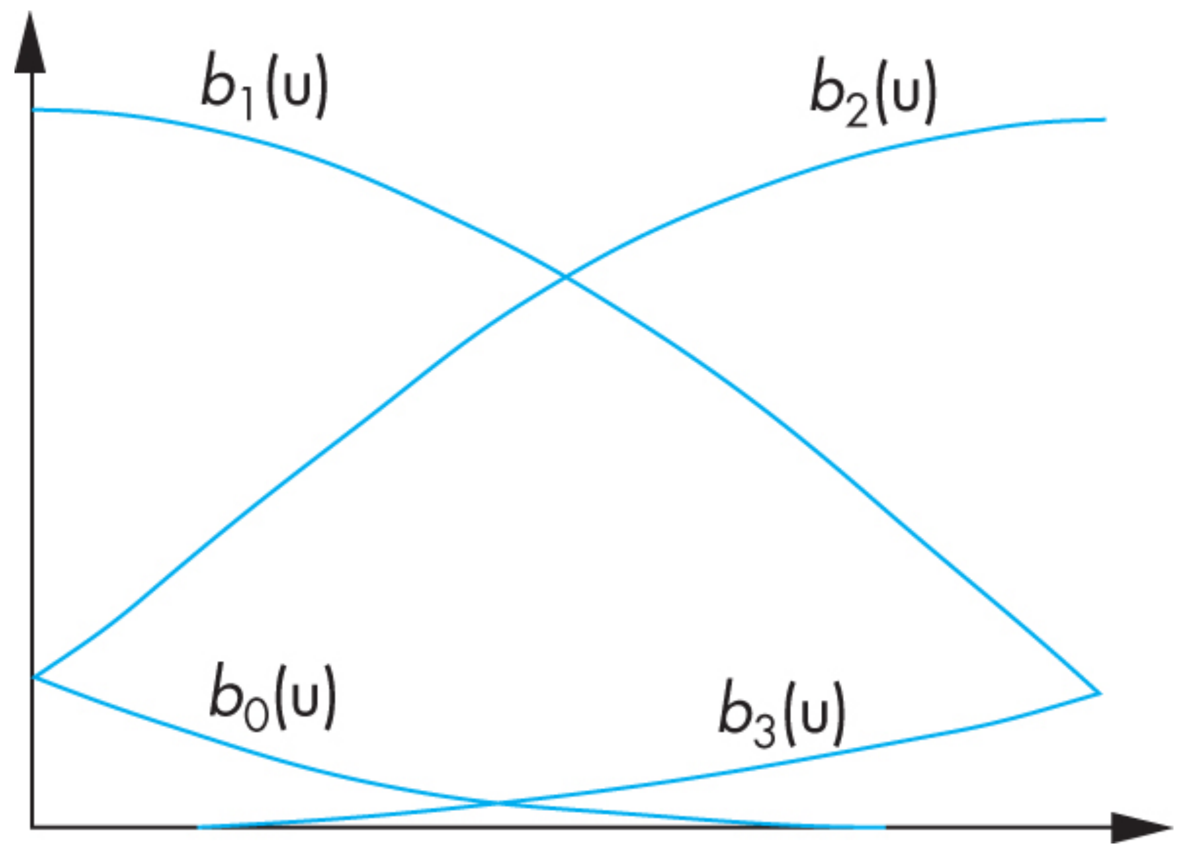
Spline blending functions

$$b_0(u) = \frac{1}{6}(1-u)^3$$

$$b_1(u) = \frac{1}{6}(4 - 6u^2 + 3u^3)$$

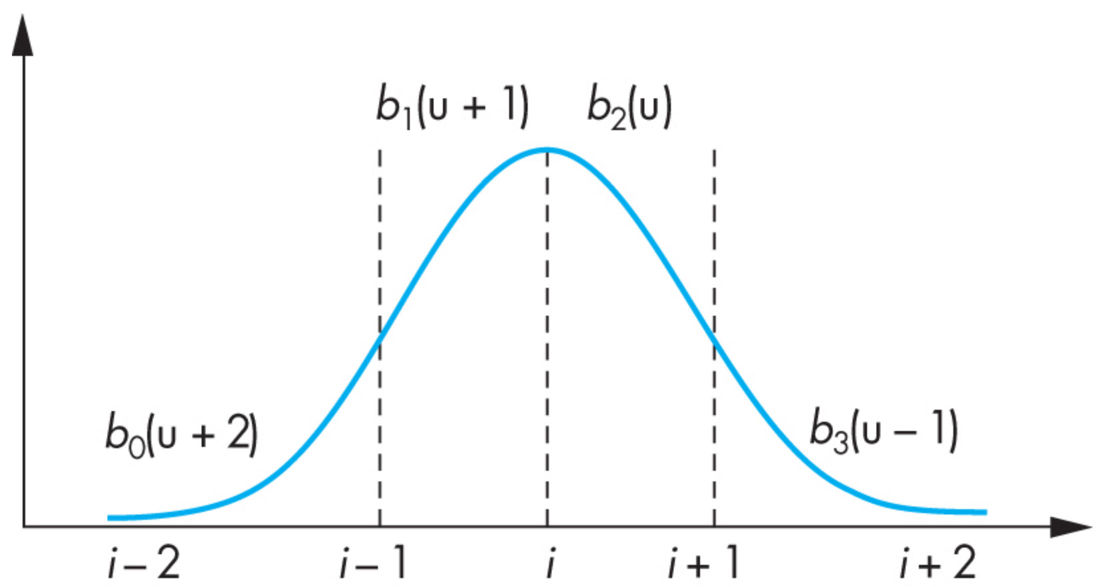
$$b_2(u) = \frac{1}{6}(1 + 3u + 3u^2 - 3u^3)$$

