## Lecture 10

- project one vector onto a direction given by a unit vector
- project one vector onto a direction given by an arbitrary vector
- Gram-Schmidt orthogonalization
- Modified Gram-Schmidt orthogonalization
- QR factorization
  - by Gram-Schmidt
  - by Householder
- Frobenius norm

## Lecture 11

- least squares
- overdetermined system
- normal equations
  - uniqueness of solution to normal equations
  - geometric interpretation
- Least squares solution by pseudo-inverse
  - minimum norm solution

## Lecture 12

- least squares solution by QR (for A with linearly independent columns)
- least squares and Tikhonov regularization
  - formulation
  - makes the problem full rank
  - makes the problem better conditioned
- weighted least squares formulation
- condition number of $f$
- condition number of a matrix
  - definition
  - properties
  - well-conditioned vs. ill-conditioned
  - condition number in 2-norm
  - condition number in 2-norm of symmetric matrix

## Lecture 13

- iterative methods
- matrix splitting
  - Jacobi
  - Gauss-Seidel
- convergence rate
- eigenvalue problems
  - power method
  - normalize power iteration
  - powert method and shifting
  - inverse iteration
  - Rayleigh quotient iteration
  - QR algorithm for eigenvalues and eigenvectors

## Lecture 14

- QR algorithm
  - write it down
  - similar matrix at each iteration
  - converging to Schur form (revealing eigenvalues)
- shifted QR algorithm
- reduction to upper Hessenberg via Householder
  - make QR more efficient
- Krylov subspaces
  - good for large, sparse A
- Arnoldi iteration
  - generates projections of A onto Krylov subspaces
  - generates upper Hessenberg matrix
  - then do QR on upper Hessenberg matrix to find approximate eigenvalues of A
- Arnoldi reduces to Lanczos for symmetric matrices
- upper Hessenberg reduces to tridiagonal for symmetric matrices

## Lecture 15

- residual
  - relation to error
  - and stopping criteria for iterative methods
- solvers based on Krylov subspaces
- Ax = b by GMRES
  - minimizes 2-norm of residual over each Krylov subspace
- spd A, Ax = b by Conjugate Gradients (CG)
  - recast as minimization of quadratic function
  - minimizes A-norm of error over each Krylov subspace
- line search method
- steepest descent method

## Lecture 16

- step size for exact line search on quadratic function
- CG and A-orthogonal search directions
  - compared with steepest descent directions
  - termination in n steps (theoretical)
- Gram-Schmidt A-orthogonalization
- preconditioning

## Lecture 17

- nonlinear equations
- root-finding
- fixed point iteration
- convergence of fixed point iteration
- Newton's method for roots
- unconstrained optimization
  - optimality conditions
- Newton's method for optimization
  - scalar and multidimensional