Homework 3
CS 210

Multiple Choice and T/F

1. Let A be an invertible, ill-conditioned matrix. Which of the following matrices might be better conditioned than A?

   (a) cA where c is a scalar.

   (b) DA where D is a diagonal matrix.

   (c) A^{-1}

   (d) PA where P is a permutation matrix.

   (e) None of the above.

2. Consider the least squares problem $\min_{\mathbf{x}} ||\mathbf{b} - A\mathbf{x}||_2$. Which of the following statements are necessarily true?

   (a) If $\mathbf{x}$ is a solution to the least squares problem, then $A\mathbf{x} = \mathbf{b}$.

   (b) If $\mathbf{x}$ is a solution to the least squares problem, then the residual vector $\mathbf{r} = \mathbf{b} - A\mathbf{x}$ is in the nullspace of A^T.

   (c) The solution is unique.

   (d) A solution may not exist.

   (e) None of the above.

3. Which of the following statements about eigenvalue problems are true? Circle each true statement.

   (a) A defective eigenvalue is one where the geometric multiplicity is greater the algebraic multiplicity.

   (b) A good way to compute eigenvalues is by finding roots of the associated characteristic polynomial.

   (c) An orthogonal projection matrix has one eigenvalue equal to 0 and the other eigenvalues equal to 1.

   (d) Symmetric matrices have orthogonal set of eigenvectors.

   (e) A projection matrix must have at least one eigenavlue equal to 0.

   (f) A matrix that has an orthogonal set of eigenvectors can be decomposed as $A = U\Lambda U^T$ where U is orthogonal and Λ is diagonal.

4. A Householder matrix H

   (a) has condition number 1.

   (b) has the property $||H||_2 = 1$.

   (c) is uniquely defined by $H\mathbf{x} = \mathbf{b}$ for two vector $\mathbf{x}$ and $\mathbf{b}$ such that $||\mathbf{x}||_2 = ||\mathbf{b}||_2$.

   (d) Both (a) and (b).

   (e) All of the above.

Written Problems

1. Given a symmetric $n \times n$ matrix A, show that two eigenvectors corresponding to two distinct eigenvalues must be orthogonal.

2. Show that a $n \times n$ Householder matrix $H = I - 2\mathbf{v}\mathbf{v}^T/\mathbf{v}^T\mathbf{v}$ has an eigenvalue of 1 with multiplicity $n-1$ and an eigenvalue of -1 with multiplicity 1.

3. Compare Newton's method and the Secant Method for solving a scalar nolinear equation. What are the advantages and disadvantages of each?