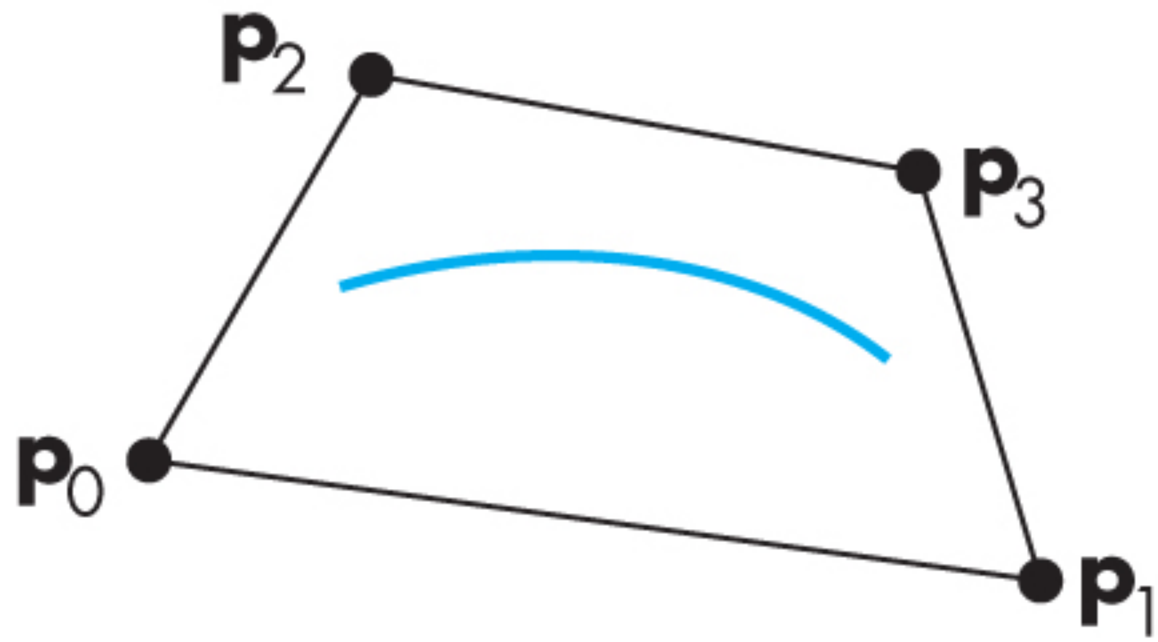
$$b_3(u) = \frac{1}{6}u^3$$



Spline properties

Basis functions





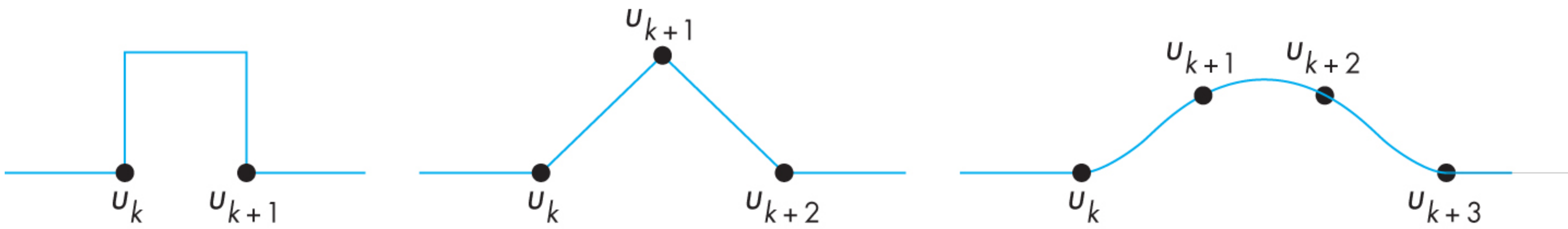
convexity

General Splines

- Defined recursively by *Cox-de Boor recursion formula*

$$b_{j,0}(t) = \begin{cases} 1 & \text{if} \quad t_j \leq t \\ 0 & \text{otherwise} \end{cases}$$

$$b_{j,n}(t) := \frac{t - t_j}{t_{j+n} - t_j} b_{j,n-1}(t) + \frac{t_{j+n+1} - t}{t_{j+n+1} - t_{j+1}} b_{j+1,n-1}(t)$$



Curve and Surface Rendering

Sampling

- Sample the curve and render resulting flat polygons
- Evaluate the curve at several sample points (n mults)

$$p(u) = c_0 + u(c_1 + u(c_2 + u(\ldots + c_n u)))$$

- Use a *divided difference table* (O(n) additions, no mults)

t	0	1	2	3	4	5
p	1	7	23	55	109	191
$\Delta^{(1)}\mathbf{p}$	6	16	32	54	82	
$\Delta^{(2)}\mathbf{p}$	10	16	22	28		
$\Delta^{(3)}\mathbf{p}$	6	6	6			

