Lighting and Shading

Why we need shading

- Suppose we build a model of a red sphere
- We get something like

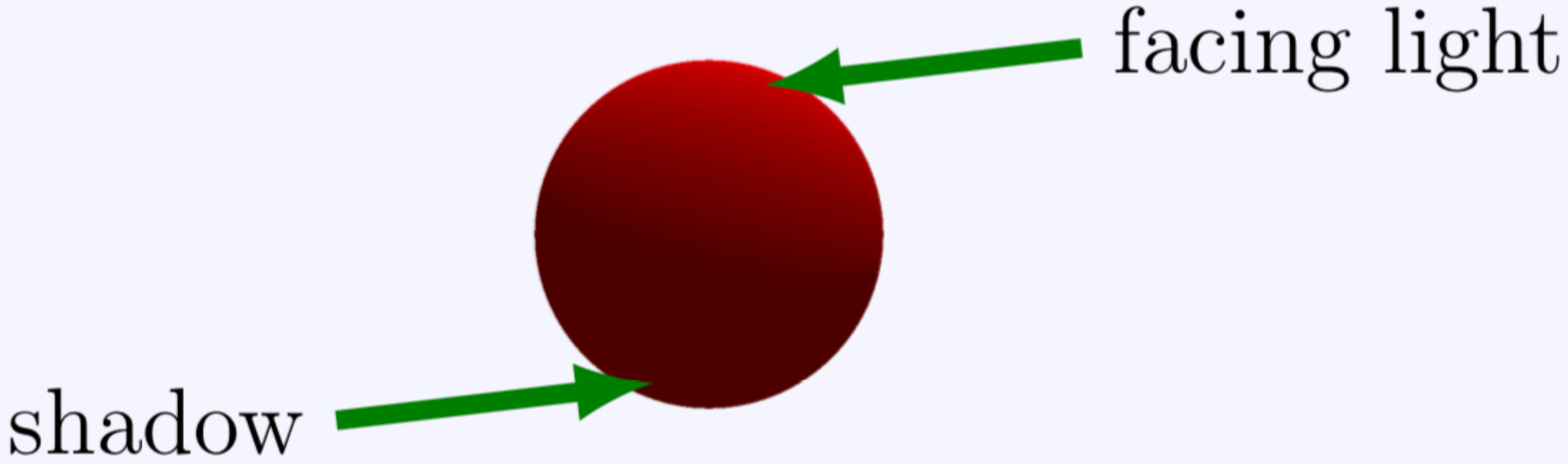
- But we want

Shading

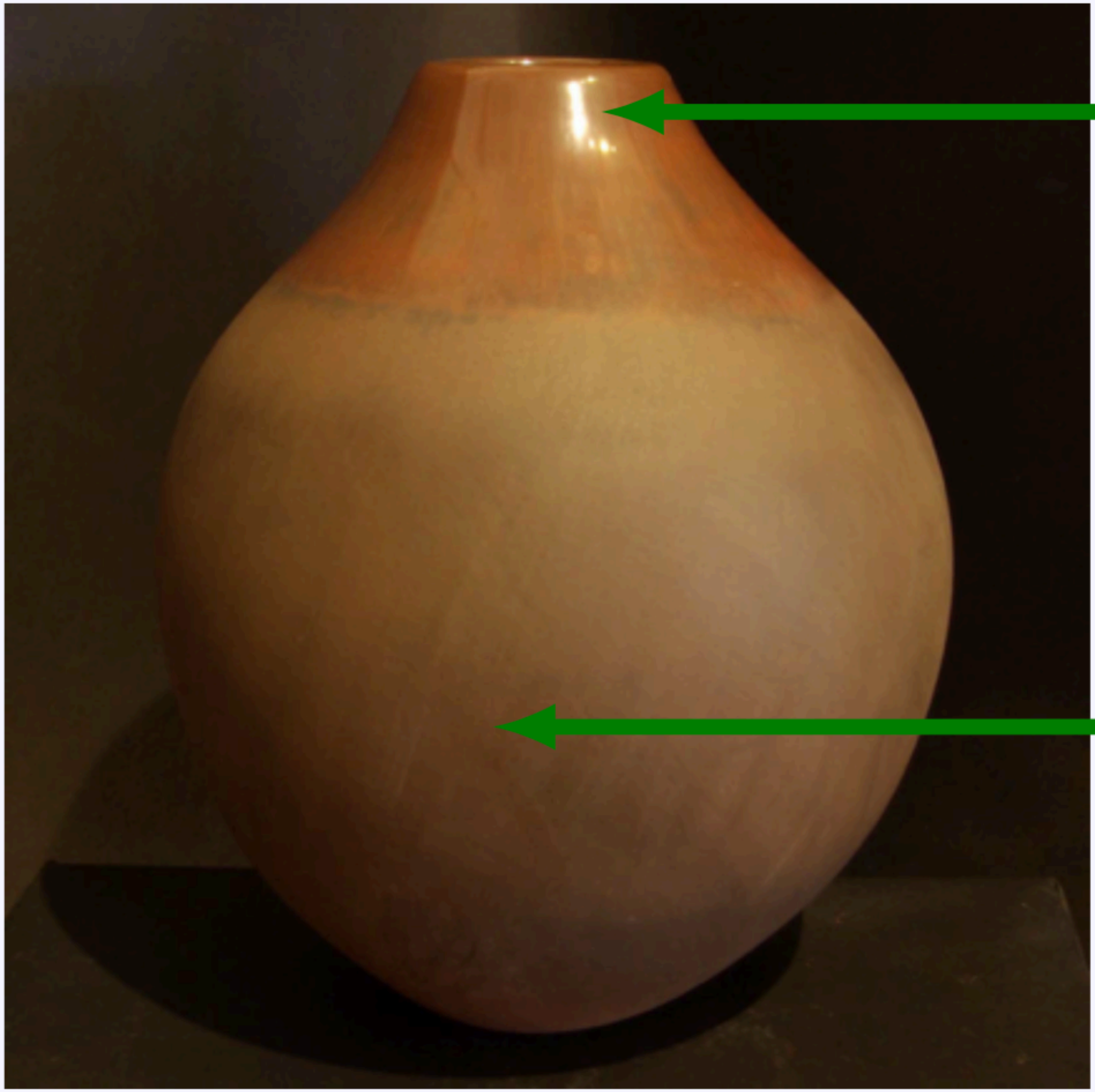
- Why does a real sphere look like this?

Shading - lighting

- Why does a real sphere look like this?



