

CS230 : Computer Graphics

Lecture 5

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Computer Science & Engineering

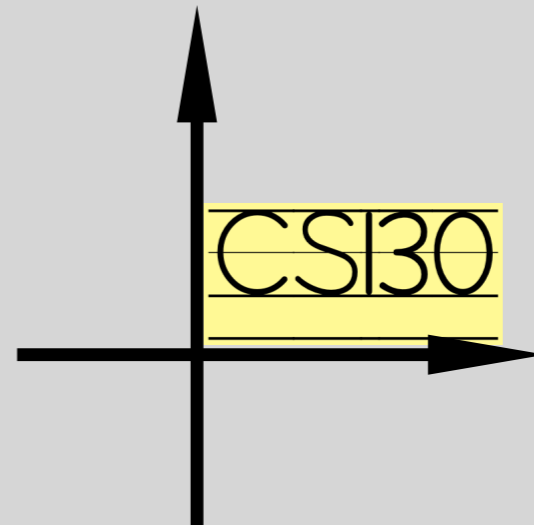
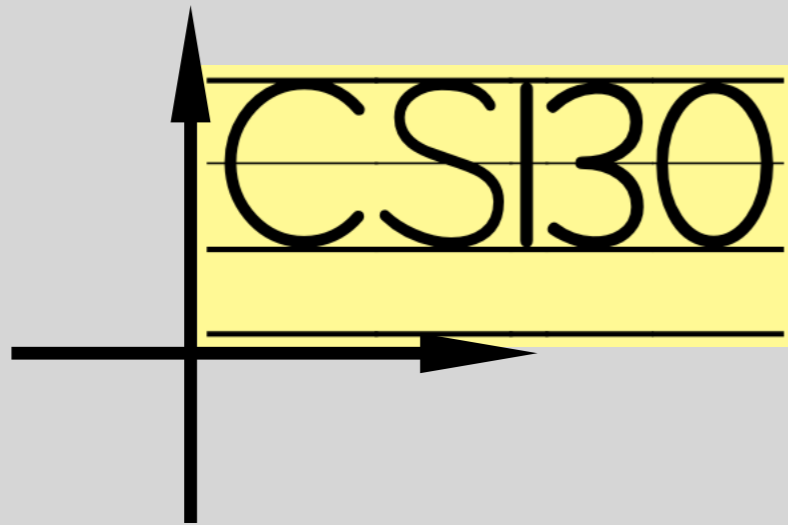
UC Riverside

2D Transformations

Uniform Scale

$$\begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} sx \\ sy \end{pmatrix}$$

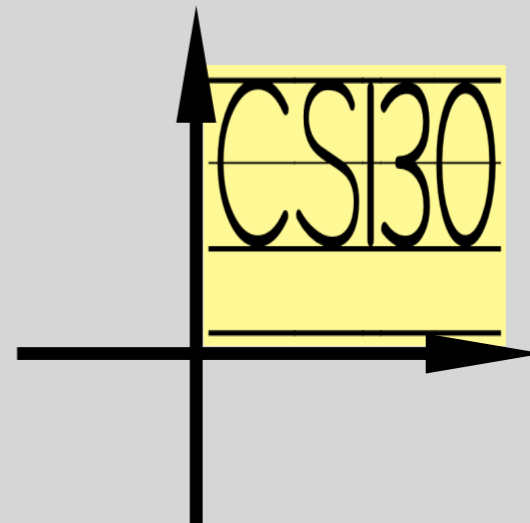
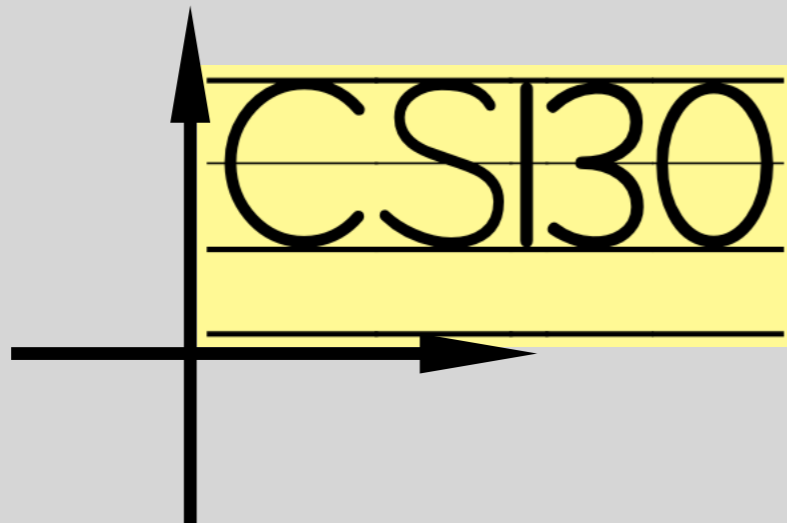
$$\begin{pmatrix} .5 & 0 \\ 0 & .5 \end{pmatrix}$$



