CS230 : Computer Graphics

Lecture 7

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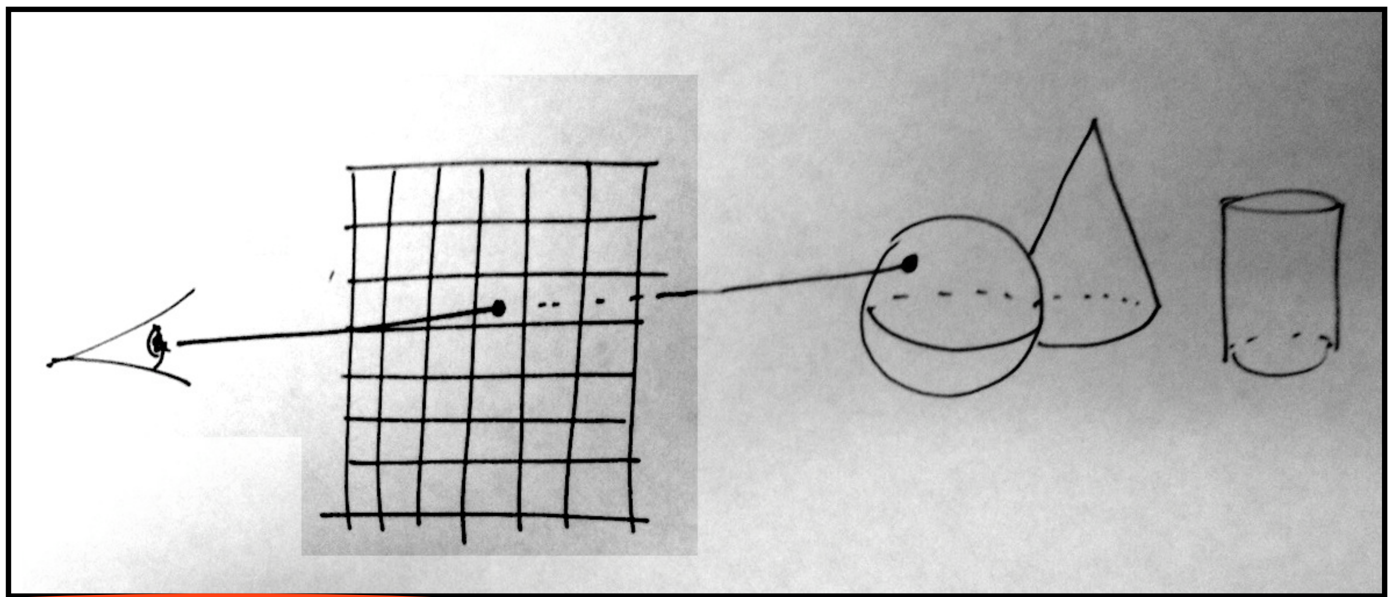
Rendering approaches

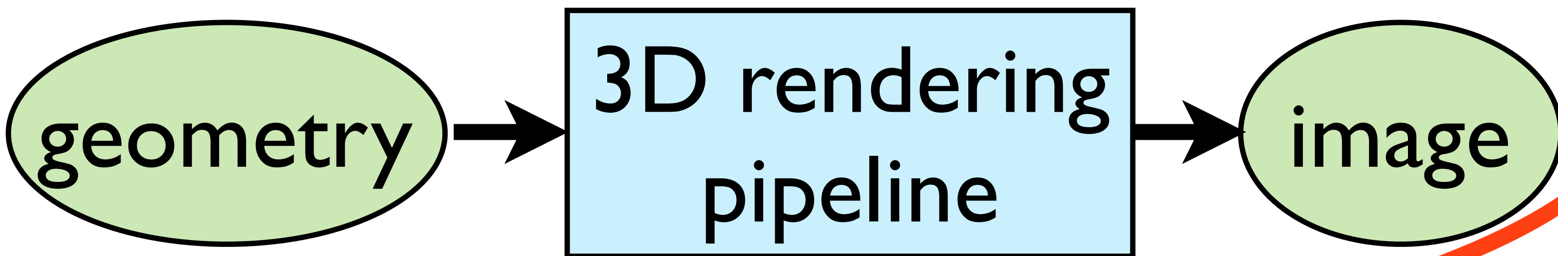
1. image-oriented

foreach pixel ...



2. object-oriented

foreach object ...

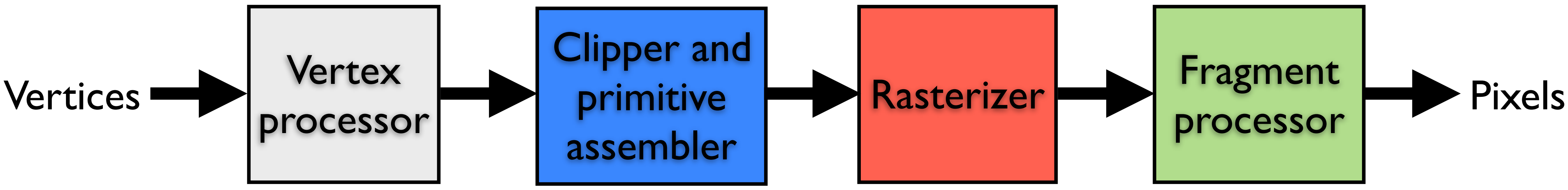


object-oriented rendering

- e.g., hardware: OpenGL graphics pipeline, Direct3D
- software: Renderman (REYES)

task: figure out where a point in the geometry will land on the final image pixels

3D graphics pipeline



Vertex processing: coordinate transformations and color

Clipping and primitive assembly: output is a set of primitives

Rasterization: output is a set of fragments for each primitive

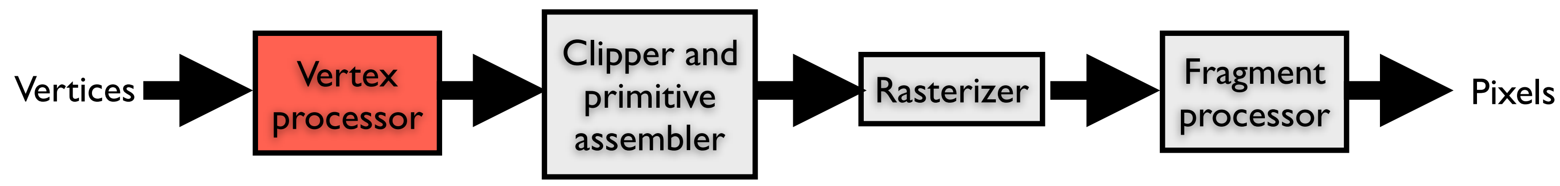
Fragment processing: update pixels in the frame buffer

the pipeline is best when we are doing the same operations on many data sets
-- good for computer graphics!! where we process larges sets of vertices and pixels in the same manner

1. **Geometry**: objects - made of primitives - made of vertices
2. **Vertex processing:** coordinate transformations and color
3. **Clipping and primitive assembly:** use clipping volume. must be primitive by primitive rather than vertex by vertex. therefore vertices must be assembled into primitives before clipping can take place. Output is a set of primitives.
4. **Rasterization:** primitives are still in terms of vertices -- must be converted to pixels. E.g., for a triangle specificied by 3 vertices, the rasterizer must figure out which pixels in the frame buffer fill the triangle. Output is a set of **fragments for each primitive**. A fragment is like a **potential pixel.** Fragments can carry depth information used to figure out if they lie behind other fragments for a given pixel.
5. **Fragment processing:** update pixels in the frame buffer. some fragments may not be visible. texture mapping and bump mapping. blending.

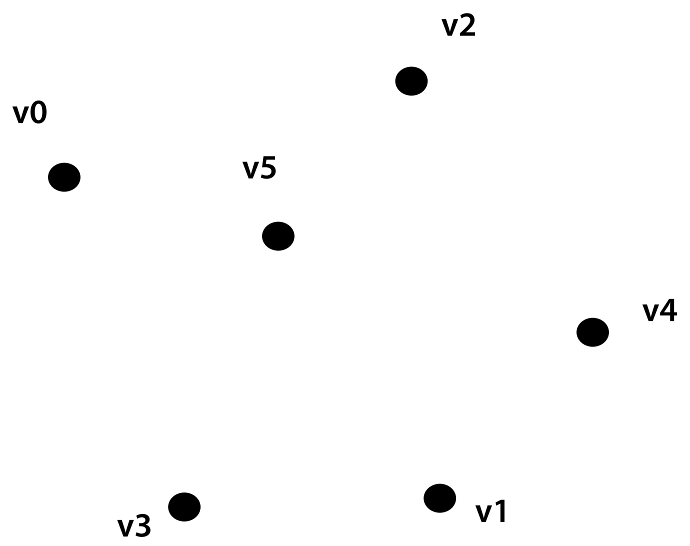
Graphics Pipeline

(slides courtesy K. Fatahalian)

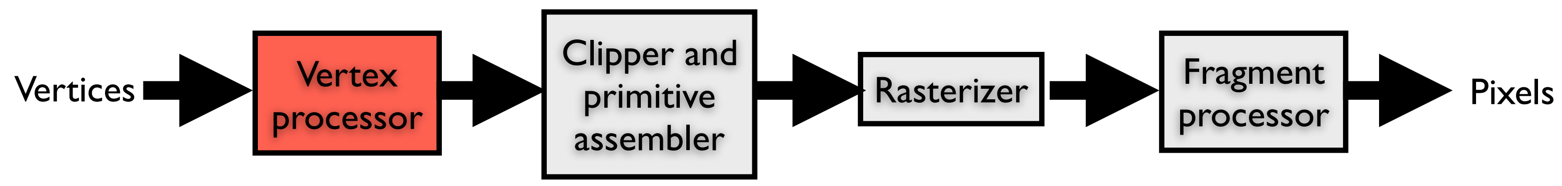


Vertex processing

Vertices are transformed into "screen space"



Vertices



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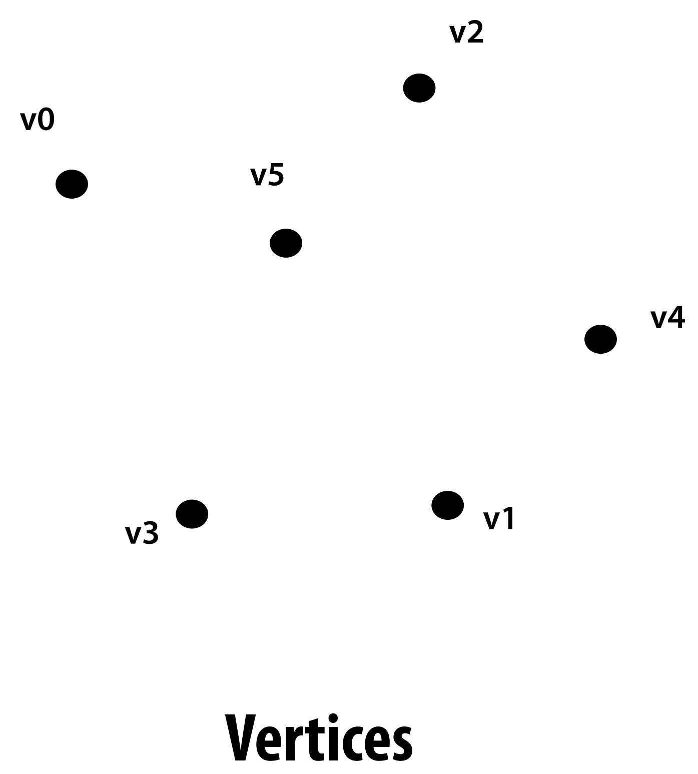
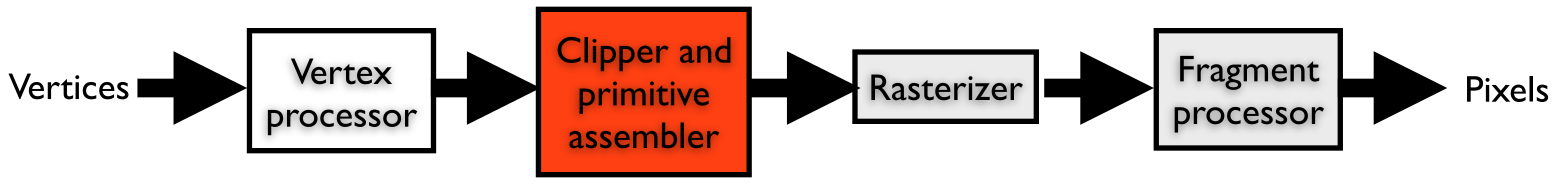
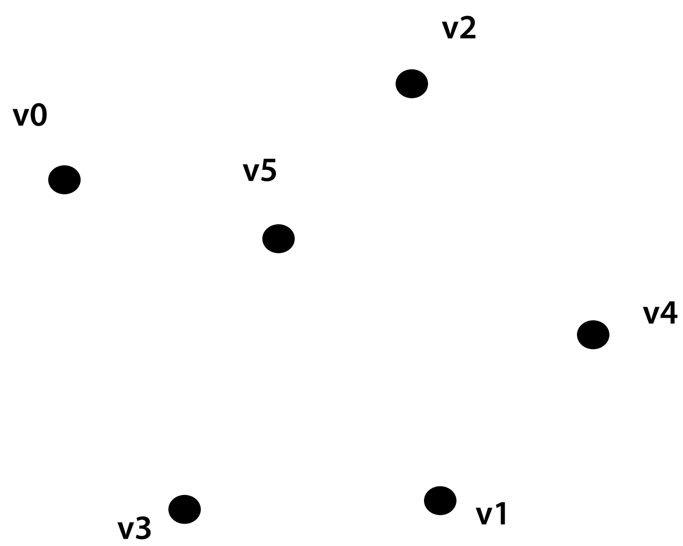


EACH VERTEX IS TRANSFORMED INDEPENDENTLY

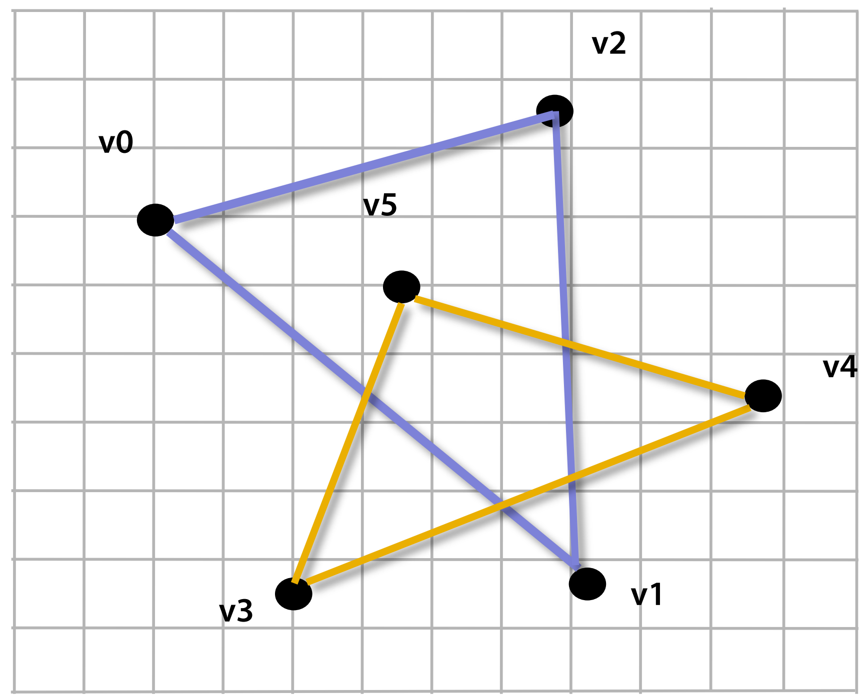


Primitive processing

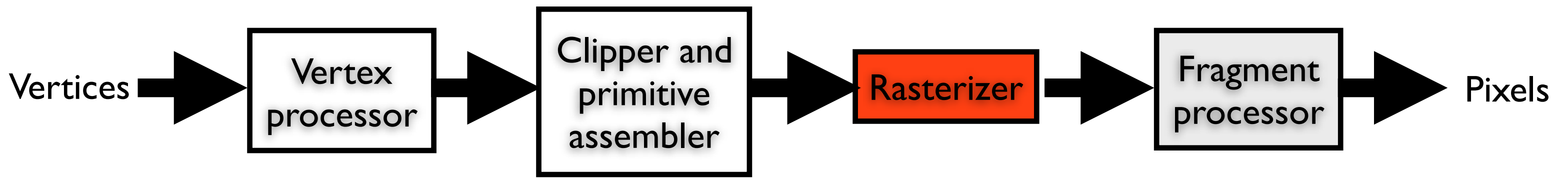
Then organized into primitives that are clipped and culled…