Shading - material properties

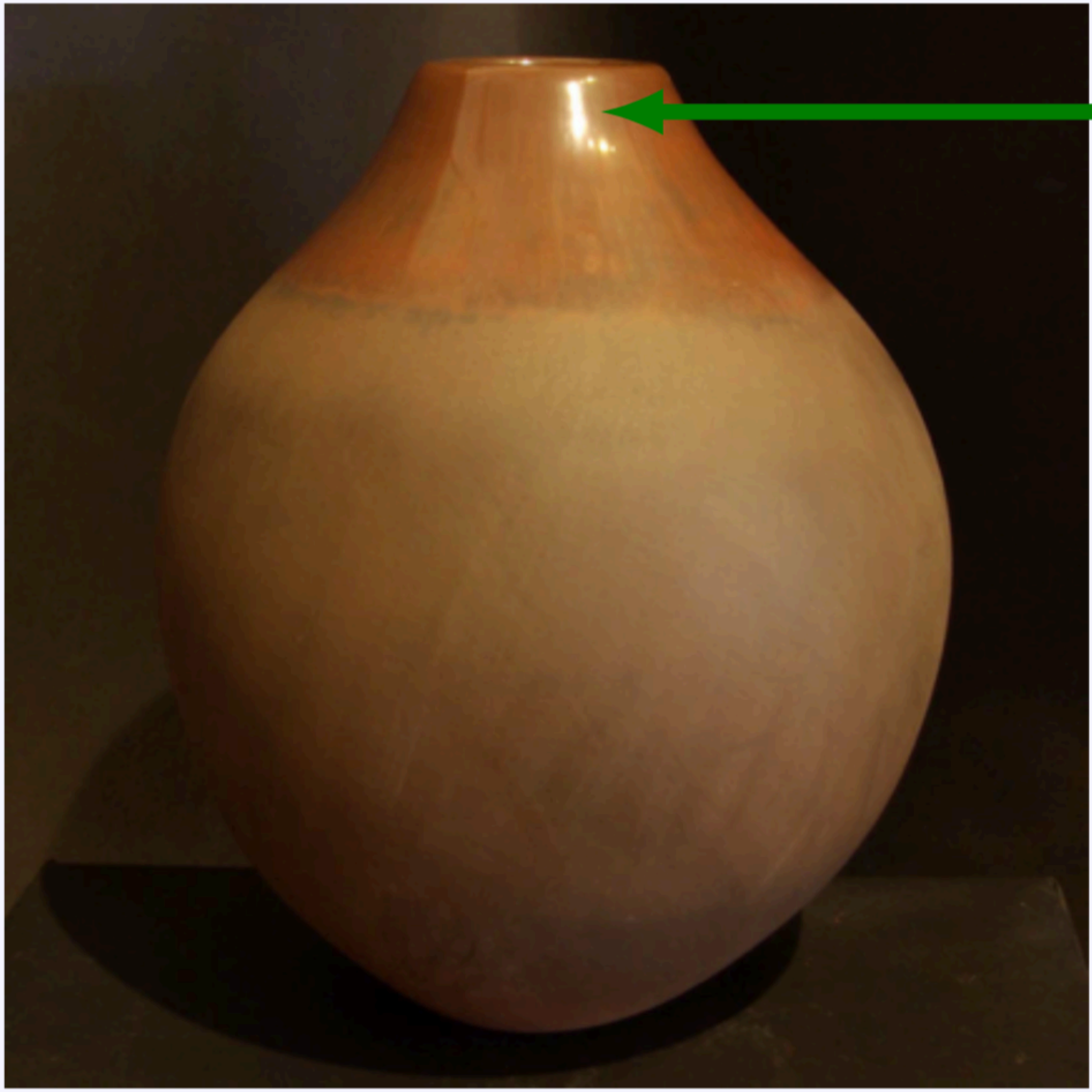


shiny

matte

Image source: [1]