Nonuniform Scale

$$\begin{pmatrix} s_x & 0 \\ 0 & s_y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} s_x x \\ s_y y \end{pmatrix}$$

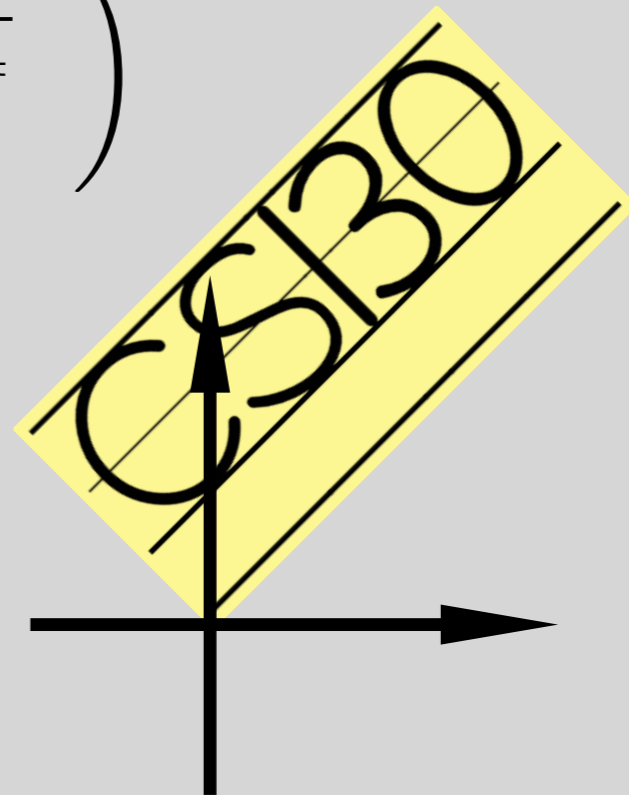
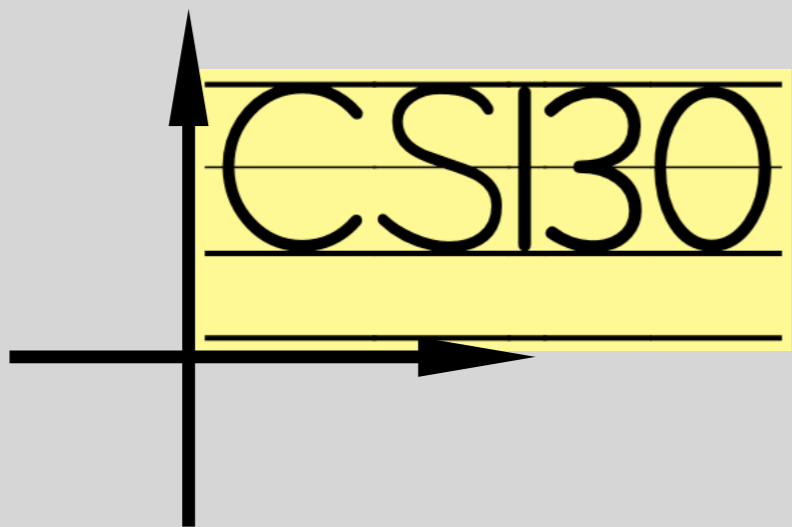
$$\begin{pmatrix} .5 & 0 \\ 0 & 1 \end{pmatrix}$$