t	0	1	2	3	4	5
p	1	7	23	55	109	191
$\Delta^{(1)}\mathbf{p}$	6	16	32	54	82	
$\Delta^{(2)}\mathbf{p}$	10	16	22	28		
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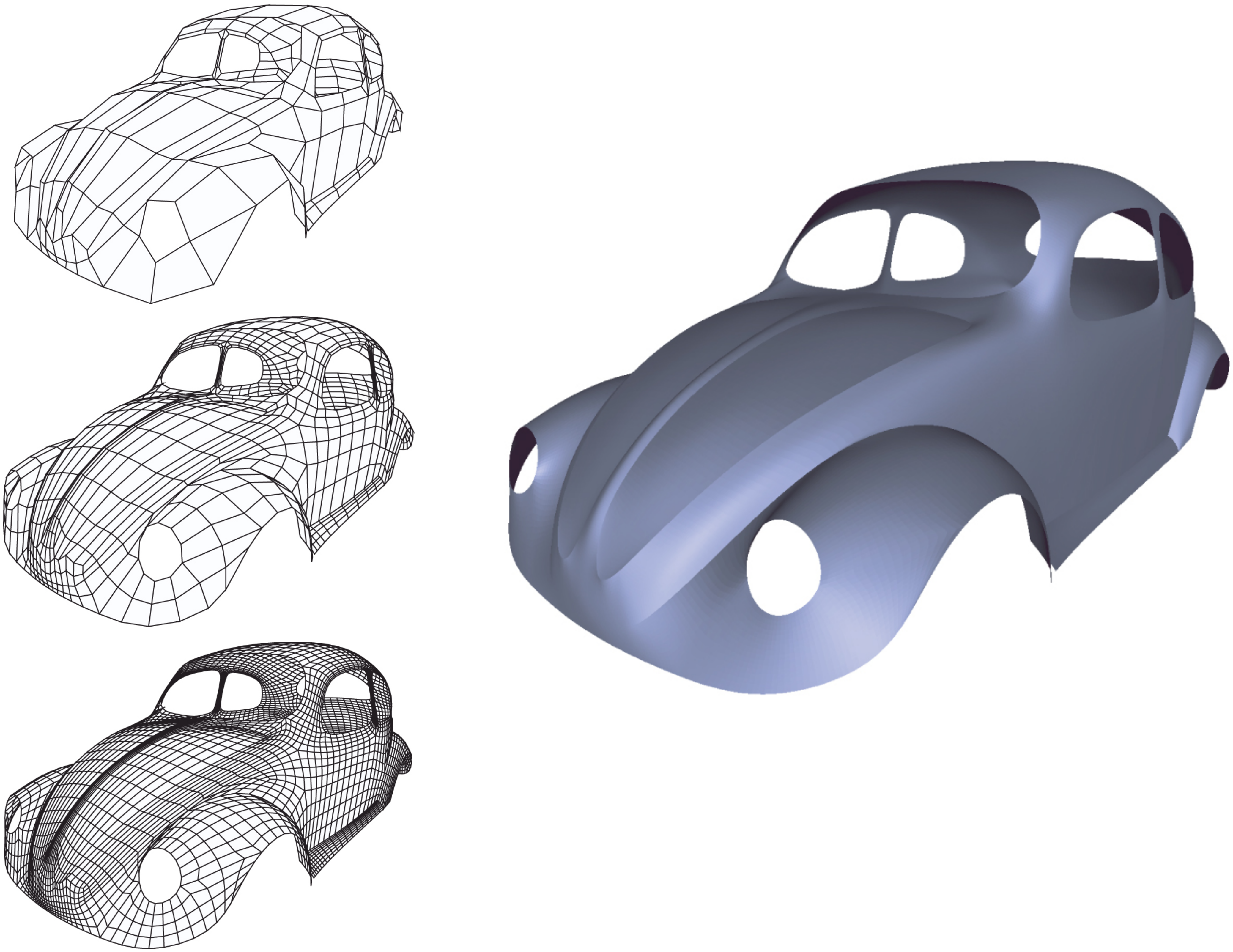
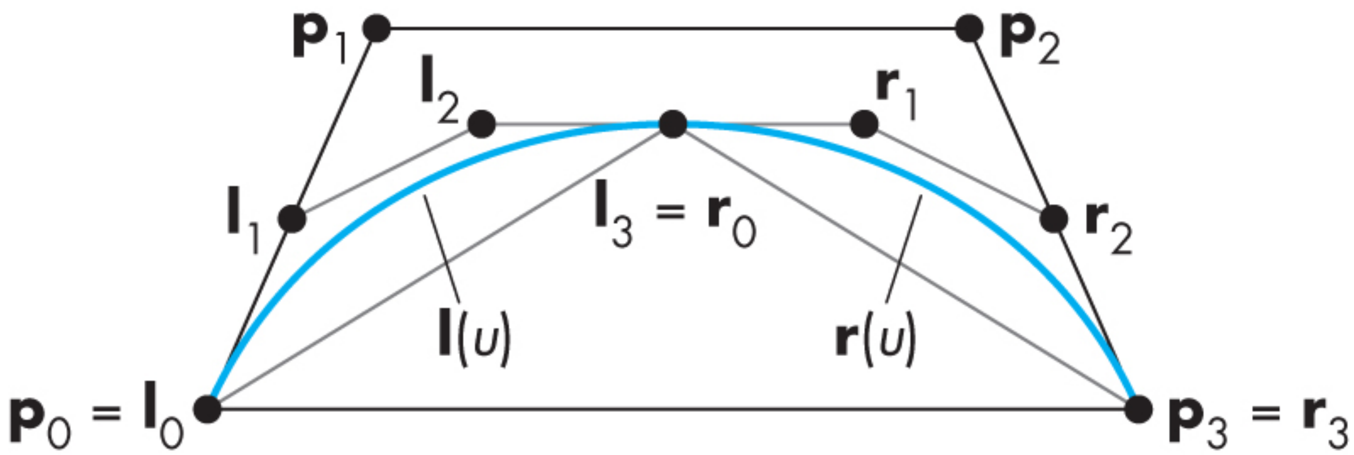
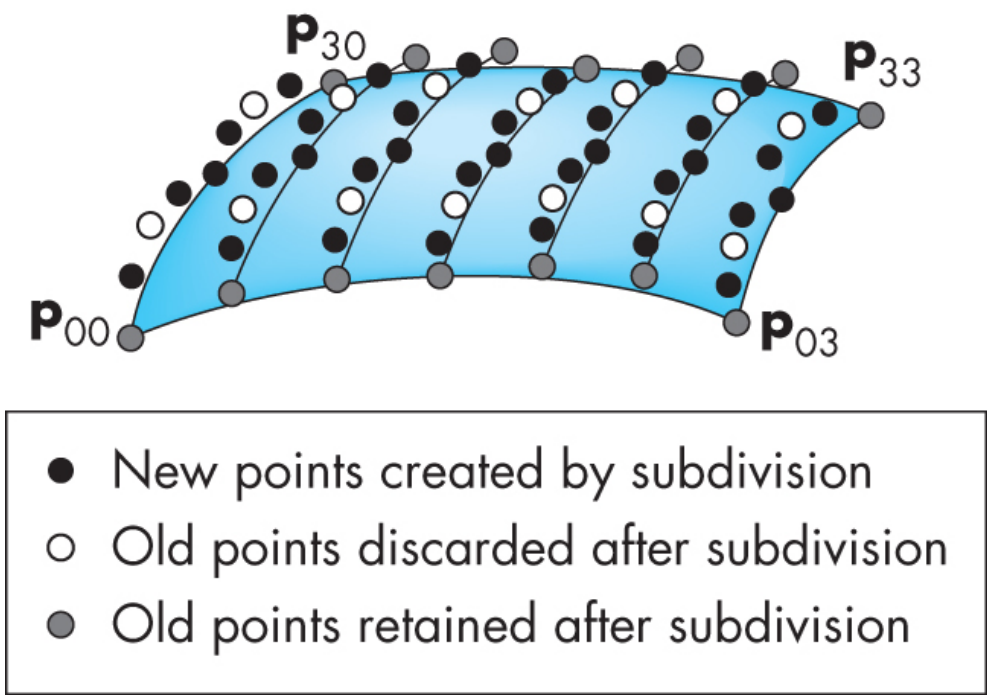
Recursive Subdivision



