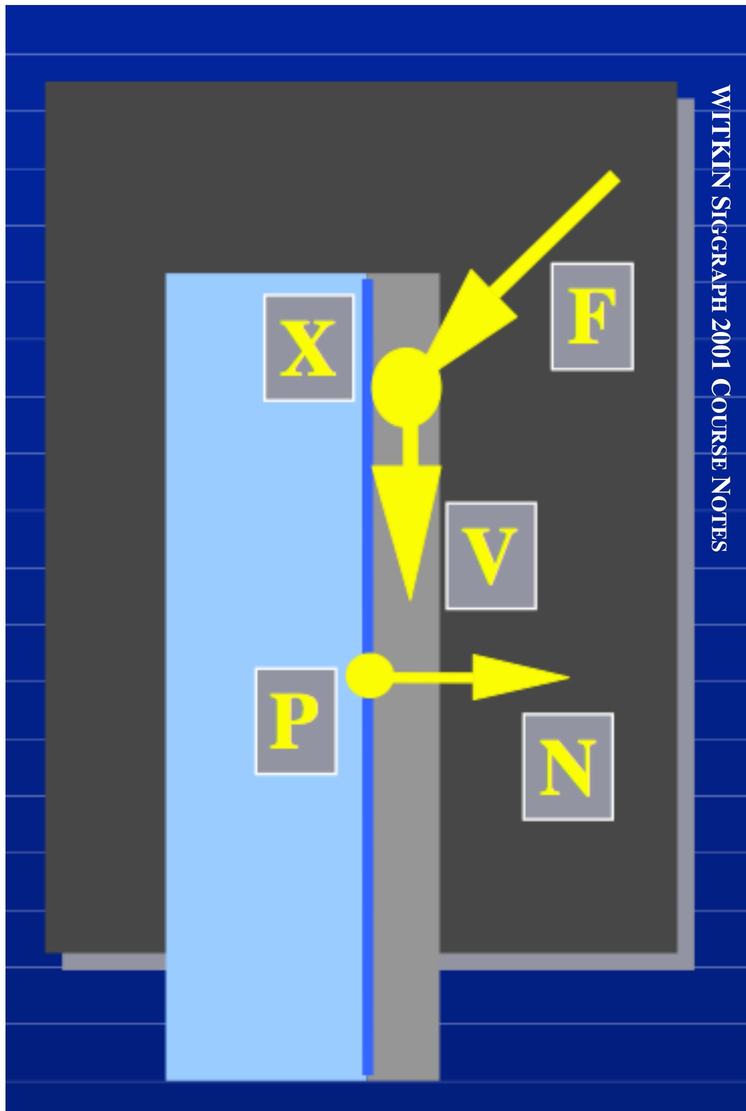
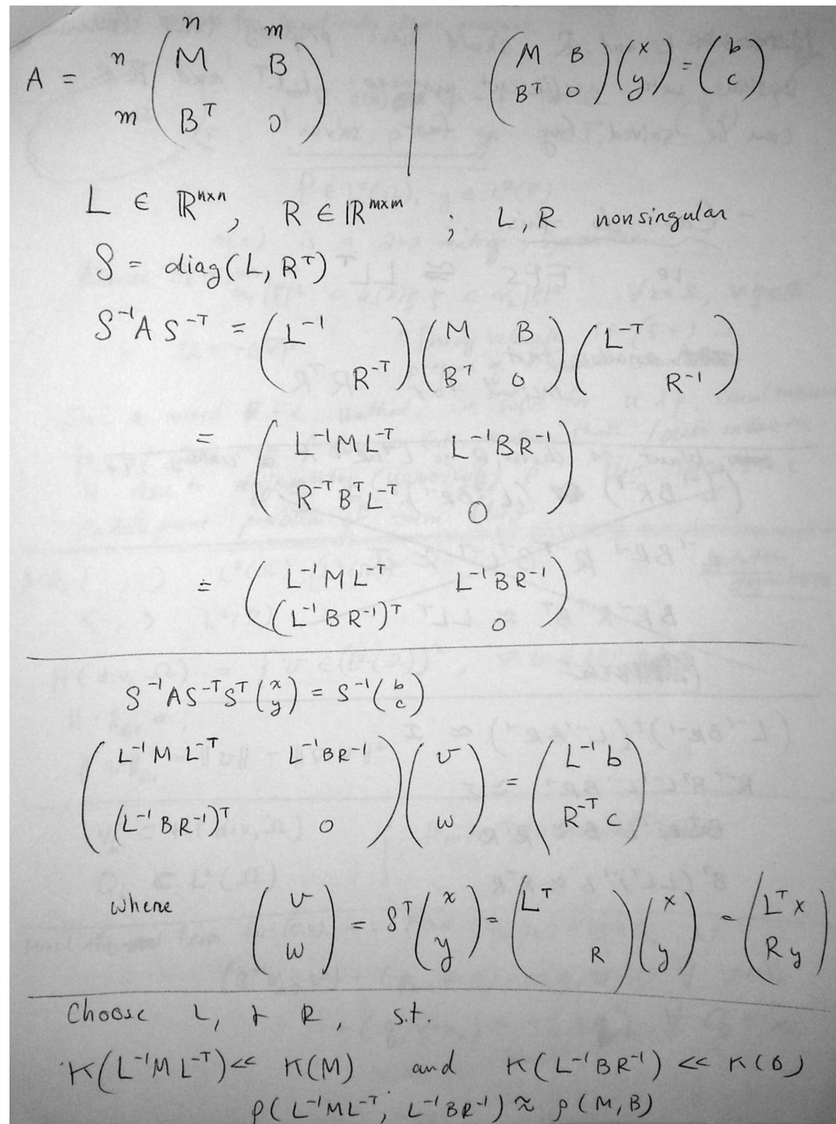
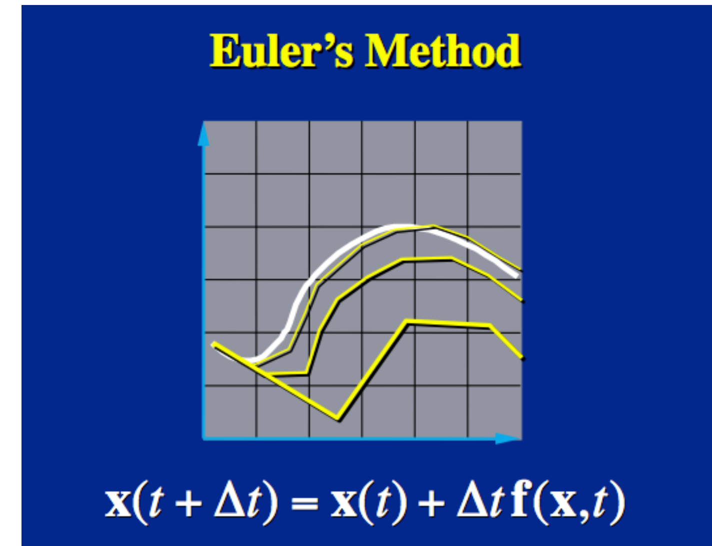
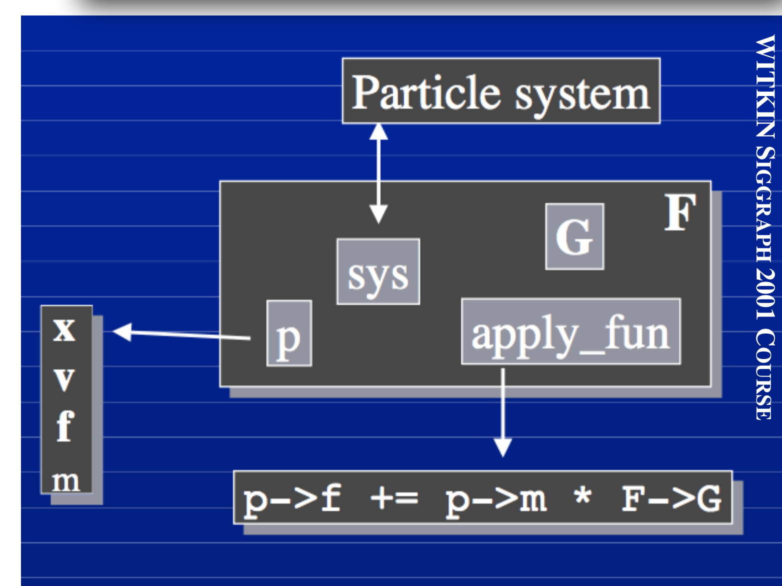
physics-based animation