Rotation

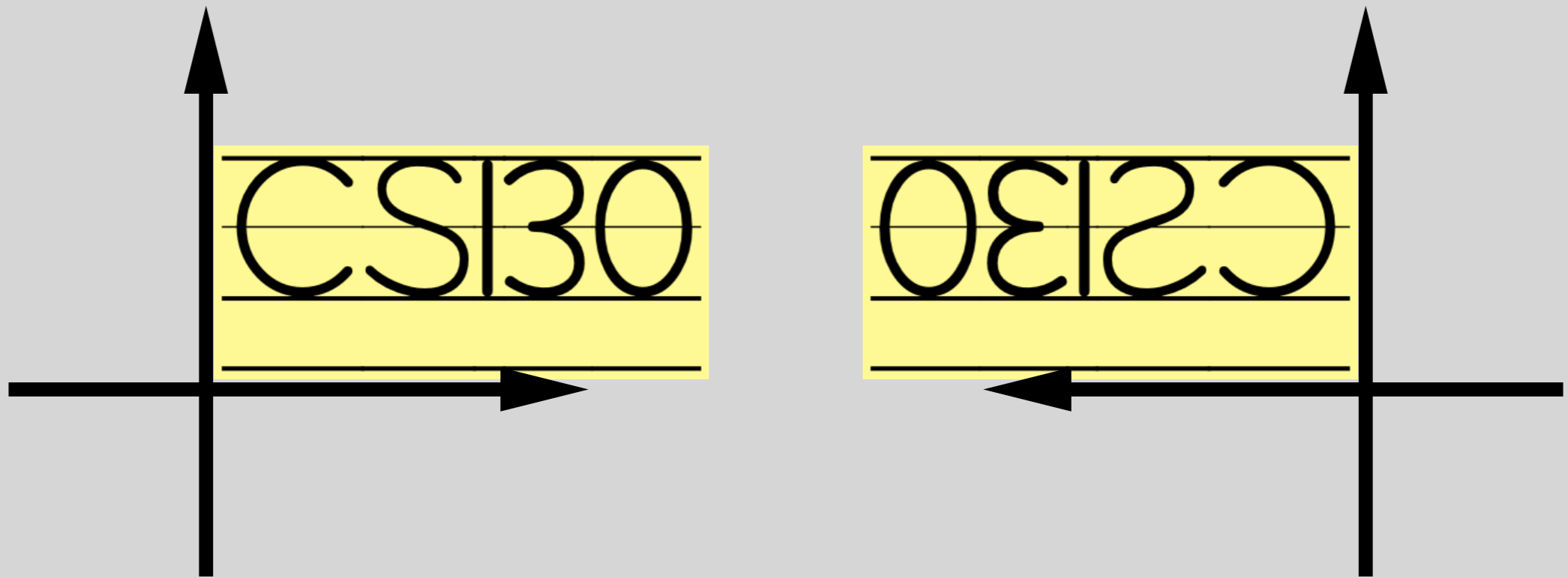
$$\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \cos \theta - y \sin \theta \\ x \sin \theta + y \cos \theta \end{pmatrix}$$

$$\begin{pmatrix} \cos \frac{\pi}{4} & -\sin \frac{\pi}{4} \\ \sin \frac{\pi}{4} & \cos \frac{\pi}{4} \end{pmatrix}$$