Recursive Subdivision

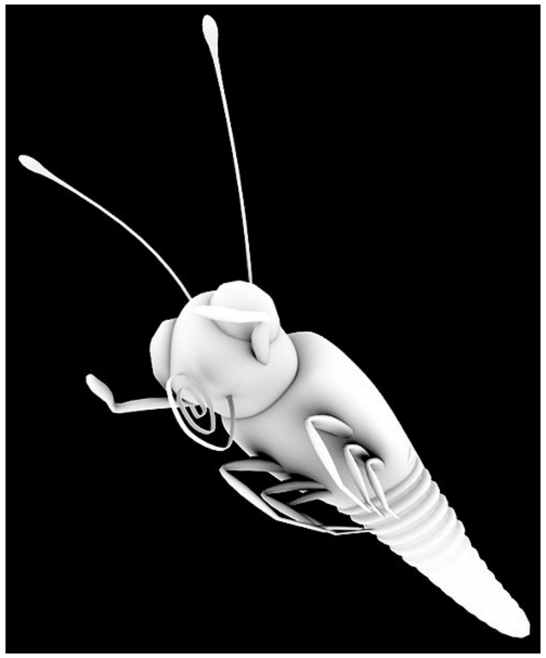
- work with convex hull, does not require evaluating the polynomial
- Bezier curves most convenient -- other curves can be transformed to Bezier
- same approach for surfaces