Physics-Based Animation





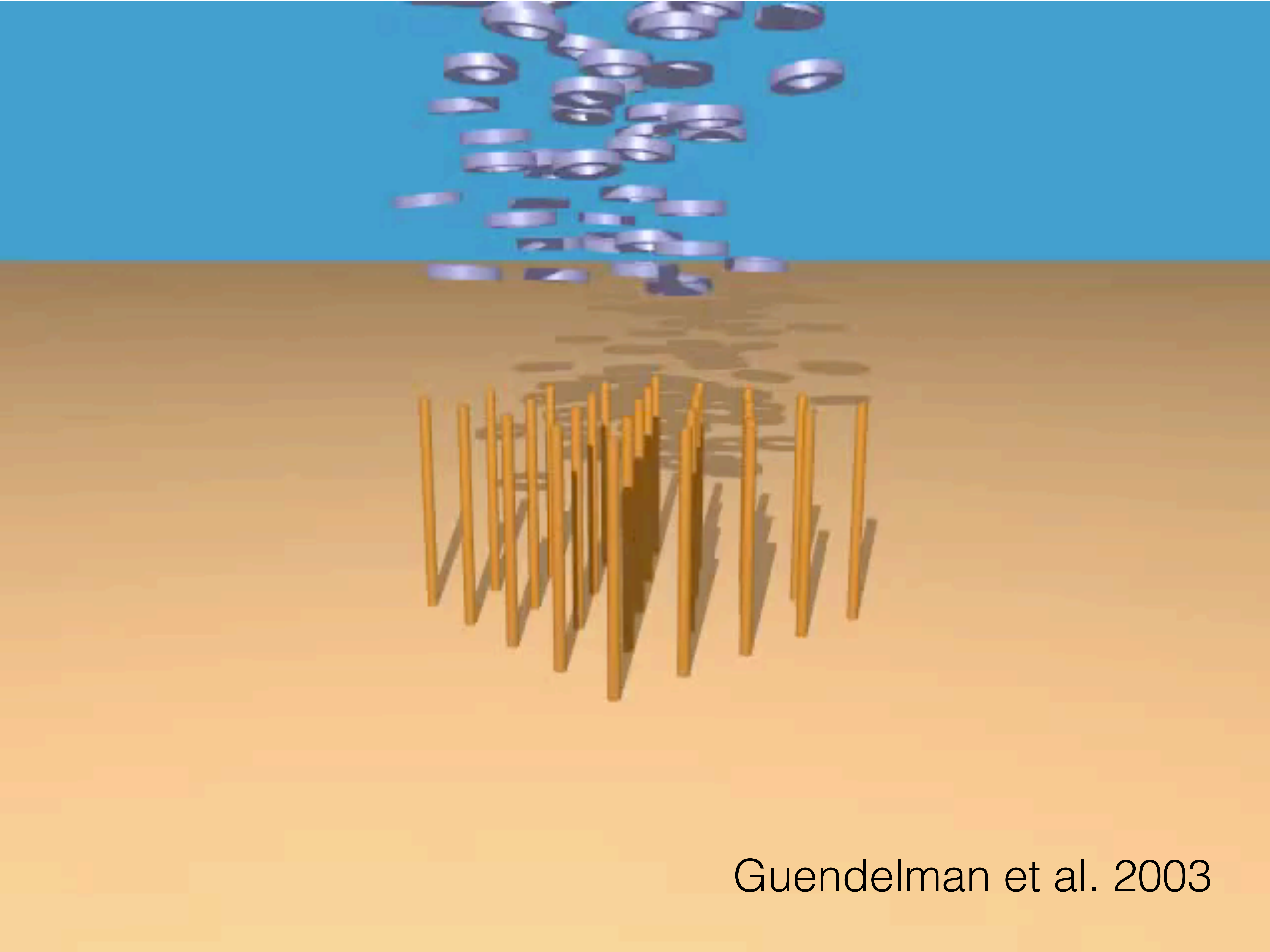




Physics

Mathematics

Numerical Methods and Algorithms

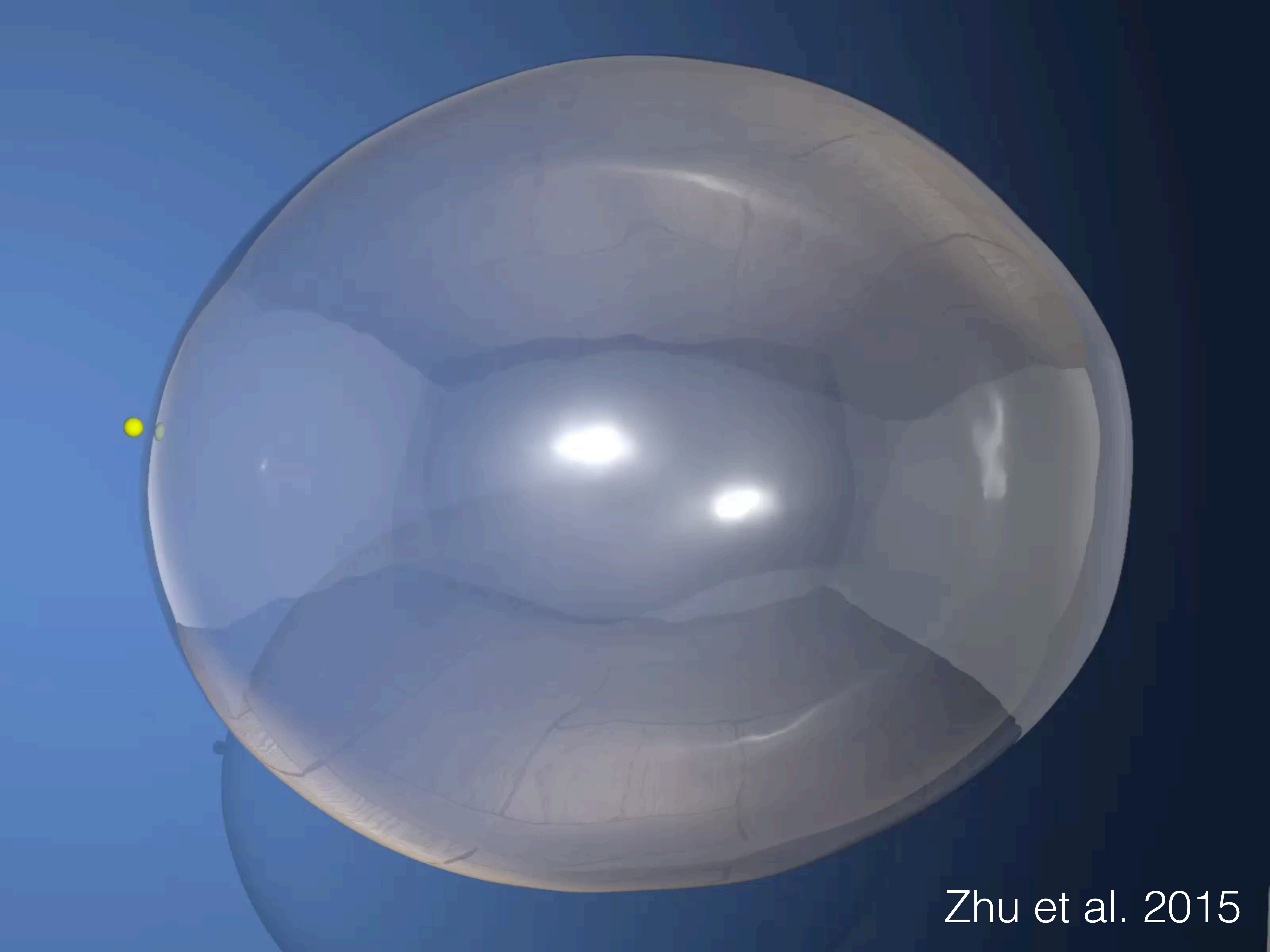


Guendelman et al. 2003


Shinar et al. 2008

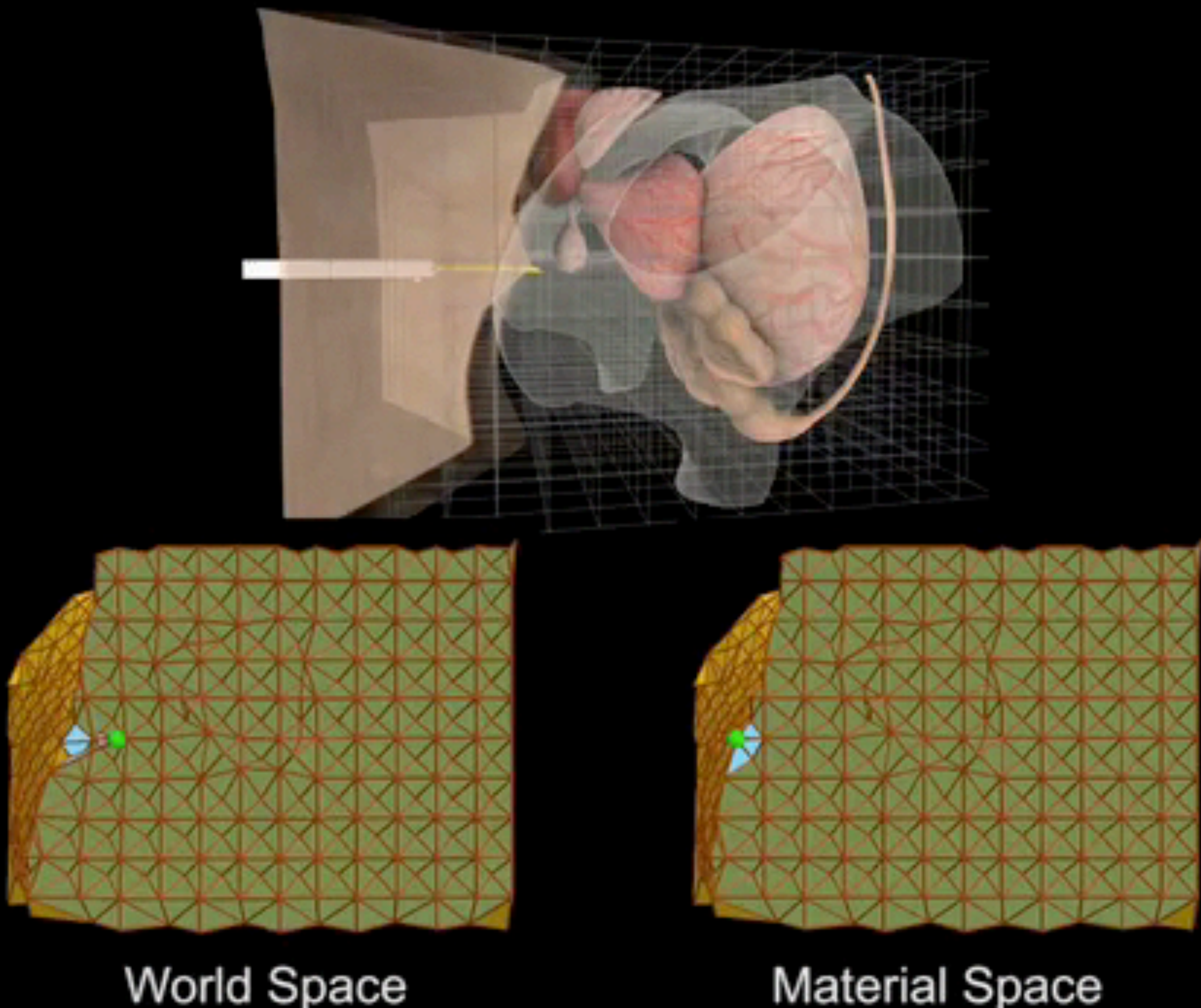
Hong et al. 2007

Clausen et al. 2013



Zhu et al. 2015

Patkar et al. 2013



Chentanez et al., 2009

Physics of Natural Phenomena

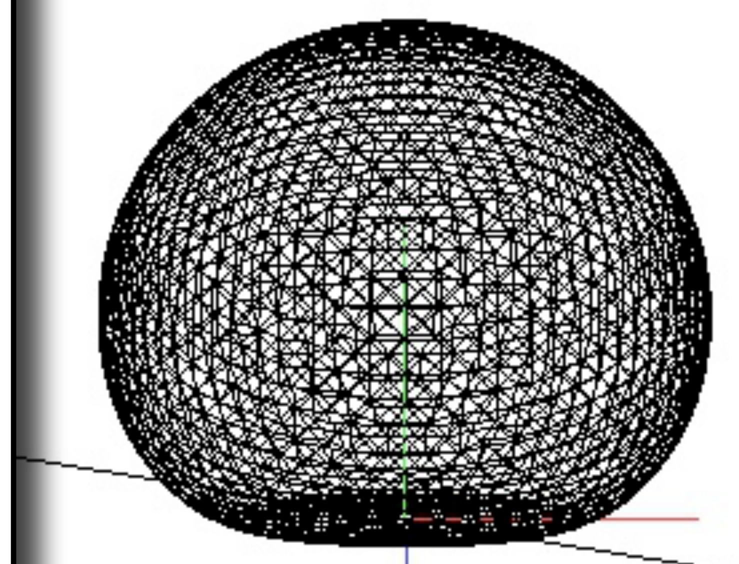
- **Newton's Second Law (*F* = *ma*)**

  *The acceleration **a** of a body is parallel and directly proportional to the net force **F** acting on the body, is in the direction of the net force, and is inversely proportional to the mass **m** of the body.*

- **Newton's Third Law (Action/Reaction)**

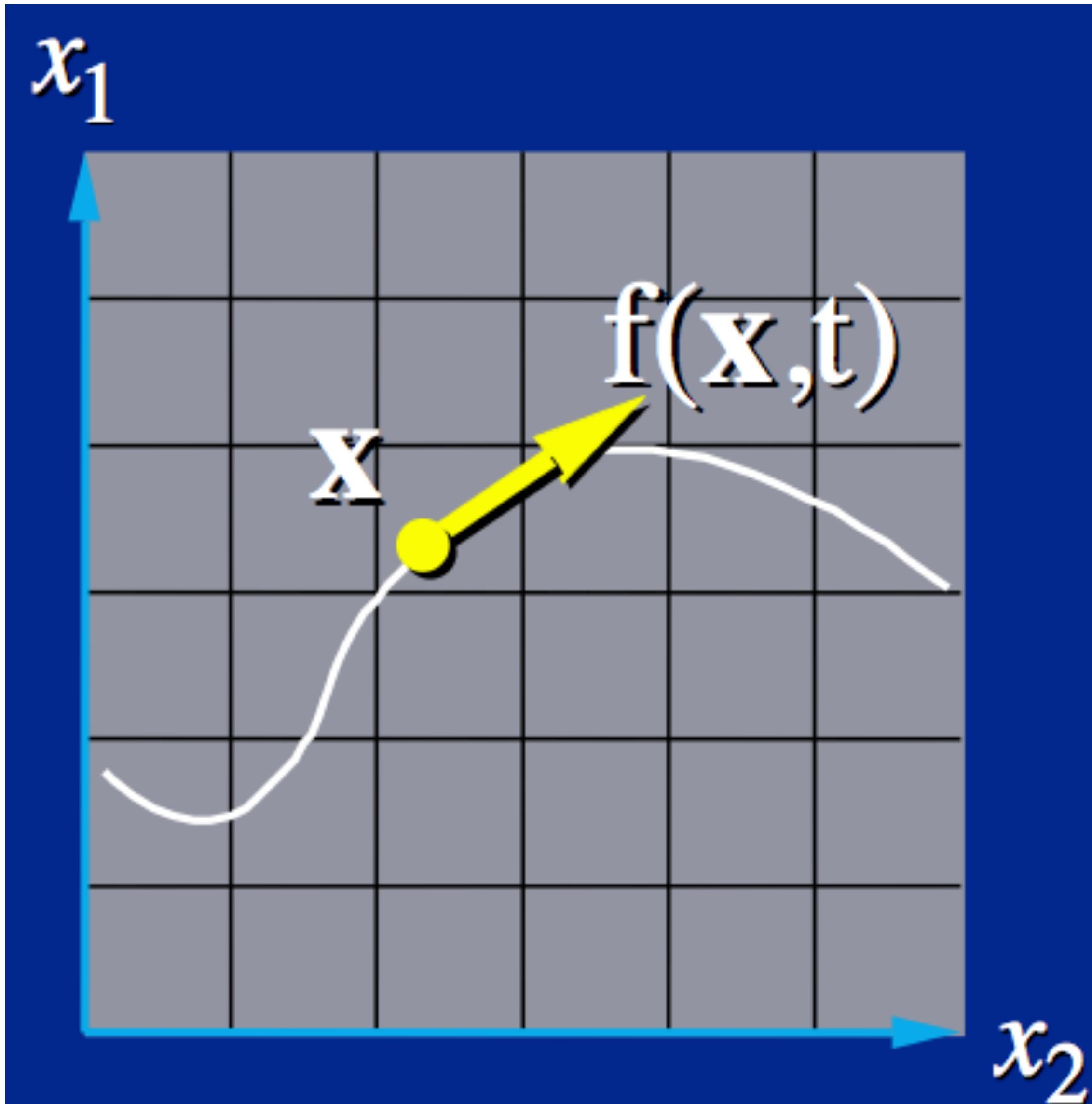
  When a body exerts a force $\mathbf{F_1}$ on a second body, the second body simultaneously exerts a force $\mathbf{F_2}=-\mathbf{F_1}$ on the first body. This means that $\mathbf{F_1}$ and $\mathbf{F_2}$ are equal in magnitude and opposite in direction.