Vertices

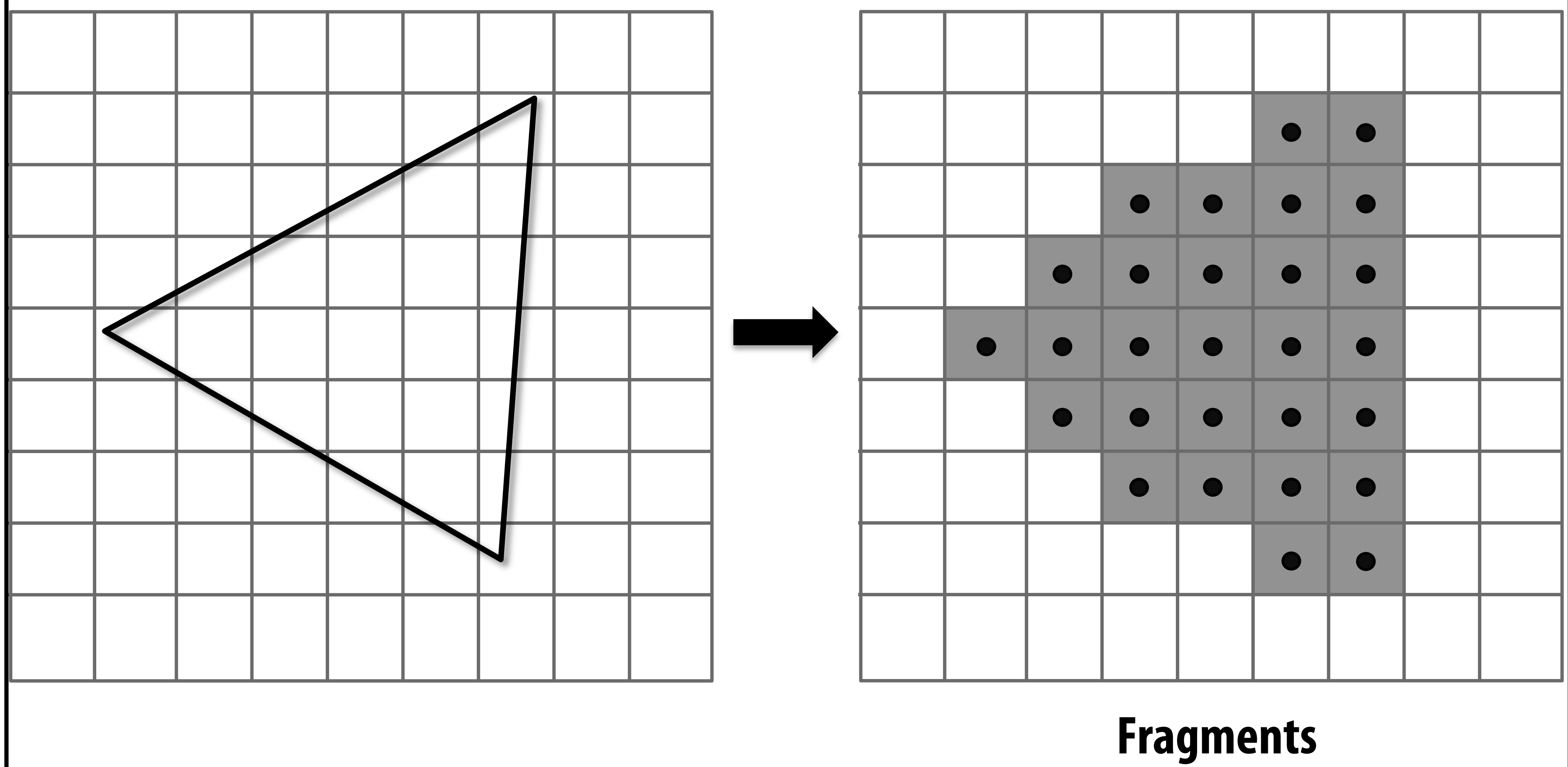


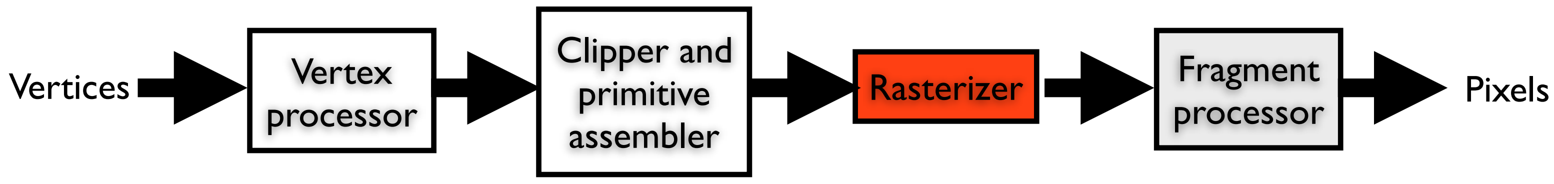
Primitives (triangles)



Rasterization

Primitives are rasterized into “pixel fragments”





Rasterization

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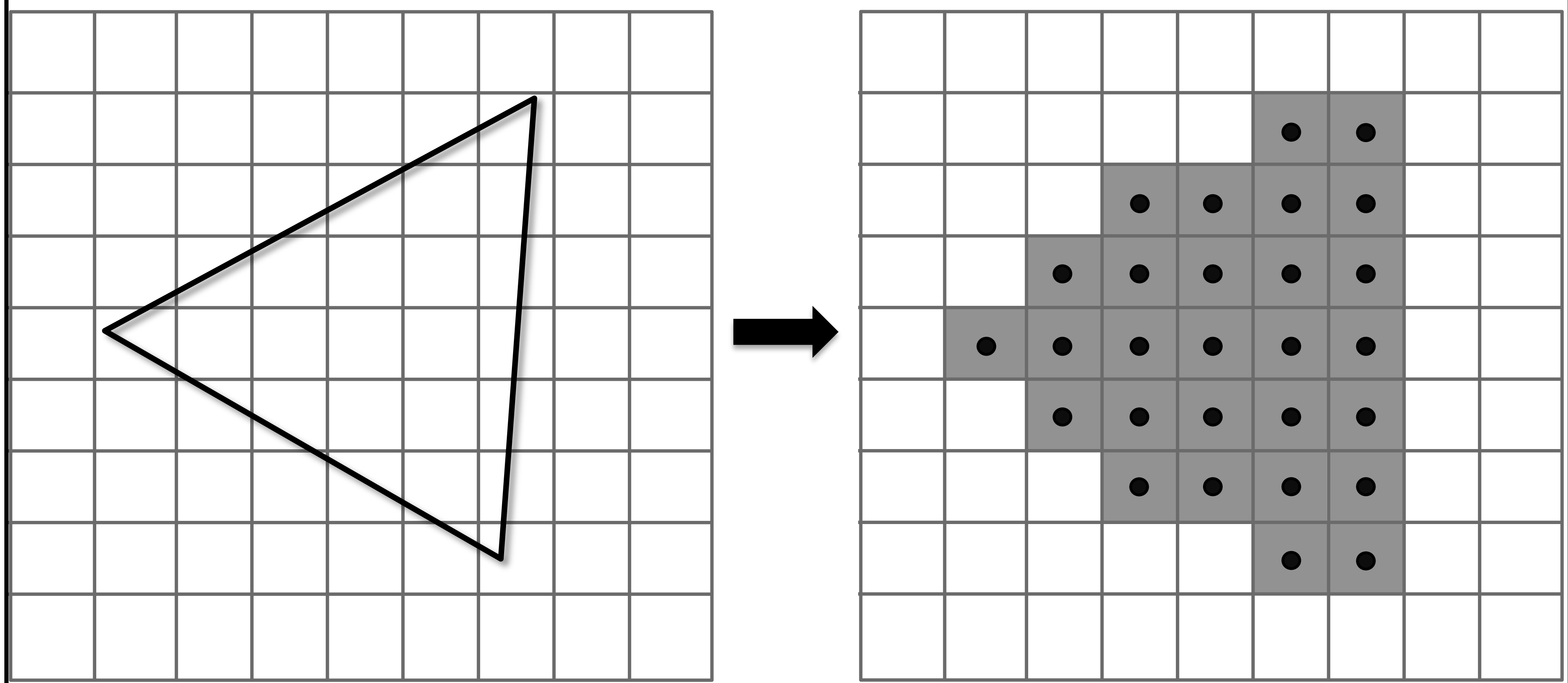
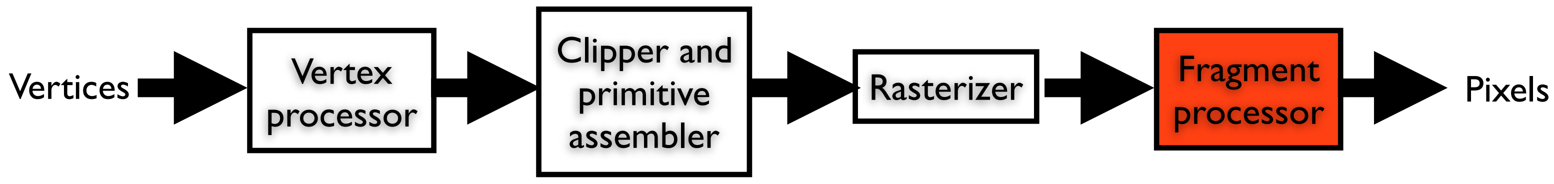
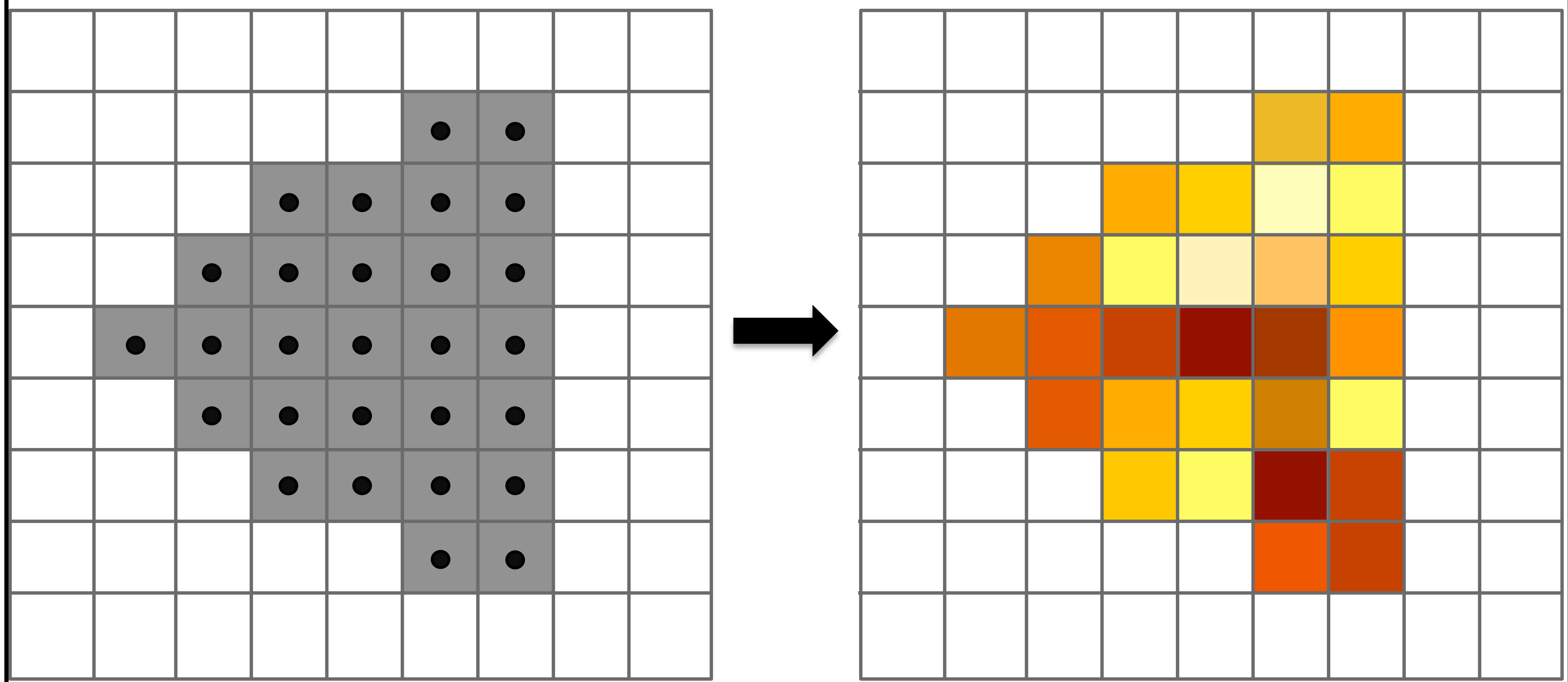


EACH PRIMITIVE IS RASTERIZED INDEPENDENTLY

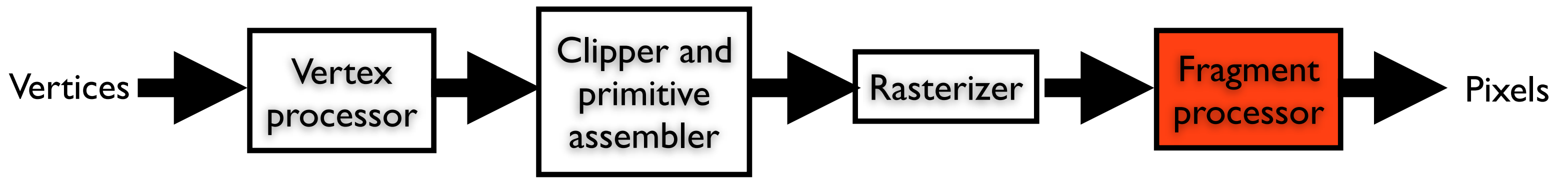


Fragment processing

Fragments are shaded to compute a color at each pixel



Shaded fragments



Fragment processing

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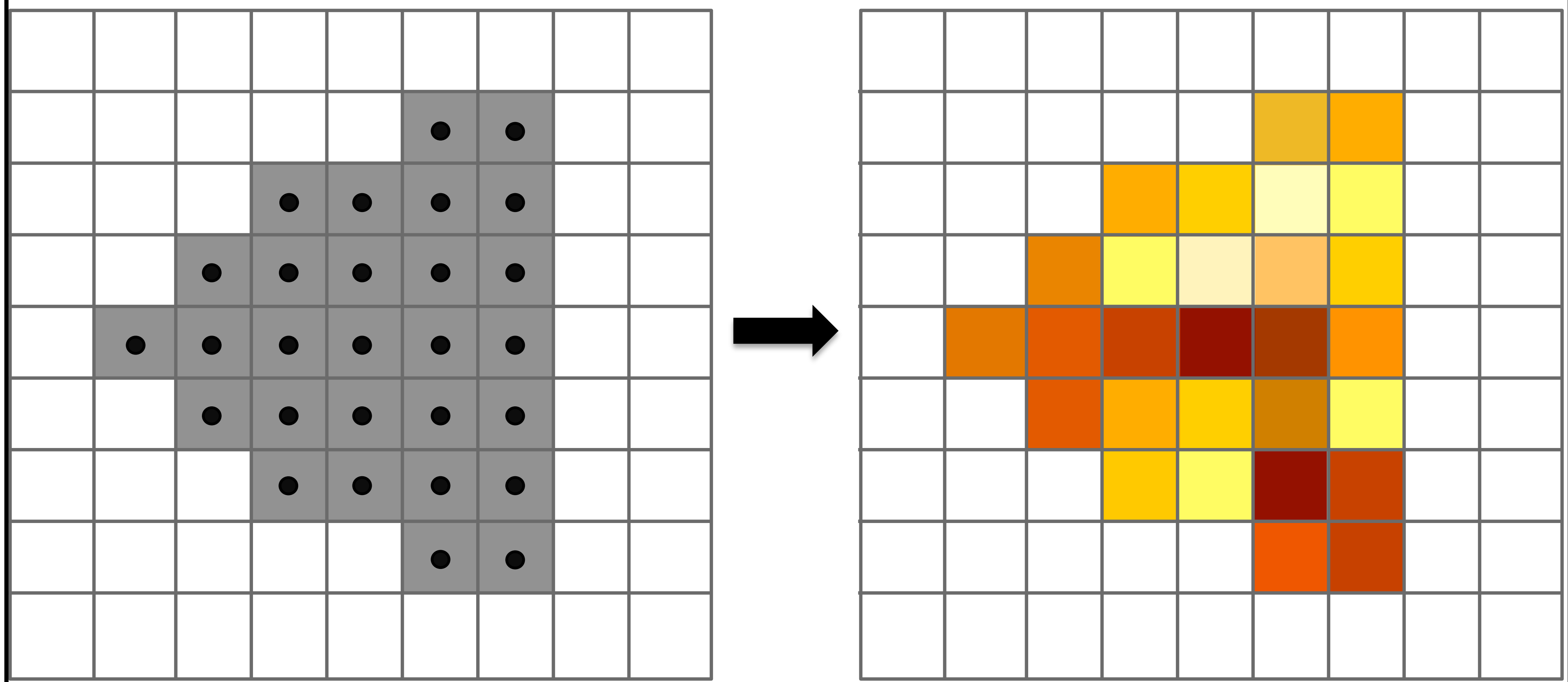
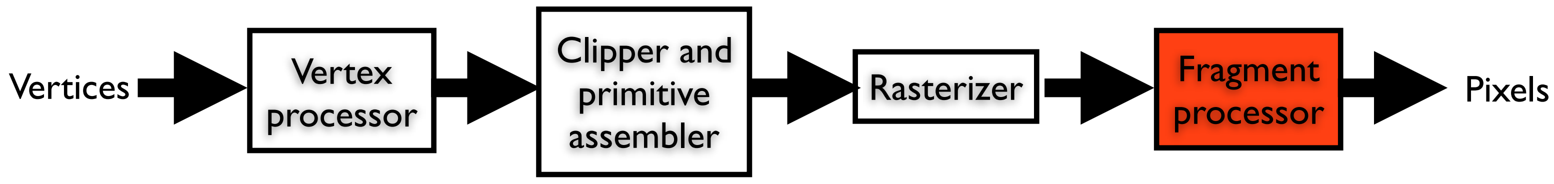
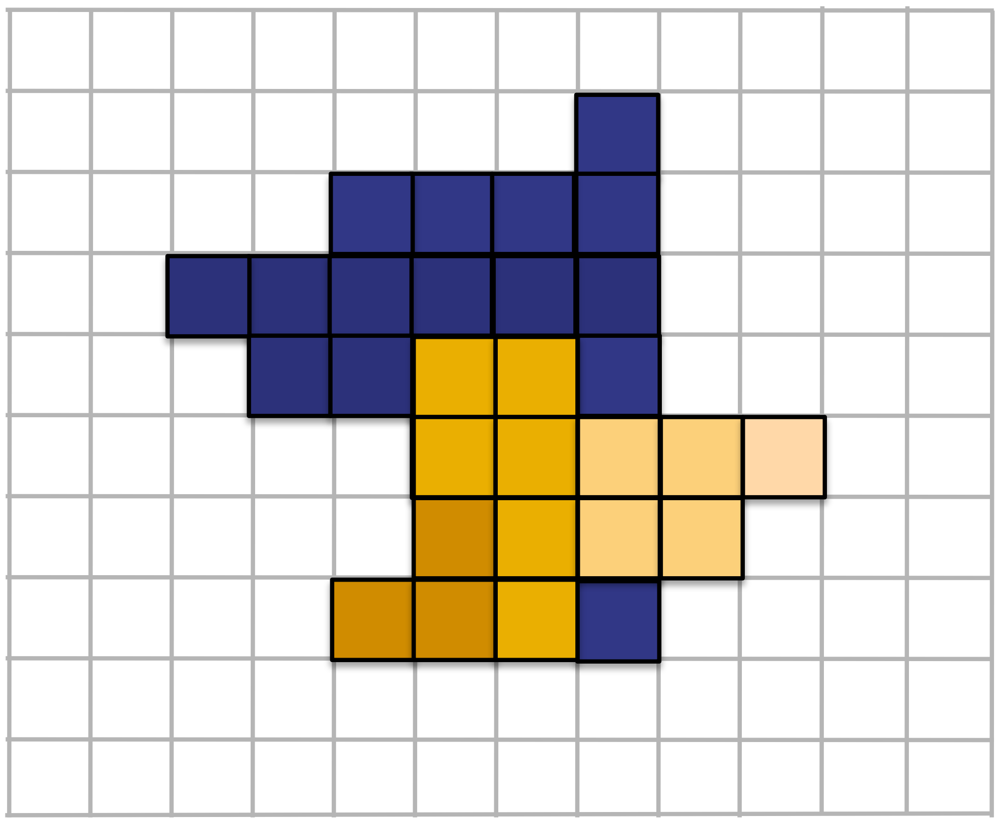


EACH FRAGMENT IS PROCESSED INDEPENDENTLY



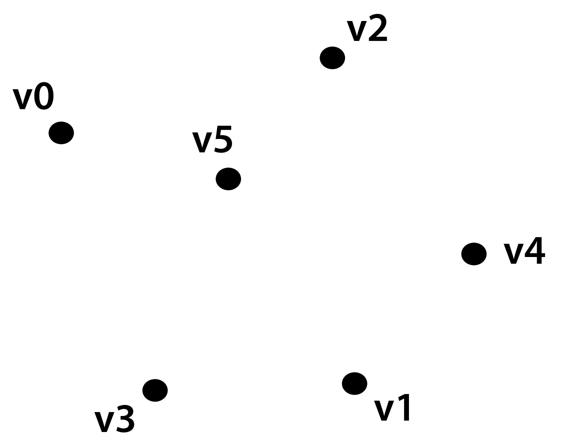
Pixel operations

Fragments are blended into the frame buffer at their pixel locations (z-buffer determines visibility)