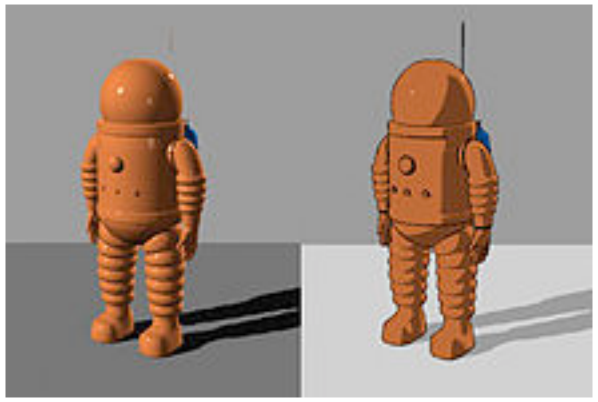


Project ideas

Rendering



Ambient occlusion rendering



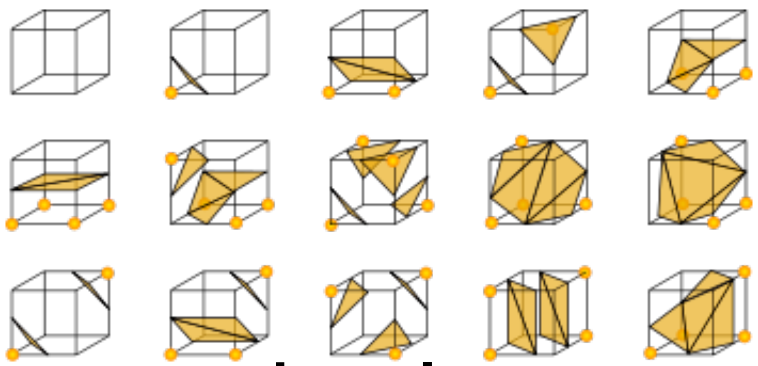
NPR

high dynamic range rendering

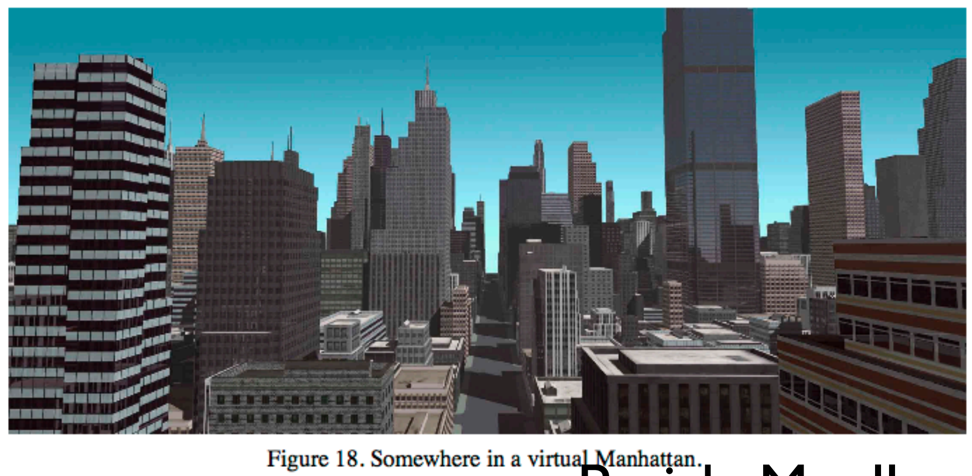
photon mapping

Other possibilities: motion blur, water rendering, atmospheric scattering, advanced shadow techniques - shadow volumes and soft shadows, volumetric effects, displacement mapping, volume rendering in OpenGL, other ideas from book Real-Time Rendering, Shader in GLSL

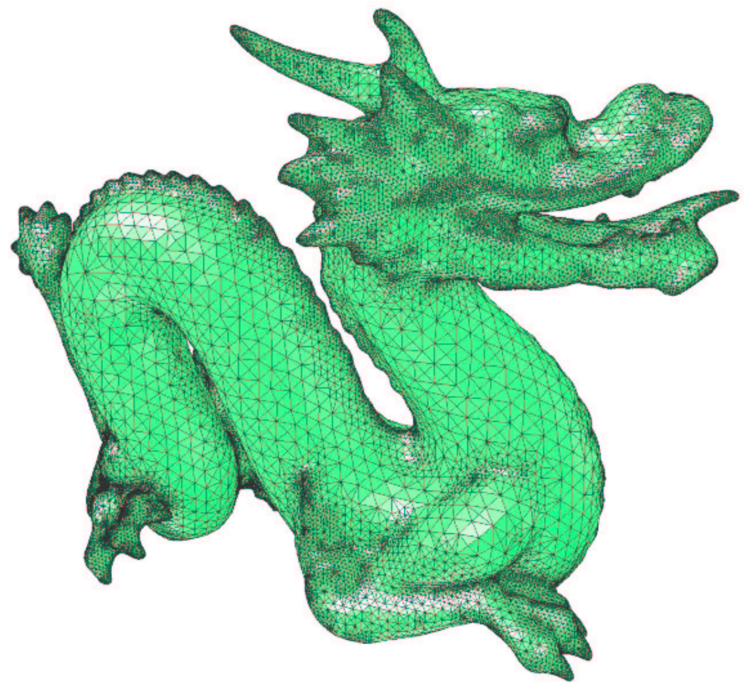
Modeling



- procedural content generation
- implicit surface tessellation
- subdivision surfaces
- mesh generation
- mesh cutting
- interactive mesh deformation
- terrain modeling