  [Wikipedia]



Math of Natural Phenomena

Ordinary Differential Equations



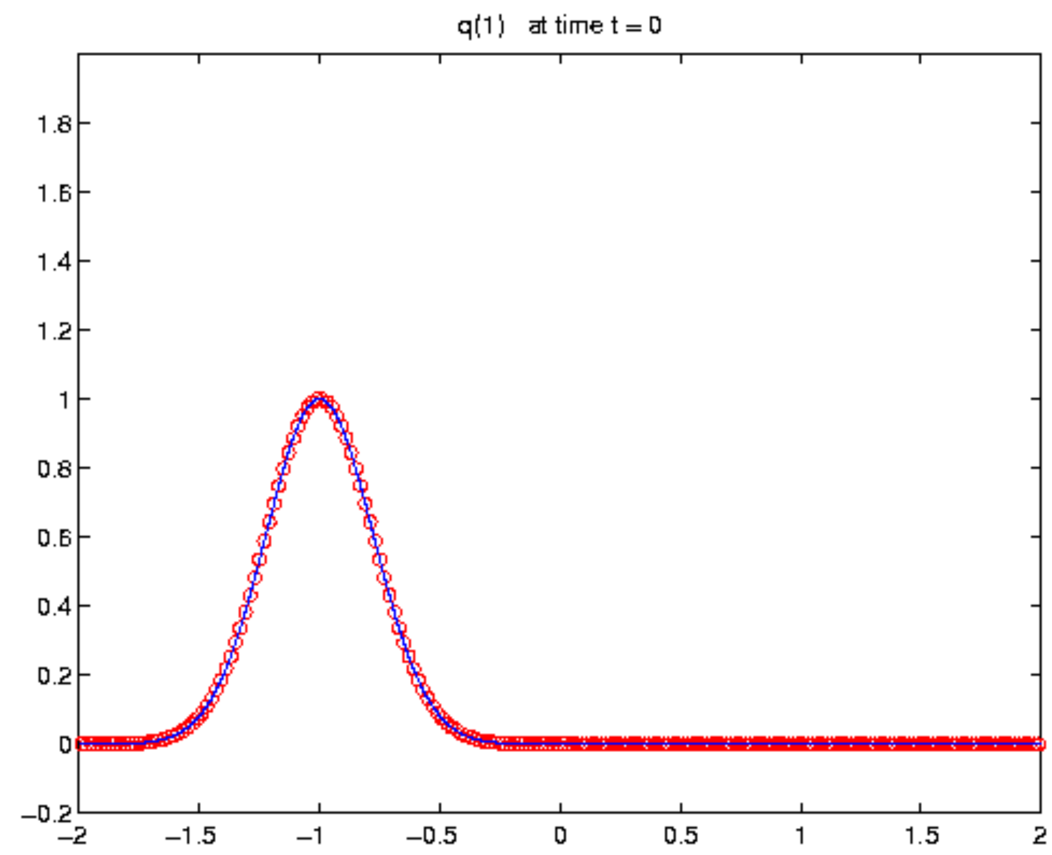
$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t)$$

- **x(*t*): a moving point.**
- **f(x,*t*): x's velocity.**

Math of Natural Phenomena

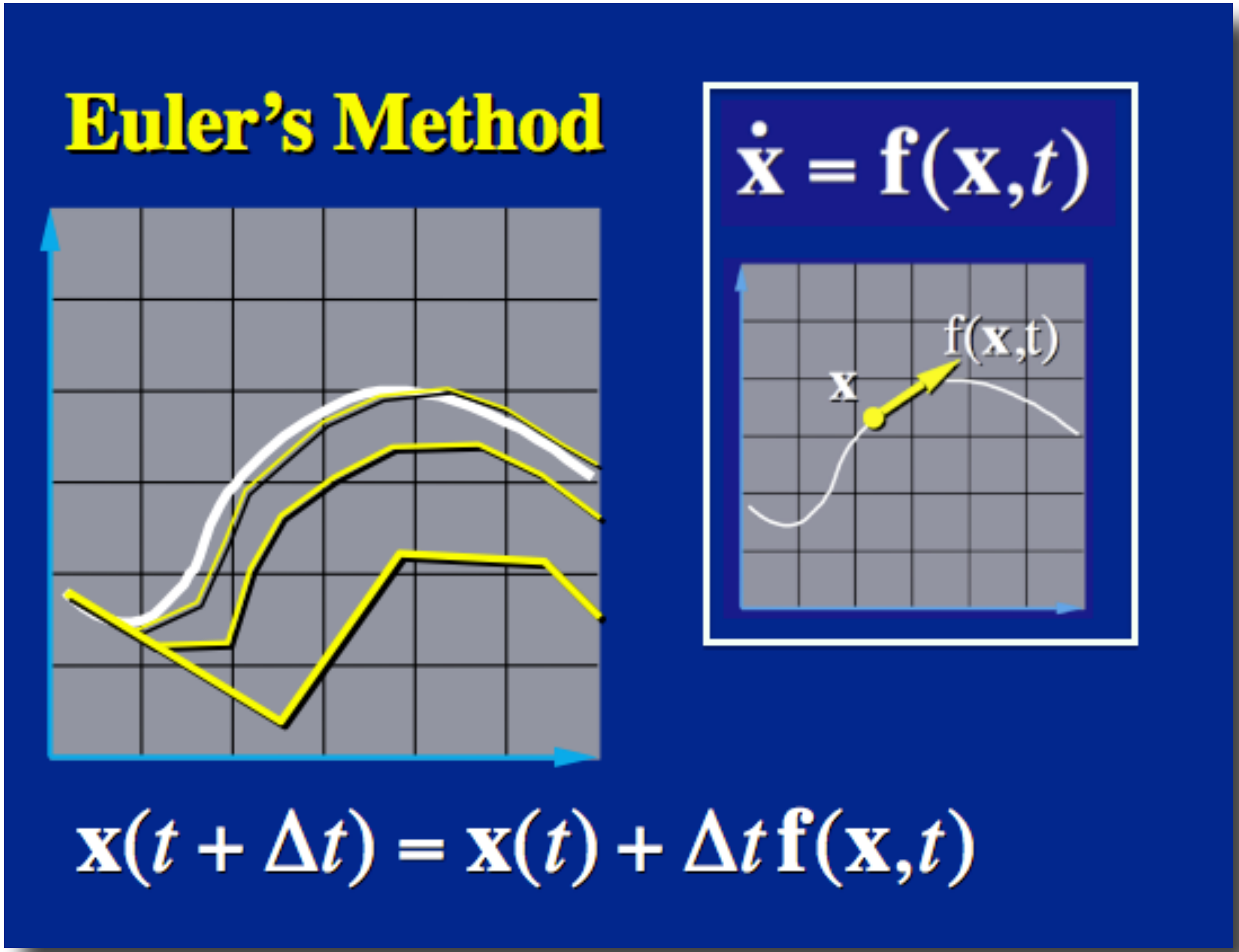
Partial Differential Equations

$$c_t + \vec{v} \cdot \nabla c = f(t)$$

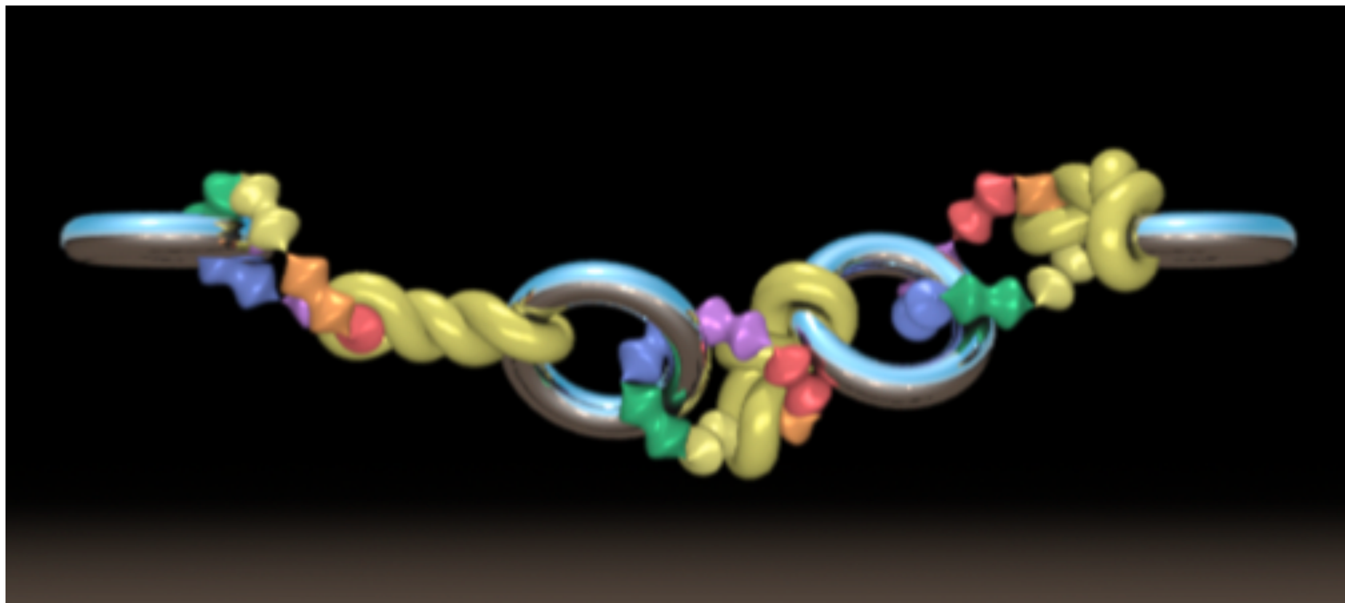


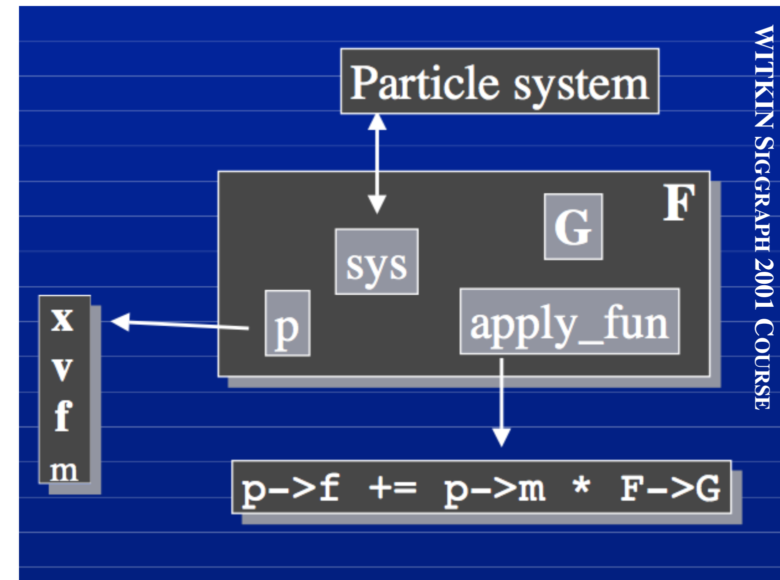
CLAWPACK

Numerical Solution of Diff. Eq.



Data Structures and Algorithms





I. Advance velocity $\mathbf{v}^n \longrightarrow \tilde{\mathbf{v}}^{n+\frac{1}{2}}$

II. Apply collisions $\mathbf{v}^n \longrightarrow \hat{\mathbf{v}}^n$, $\tilde{\mathbf{v}}^{n+\frac{1}{2}} \longrightarrow \hat{\mathbf{v}}^{n+\frac{1}{2}}$

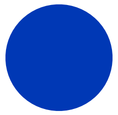
III. Apply contact and constraint forces $\hat{\mathbf{v}}^{n+\frac{1}{2}} \longrightarrow \mathbf{v}^{n+\frac{1}{2}}$

IV. Advance positions $\mathbf{x}^n \longrightarrow \mathbf{x}^{n+1}$ using $\mathbf{v}^{n+\frac{1}{2}}$, $\hat{\mathbf{v}}^n \longrightarrow \bar{\mathbf{v}}^n$

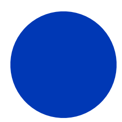
V. Advance velocity $\bar{\mathbf{v}}^n \longrightarrow \mathbf{v}^{n+1}$

