CS 130
Midterm

Winter 2020

Name	
Student ID	
Signature	

You may not ask any questions during the test. If you believe that there is something wrong with a question, write down what you think the question is trying to ask and answer that.

Question	**Points**	**Score**
True/False		
1	2	
2	2	
3	2	
4	2	
5	2	
6	2	
7	2	
8	2	
9	2	
10	2	
Multiple Choice/		
11	3	
12	3	
Short Answer		
13	6	
14	6	
Written		
15	10	
16	10	
Total	58	

1 True/False

For each question, indicate whether the statement is true or false by circling T or F, respectively. You get 2 points for answering a question correctly, -0.5 points for answering the question incorrectly, and 1 point for leaving it blank. (It is statistically to your advantage to answer only if you are at least 60% percent confident that your answer is correct).

1. ($\boxed{\text{T}}$/F) The dot product of two vectors that are orthogonal is zero.

2. (T/$\boxed{\text{F}}$) The cross product of two vectors that are orthogonal is the zero vector.

3. ($\boxed{\text{T}}$/F) Consider the implicit sphere equation $f(\mathbf{x}) = (\mathbf{x} - \mathbf{c}) \cdot (\mathbf{x} - \mathbf{c}) - r^2 = 0$. $f(\mathbf{x}) < 0$ for points strictly inside the sphere.

4. ($\boxed{\text{T}}$/F) Let $\mathbf{n}$ be a unit vector. Then $(\mathbf{n} \cdot \mathbf{v})\mathbf{n}$ is a projection of the vector $\mathbf{v}$ onto the direction $\mathbf{n}$.

5. ($\boxed{\text{T}}$/F) In Lambert's cosine law, the color of an object depends on the angle between the surface normal and the unit vector pointing to the light.

6. (T/$\boxed{\text{F}}$) The shadow cast by a point light will include both an umbra and penumbra region.

7. (T/$\boxed{\text{F}}$) Consider a nondegenerate triangle and the plane that contains that triangle. If a point on that plane is outside the triangle, then its barycentric coordinates are all negative.

8. ($\boxed{\text{T}}$/F) When antialiasing is used in a ray tracer, then more than one view ray is cast per pixel.

9. ($\boxed{\text{T}}$/F) To create a depth of field effect in a ray tracer, we should cast multiple view rays for each pixel, with the view ray endpoints sampled over an area representing a lens.

10. (T/$\boxed{\text{F}}$) Solving for the intersection of a ray with a plane leads to a quadratic equation in the ray parameter t.

2 Multiple Choice

For each question, circle exactly one of (a)-(e), unless otherwise stated.

11. Using the Phong reflectance model, the strength of the specular highlight is determined by the angle between

    (a) the view vector and the normal vector.

    (b) the light vector and the normal vector.

    (c) the light vector and the reflected light vector.

    $\boxed{\text{(d)}}$ the reflected light vector and the view vector.

    (e) none of the above.