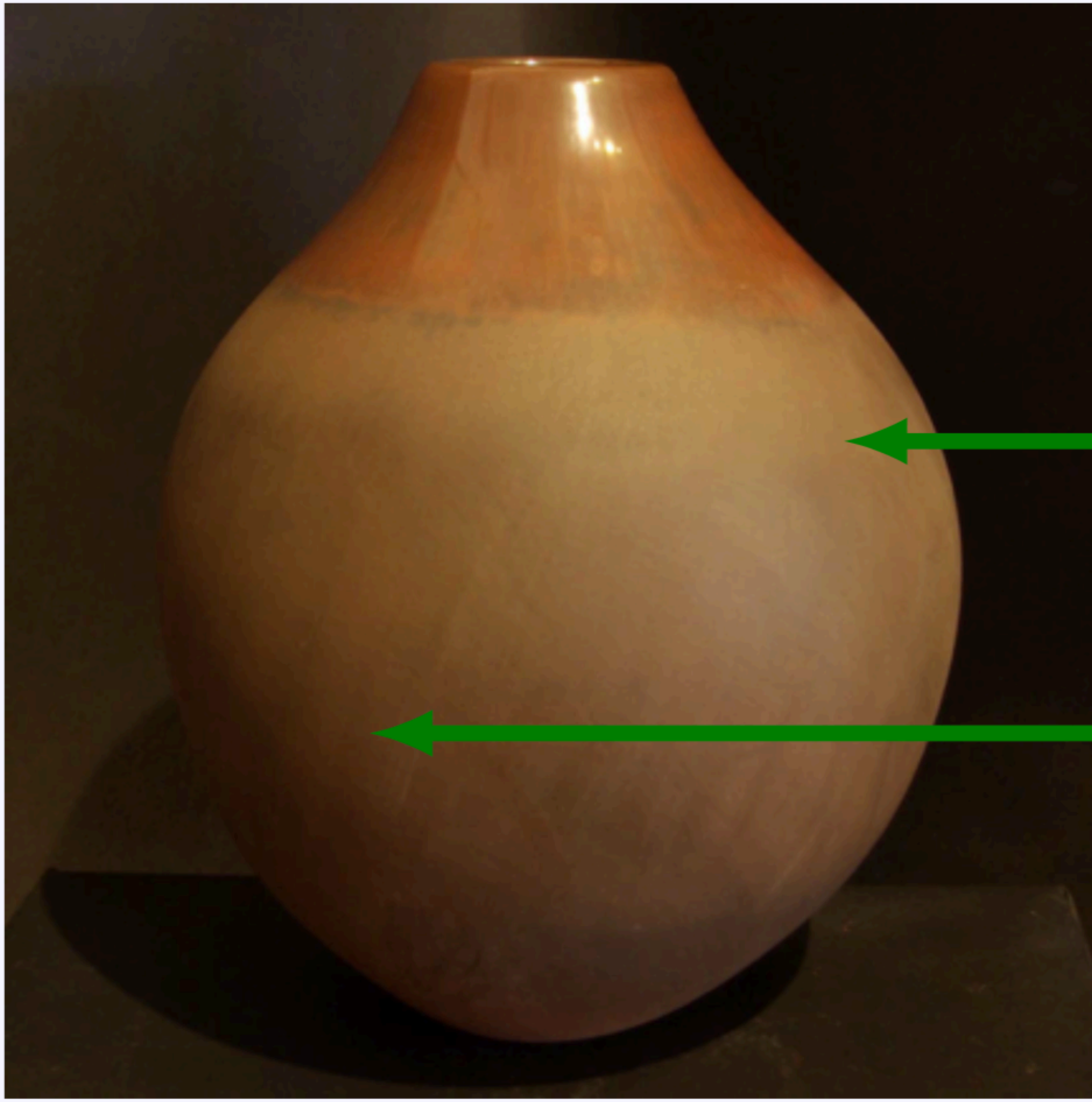
Shading - viewing location



What if I move?

Image source: [1]