Particles

Particle: basic dynamic object

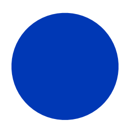


Particle: basic dynamic object



mass m

Particle: basic dynamic object

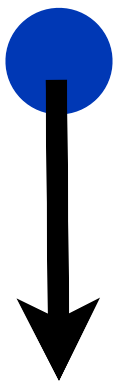


mass m

3 dof

$$\vec{X} = (x, y, z)$$

Particle: basic dynamic object



mass

m

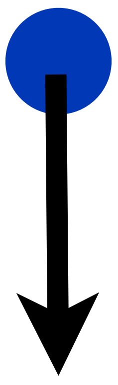
3 dof

$$\vec{X} = (x, y, z)$$

forces: e.g., gravity

$$\vec{F} = -m\vec{g}$$

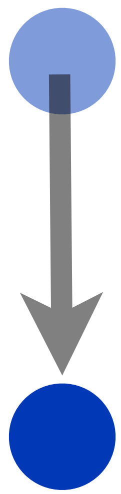
Particle: basic dynamic object



Equations of motion:
Newton's 2nd Law

$$\vec{F} = m\vec{a}$$

Particle: basic dynamic object



Equations of motion:
Newton's 2nd Law

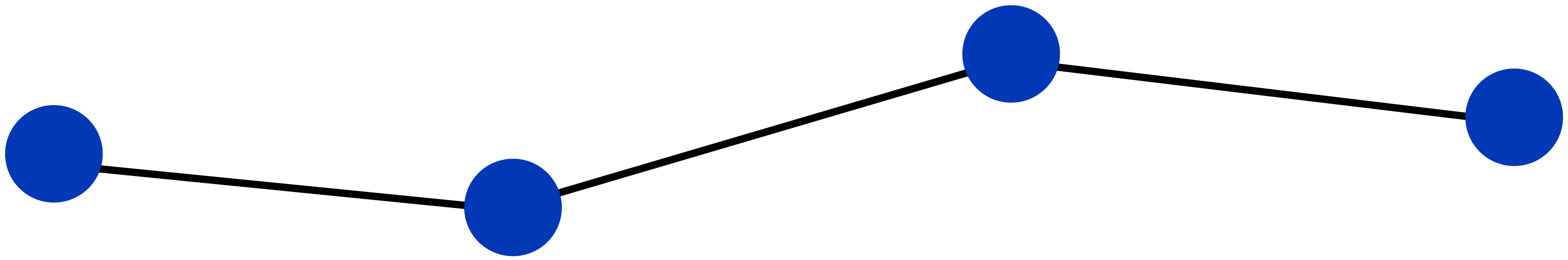
$$\vec{F} = m\vec{a}$$

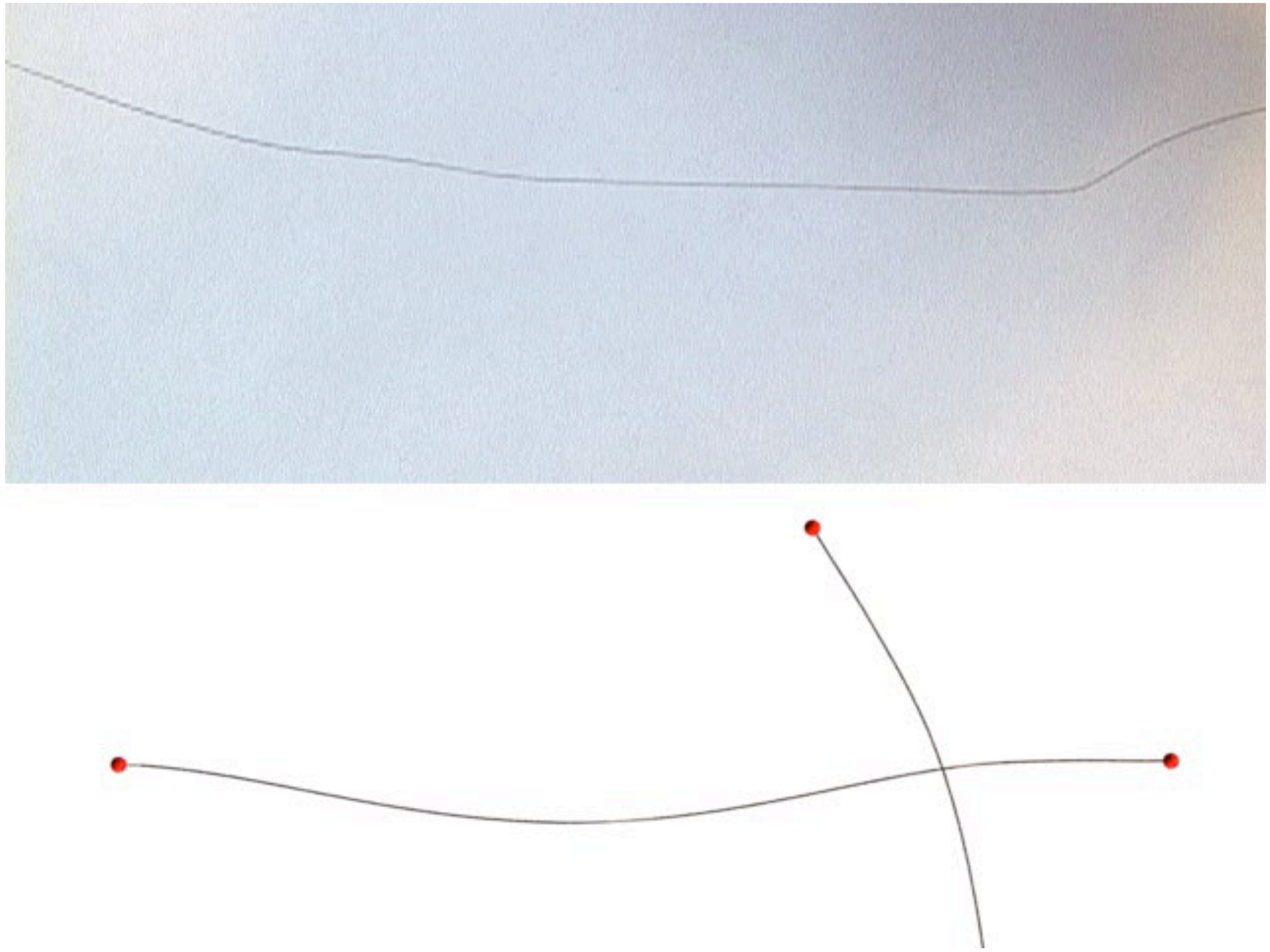
$$\frac{d\vec{x}}{dt} = \vec{v}$$
$$m\frac{d\vec{v}}{dt} = \vec{F}$$

System of ODEs

Deformable bodies

Connect a bunch of particles into a 1D line segment with springs

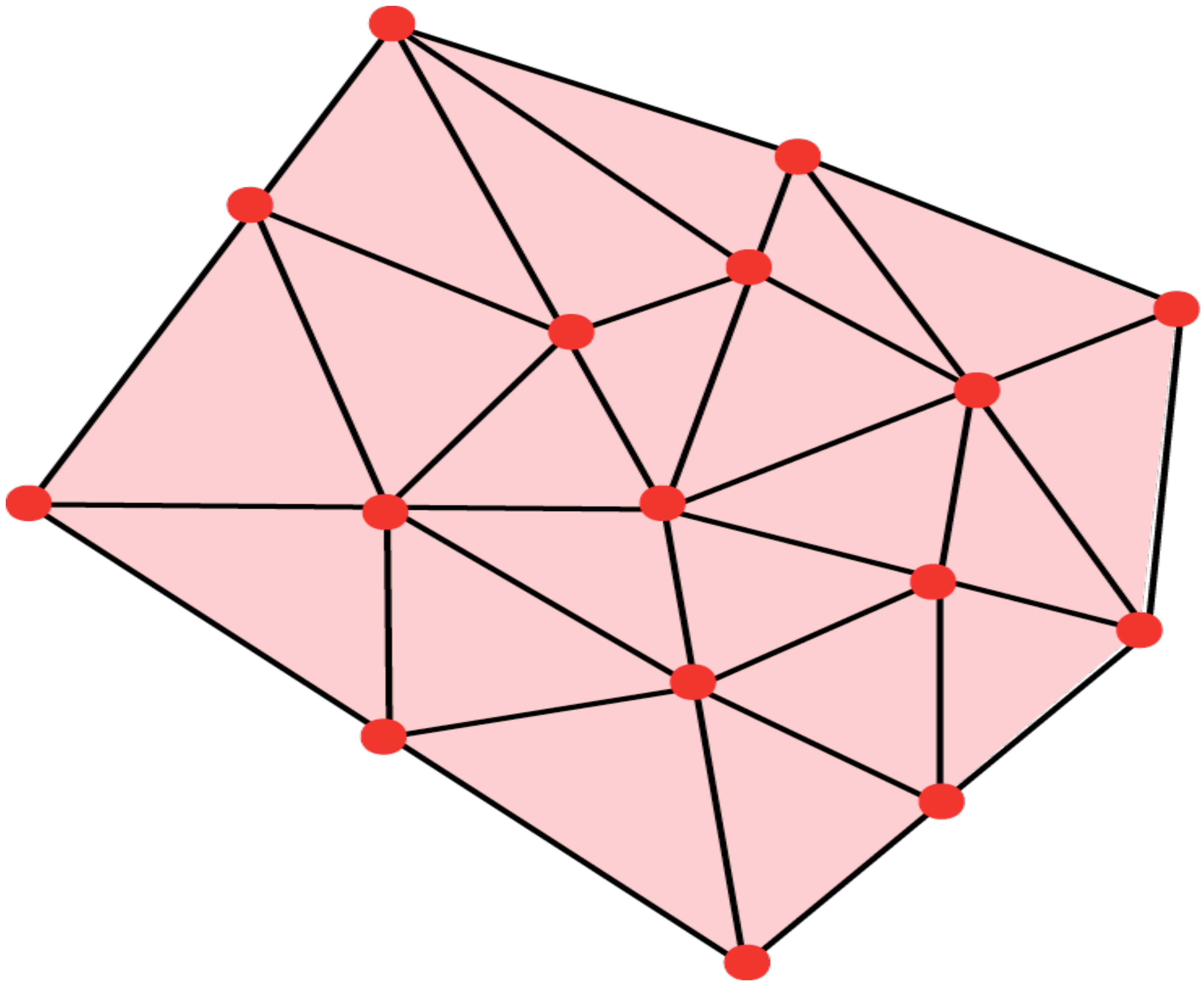




A Mass Spring Model for Hair Simulation

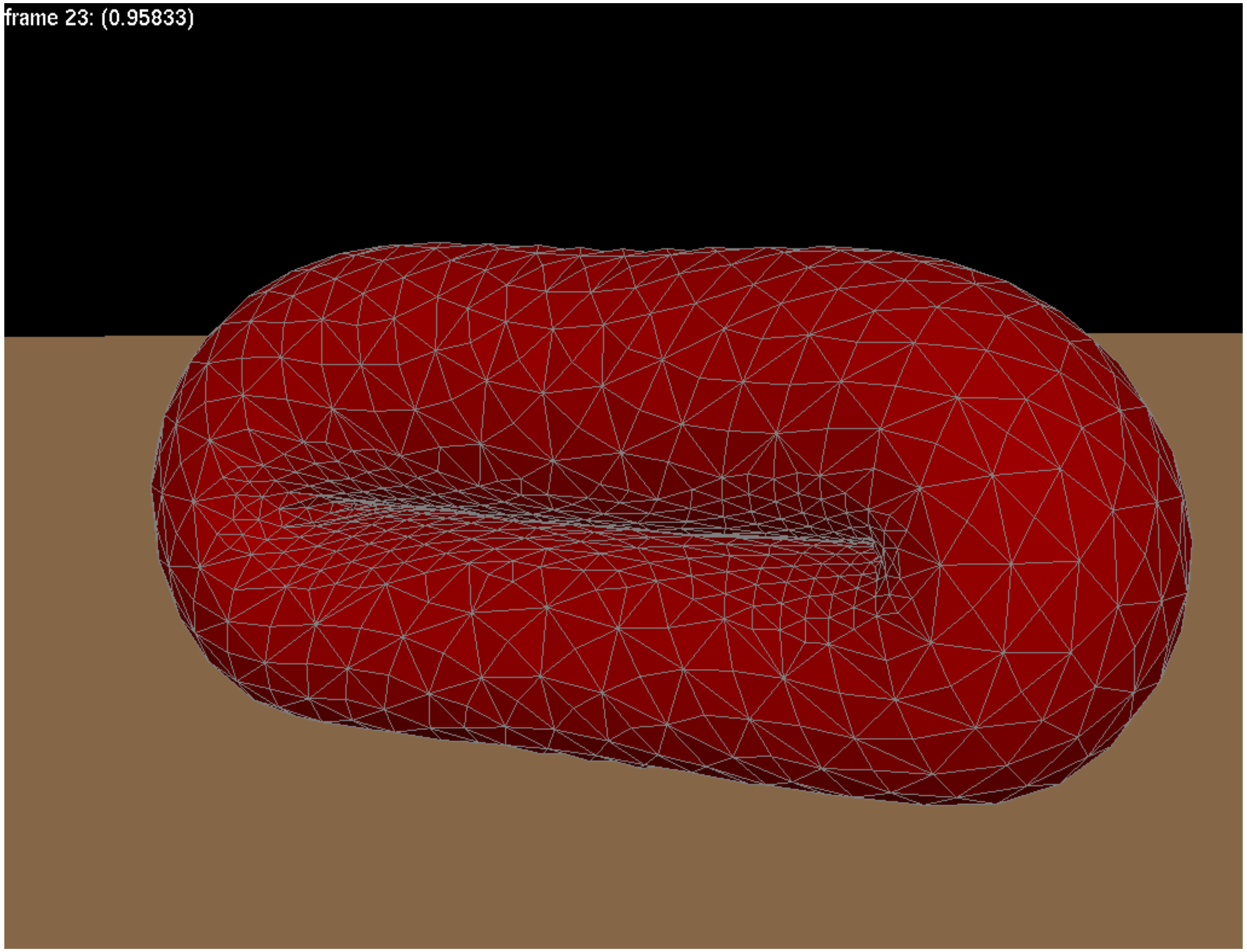
Selle, A., Lentine, M., G., and Fedkiw, R. ACM Transactions on Graphics SIGGRAPH 2008, ACM TOG 27, 64.1-64.11 (2008)