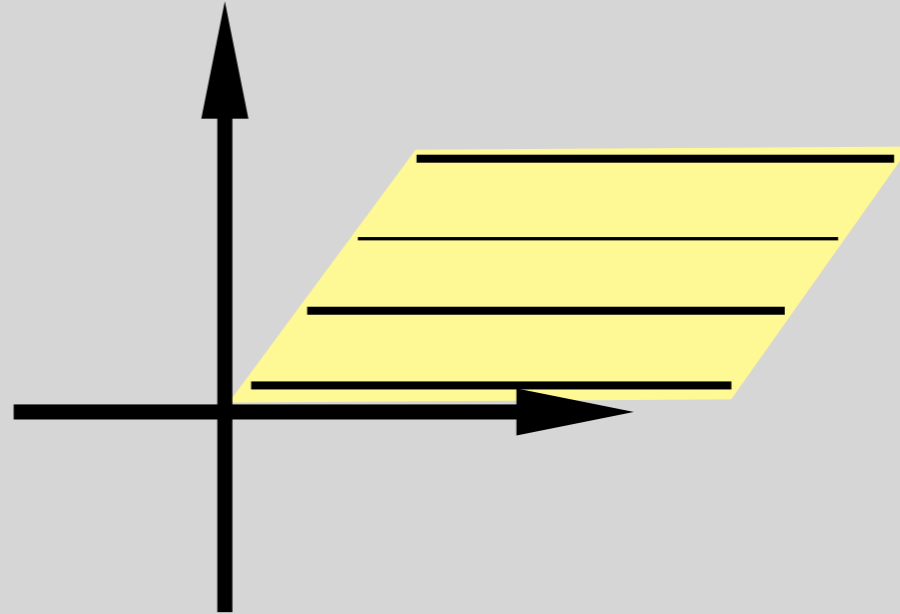
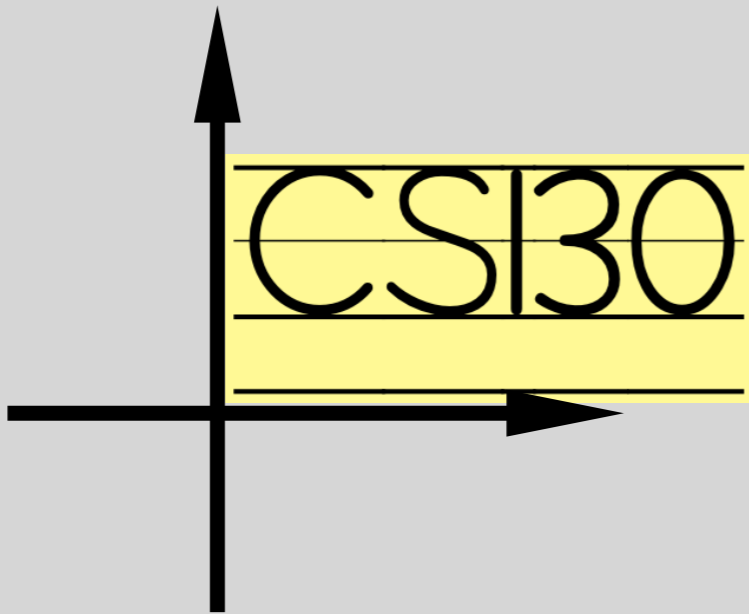
Reflection

