True/False

For each question, indicate whether the statement is true or false by circling T or F, respectively.

1. (T/F) Addition of two positive floating point numbers may cause underflow.
2. (T/F) Floating point cancellation errors can be a source of instability in an algorithm.
3. (T/F) A condition number of 1 is a good condition number.
4. (T/F) If $A\mathbf{x} = \mathbf{b}$ then $\mathbf{x}$ is necessarily in the range of A.
5. (T/F) Cholesky factorization without pivoting is a stable algorithm.

Multiple Choice

Instructions: For the multiple choice problems, circle exactly one of (a) - (e).

6. Which of the following statements are necessarily true?

   I. A small residual implies a small error. F

   II. It is possible for rounding errors to catastrophically destroy the accuracy of an algorit

   III. A large absolute error implies a large relative error. F

   (a) I only
   (b) II only
   (c) III only
   (d) I and II only
   (e) II and III only

7. Which of the following statements are true?

   I. A triangular matrix with a zero on its diagonal is singular. T

   II. A matrix with a zero on its diagonal is singular. F

   III. If $A\mathbf{z} = \mathbf{0}$ for a non-zero vector $\mathbf{z}$, then $\det(A) = 0$. T

   (a) I only
   (b) II only
   (c) I and III only
   (d) II and III only
   (e) I, II and III

8. Consider an orthogonal matrix $Q \in \mathbb{R}^{n \times n}$. Which statement is false?

   (a) Q must be nonsingular. T
   (b) $||Q\mathbf{x}||_2 = ||\mathbf{x}||_2$ for any vector $\mathbf{x} \in \mathbb{R}^n$. T
   (c) If $\{\mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_n\}$ are the columns of Q, then for all $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{x} = \mathbf{q}_1\mathbf{q}_1^T\mathbf{x} + \ldots + \mathbf{q}_n\mathbf{q}_n^T\mathbf{x}$.
   (d) $Q = Q^T$. F
   (e) The condition number of Q with respect to the 2-norm is 1. T

9. Which of the following statements about the Cholesky factorization $A = LL^T$ are true?

T I. When applicable, a Cholesky factorization can be computed in fewer operations than an LU factorization.

F II. Cholesky factorization works on any symmetric matrix.

T III. $a_{11} = l_{11}^2$, where a_{11} is the element in the first row and first column of A, and l_{11} is defined analogously.

(a) I only

(b) II only

(c) I and II only

(d) I and III only

(e) I, II and III

Written Response

10. Consider a normalized floating point number system with p digits of precision, base β and integer exponent E, $L \leq E \leq U$. Let x be a given nonzero floating-point number in this system and let y be an adjacent floating-point number, also nonzero.

    (a) What is the minimum possible distance ($|x - y|$) between x and y?

    (b) What is the maximum possible distance between x and y?

    (c) If we allow denormalization, what will be the distance between the denormalized numbers?

11. Let $\mathbf{x}^T = (1, 2, 3)$, $\mathbf{y}^T = (0, 1, -2)$.

    (a) Write down the values of $\mathbf{x}^T\mathbf{y}$ and $\mathbf{x}\mathbf{y}^T$.

    (b) What is $\text{rank}(\mathbf{x}\mathbf{y}^T)$?

    (c) Give a basis for $\text{range}(\mathbf{x}\mathbf{y}^T)$?

~~10. (a)~~

(10)

0 -1 -2 -3 -4

— . — — — —

|← p →|

digits

5 digits

$\beta^{-(p-1)}$

least significant

least significant digit has value of

(a) $\beta^{-(p-1)} \times \beta^{L} = \beta^{L-p+1}$

(b) $\beta^{-(p-1)} \times \beta^{U} = \beta^{U-p+1}$

(c) same as (a)

---

(11) $x = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ $y = \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$

(a) $x^T y = 1 \cdot 0 + 2 \cdot 1 + 3 \cdot (-2) = 0 + 2 - 6 = -4$

$xy^T = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (0 \ \ 1 \ \ -2) = \begin{pmatrix} 0 & 1 & -2 \\ 0 & 2 & -4 \\ 0 & 3 & -6 \end{pmatrix}$

(b) $\text{rank}(xy^T) = 1$

because span$\{x\}$ = range(xy^T)

$\dim(\text{range}(xy^T)) = 1$

(c) $\{\vec{x}\}$