Connect a bunch of particles into a 2D mesh

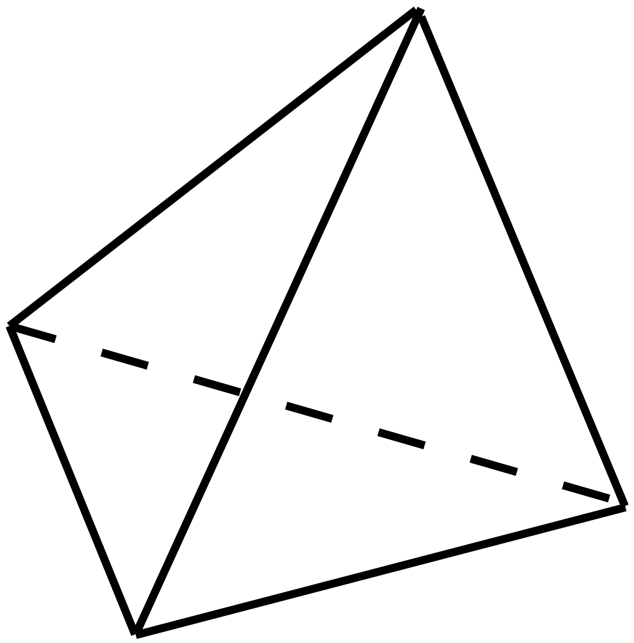


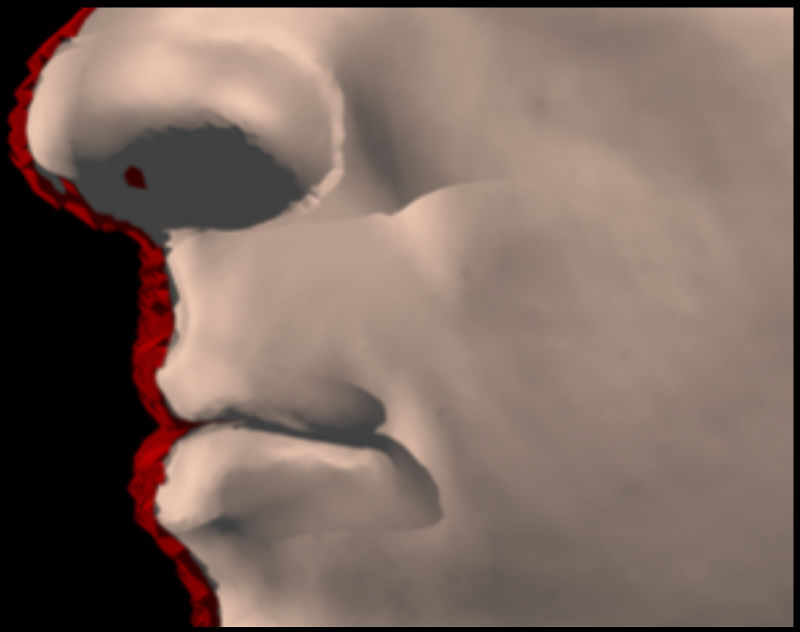
Selle et al. 2007

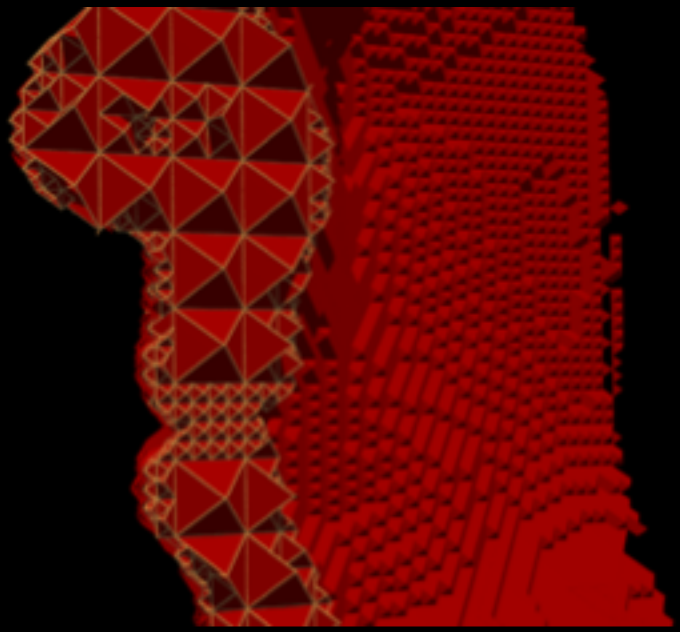
Connect a bunch of particles into a 3D mesh