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x \\ y \end{pmatrix}$$



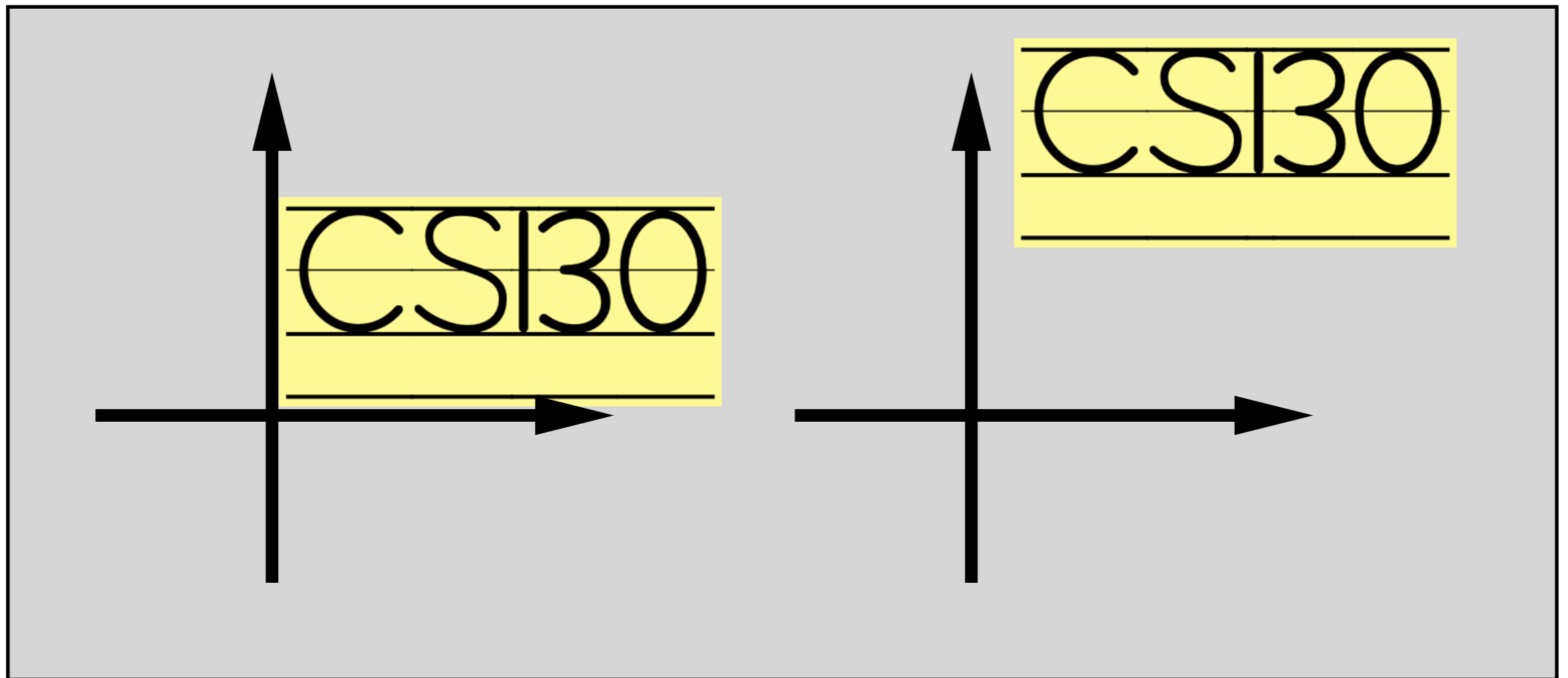
Shear

$$\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + ay \\ y \end{pmatrix}$$