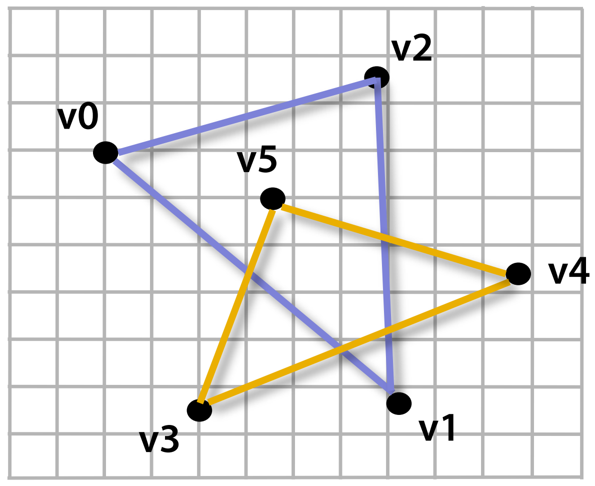


Pixels

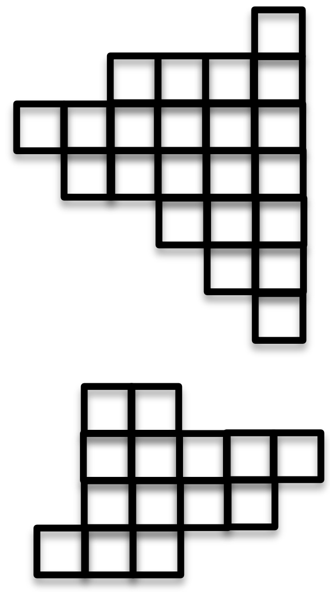
Pipeline entities



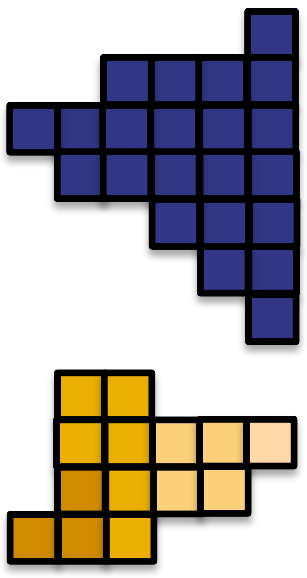
Vertices



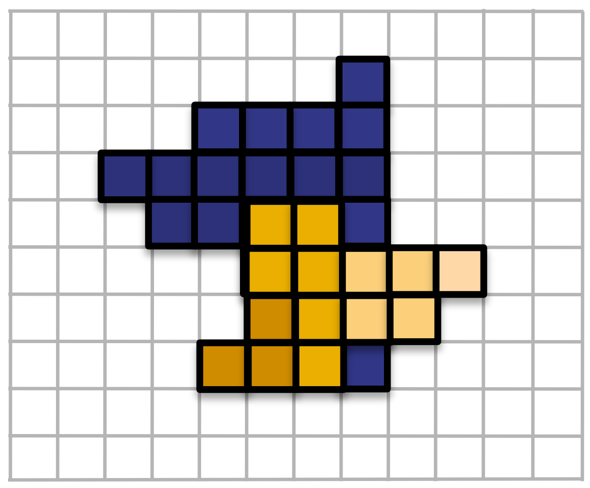
Primitives



Fragments



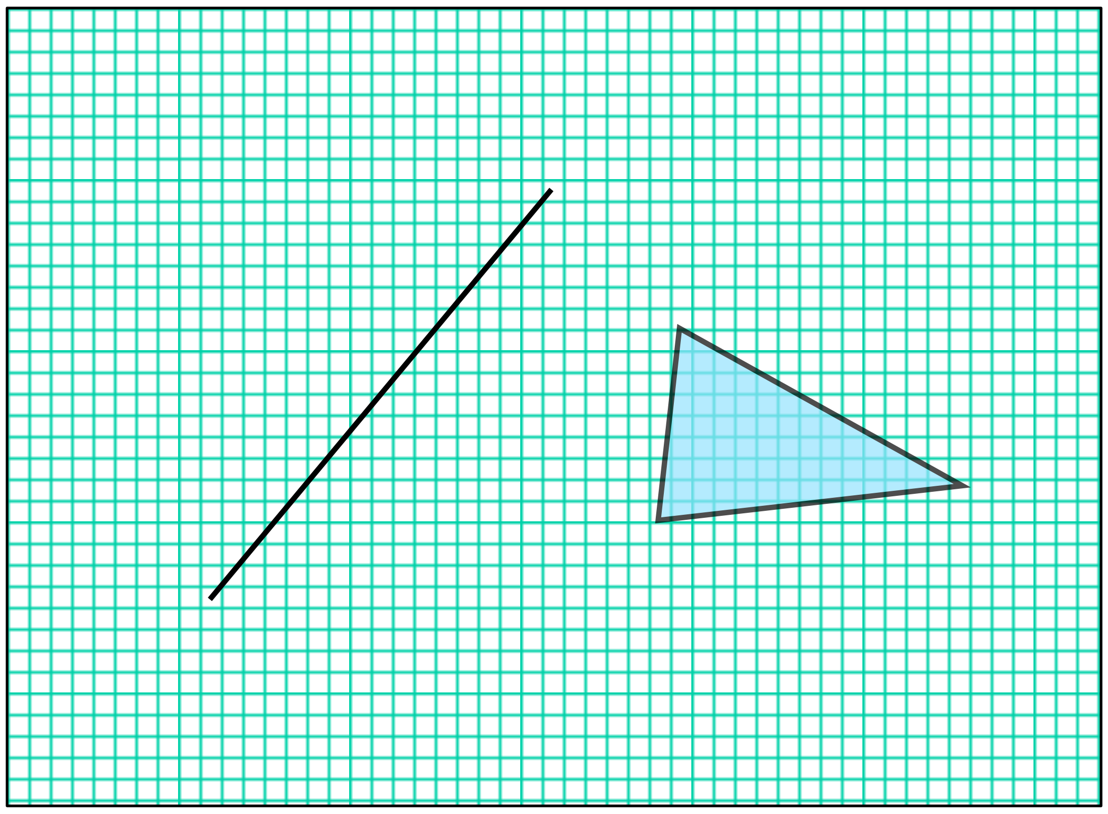
Fragments (shaded)



Pixels

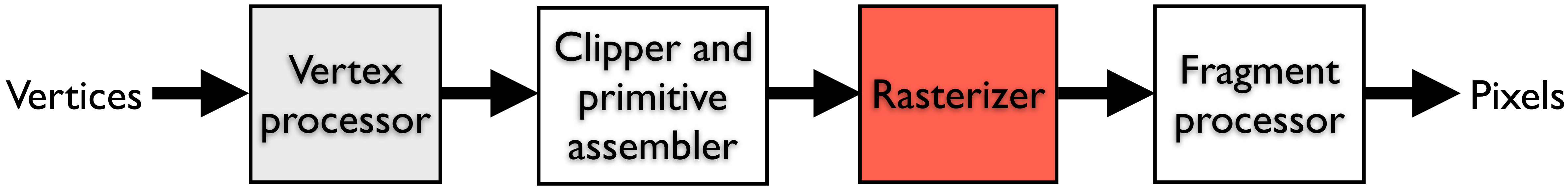
Rasterization

What is rasterization?



Rasterization is the process of determining which pixels are "covered" by the primitive

What is rasterization?



input: primitives **output**: fragments

enumerate the pixels covered by a primitive

interpolate attributes across the primitive

- **output** 1 fragment per pixel covered by the primitive

Rasterization

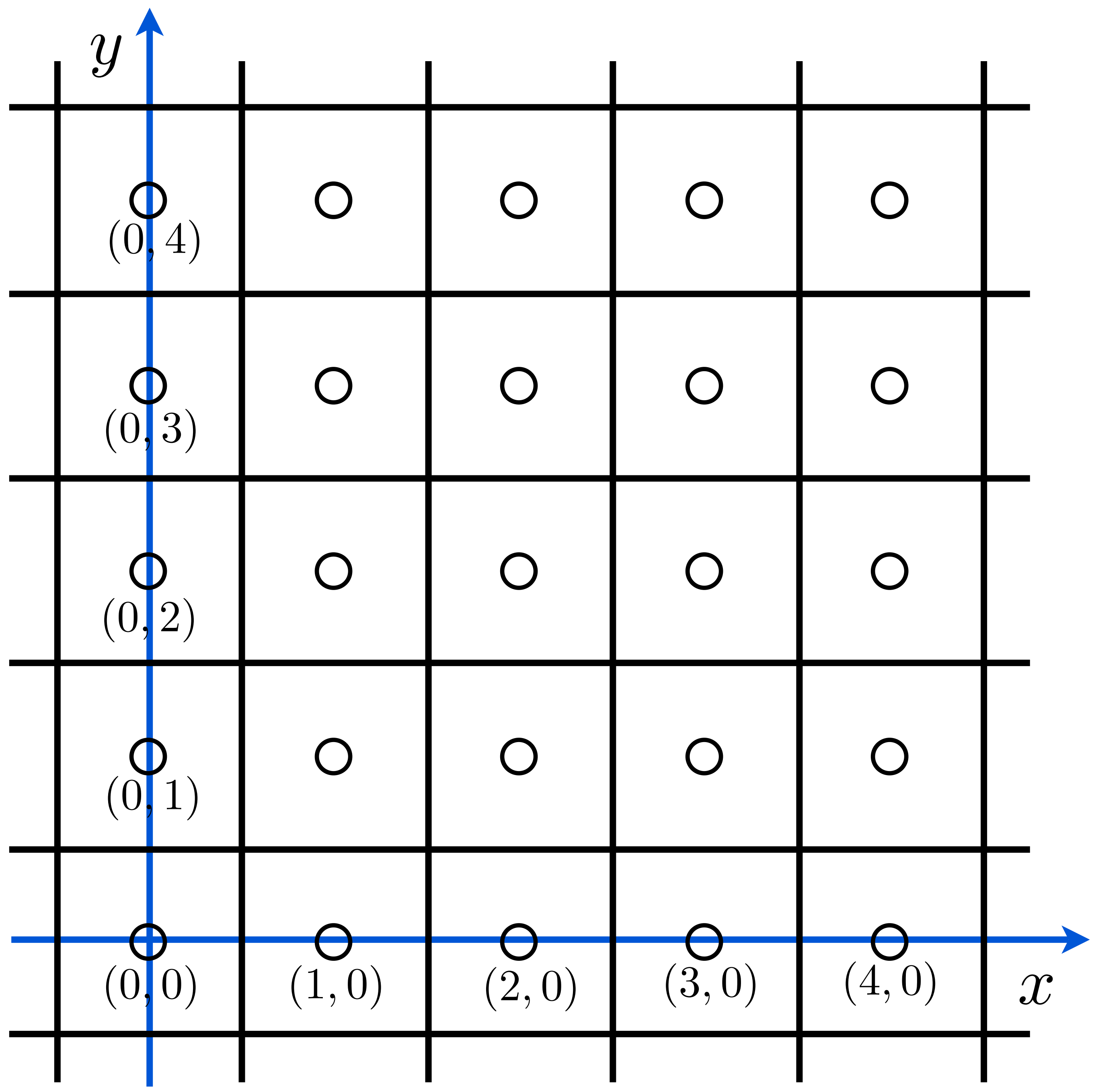
Compute integer coordinates for pixels covered by the 2D primitives

Algorithms are invoked many, many times and so must be efficient

Output should be visually pleasing, for example, lines should have constant density

Obviously, should be able to draw all possible 2D primitives

Screen coordinates



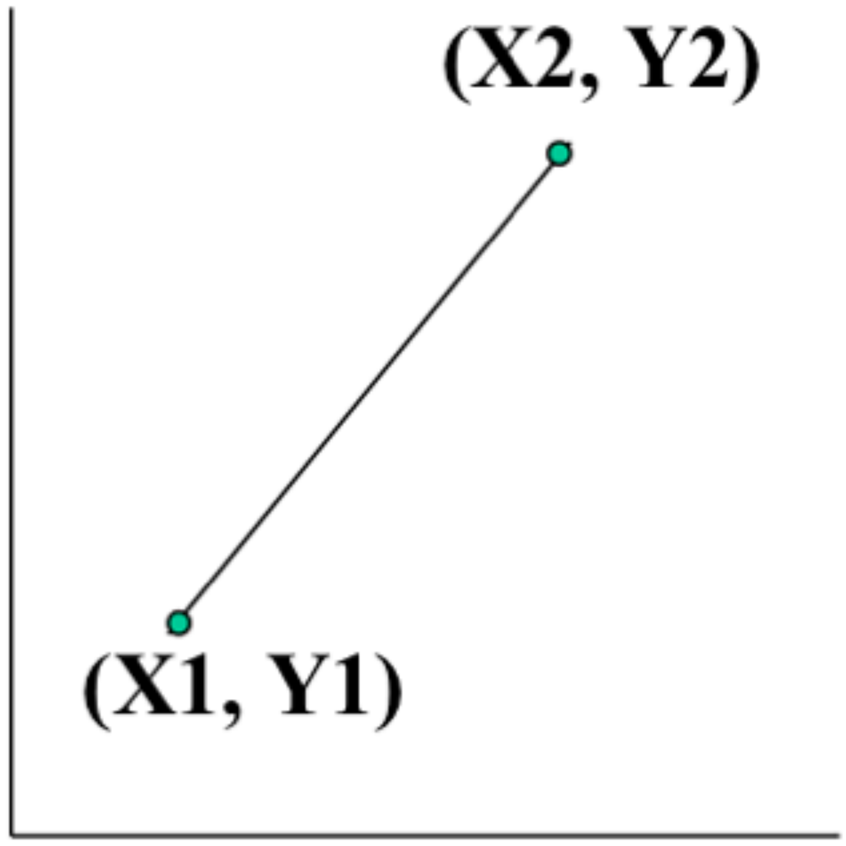
we'll assume stuff has been converted to **screen coordinates**

Line Representation

Math Review

- 2D math for lines

How do we determine the equation of the line?



Math Review

- 2D math for lines

  Slope-Intercept formula for a line

  Slope = (Y2 – Y1)/(X2 – X1)

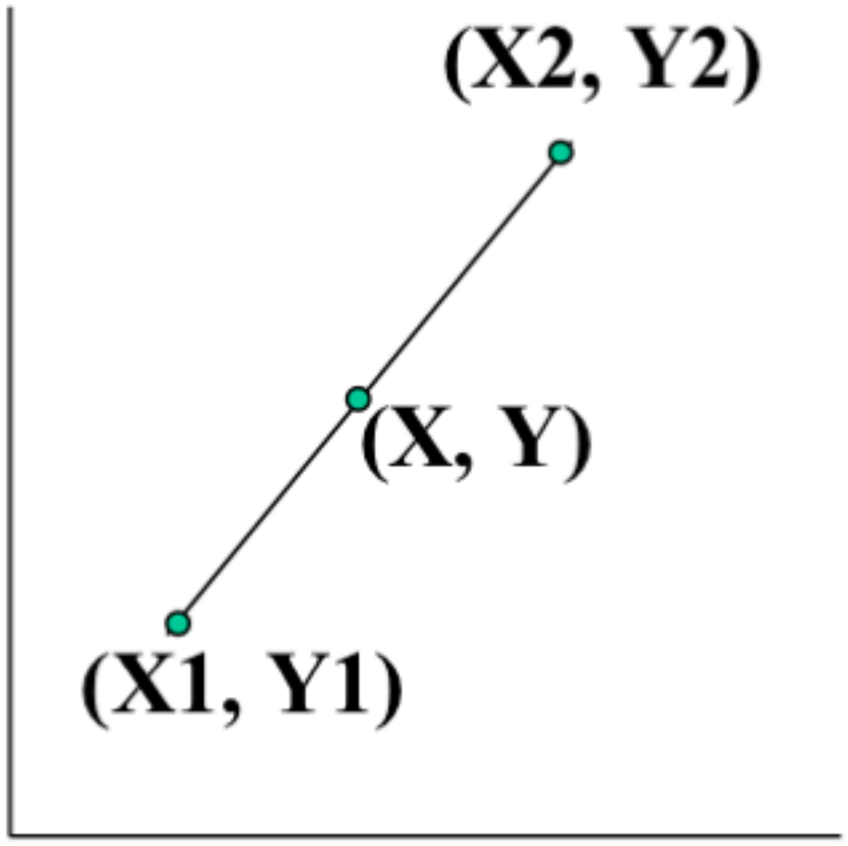
  (Y - Y1)/(X - X1)