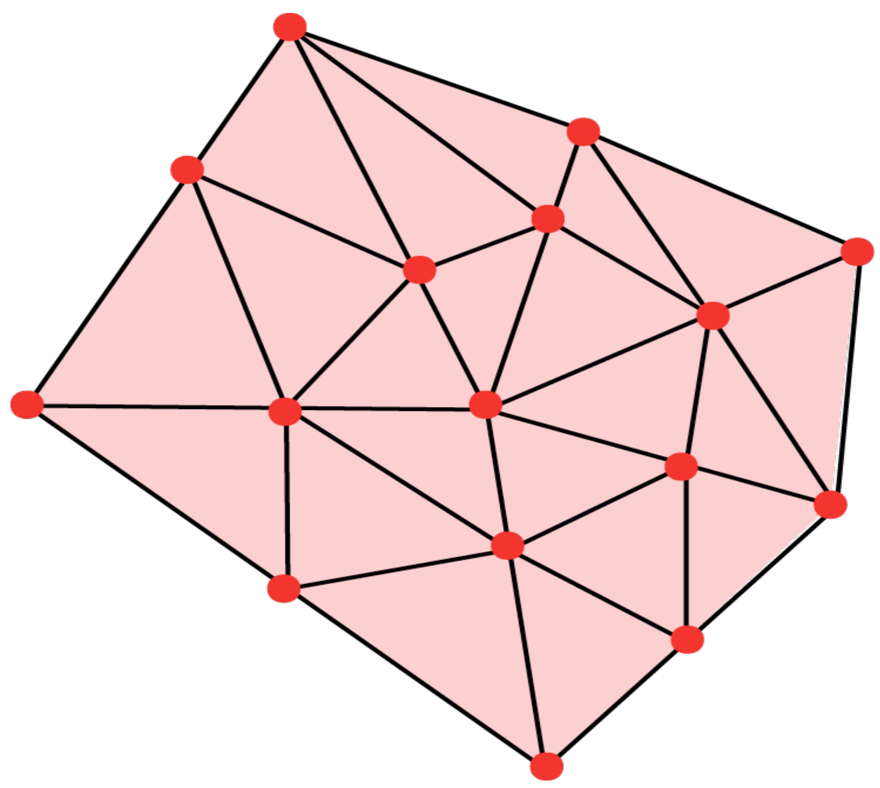
tetrahedron







Deformable bodies: equations of motion



Equations of motion:
Newton's 2nd Law

$$\vec{F} = m\vec{a}$$

$$\frac{d\vec{x}}{dt} = \vec{v}$$

$$m\frac{d\vec{v}}{dt} = \vec{F}$$

System of PDEs

contains spatial derivatives

Rigid bodies

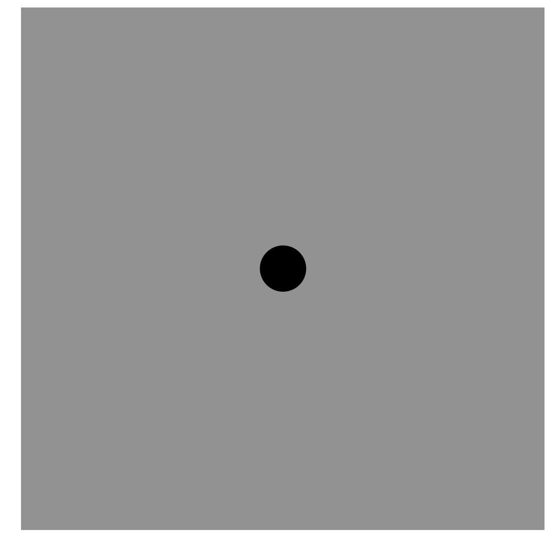
Rigid bodies

6 dofs

forces and torques

elastic collisions

ODEs



$(\vec{X}, \vec{\Omega})$

$(\vec{F}, \vec{\tau})$

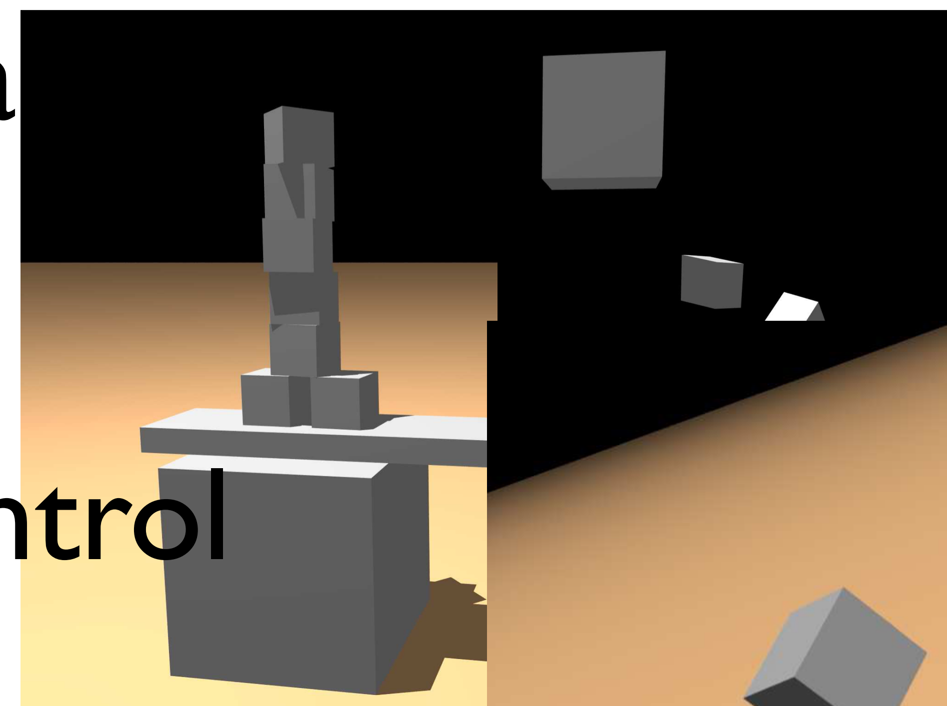
Rigid body phenomena

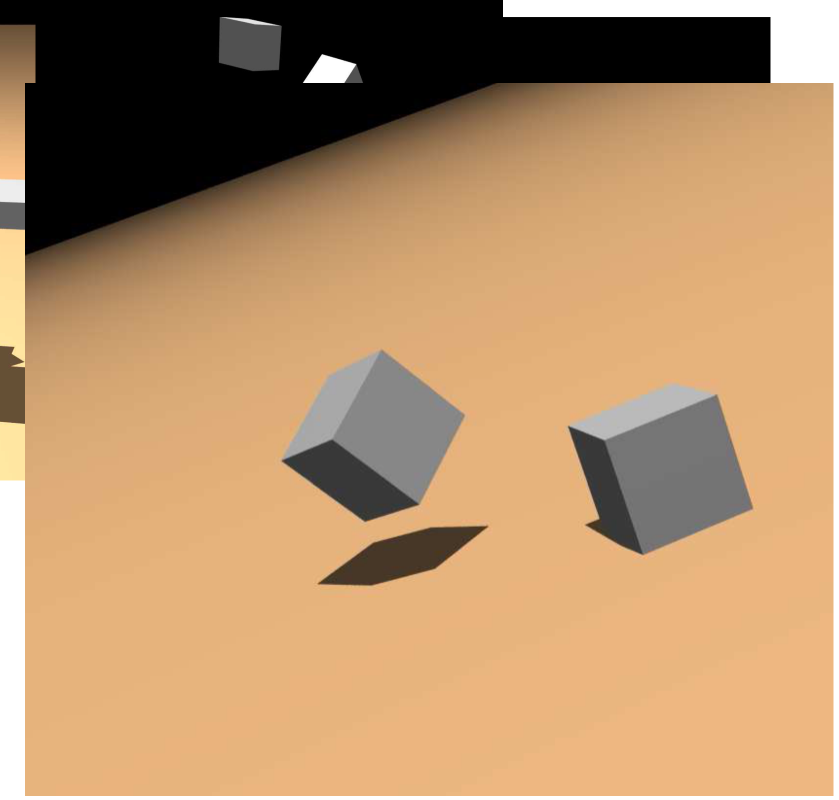
stacking

collisions, conta

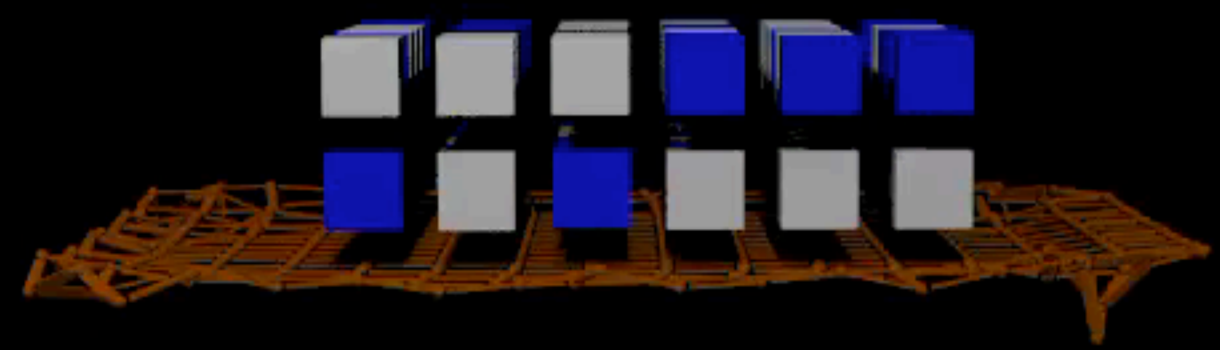
friction

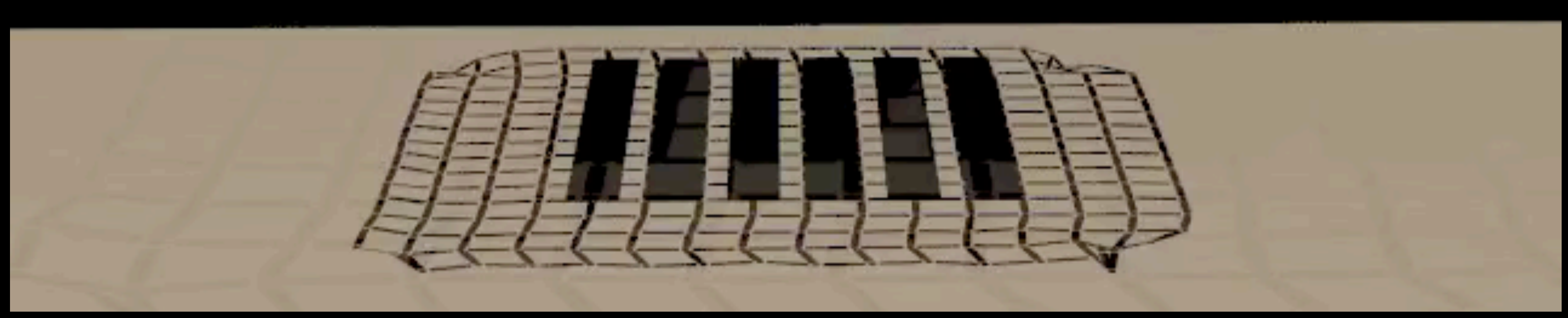
articulation, control





Articulated rigid bodies

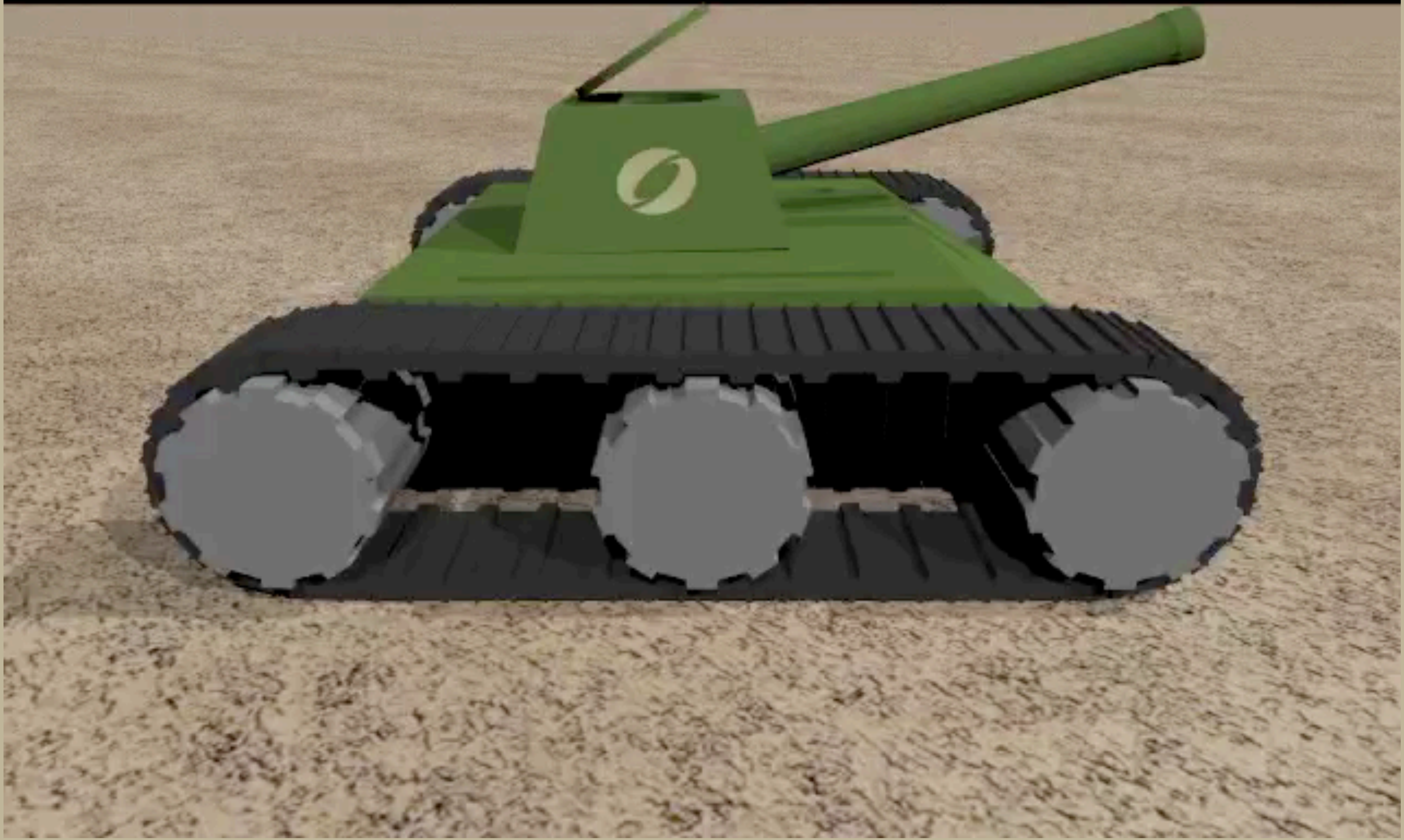




Rachel Weinstein, Joey Teran and Ron Fedkiw

Rigid body simulation

[Weinstein et al 2006]



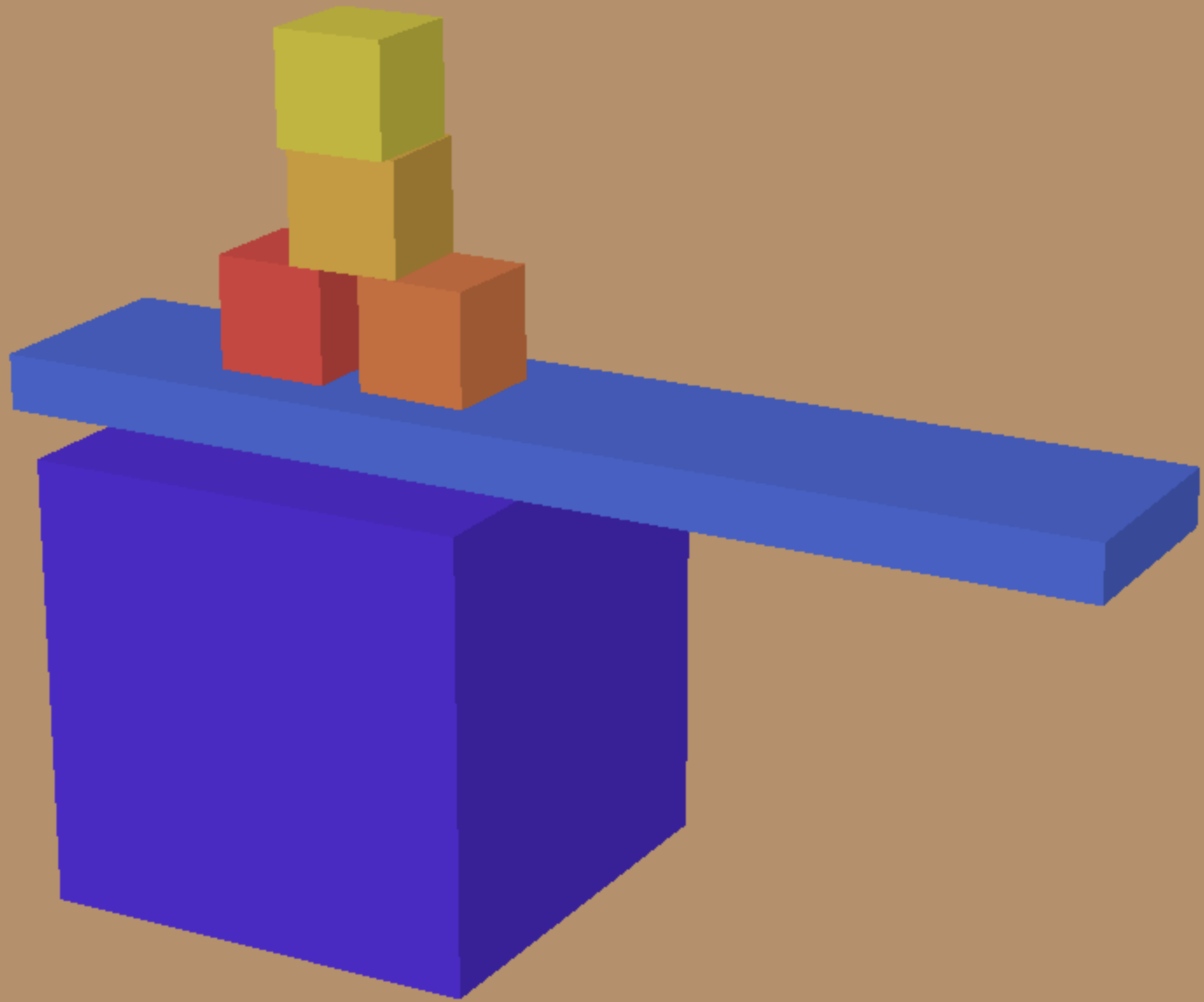
Rigid and deformable solids coupled together...

