(Lectures 12-19)

Lecture 12

- overdetermined system
- least squares
- normal equations
  - uniqueness of solution to normal equations
  - geometric interpretation
- Least squares solution by pseudo-inverse
  - minimum norm solution

Lecture 13

- least squares solution by QR (for A with linearly independent columns)
  - case 1: A has linearly indepedent columns
  - case 2: A has linearly dependent columns
- least squares and Tikhonov regularization
  - formulation
  - makes the problem full rank
  - makes the problem better conditioned
- weighted least squares formulation
- condition number of f
- condition number of a matrix
  - definition
  - properties
  - well-conditioned vs. ill-conditioned
  - condition number in 2-norm
  - condition number in 2-norm of symmetric matrix
  - and geometric interpretation
- normal equations square the condition number

Lectures 14-15

- conditioning and stability
- backward error
- condition number
- stability and accuracy
- Floating Point
  - finite precision
  - general floating point system: base, precision, exponent range
  - example system
  - normalization

Lecture 16

- Floating Point Numbers
  - underflow level
  - overflow level
  - picture of representable floating point numbers
  - subnormals
  - exceptional values: Inf and Nan
- Floating Point Math
  - Rounding: chop, nearest, even
  - Machine epsilon
  - addition/subtraction
  - multiplication/division
  - Rounding Error analysis
  - Floating Point Issues
    - Finite Precision
    - Overflow
    - Cancellation error
  - Examples of floating point issues

Lecture 17

- iterative methods
- matrix splitting
  - Jacobi
  - Gauss-Seidel
- convergence rate
- eigenvalue problems
  - power method
  - normalized power iteration
  - powert method and shifting
  - inverse iteration
  - Rayleigh quotient iteration
  - QR algorithm for eigenvalues and eigenvectors

Lecture 18

- QR algorithm
  - basic algorithm
  - generates similar matrix at each iteration
  - converges to Schur form (revealing eigenvalues)
- optimizations
  - first reduce matrix to upper Hessenberg/Tridiagonal via Householder, cuts cost of QR decomposition
  - shifted QR algorithm to accelerate convergence
- Krylov subspaces
  - good for large, sparse A
- Arnoldi iteration
  - generates projections of A onto Krylov subspaces
  - generates upper Hessenberg matrix
  - then do QR on upper Hessenberg matrix to find approximate eigenvalues of A
- Arnoldi reduces to Lanczos for symmetric matrices
- upper Hessenberg reduces to tridiagonal for symmetric matrices

- residual
  - relation to error
  - and stopping criteria for iterative methods
- solvers based on Krylov subspaces
- $Ax = b$ by GMRES
  - minimizes 2-norm of residual over each Krylov subspace

Lecture 19

- line search method
- step size for exact line search on quadratic function
- steepest descent method
- spd A, $Ax = b$ by Conjugate Gradients (CG)
  - recast as minimization of quadratic function
  - minimizes A-norm of error over each Krylov subspace
  - A-orthogonal search directions
  - compared with steepest descent directions
  - termination in n steps (theoretical)
  - Gram-Schmidt A-orthogonalization