  Solving For Y

  Y = [(Y2 – Y1)/(X2 – X1)]X

  + [-(Y2 – Y1)/(X2 - X1)]X1 + Y1 or

  Y = m X + b



Math Review

- Explicit (functional) representation

  $y = f(x)$

  y is the dependent, x independent variable

  Find value of y from value of x

  Example, for a line: $y = mx + b$

  for a circle: $x^2 + y^2 = r^2$

Math Review

- Parametric Representation

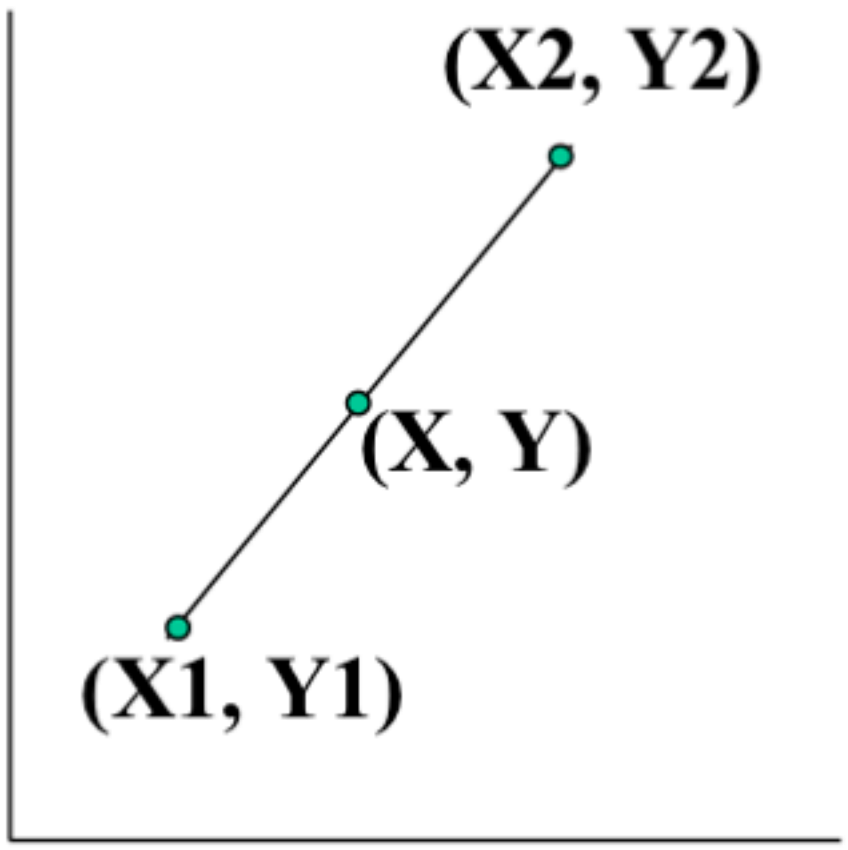
$$x = x(u),\ y = y(u)$$

where new parameter u (or often t) determines the value of x and y (and possibly z) for each point

x,y treated the same, axis invariant

Math Review

Parametric formula for a line



X = X1 + t(X2 – X1)
Y = Y1 + t(Y2 – Y1)

for parameter t from 0 to 1

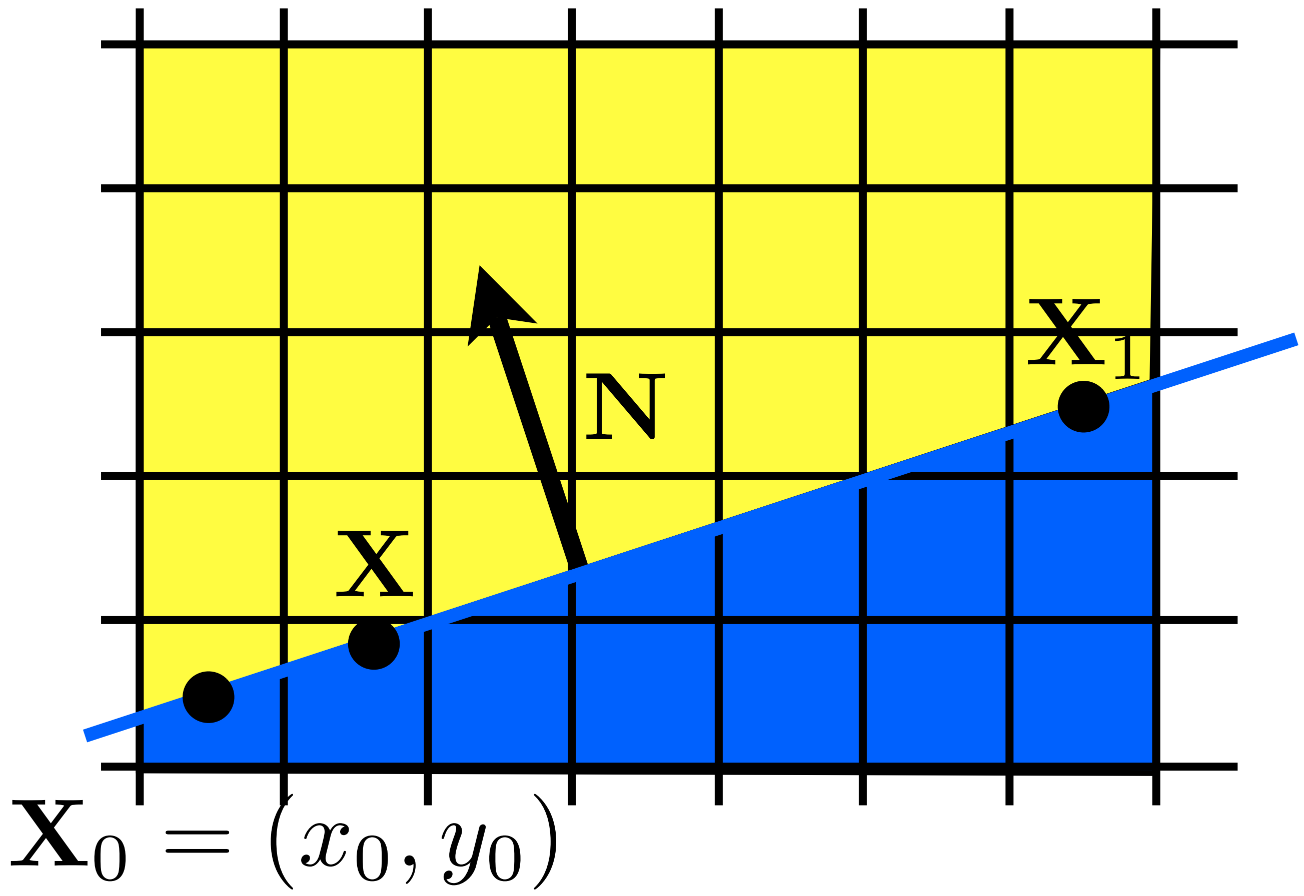
Therefore, when

t = 0 we get (X1,Y1)

t = 1 we get (X2,Y2)

Varying t gives the points along the line segment

Implicit Line Equation

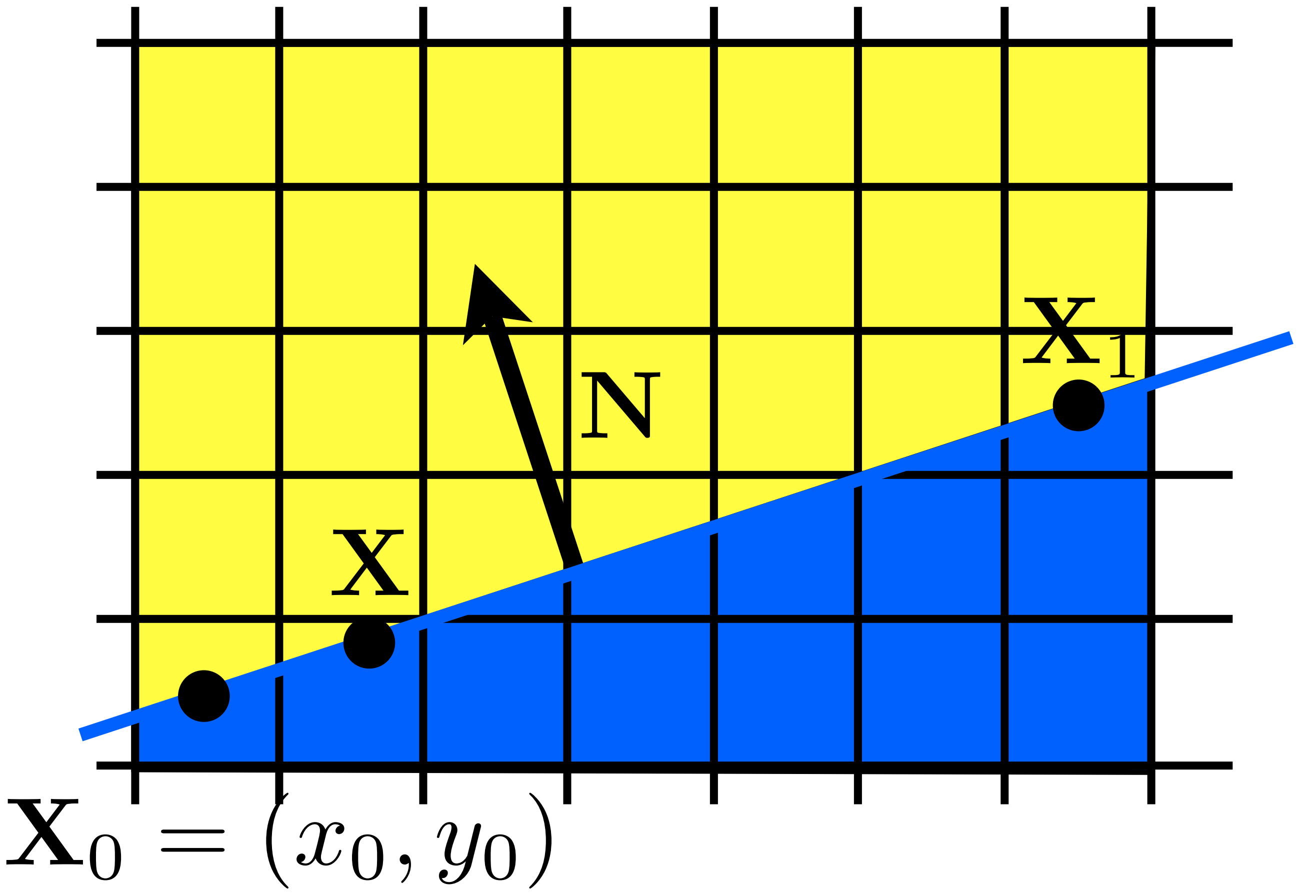


$$f(\mathbf{X}) = \mathbf{N} \cdot (\mathbf{X} - \mathbf{X}_0) = 0$$

<whiteboard>

<whiteboard>: work out the implicit line equation in terms of X0 and X1

Implicit Line Equation



decision variable, d

$$f(\mathbf{X}) = \mathbf{N} \cdot (\mathbf{X} - \mathbf{X}_0) = d$$

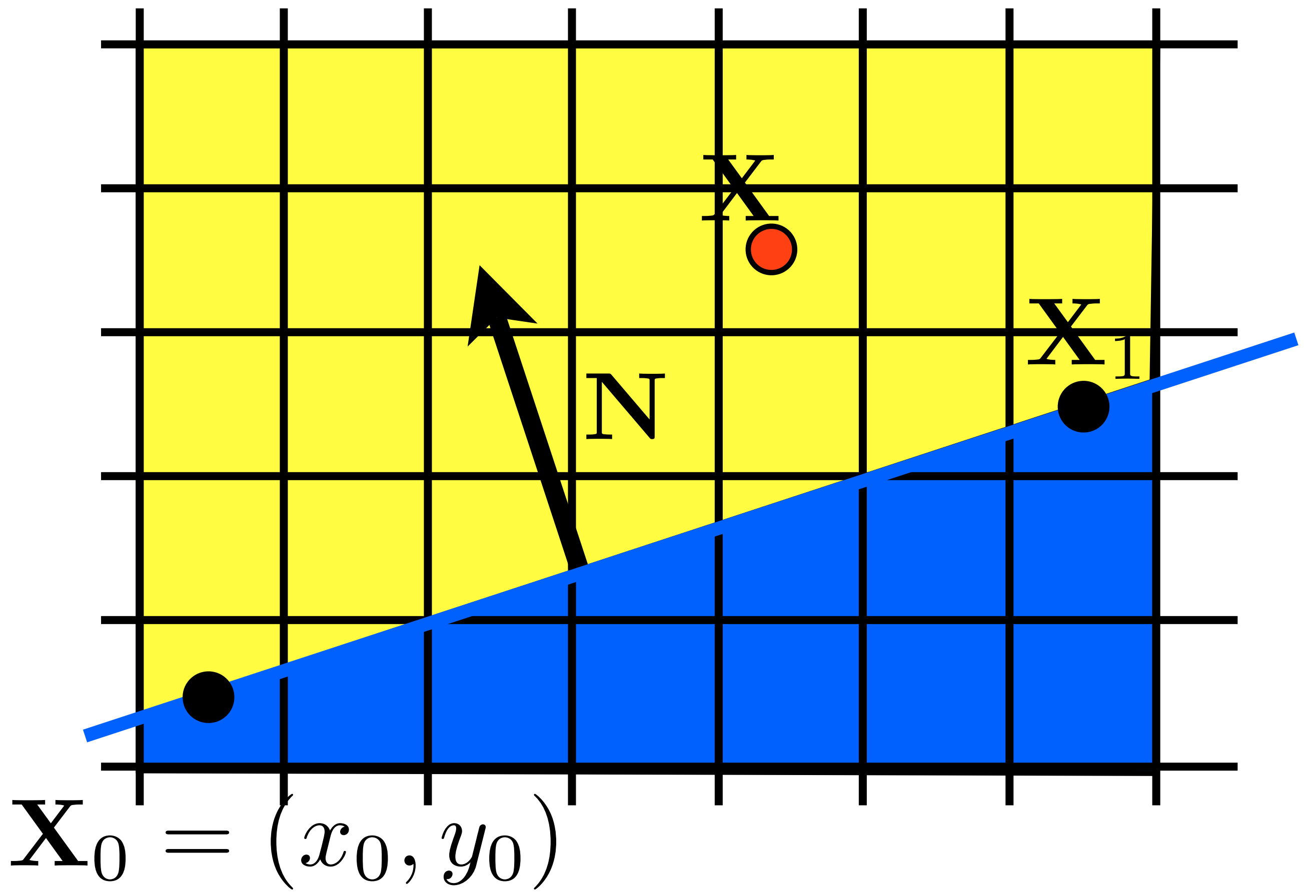
$$d > 0$$

$$d < 0$$

$$d = 0$$

<whiteboard>: work out the implicit line equation in terms of X0 and X1

Implicit Line Equation



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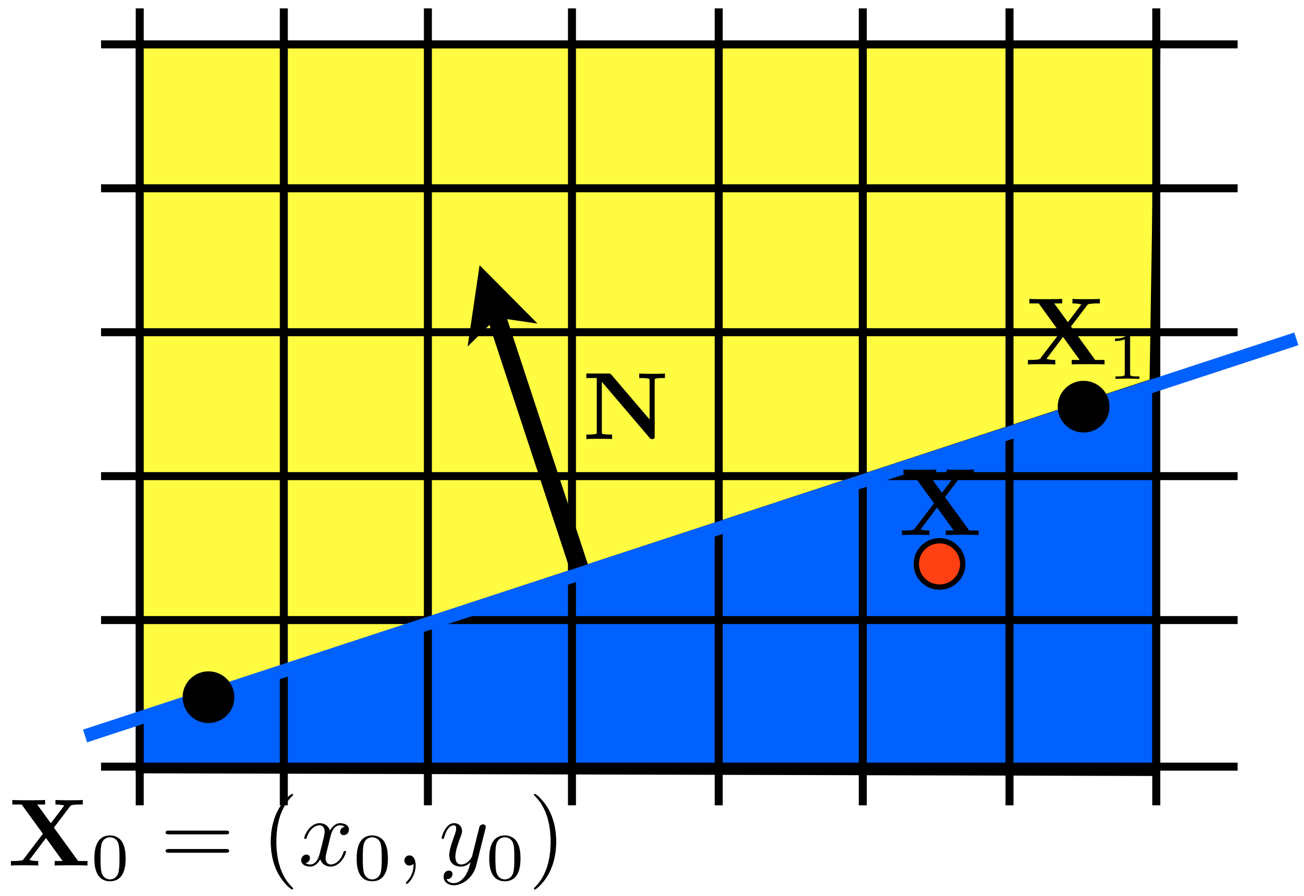
Implicit Line Equation



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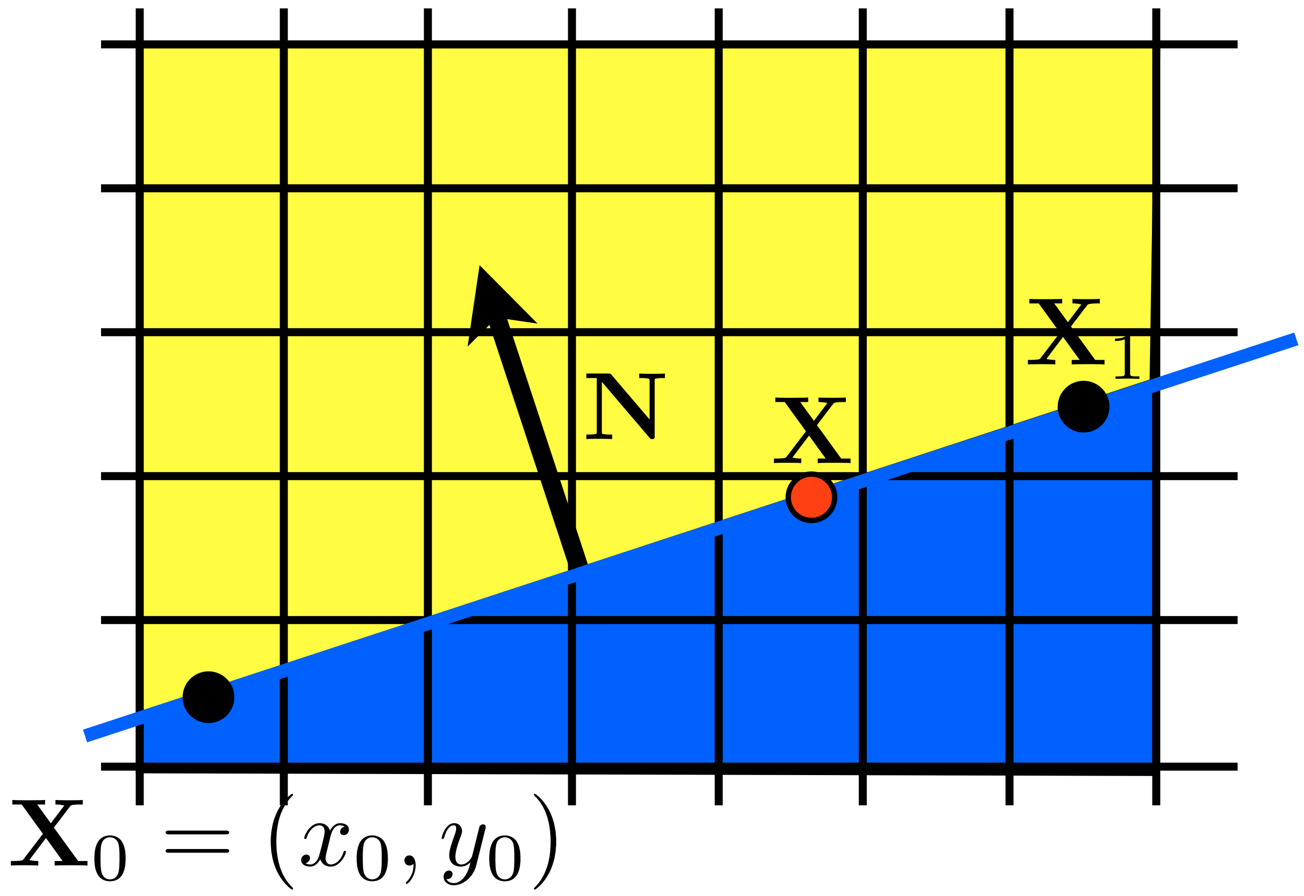
Implicit Line Equation