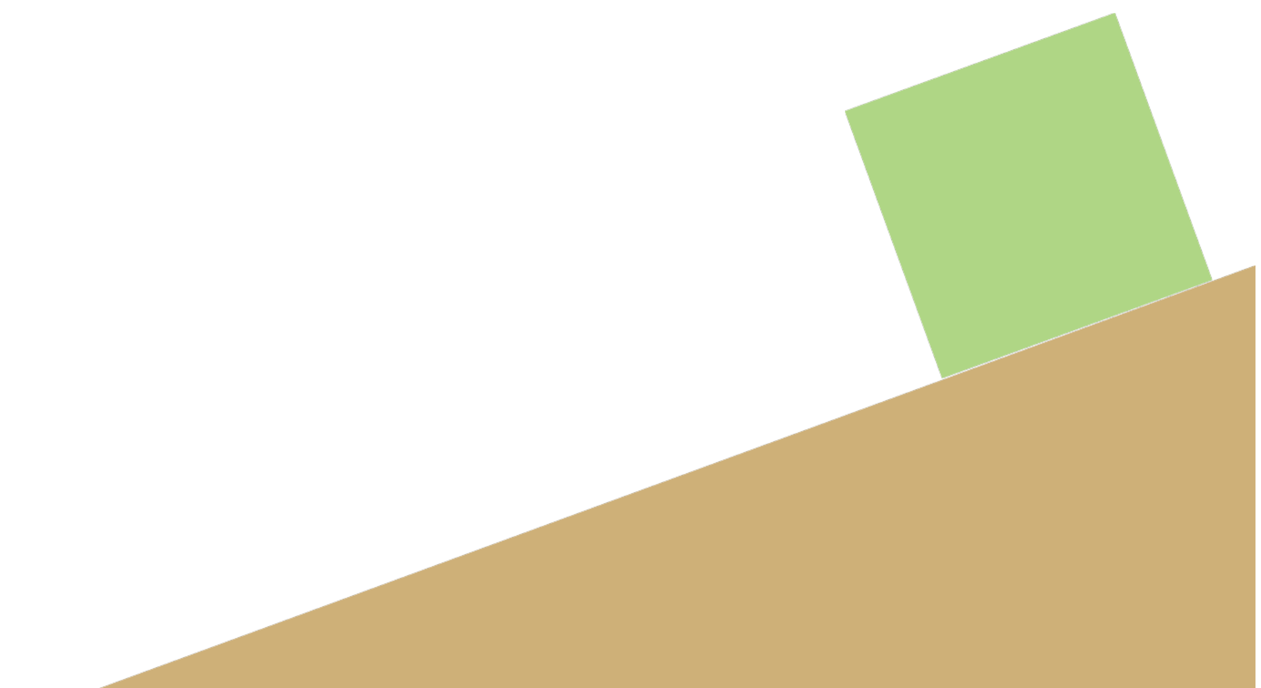
Fracture

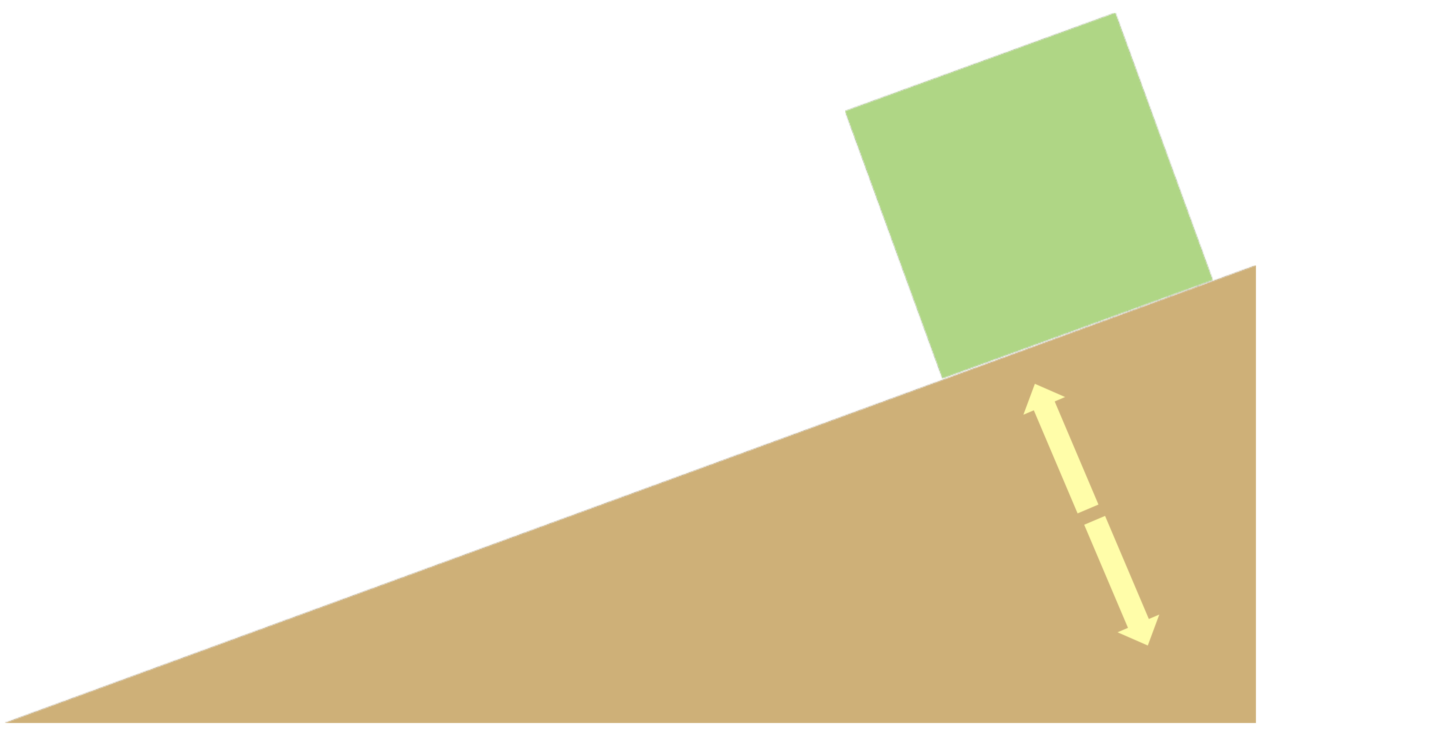
[Molino et al. 2004]

Contact and collision

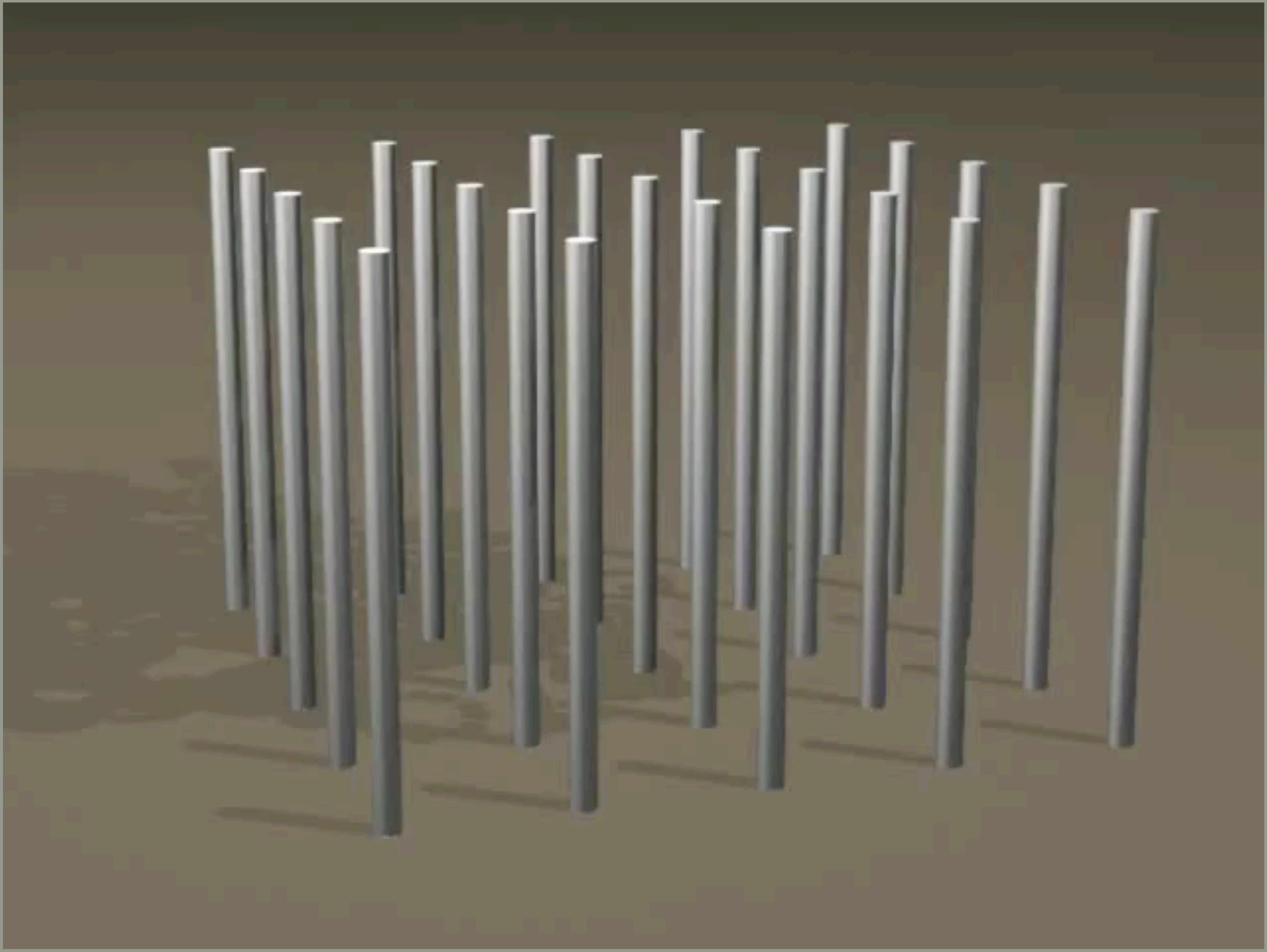
frame 25: (1.04167)

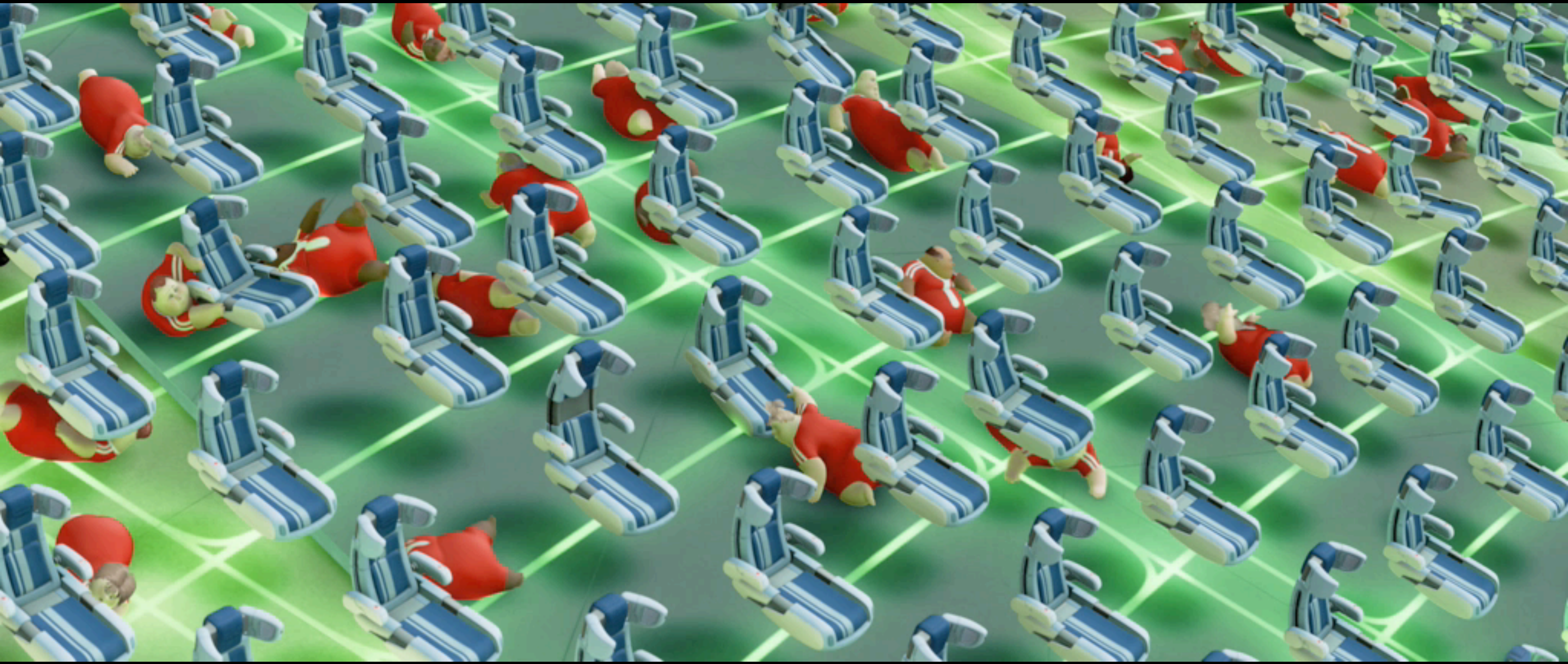






Simultaneous resolution of contact, elastic deformation, articulation constraints



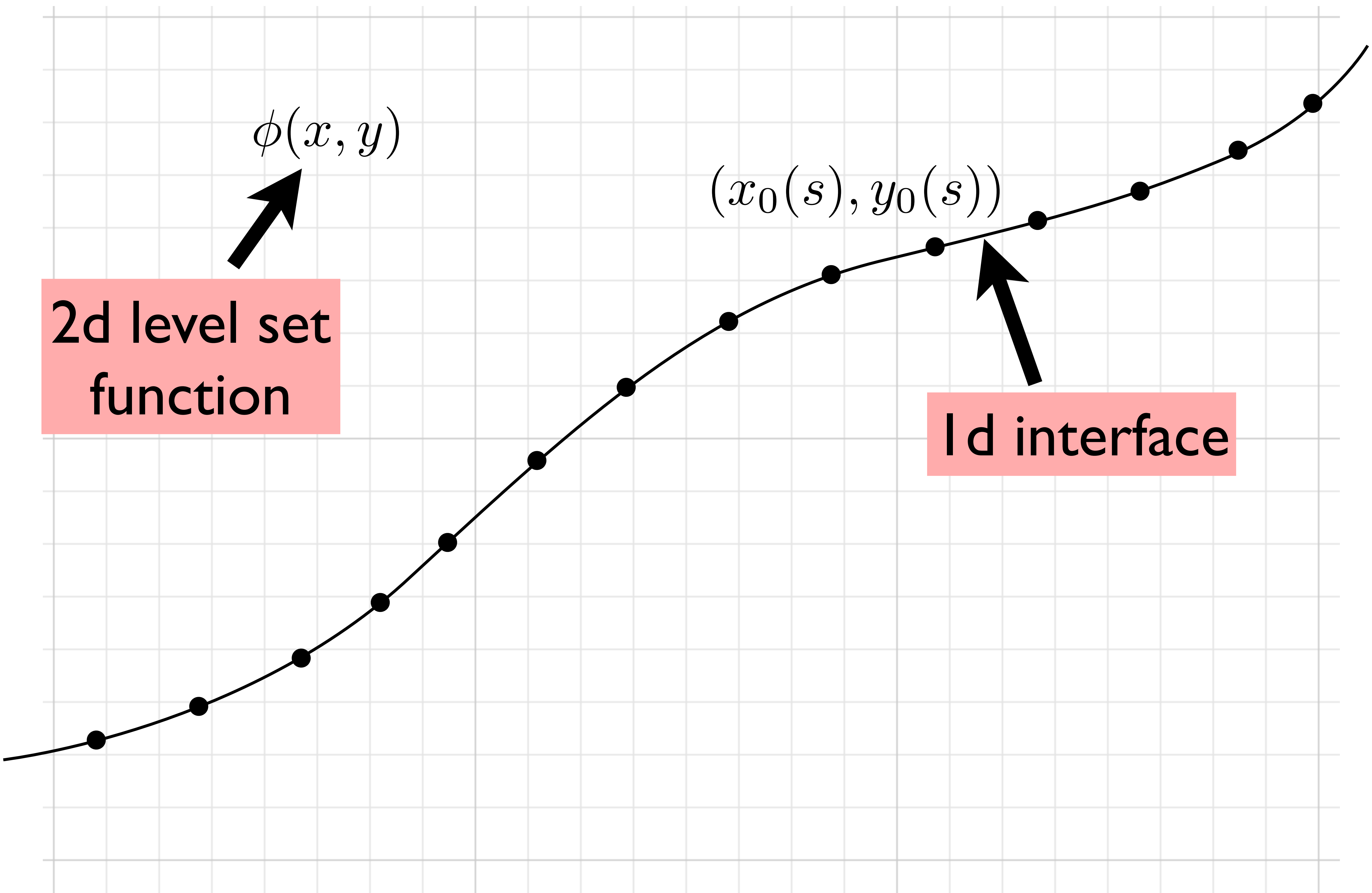


our rigid/deformable simulator in Pixar's *WALL-E*

Selle et al. 2007

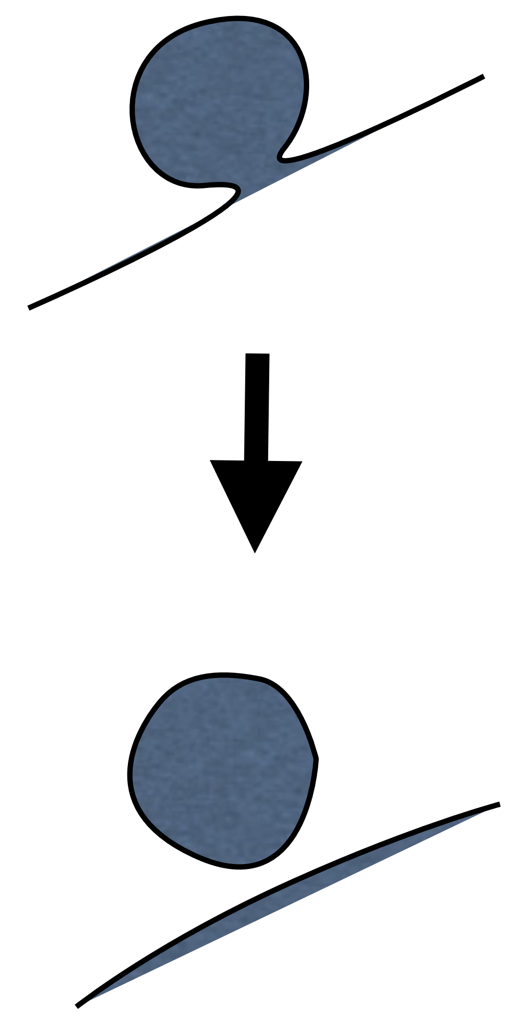
Fluid simulation

In fluid simulation, we often use a grid-based representation



An implicit representation has certain advantages over an explicit representation