Shading - surface orientation



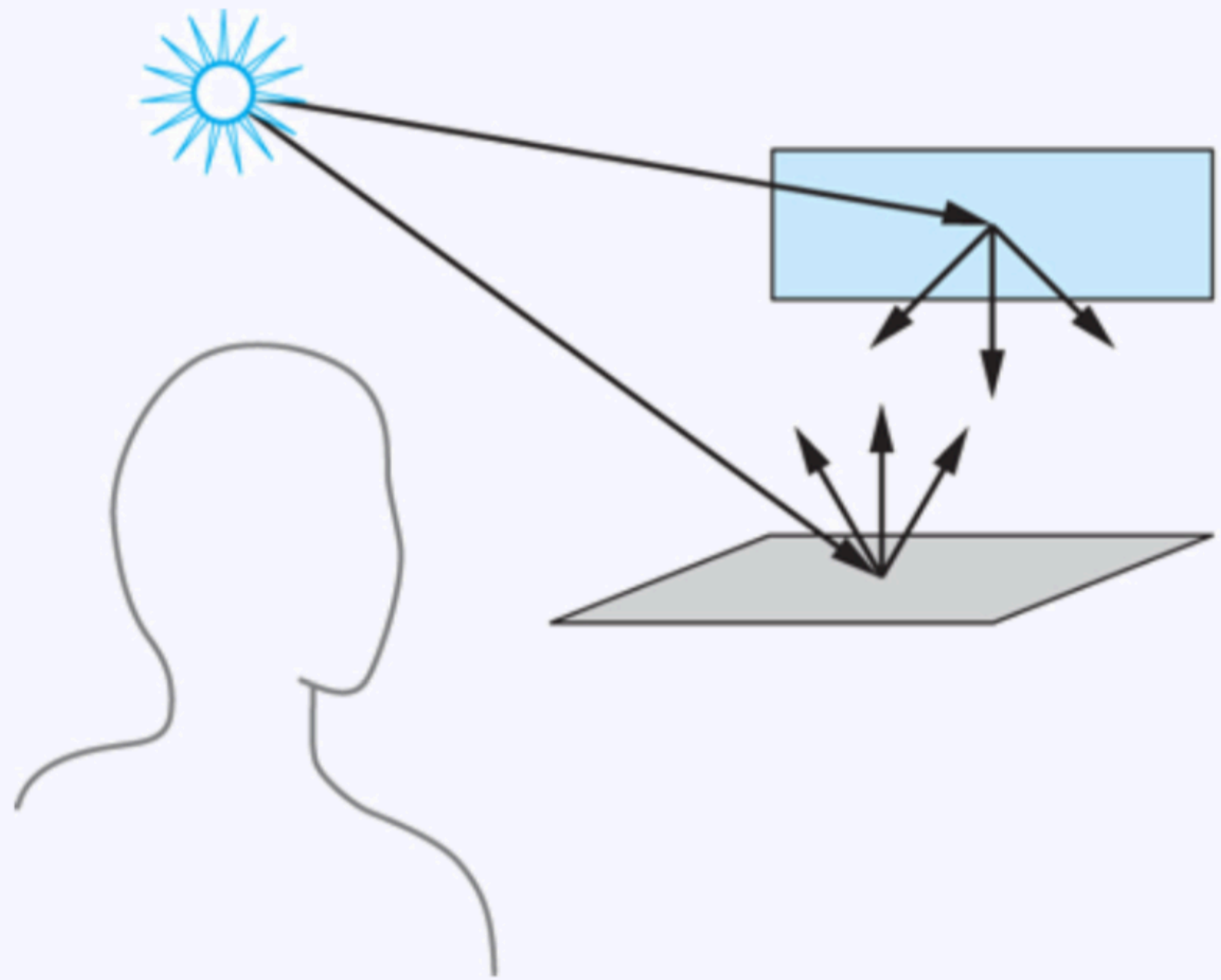
well lit

poorly lit

Image source: [1]

