Homework 5
CS 210

Multiple Choice and T/F

1. Consider minimizing a real-valued function of one variable. Circle each true statement.
   (a) If a function is unimodel on a closed interval, then it has exactly one minimum in that interval.
   (b) The discarded point in Golden Section search is always the point with the highest function value.
   (c) Golden Section search has a linear convergence rate.
   (d) When convergent, Newton's Method for minimization has a linear convergence rate unlike Newton's Method for root-finding which has a quadratic convergence rate.

2. Consider minimizing a real-valued function of n variables. Which one statement is true?
   (a) The Steepest Descent Method converges more quickly than Newton's Method.
   (b) The Conjugate Gradient Method converges in exactly n iterations.
   (c) Newton's Method fits a quadratic to the function and then minimizes that quadratic in each step.
   (d) None of the above.

3. Consider a quadratic function of n variables for which a minimum exists. Which one statement is true?
   (a) In exact arithmetic, the Conjugate Gradient Method converges in at most n iterations.
   (b) In exact arithmetic, Newton's Method requires at least n steps for convergence.
   (c) The Hessian of the function has both positive and negative eigenvalues.
   (d) None of the above.

4. Given n distinct points $(x_i, y_i),\ 1 \le i \le n$,
   (a) there is a unique polynomial of degree n that interpolates the points.
   (b) using Lagrange basis functions leads to a dense system for the coefficients of the polynomial.
   (c) using Monomial basis functions leads to a dense system for the coefficients of the polynomial.
   (d) None of the above.

5. Consider numerical integrating a real-valued function of one variable. Which one statement is true?
   (a) Using k quadrature points, we can get at most a k^{th} order integration scheme.
   (b) Newton-Cotes quadrature uses specially places points that maximize the accuracy for a given number of quadrature points.
   (c) Both the Midpoint Rule and Trapezoidal Rule are second order accurate.
   (d) None of the above.

Written Problems

1. Let A be a symmetric positive definite $n \times n$ matrix, and let $vecx, \mathbf{y}, \mathbf{z} \in \mathcal{R}^n$. Show that $< \mathbf{x}, \mathbf{y} >_A = \mathbf{x}^T A \mathbf{y}$ is an *inner product* on $\mathcal{R}^n$. I.e., show that

   (a) **Symmetry.** Show that $< \mathbf{x}, \mathbf{y} >_A = < \mathbf{y}, \mathbf{x} >_A$.

   (b) **Linearity.** Show that

   $$< \alpha \mathbf{x}, \mathbf{y} >_A = \alpha < \mathbf{x}, \mathbf{y} >_A, \quad \alpha \in \mathcal{R}$$
   $$< \mathbf{x} + \mathbf{z}, \mathbf{y} >_A = < \mathbf{x}, \mathbf{y} >_A + < \mathbf{z}, \mathbf{y} >_A$$

   (c) **Positive Definiteness.** $< \mathbf{x}, \mathbf{x} >_A \geq 0$ with equality only for $\mathbf{x} = 0$.

2. Let f be a smooth, real-valued function defined on the interval $[a, b]$. Show that Trapezoidal Rule for computing $\int_a^b f(x)dx$ is second order accurate.