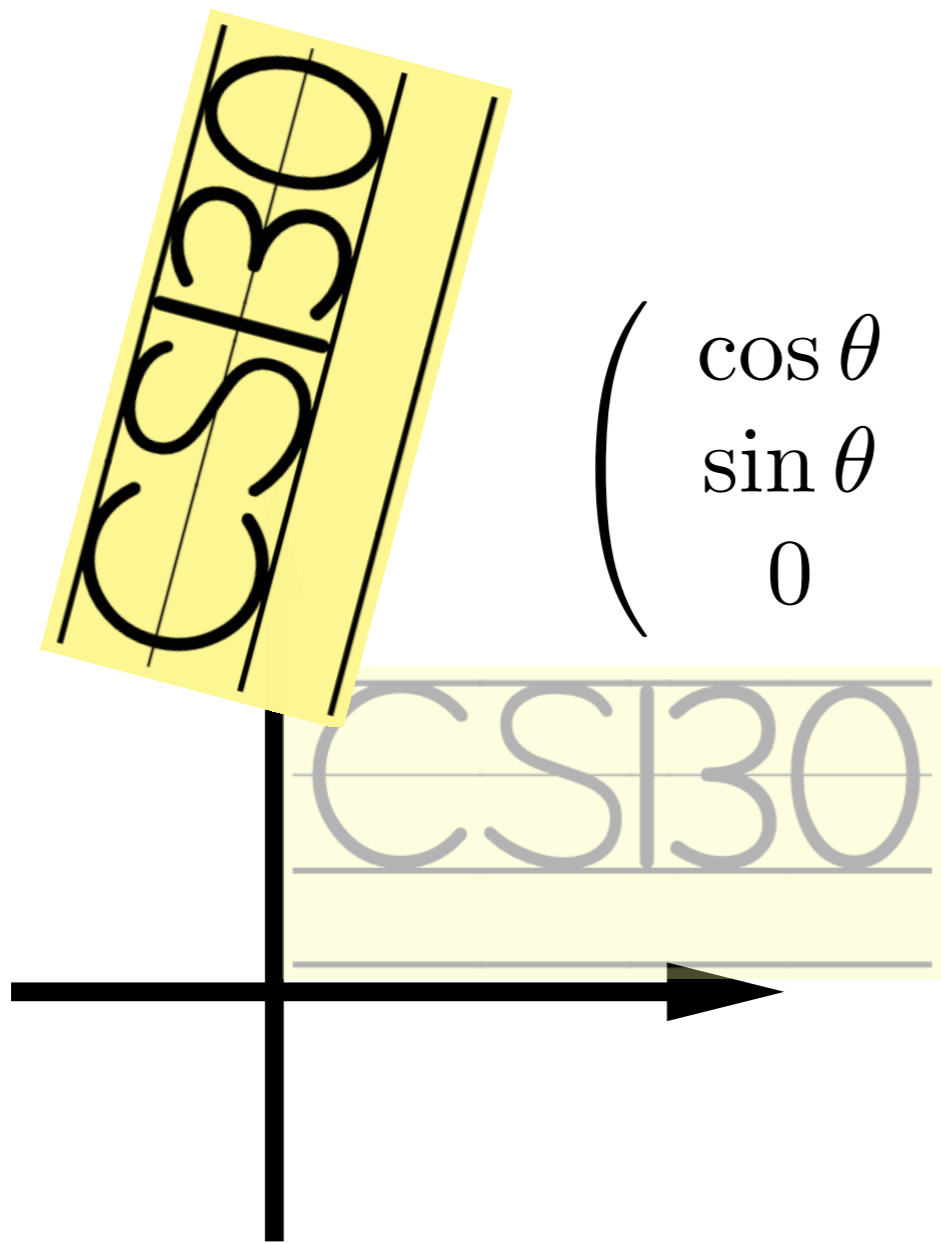


Translation

$$\begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x + t_x \\ y + t_y \\ 1 \end{pmatrix}$$