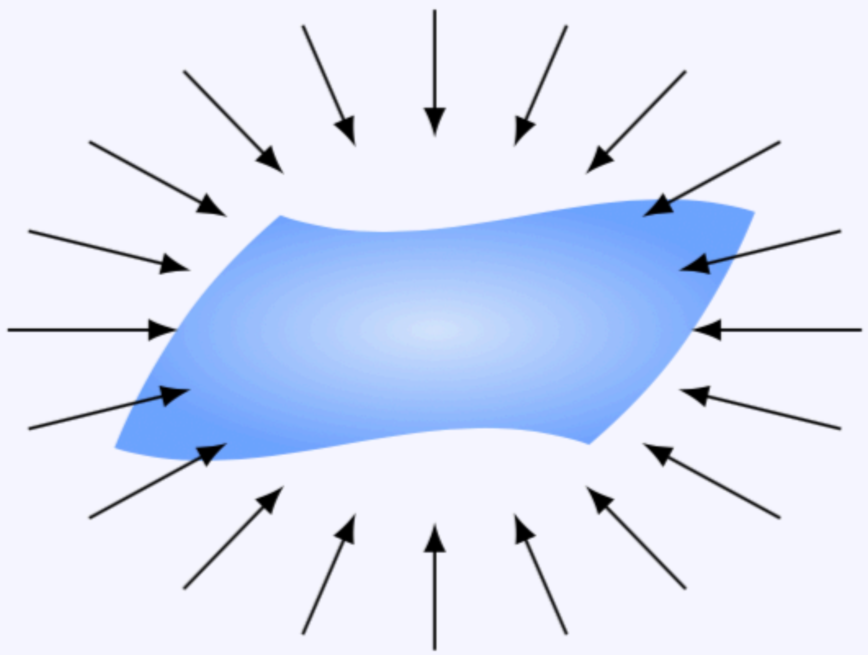
General rendering

- Based on physics
  - conservation of energy
- Surfaces can
  - absorb light
  - emit light
  - reflect light
  - transmit light