12. Which type of shading would lead to recursion in a ray tracer?

    I. Phong shading

    II. Translucent shading

    III. Reflective shading

    (a) III only

    (b) I and II only

    (c) I and III only

    $\boxed{\text{(d)}}$ II and III only

    (e) I, II, and III

3 Short Answer

13. Given two vectors, **u** and **v**,

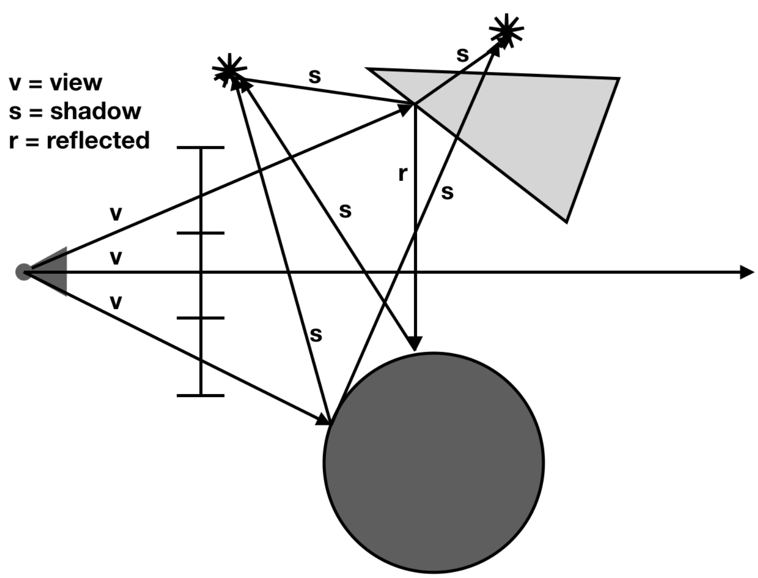
    (a) how do you determine if the vectors are orthogonal?

    Solution: If $\mathbf{u} \cdot \mathbf{v} = 0$, then **u** and **v** are orthogonal. Otherwise, they are not.

    (b) how do you generate a third vector that is normal to both? Assume the angle between the vectors is not 0 or 180 degrees.

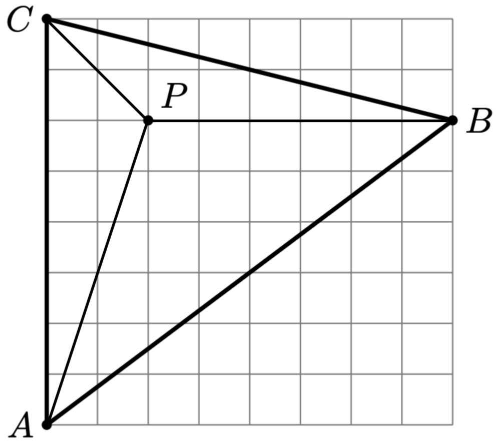
    Solution: We are assuming these are three-dimensional vectors. In this case, you can use the cross product to generate a vector orthogonal to both. Specifically, the vector **w** defined as $\mathbf{w} = \mathbf{u} \times \mathbf{v}$, or $\mathbf{w} = \mathbf{v} \times \mathbf{u}$ is orthogonal to **u** and **v**.

14. The image below depicts a simple 2D raytracing setup. The 1D images has 3 pixels. The light grey triangle is reflective. The dark grey sphere is not reflective. The two yellow stars represent point lights. Draw all view rays, shadow rays, and reflected rays that would be cast by the ray tracer. (You do not need to draw the rays created for the Phong shading computation). Label your rays as "view", "shadow", or "reflected".

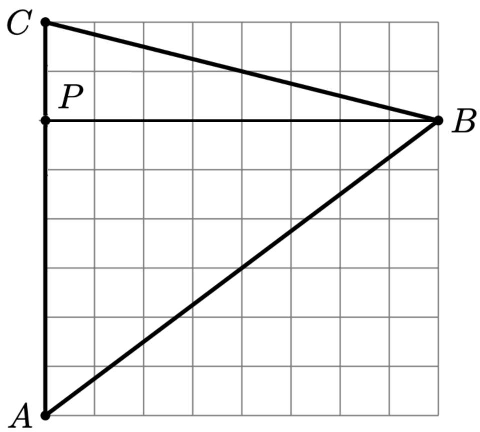


4 Written Response

15. For both cases below, find the barycentric coordinates of the point P with respect to the triangle ABC depicted. Specifically, give the numerical value of α, β, and γ for each case.