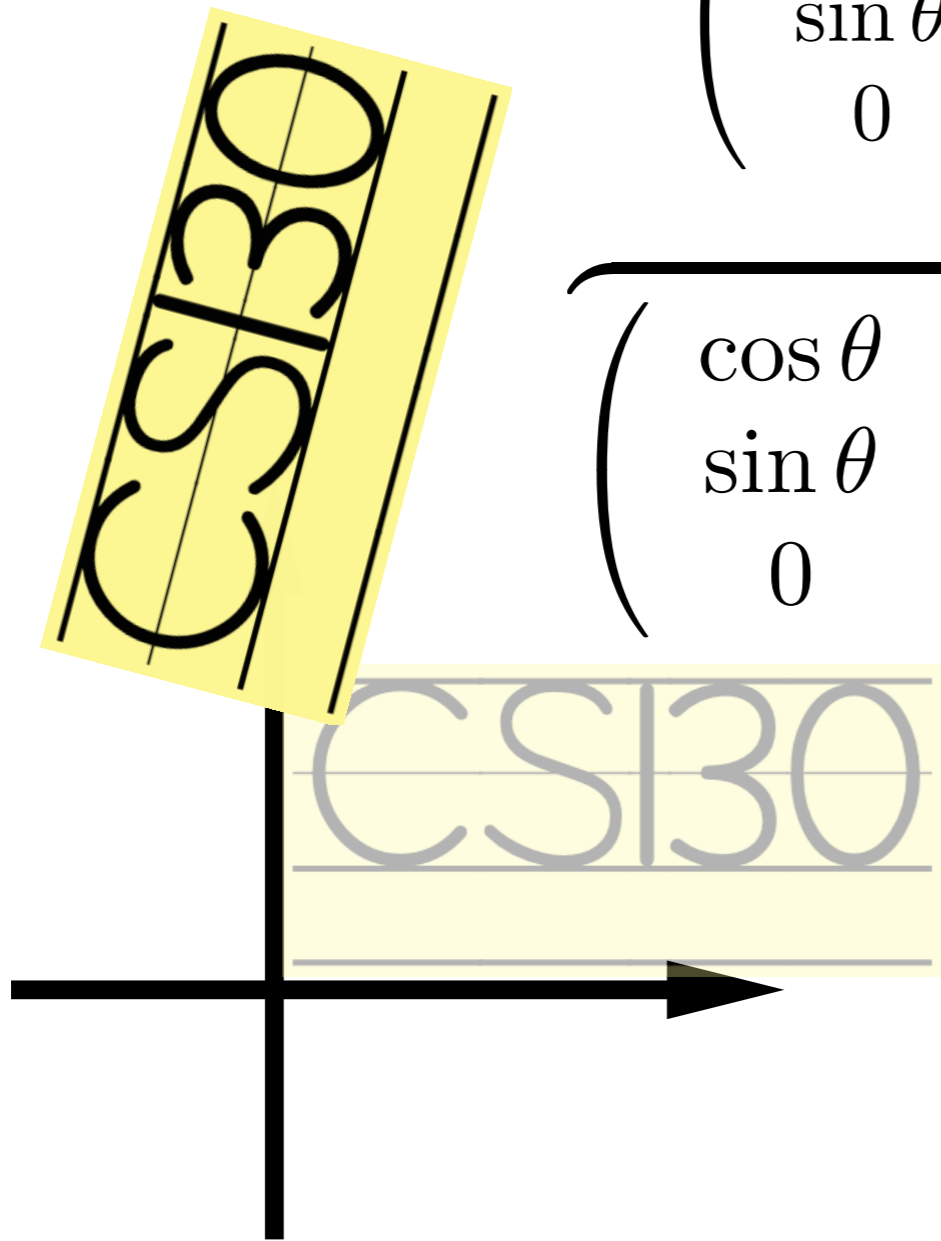


Composition



$$\begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} \cos \theta(x + t_x) - \sin \theta(y + t_y) \\ \sin \theta(x + t_x) + \cos \theta(y + t_y) \\ 1 \end{pmatrix}$$