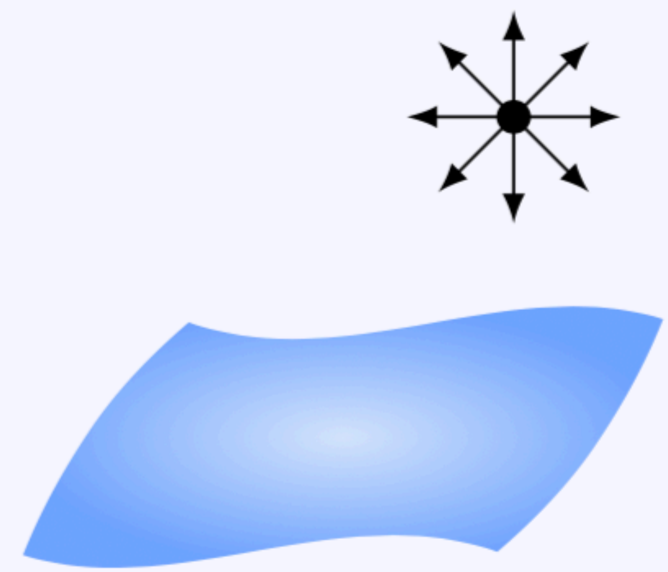


Light Sources

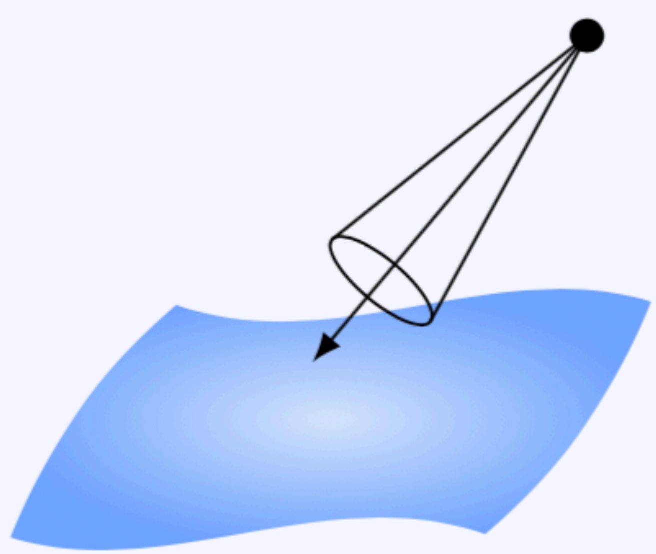
Idealized light sources



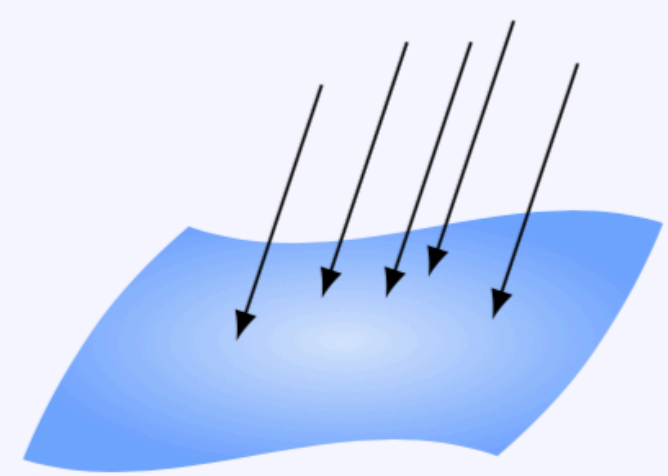