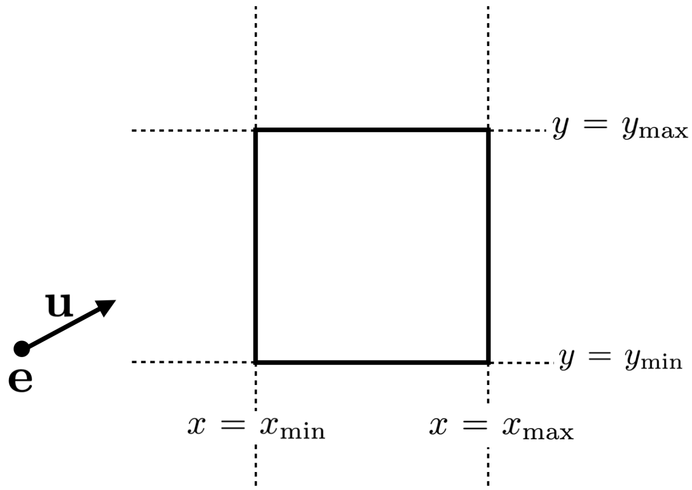


Solution: We can use areas to the compute the barycentric weights. We use the formula $\frac{1}{2}$ base * height, getting a base and height for each triangle depicted from the grid lines. The whole triangle as area $\frac{1}{2}8 * 8 = 32$. The triangles opposite A, B, and C, have areas $\frac{1}{2}6 * 2 = 6$, $\frac{1}{2}8 * 2 = 8$, and $\frac{1}{2}6 * 6 = 18$, respectively. Hence the barycentric weights are $\boxed{\alpha = \frac{6}{32}, \beta = \frac{8}{32}, \gamma = \frac{18}{32}}$. Alternatively, you can use the grid lines to observe that P is $\frac{1}{4}$ of the distance from AC to B, giving $\beta = \frac{1}{4}$, and that the remaining $\frac{3}{4}$ should be assigned to A and C as $\alpha = \frac{1}{4} * \frac{3}{4} = \frac{3}{16}$ and $\beta = \frac{3}{4} * \frac{3}{4} = \frac{9}{16}$, yielding the same results.



Solution: We can observe that $\beta = 0$ because P is on the segment opposite B. Then the remaning weight of 1 is divided among A and C as $\alpha = \frac{1}{4}$ and $\gamma = \frac{3}{4}$, respectively. Alternatively, one can also use triangle areas to arrive at the same result, $\boxed{\alpha = \frac{1}{4}, \beta = 0, \text{ and } \gamma = \frac{3}{4}}$.

16. Give a detailed algorithm for determining if a ray intersects an axis-aligned box in 2D. Assume the box has left, right, bottom, and top edges at $x = x_{\min}$, $x = x_{\max}$, $y = y_{\min}$, $y = y_{\max}$, respectively. Let the ray be given by $\mathbf{r}(t) = \mathbf{e} + t\mathbf{u}$, $t \geq 0$. An example case is depicted below.



(Hint: First find the intersection of the ray with each of the four planes $x = x_{\min}$, $x = x_{\max}$, $y = y_{\min}$, $y = y_{\max}$.)

Solution: We first find the intersection of the ray with each plane. Let $\mathbf{e} = (e_x, e_y, e_z)$ and $\mathbf{u} = (u_x, u_y, u_z)$.

The ray intersects the plane $x = x_{\min}$ with the x component of the ray is $x_{\min}$, i.e., when

$$e_x + t_{xmin} u_x = x_{\min}.$$

If $u_x \neq 0$, then $t_{xmin} = \frac{x_{\min} - e_x}{u_x}$. Analogously, $t_{xmax} = \frac{x_{\max} - e_x}{u_x}$. Let $t_{x1} = \min(t_{xmin}, t_{xmax})$, and $t_{x2} = \max(t_{xmin}, t_{xmax})$. Then the ray is between $x_{\min}$ and $x_{\max}$ for the interval $I_x = [t_{x1}, t_{x2}]$. On the other hand, if $u_x = 0$, then the ray is parallel to the y-axis. We check if $e_x \in [x_{\min}, x_{\max}]$. In this case $I_x = [-\infty, \infty]$. Otherwise $I_x = \emptyset$, the empty set.

We define the interval I_y analogously for the y direction.

The intersection of the ray with the box then occurs in the interval $I = I_x \cap I_y$. If this intersection contains a $t \geq 0$, then the ray intersects the box. Otherwise, the ray does not intersect the box.