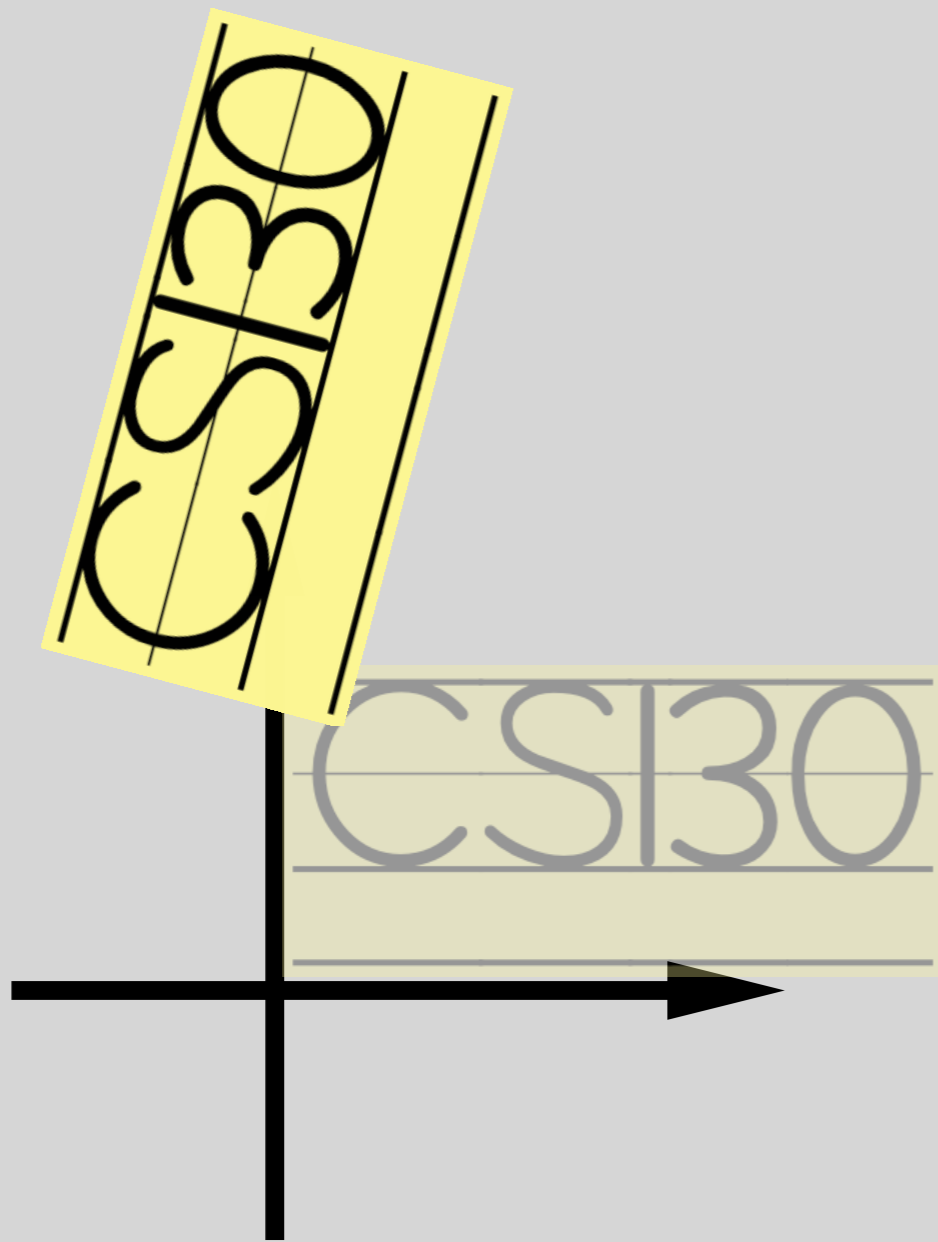
Composition



$$\begin{pmatrix} \cos \theta & -\sin \theta & t_x \cos \theta - t_y \sin \theta \\ \sin \theta & \cos \theta & t_x \sin \theta + t_y \cos \theta \\ 0 & 0 & 1 \end{pmatrix}$$
$$\begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} =$$
$$\begin{pmatrix} \cos \theta(x + t_x) - \sin \theta(y + t_y) \\ \sin \theta(x + t_x) + \cos \theta(y + t_y) \\ 1 \end{pmatrix}$$

Noncommutativity

translate, rotate



rotate, translate