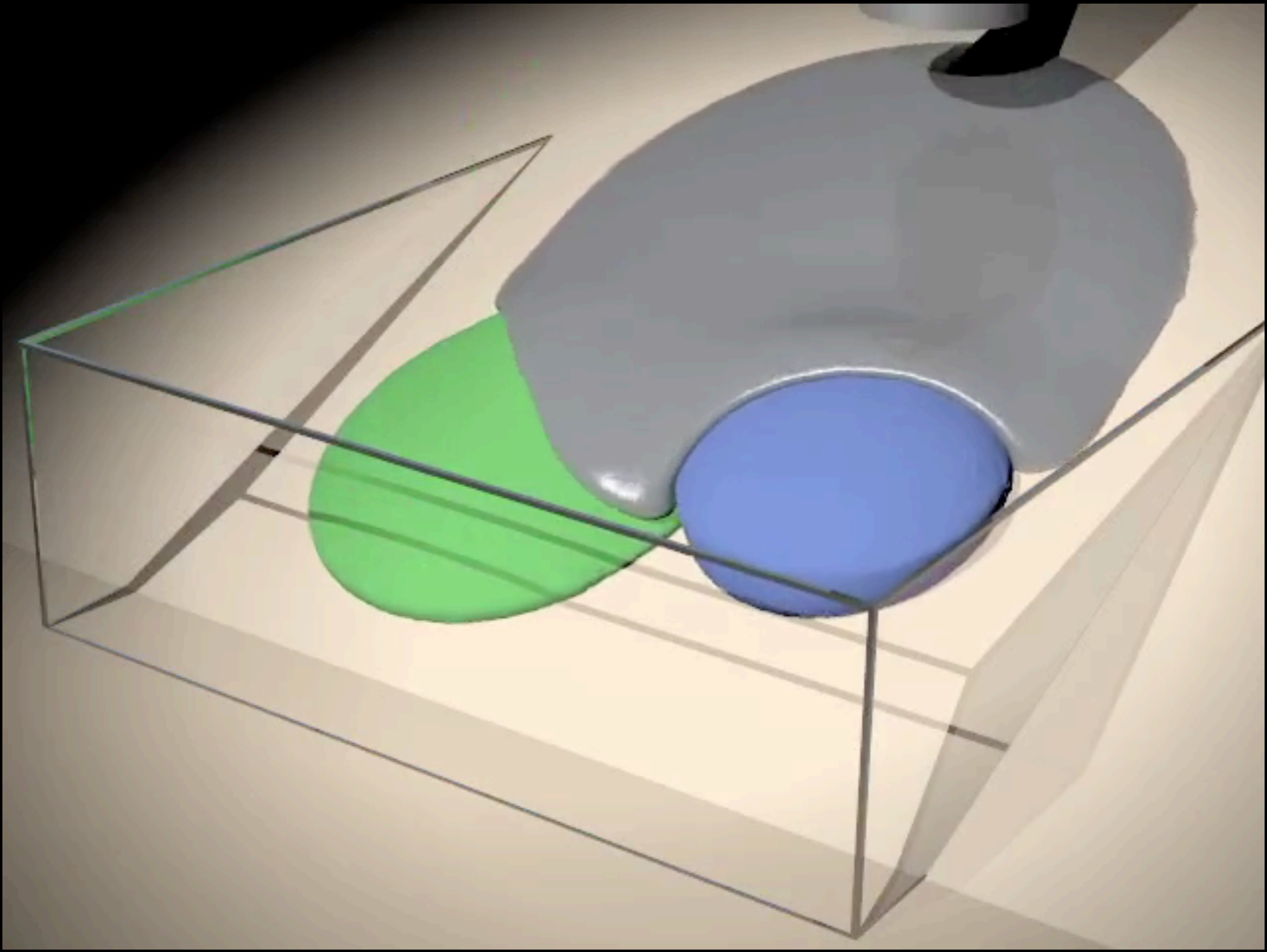
- naturally handles topological changes
- very easy to extend from 2D to 3D



Fluid equations of motion: Navier-Stokes equations

$$\vec{F} = m\vec{a}$$

$$\rho(\mathbf{u}_t + \mathbf{u} \cdot \nabla \mathbf{u}) = \mu \triangle \mathbf{u} - \nabla p + \mathbf{f}$$

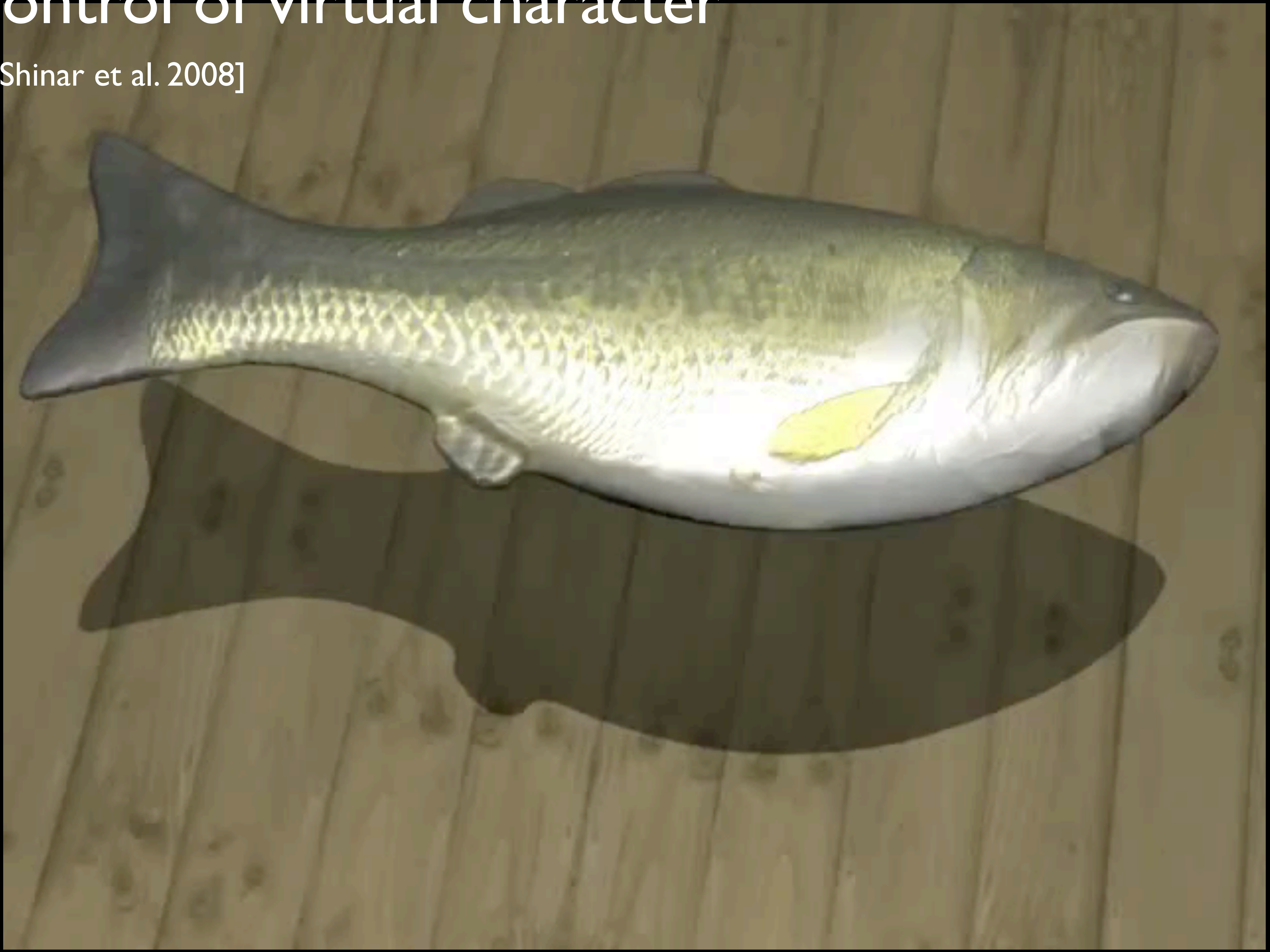


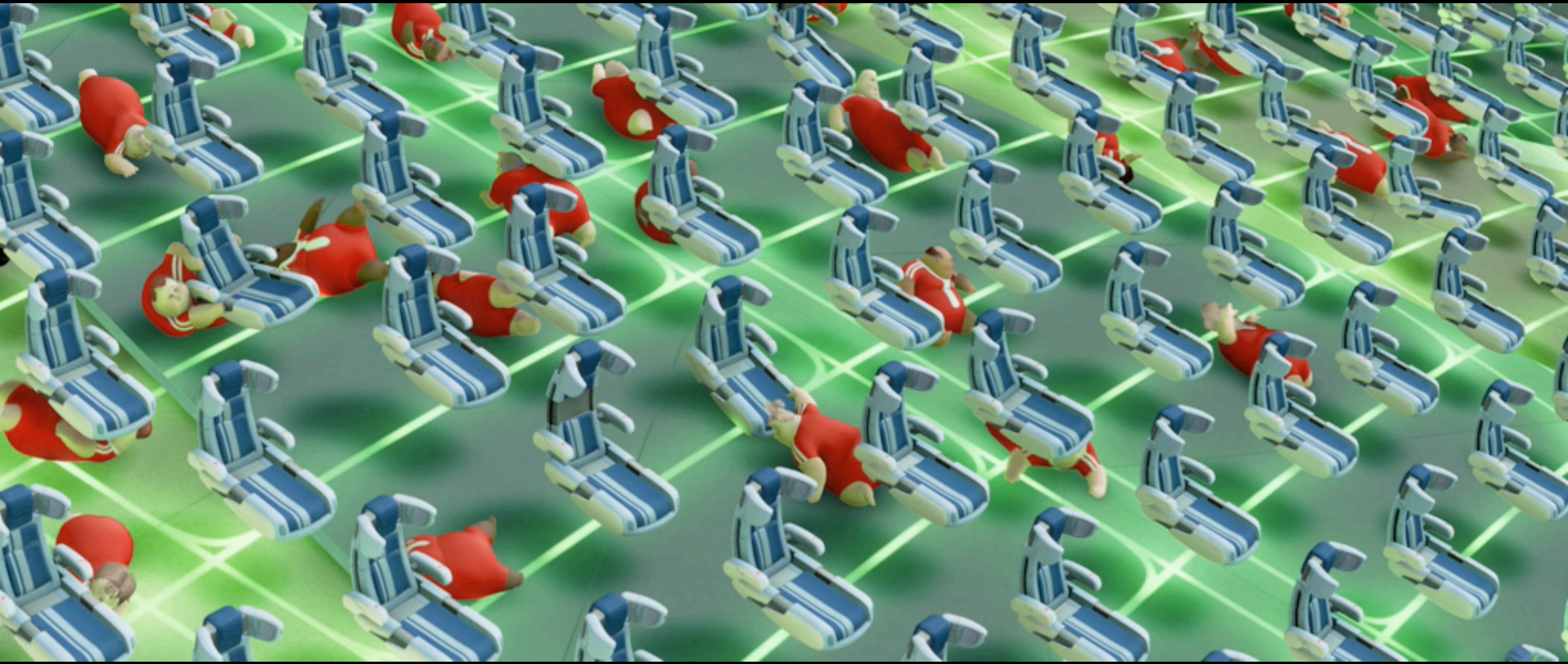
Hong et al. 2007

Losasso, F., Talton, J., Kwatra, N. and Fedkiw, R. IEEE TVCG 14, No. 4 (2008)

Control of virtual character

[Shinar et al. 2008]





rigid/deformable simulator in Pixar's *WALL-E*