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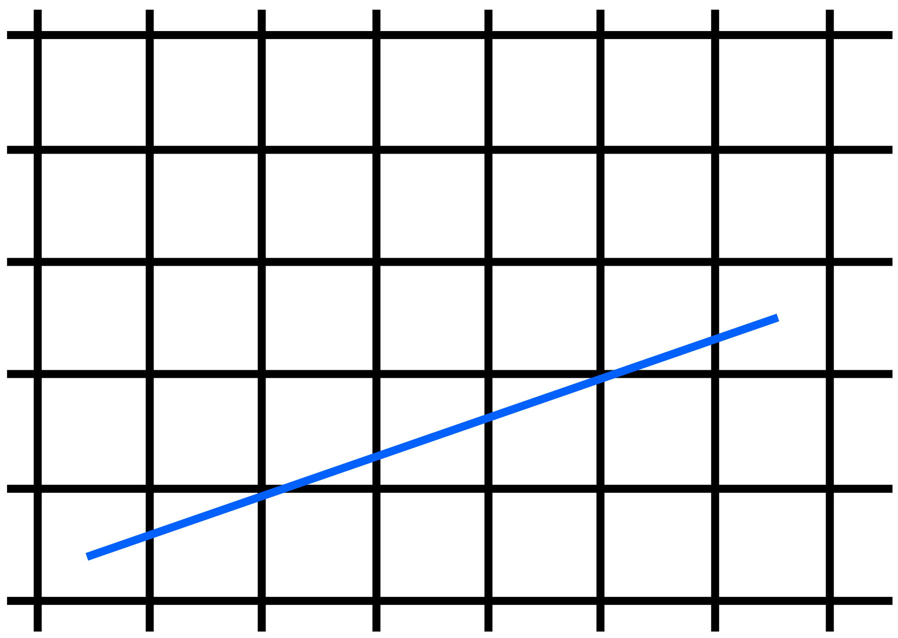
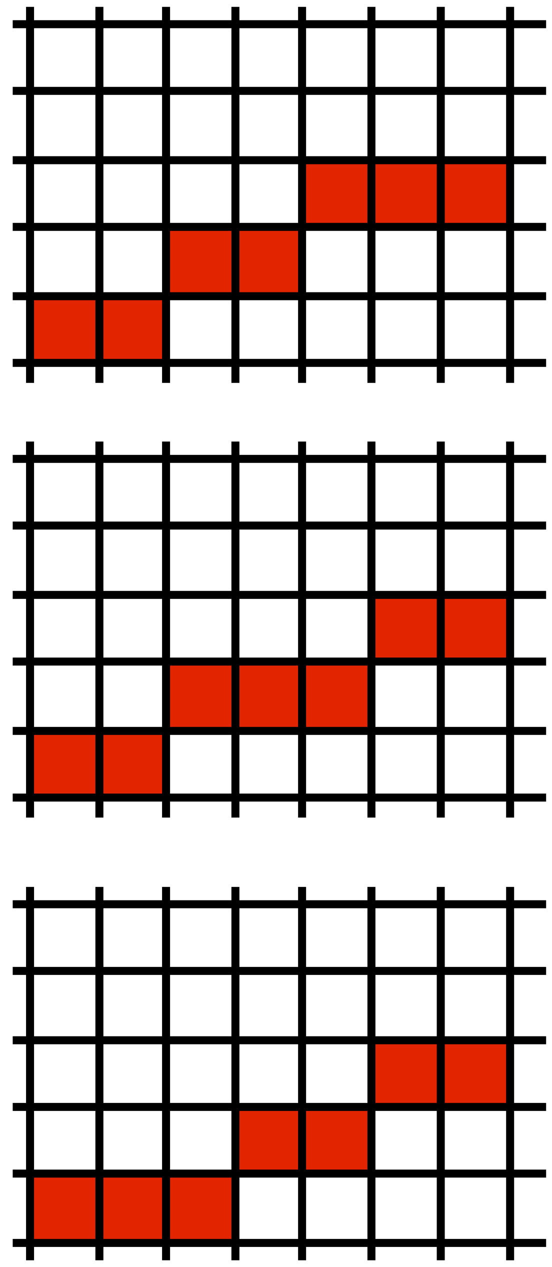
Line Drawing

Which pixels should be used to approximate a line?



Draw the thinnest possible line that has no gaps



DDA algorithm for lines

Parametric Lines: the DDA algorithm
(digital differential analyzer)

$Y_{i+1} = m\, x_{i+1} + B$

$= m(x_i + \Delta x) + B \qquad \Delta x = (x_{i+1} - x_i)$

$= y_i + m(\Delta x)$ **<- must round to find int**

If we increment by 1 pixel in X, we turn on
[xi, Round(yi)] or same for Y if m > 1

Scan conversion for lines

DDA includes Round( ); and this is fairly slow

For Fast Lines, we want to do only integer math +,-

We do this using the Midpoint Algorithm

To do this, lets look at lines with y-intercept B
and with slope between 0 and 1:

$$y = (dy/dx)x + B \quad \Longrightarrow$$

$$f(x,y) = (dy)x - (dx)y + B(dx) = 0$$

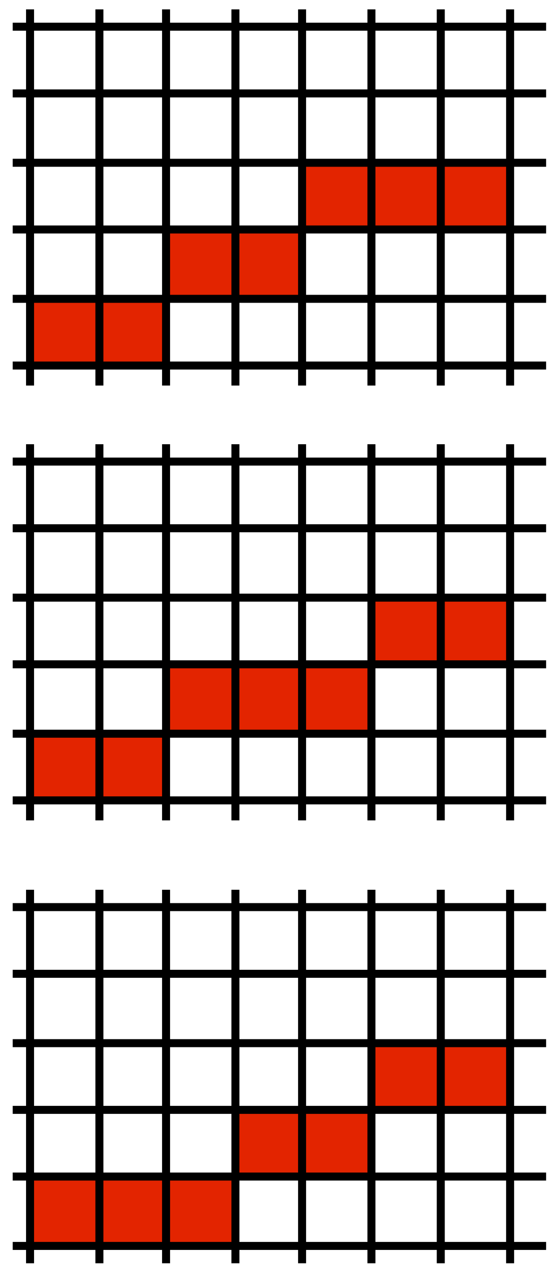
Removes the division => slope treated as 2 integers

Line drawing algorithm

(case: 0 < m <= 1)

```
y = y0
for x = x0 to x1 do
    draw(x,y)
    if (<condition>) then
        y = y+1
```

- move from left to right
- choose between (x+1,y) and (x+1,y+1)



draw pixels from left to right, occasionally move up

Line drawing algorithm

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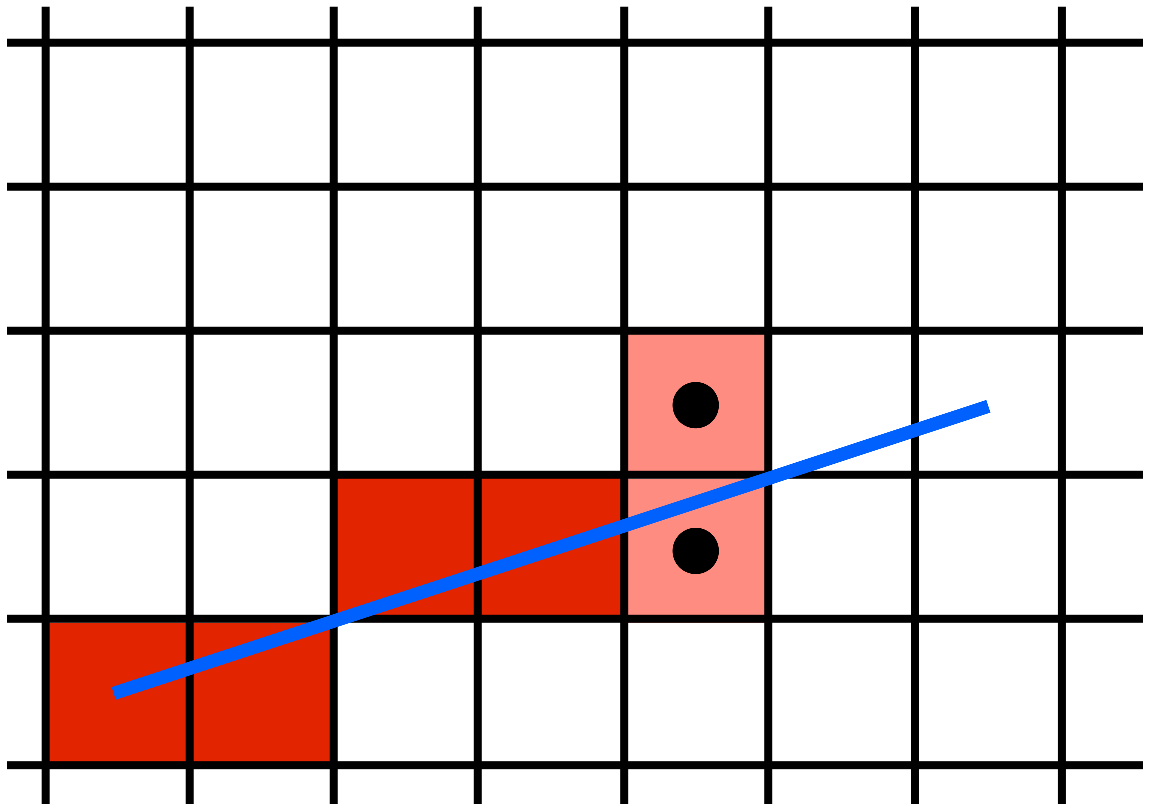
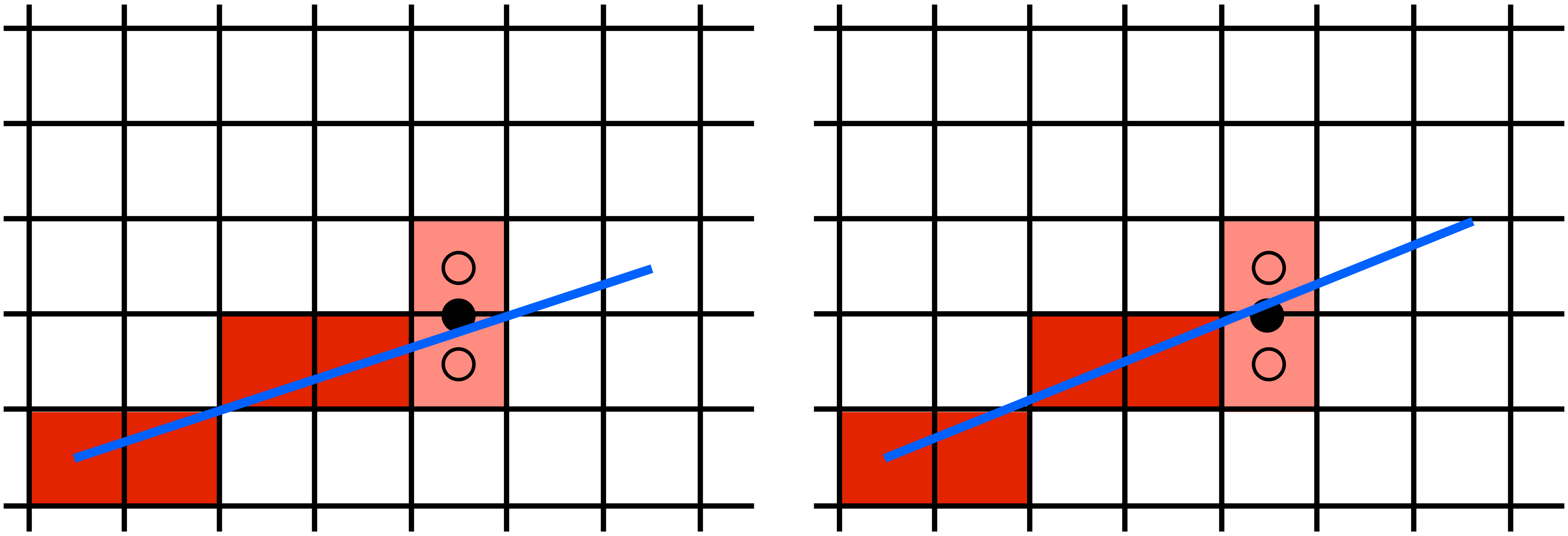


- move from left to right
- choose between (x+1,y) and (x+1,y+1)

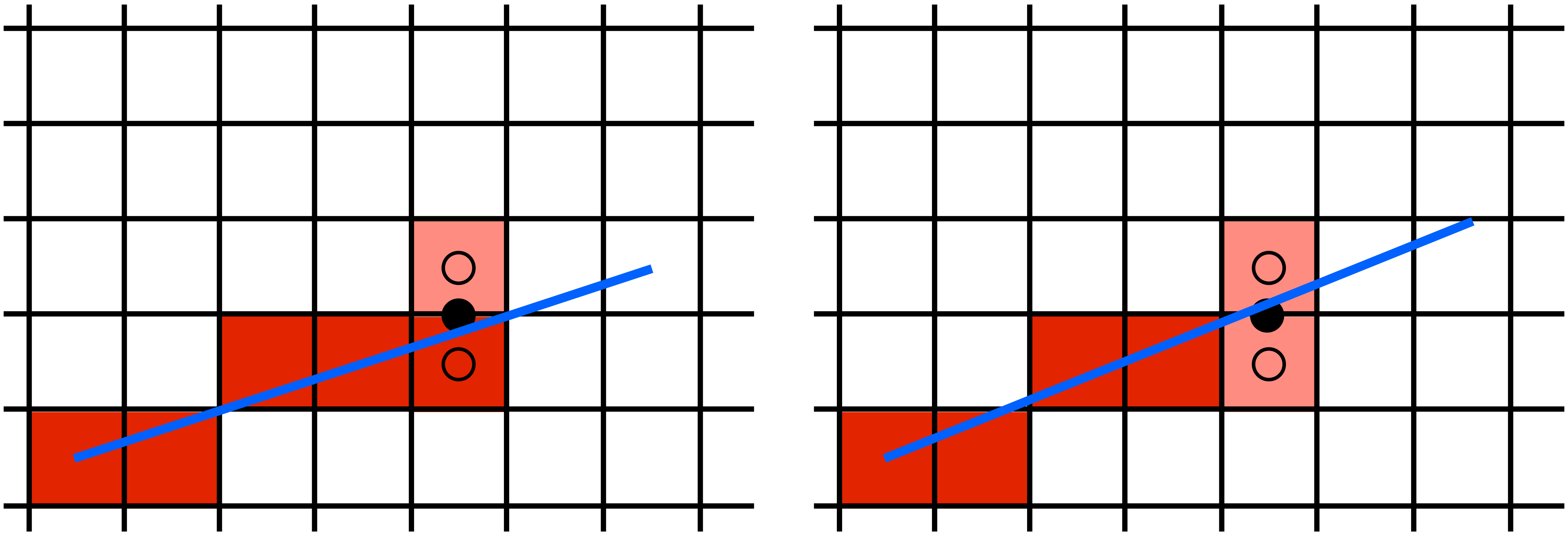
draw pixels from left to right, occasionally move up

Use the midpoint between the two pixels to choose



If the line falls **below** the midpoint, use the bottom pixel
if the line falls **above** the midpoint, use the top pixel

Use the midpoint between the two pixels to choose



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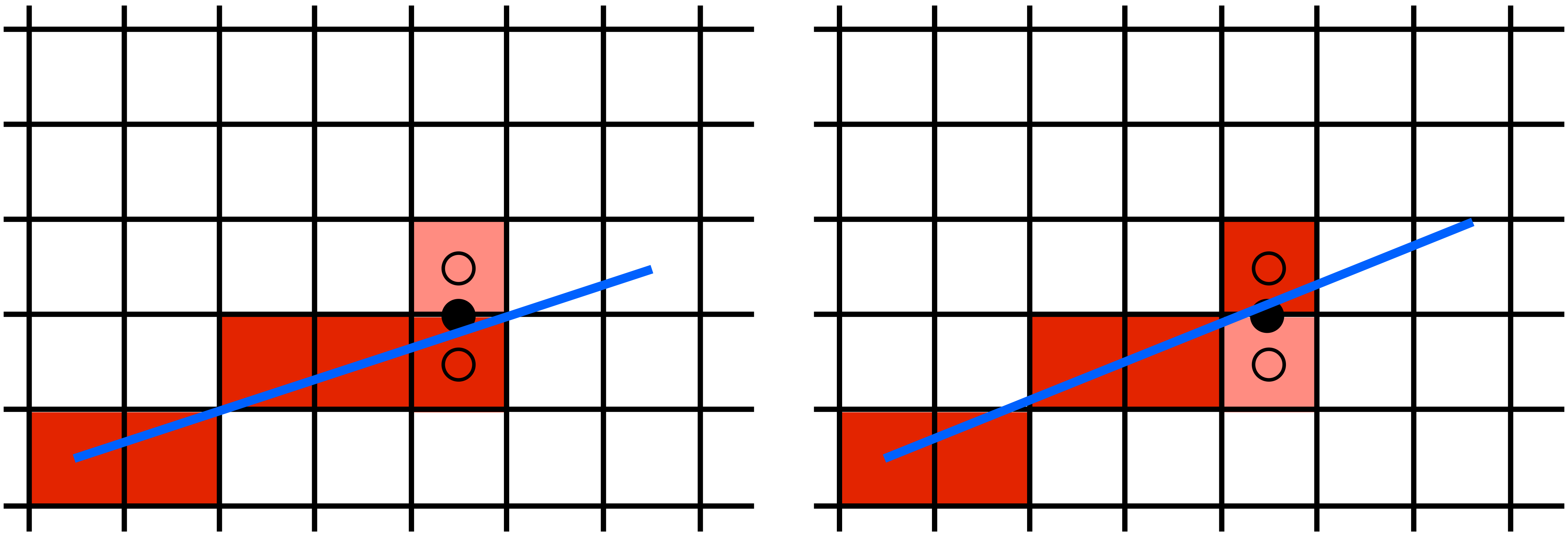

