

## True/False

For each question, indicate whether the statement is true or false by circling T or F, respectively.

1. (T) **F** Addition of two positive floating point numbers may cause underflow.
2. (T) **F** Floating point cancellation errors can be a source of instability in an algorithm.
3. (T) **F** A condition number of 1 is a good condition number.
4. (T) **F** If  $Ax = b$  then  $x$  is necessarily in the range of  $A$ .
5. (T) **F** Cholesky factorization without pivoting is a stable algorithm.

## Multiple Choice

Instructions: For the multiple choice problems, circle exactly one of (a) - (e).

6. Which of the following statements are necessarily true?

- I. A small residual implies a small error. **F**
- II. It is possible for rounding errors to catastrophically destroy the accuracy of an algorithm.
- III. A large absolute error implies a large relative error. **F**

- (a) I only  
(b) II only  
(c) III only  
(d) I and II only  
(e) II and III only

7. Which of the following statements are true?

- I. A triangular matrix with a zero on its diagonal is singular. **T**
- II. A matrix with a zero on its diagonal is singular. **F**
- III. If  $Az = 0$  for a non-zero vector  $z$ , then  $\det(A) = 0$ . **T**

- (a) I only  
(b) II only  
(c) I and III only  
(d) II and III only  
(e) I, II and III

8. Consider an orthogonal matrix  $Q \in \mathbb{R}^{n \times n}$ . Which statement is false?

- (a)  $Q$  must be nonsingular. **T**  
(b)  $\|Qx\|_2 = \|x\|_2$  for any vector  $x \in \mathbb{R}^n$ . **T**  
(c) If  $\{q_1, q_2, \dots, q_n\}$  are the columns of  $Q$ , then for all  $x \in \mathbb{R}^n$ ,  $x = q_1 q_1^T x + \dots + q_n q_n^T x$ .  
(d)  $Q = Q^T$ . **F**  
(e) The condition number of  $Q$  with respect to the 2-norm is 1. **T**

9. Which of the following statements about the Cholesky factorization  $A = LL^T$  are true?

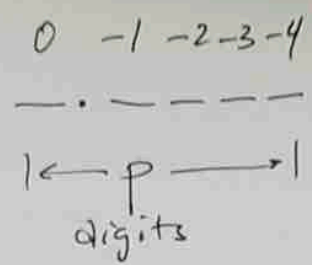
- T I. When applicable, a Cholesky factorization can be computed in fewer operations than an LU factorization.
- F II. Cholesky factorization works on any symmetric matrix.
- T III.  $a_{11} = l_{11}^2$ , where  $a_{11}$  is the element in the first row and first column of  $A$ , and  $l_{11}$  is defined analogously.

- (a) I only  
(b) II only  
(c) I and II only  
(d) I and III only  
(e) I, II and III

### Written Response

10. Consider a normalized floating point number system with  $p$  digits of precision, base  $\beta$  and integer exponent  $E$ ,  $L \leq E \leq U$ . Let  $x$  be a given nonzero floating-point number in this system and let  $y$  be an adjacent floating-point number, also nonzero.
- (a) What is the minimum possible distance ( $|x - y|$ ) between  $x$  and  $y$ ?
- (b) What is the maximum possible distance between  $x$  and  $y$ ?
- (c) If we allow denormalization, what will be the distance between the denormalized numbers?
11. Let  $\mathbf{x}^T = (1, 2, 3)$ ,  $\mathbf{y}^T = (0, 1, -2)$ .
- (a) Write down the values of  $\mathbf{x}^T \mathbf{y}$  and  $\mathbf{xy}^T$ .
- (b) What is  $\text{rank}(\mathbf{xy}^T)$ ?
- (c) Give a basis for  $\text{range}(\mathbf{xy}^T)$ ?

~~10. (a)~~



5 digits  
 $-(p-1)$   
 $\beta$   
 least significant

(10)

least significant digit has value of

(a)  $\beta^{-(p-1)} \times \beta^L = \beta^{L-p+1}$

(b)  $\beta^{-(p-1)} \times \beta^U = \beta^{U-p+1}$

(c) same as (a)

(11)

$x = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad y = \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$

(a)  $x^T y = 1 \cdot 0 + 2 \cdot 1 + 3 \cdot (-2) = 0 + 2 - 6 = -4$

$xy^T = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \begin{pmatrix} 0 & 1 & -2 \end{pmatrix} = \begin{pmatrix} 0 & 1 & -2 \\ 0 & 2 & -4 \\ 0 & 3 & -6 \end{pmatrix}$

(b)  $\text{rank}(xy^T) = 1$

because ~~span~~  $\text{span}(xy) = \text{range}(xy^T)$

$\dim(\text{range}(xy^T)) = 1$

(c)