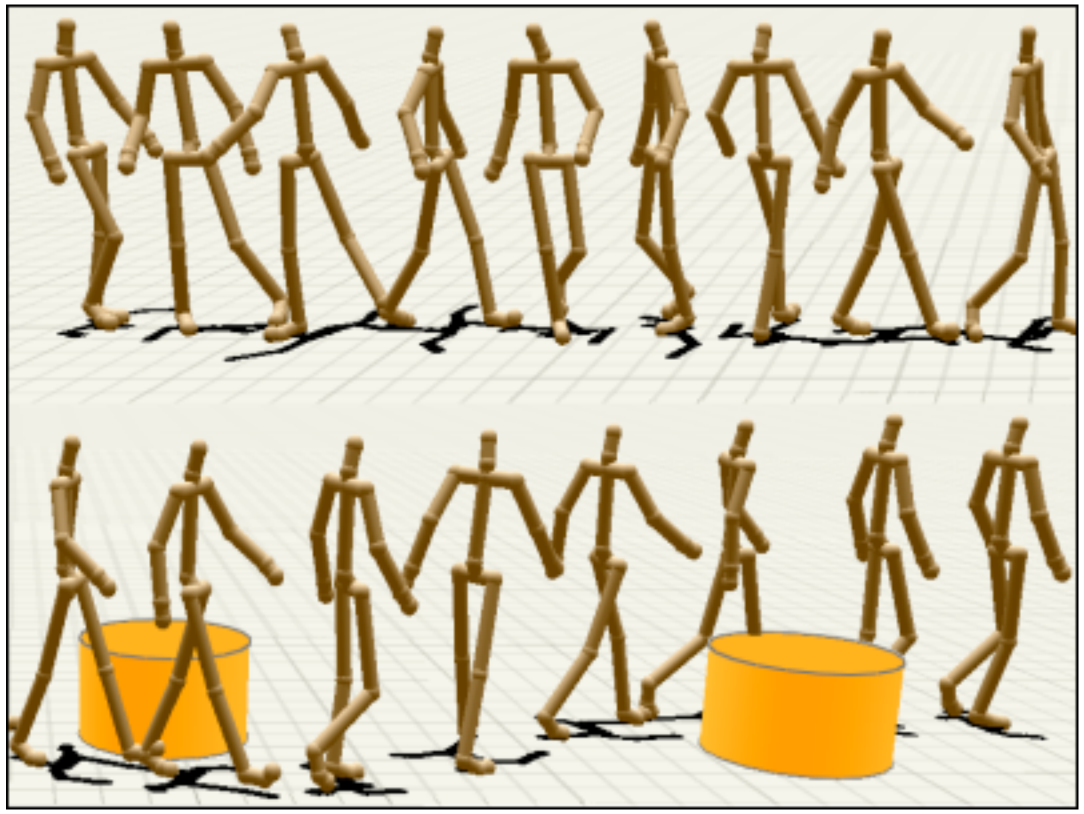
Parish, Mueller



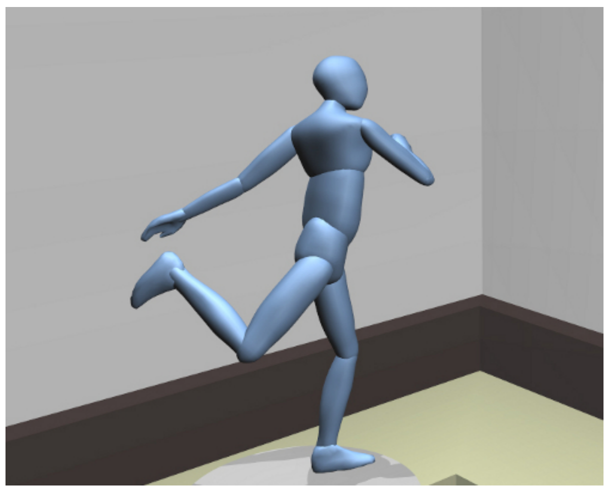
Molino et al, 2003

Character Animation

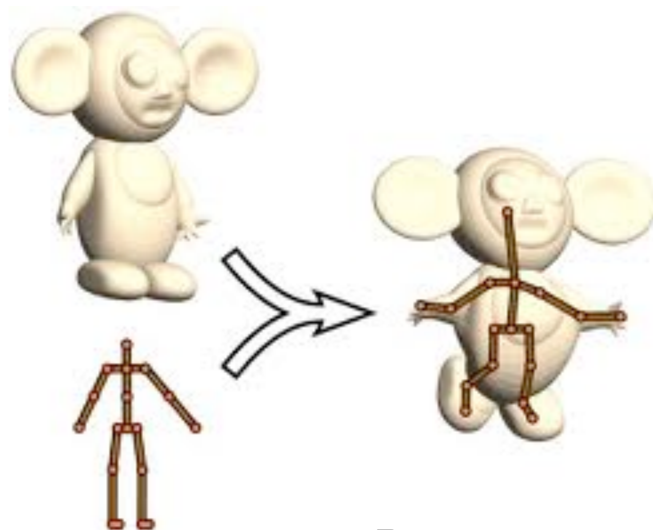
- rigging and skinning
- keyframe animation
- control algorithms



Treuille, A. Lee, Y. Popović, Z.



Macchietto et al. 2009

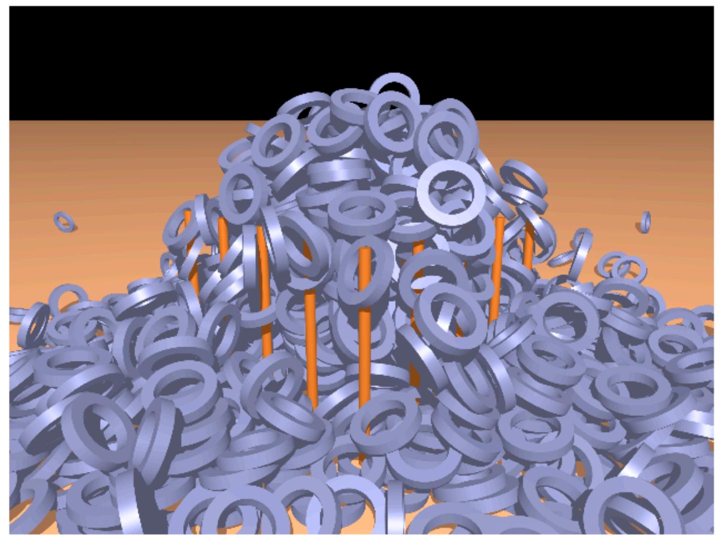


Popovic

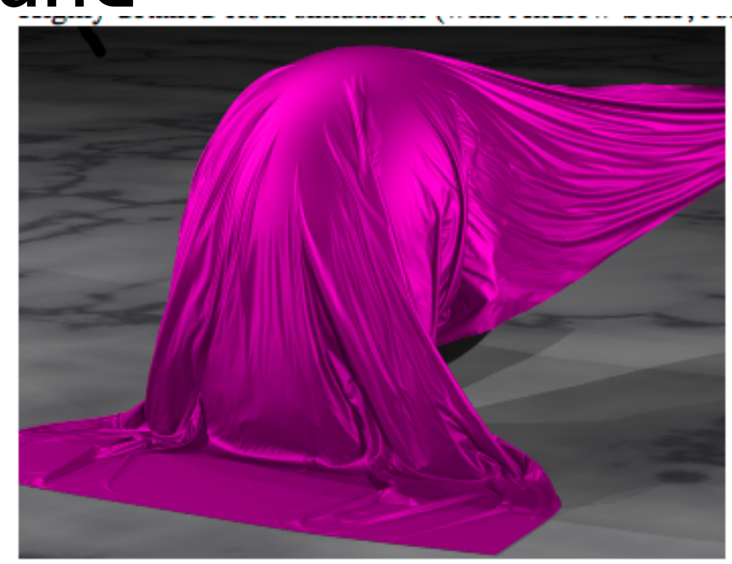
Simulation

- basic water simulator
- rigid body dynamics - integration, collisions
- cloth simulation
- fracture simulation
- particle system
- physically-based sound

Losasso et al., 2008



Guendelman et al., 2003



Selle et al., 2009

Project proposal

- *Pre-proposal: Monday, Feb. 20. A short paragraph with the main idea. I will give feedback and any changes should be agreed by Wed.*
- Proposal due Monday, Feb. 27
- 2-3 pages
- list group (1-2 people)
- background on the problem
- concrete project goals