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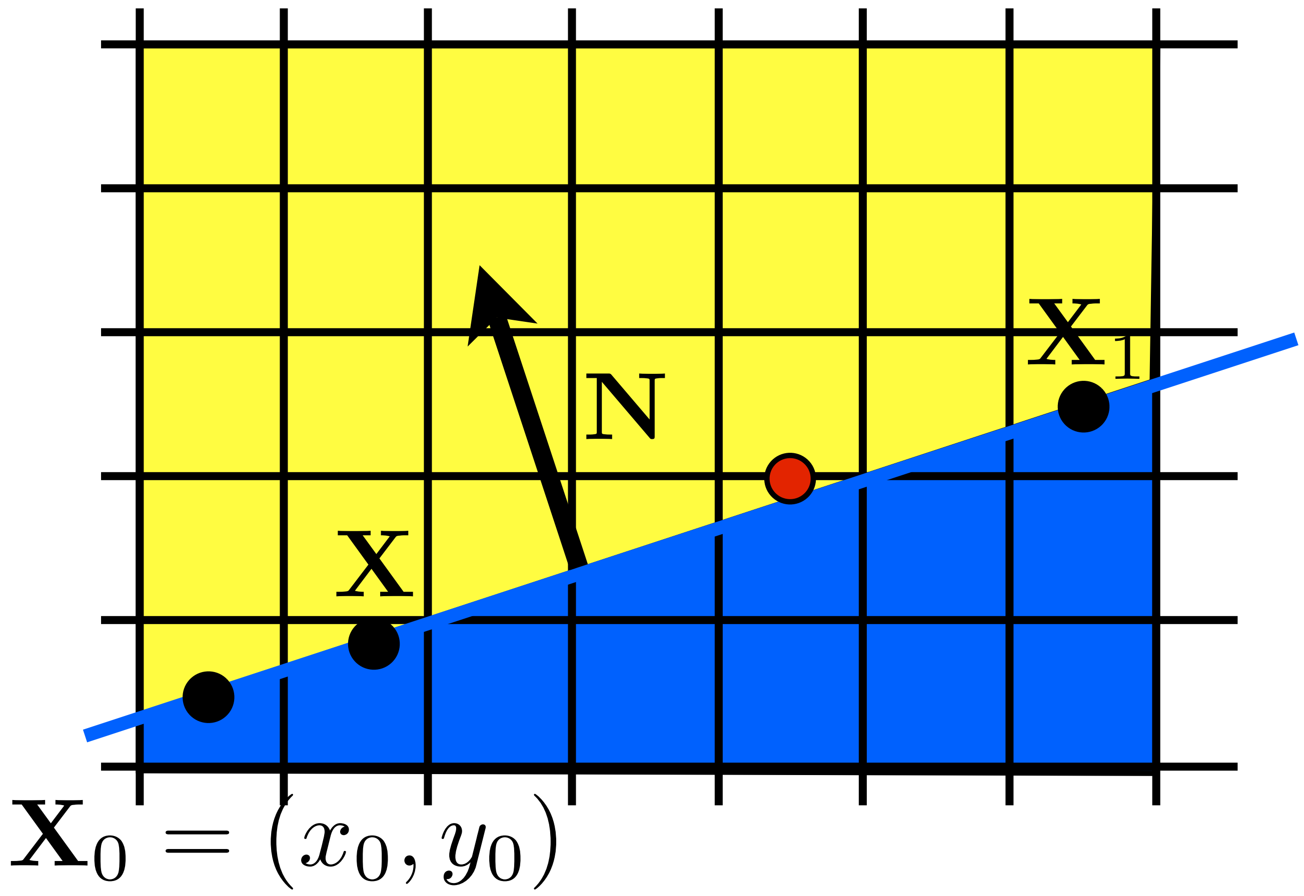


implicit line equation:

$$f(\mathbf{X}) = \mathbf{N} \cdot (\mathbf{X} - \mathbf{X}_0) = 0$$

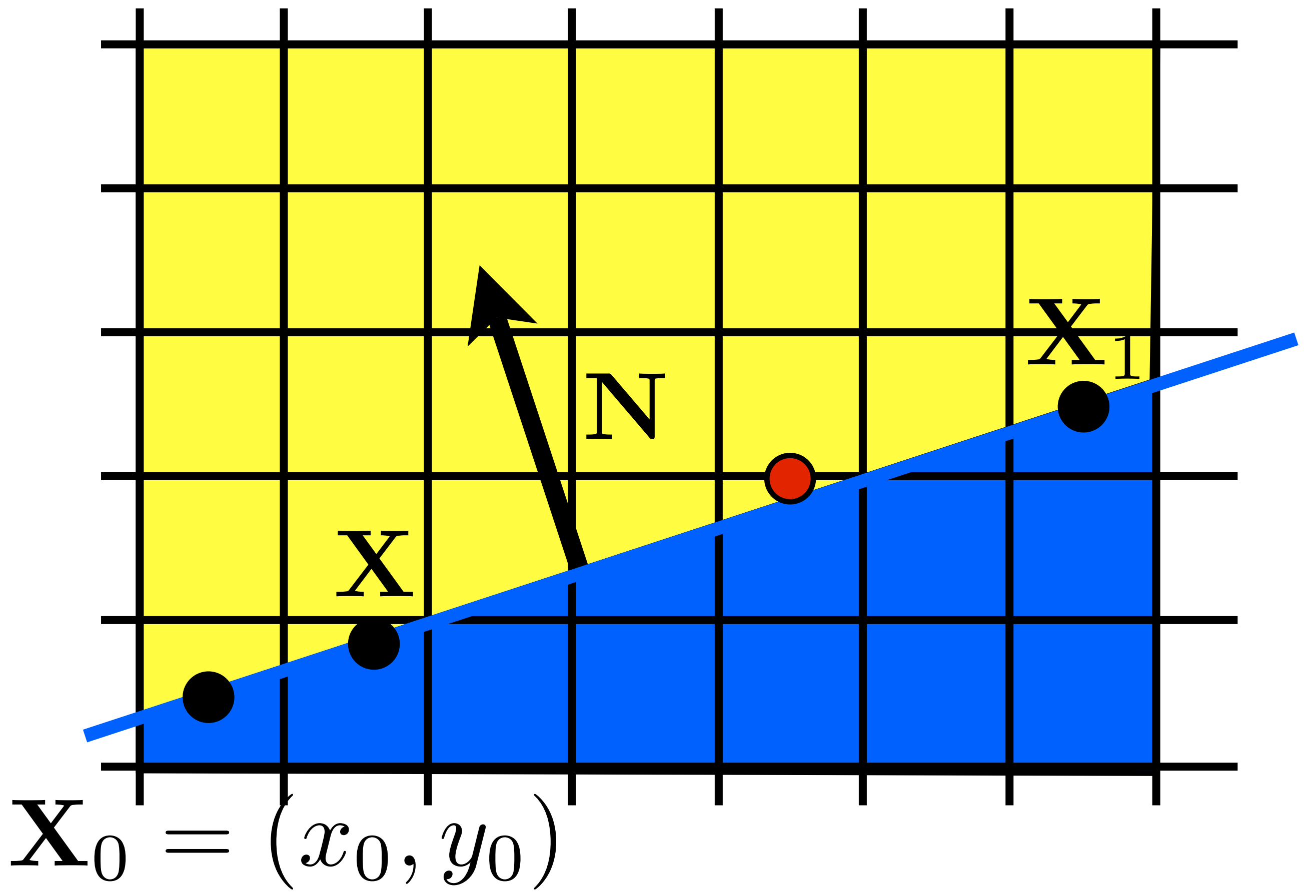
<whiteboard>

evaluate f at midpoint:

$$f(x, y + \frac{1}{2}) \ ? \ 0$$

<whiteboard>: work out the implicit line equation in terms of X0 and X1
Question: will f(x,y+1/2) be > 0 or < 0?

Use the midpoint between the two pixels to choose



implicit line equation:

$$f(\mathbf{X}) = \mathbf{N} \cdot (\mathbf{X} - \mathbf{X}_0) = 0$$

evaluate f at midpoint:

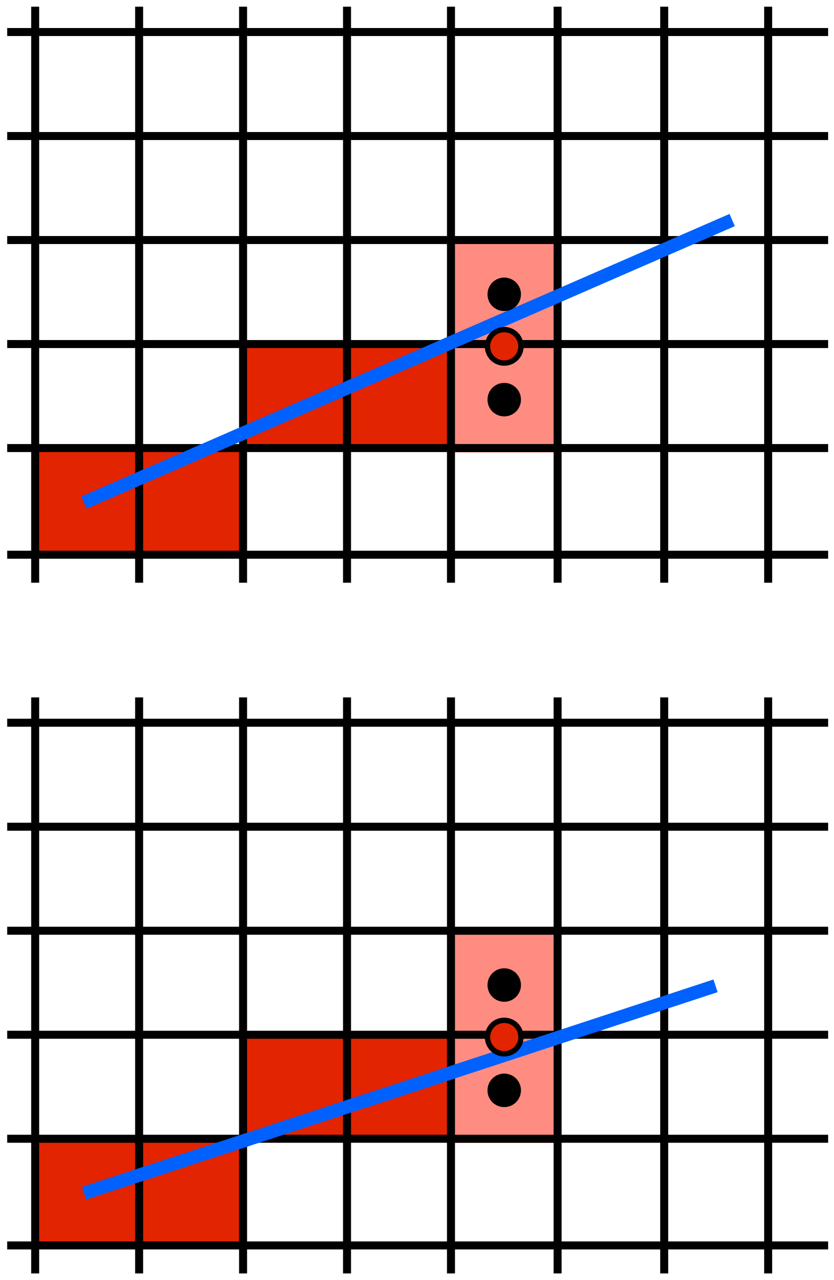
$$f(x, y + \frac{1}{2}) \ > \ 0$$

this means midpoint is above the line -> line is closer to bottom pixel

Line drawing algorithm

(case: 0 < m <= 1)

```
y = y0
for x = x0 to x1 do
    draw(x,y)
    if ($f(x+1, y+\frac{1}{2}) < 0$) then
        y = y+1
```

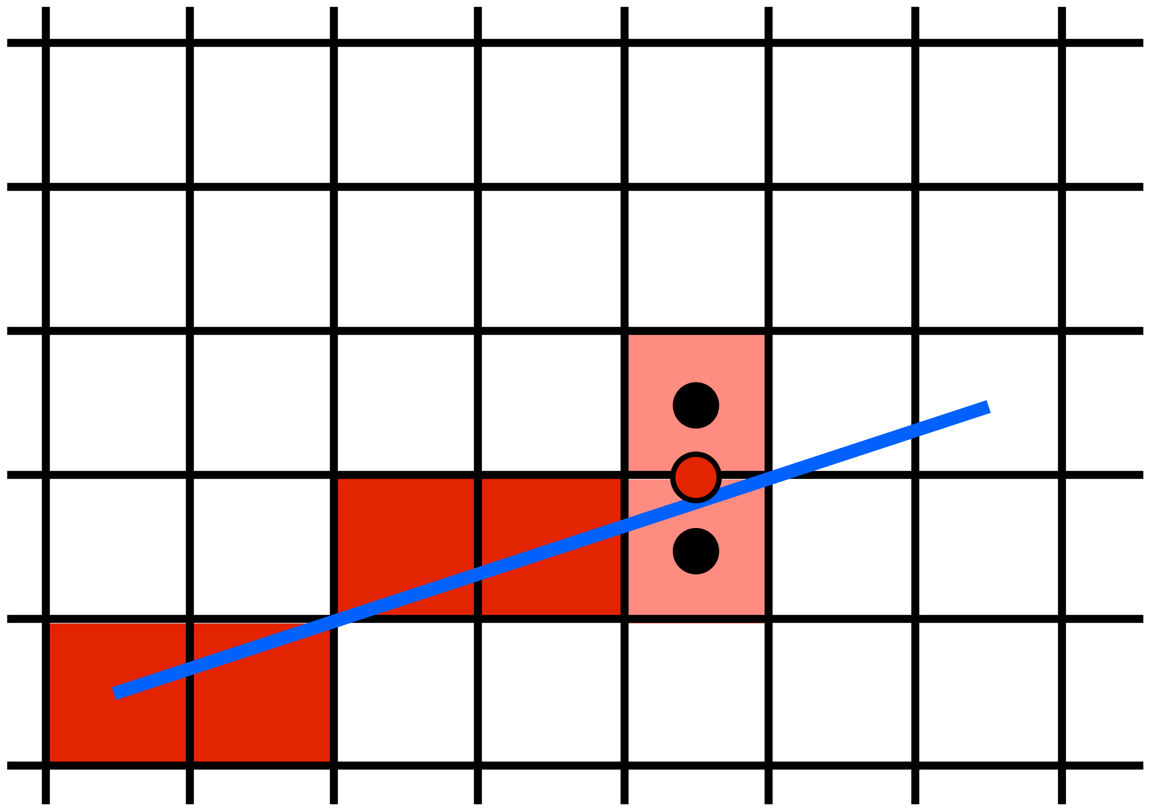


can now fill in the **condition**

We can make the Midpoint Algorithm more efficient

```
y = y0
for x = x0 to x1 do
    draw(x,y)
    if (f(x+1, y+1/2) < 0) then
        y = y+1
```

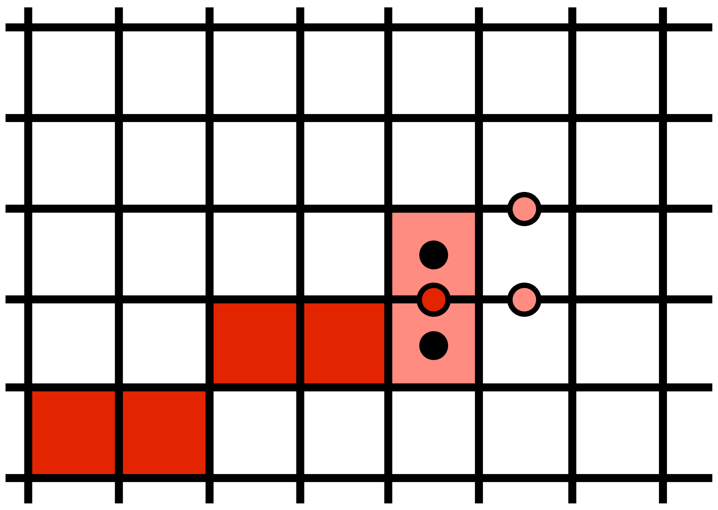
$$\text{if } \left(f(x+1, y+\tfrac{1}{2}) < 0 \right) \text{ then}$$



in each iteration we draw the **current** pixel and we evaluate the line equation at the **next** midpoint halfway above the **current** pixel

We can make the Midpoint Algorithm more efficient

by making it incremental!



$$f(x, y) = (y_0 - y_1)x + (x_1 - x_0)y + x_0y_1 - x_1y_0 = 0$$

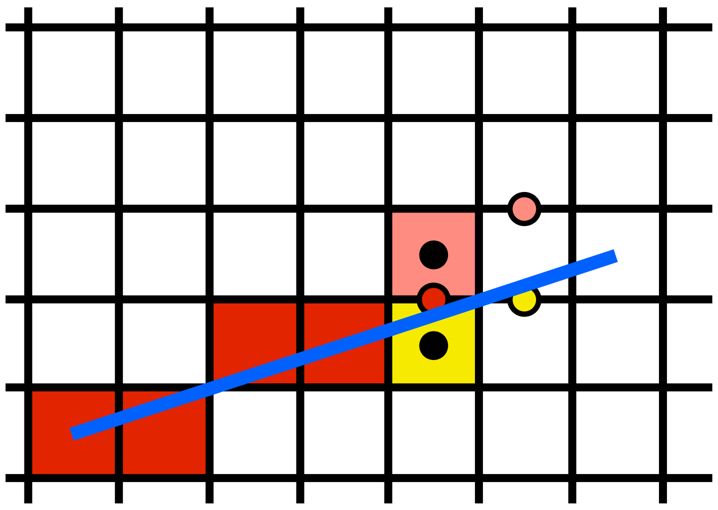
$$f(x+1, y) = f(x, y) + (y_0 - y_1)$$

$$f(x+1, y+1) = f(x, y) + (y_0 - y_1) + (x_1 - x_0)$$

Assume we have drawn the last red pixel and evaluated the line equation at the next (Red) midpoint
There are two possible outcomes:
1. we will choose the bottom pixel. In this case the next midpoint will be at the same level (x+1,y)
2. we will choose the top pixel. In this case the next midpoint will be one level up (x+1, y+1)
The line equation at these next midpoints can be evaluated incrementally using the update formulas shown.

We can make the Midpoint Algorithm more efficient

$$f(x+1, y+\frac{1}{2}) > 0$$



$$f(x,y) = (y_0 - y_1)x + (x_1 - x_0)y + x_0y_1 - x_1y_0 = 0$$

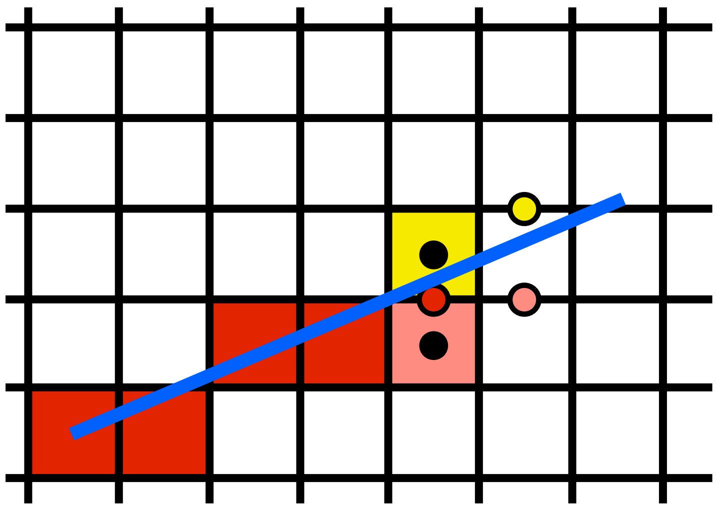
$$f(x+1,y) = f(x,y) + (y_0 - y_1)$$

$$f(x+1,y+1) = f(x,y) + (y_0 - y_1) + (x_1 - x_0)$$

As we move over one pixel to the right, we will choose either (x+1,y) (yellow) or (x+1,y+1) (pink) and the next midpoint we will evaluate will be eiterh

We can make the Midpoint Algorithm more efficient

$$f(x+1, y+\frac{1}{2}) < 0$$



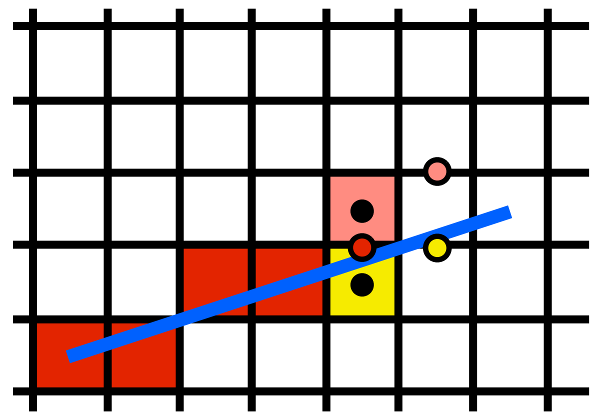
$$f(x,y) = (y_0 - y_1)x + (x_1 - x_0)y + x_0y_1 - x_1y_0 = 0$$

$$f(x+1, y) = f(x,y) + (y_0 - y_1)$$

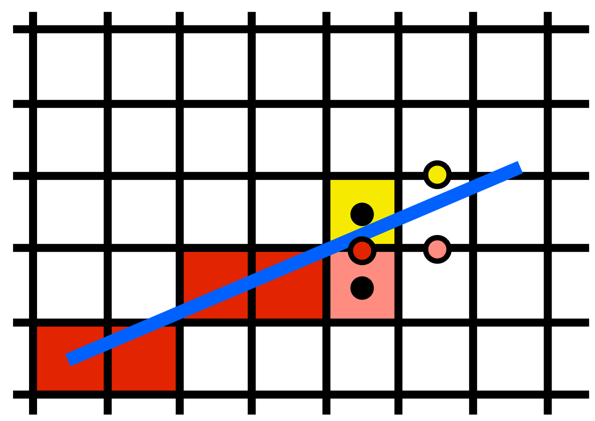
$$f(x+1, y+1) = f(x,y) + (y_0 - y_1) + (x_1 - x_0)$$

We can make the Midpoint Algorithm more efficient

```
y = y0
d = f(x0+1,y0+1/2)
for x = x0 to x1 do
    draw(x,y)
    if (d<0) then
        y = y+1
        d = d+(y0-y1)+(x1-x0)
    else
        d = d+(y0-y1)
```



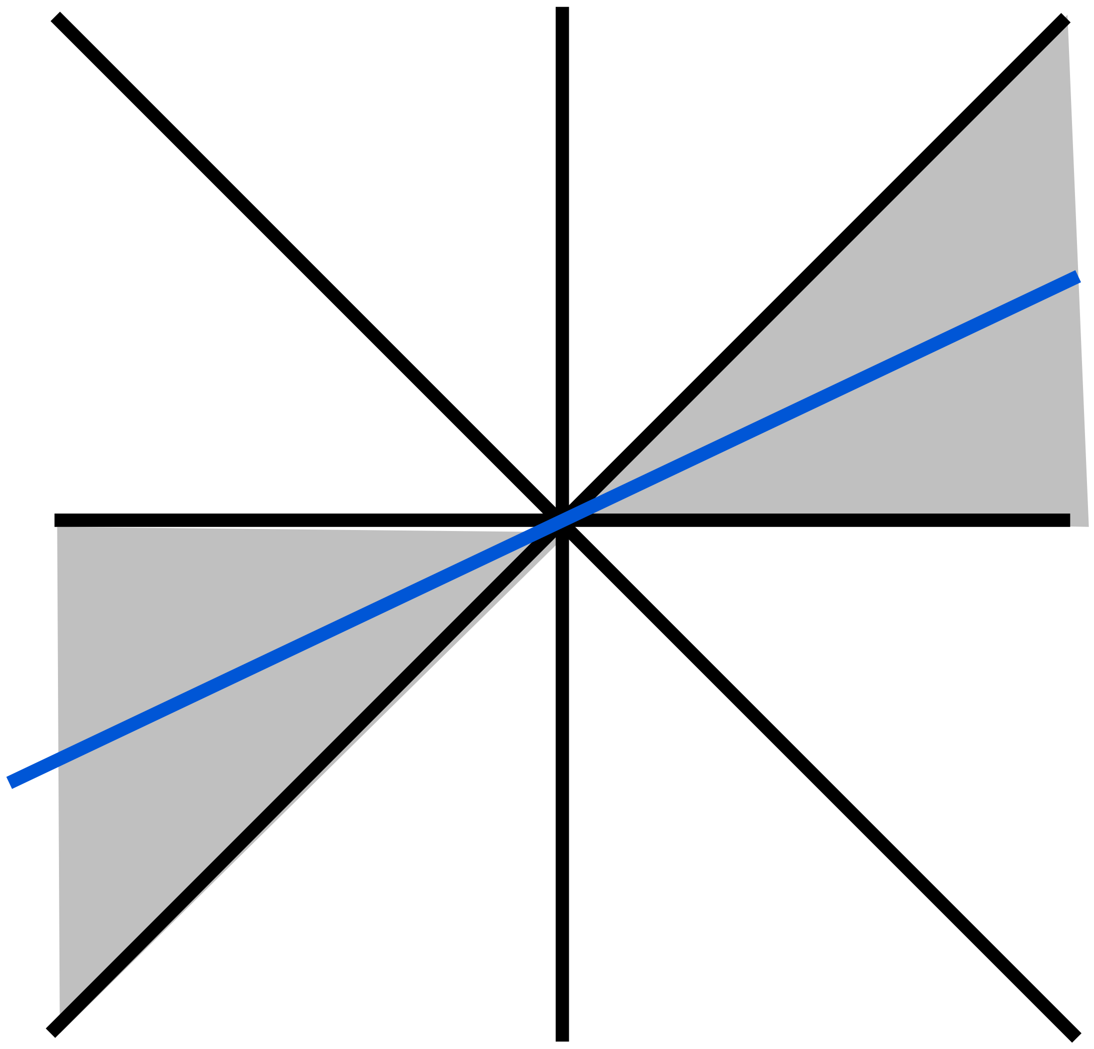
$$f(x+1, y) = f(x, y) + (y_0 - y_1)$$



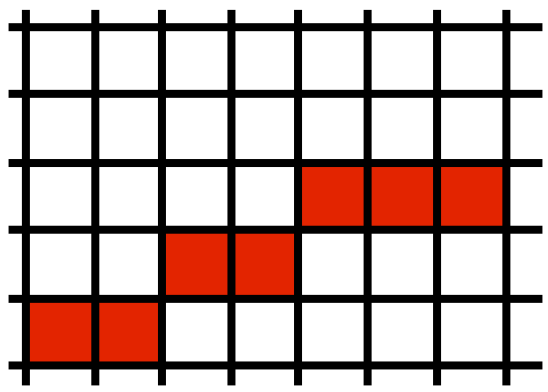
$$f(x+1, y+1) = f(x, y) + (y_0 - y_1) + (x_1 - x_0)$$

algorithm is **incremental** and uses only **integer arithmetic**

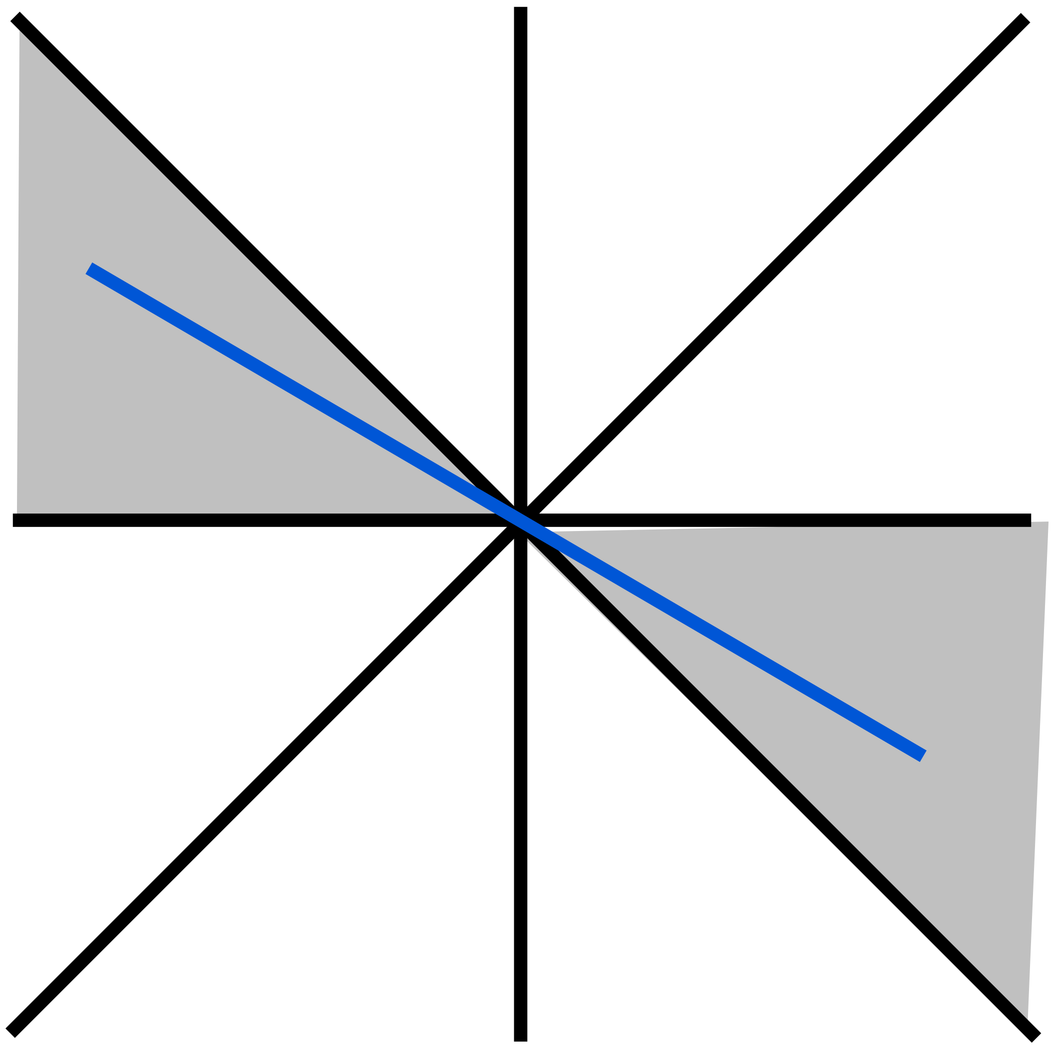
Adapt Midpoint Algorithm for other cases



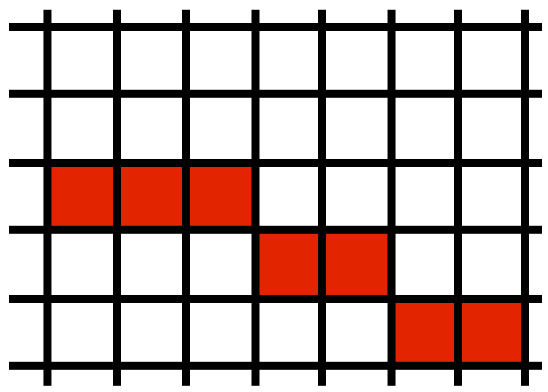
case: 0 < m <= 1



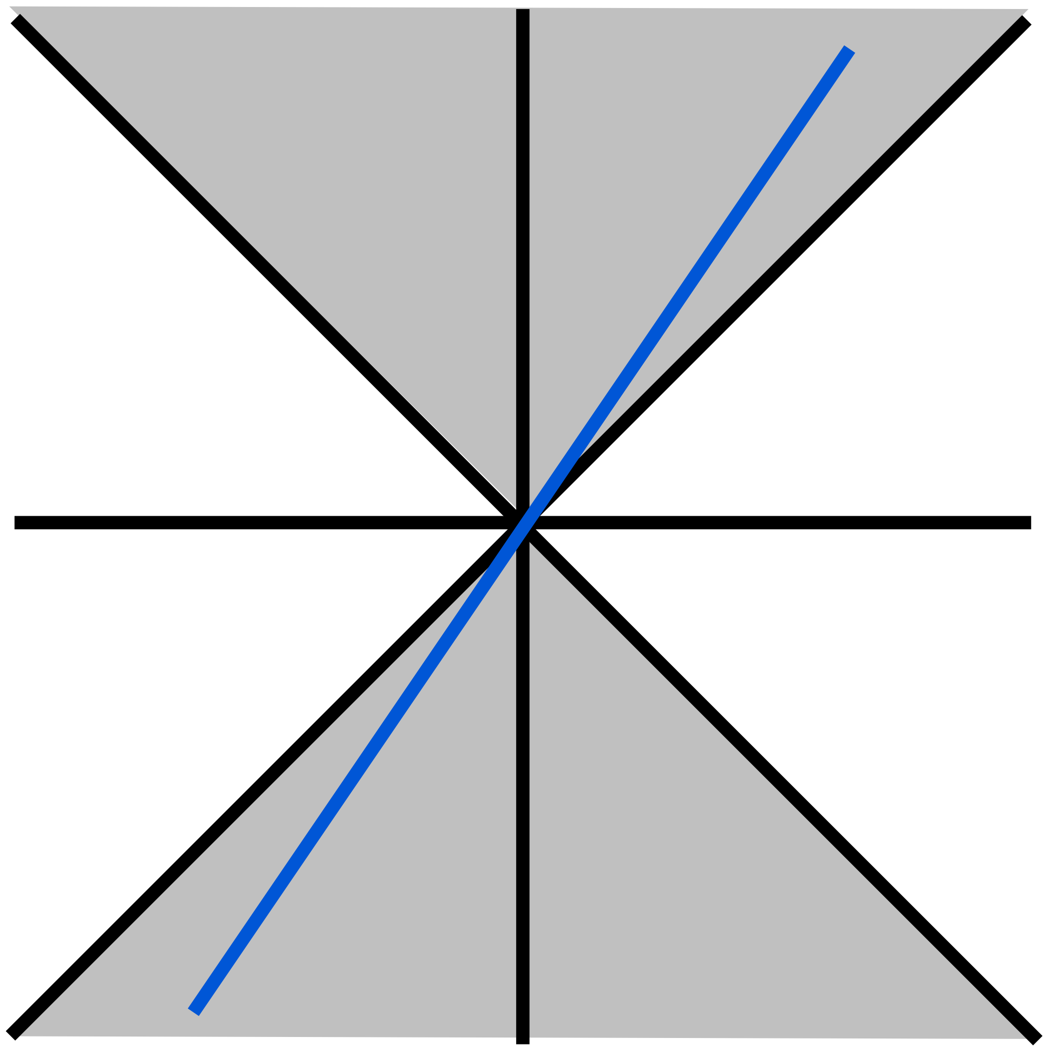
Adapt Midpoint Algorithm for other cases



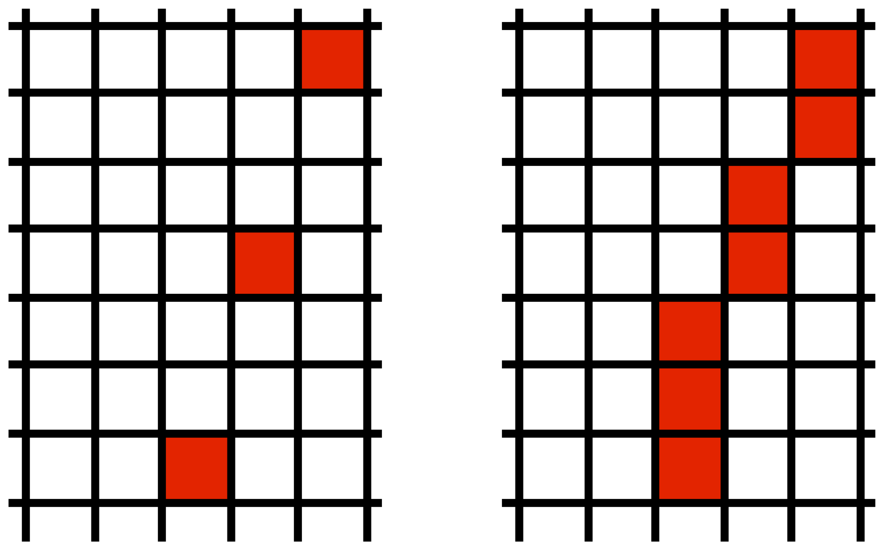
case: -1 <= m < 0



Adapt Midpoint Algorithm for other cases



case: 1 <= m
or m <= -1

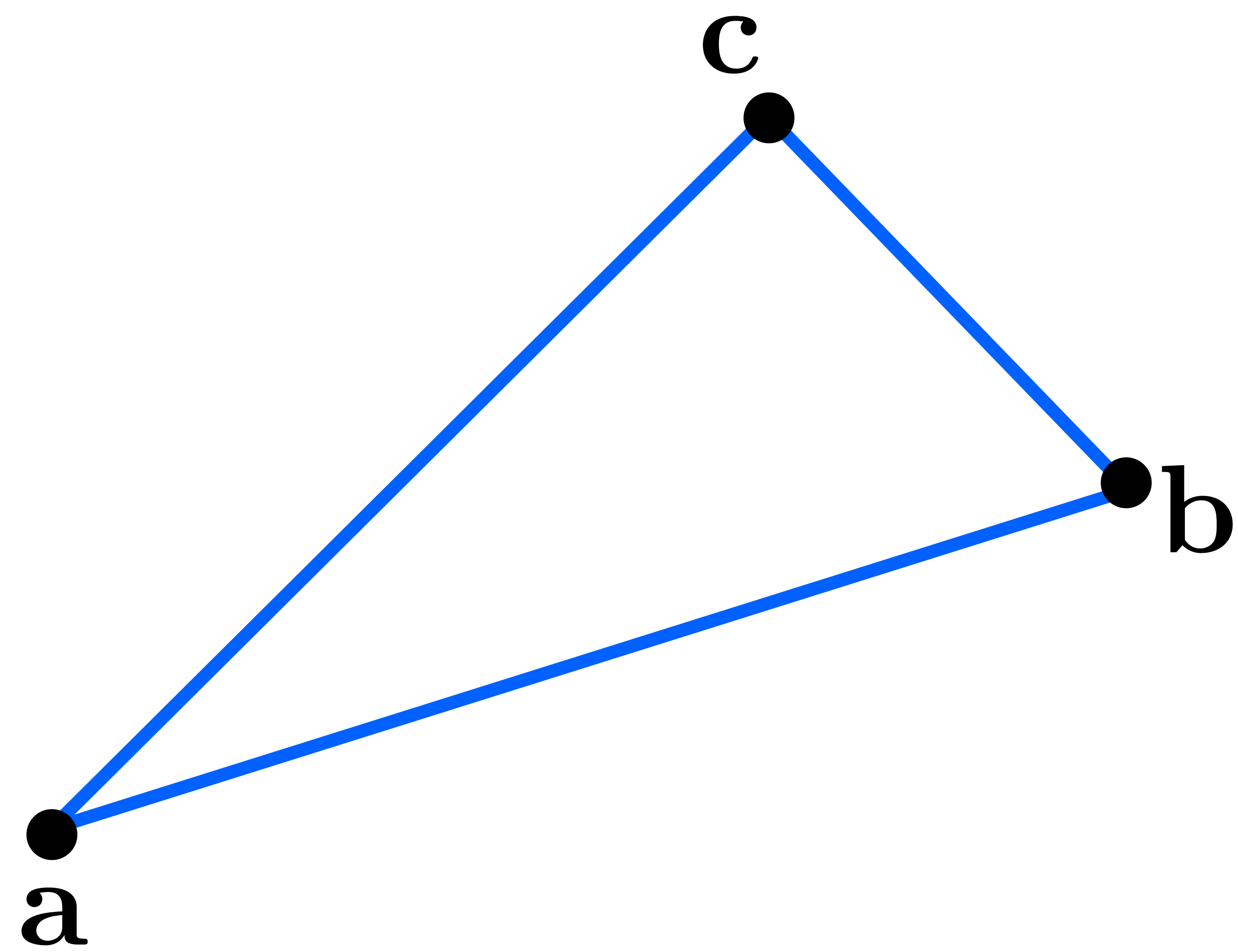


Line drawing references

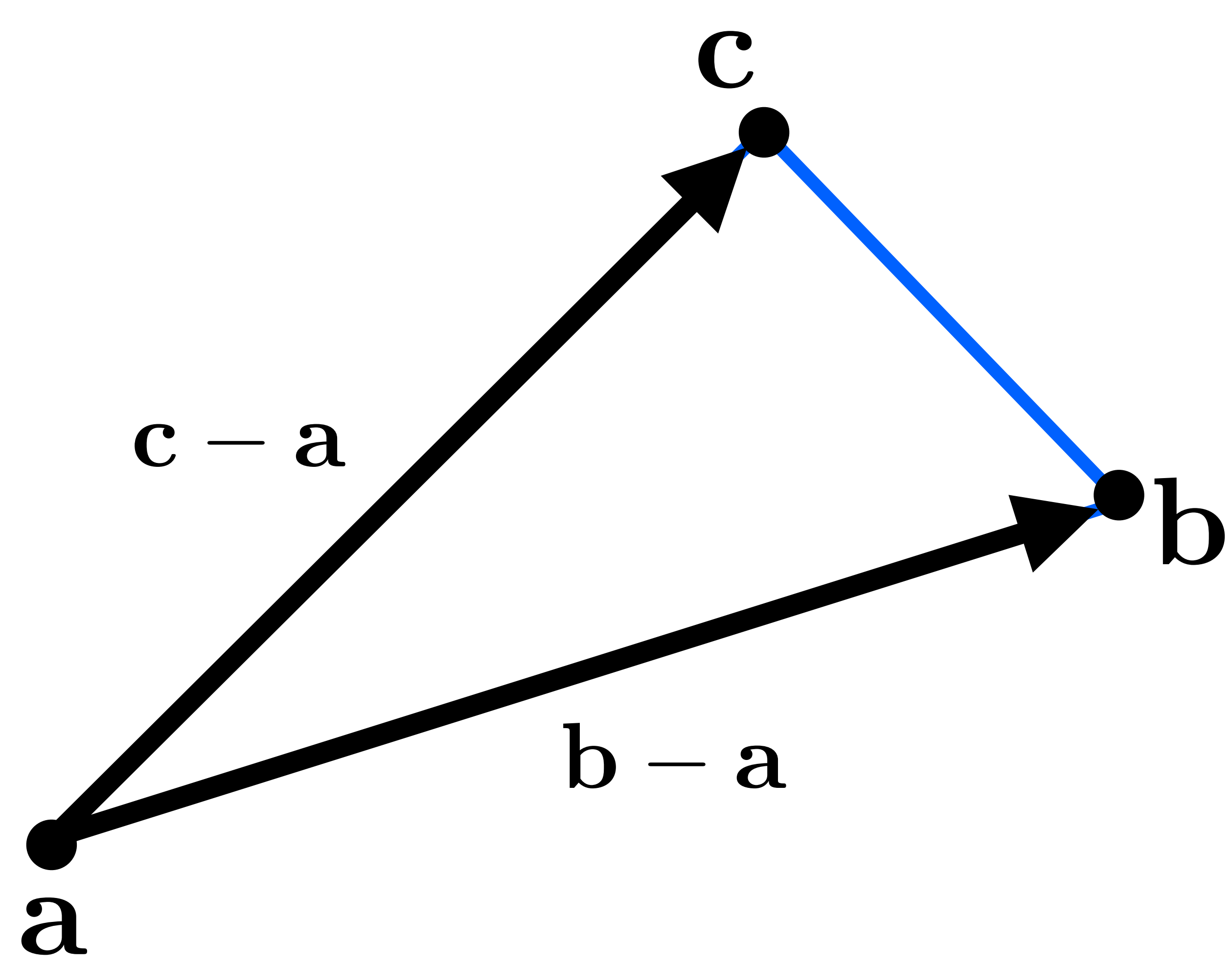
- the algorithm we just described is the *Midpoint Algorithm* (Pitteway, 1967), (van Aken and Novak, 1985)
- draws the same lines as the *Bresenham Line Algorithm* (Bresenham, 1965)

Triangles

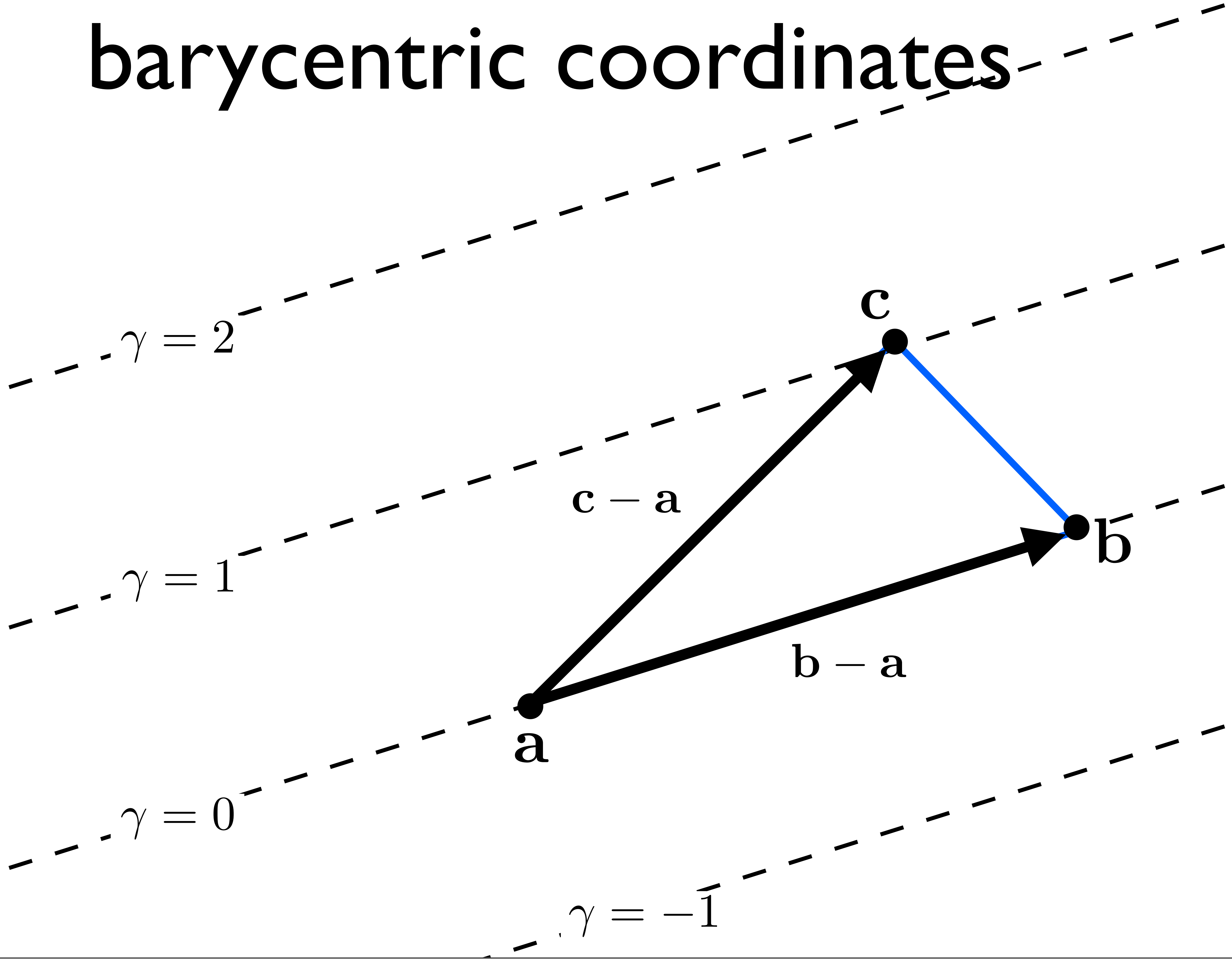
barycentric coordinates



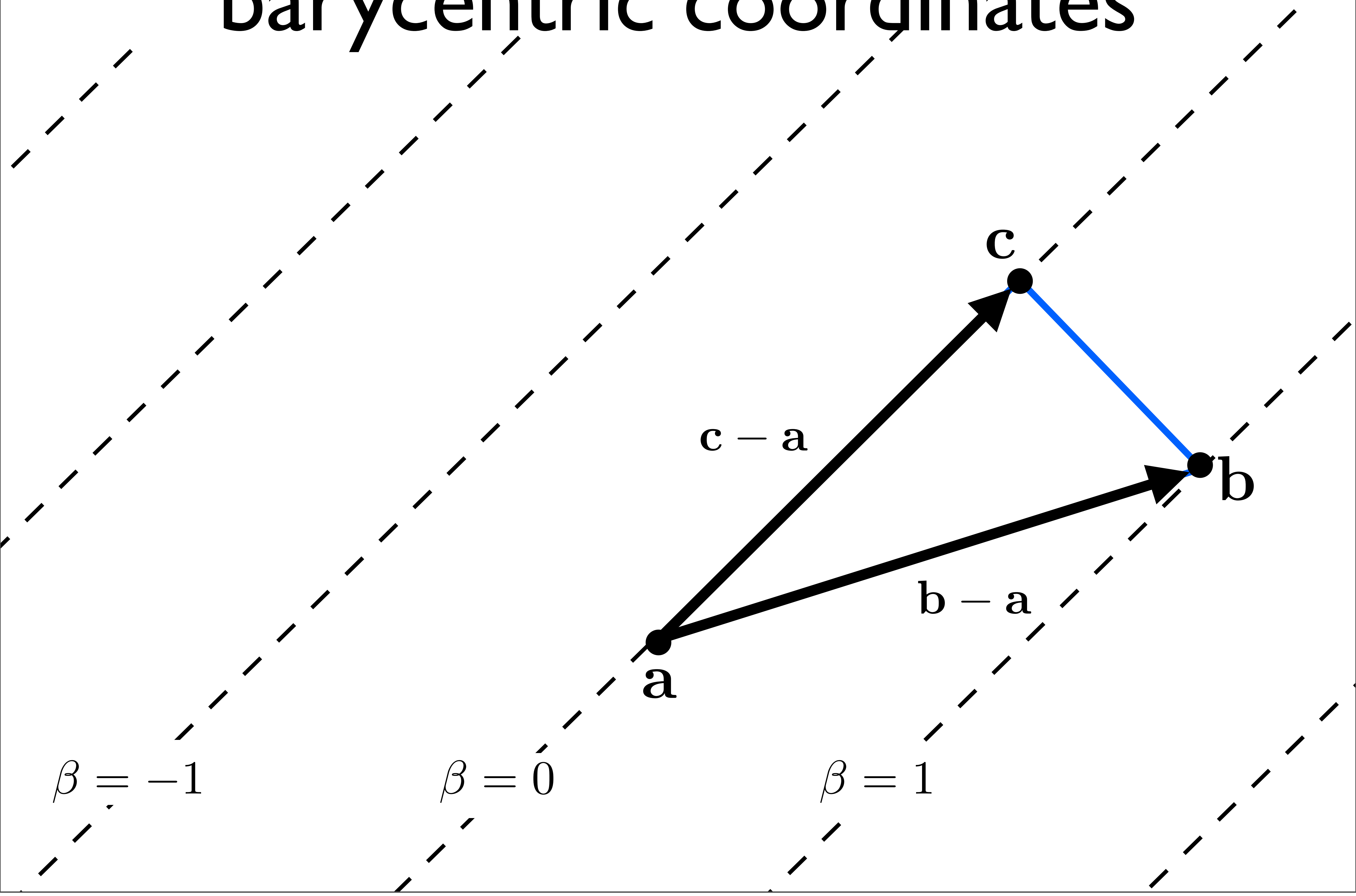
barycentric coordinates



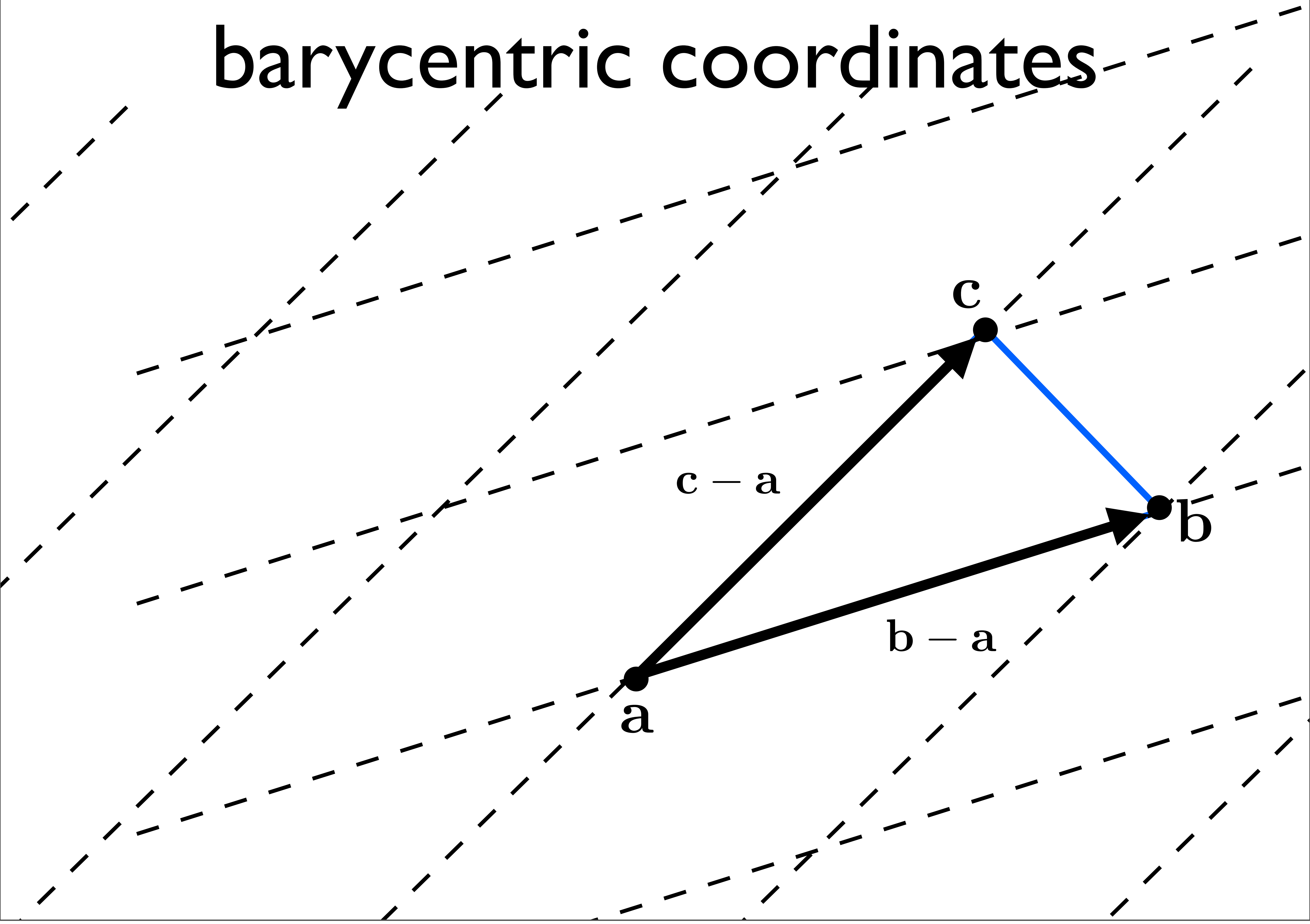
barycentric coordinates



barycentric coordinates



barycentric coordinates



barycentric coordinates

$$\mathbf{p} = \alpha\mathbf{a} + \beta\mathbf{b} + \gamma\mathbf{c}$$

What are (α, β, γ) ?

<whiteboard>