ambient



point light



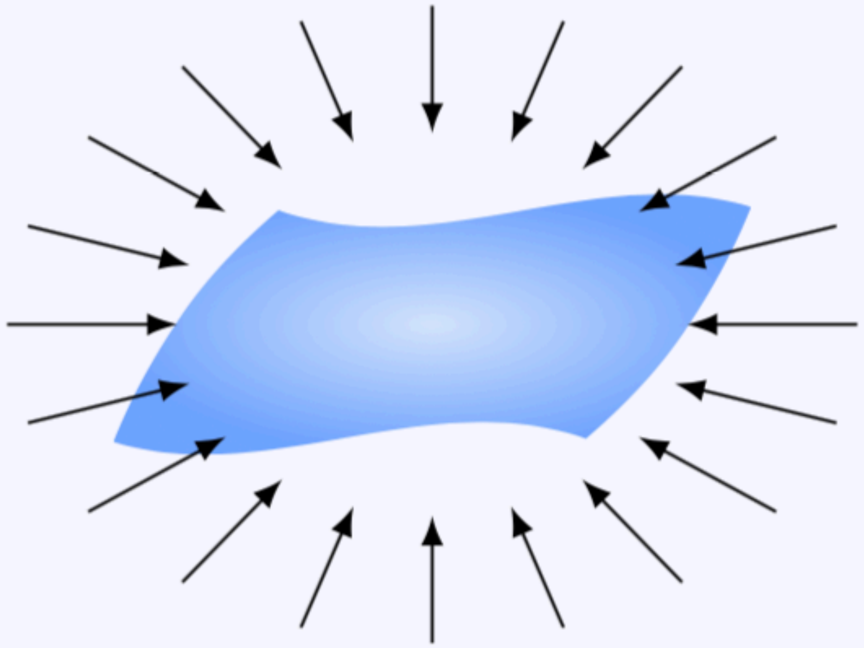
spotlight



directional light

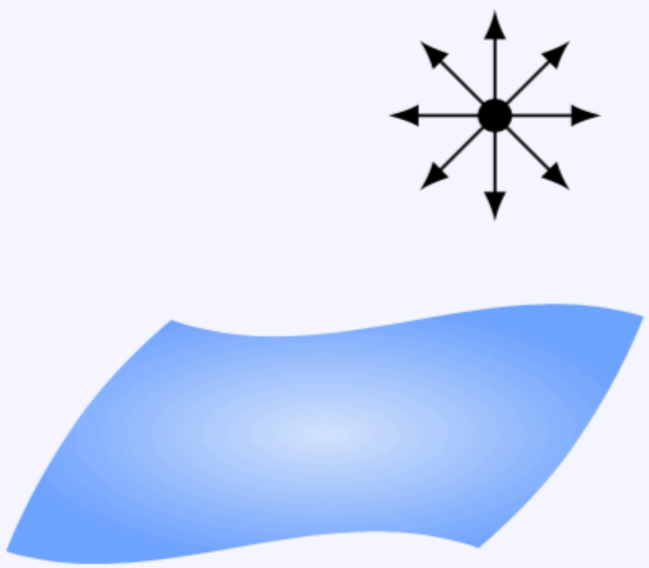
Ambient light

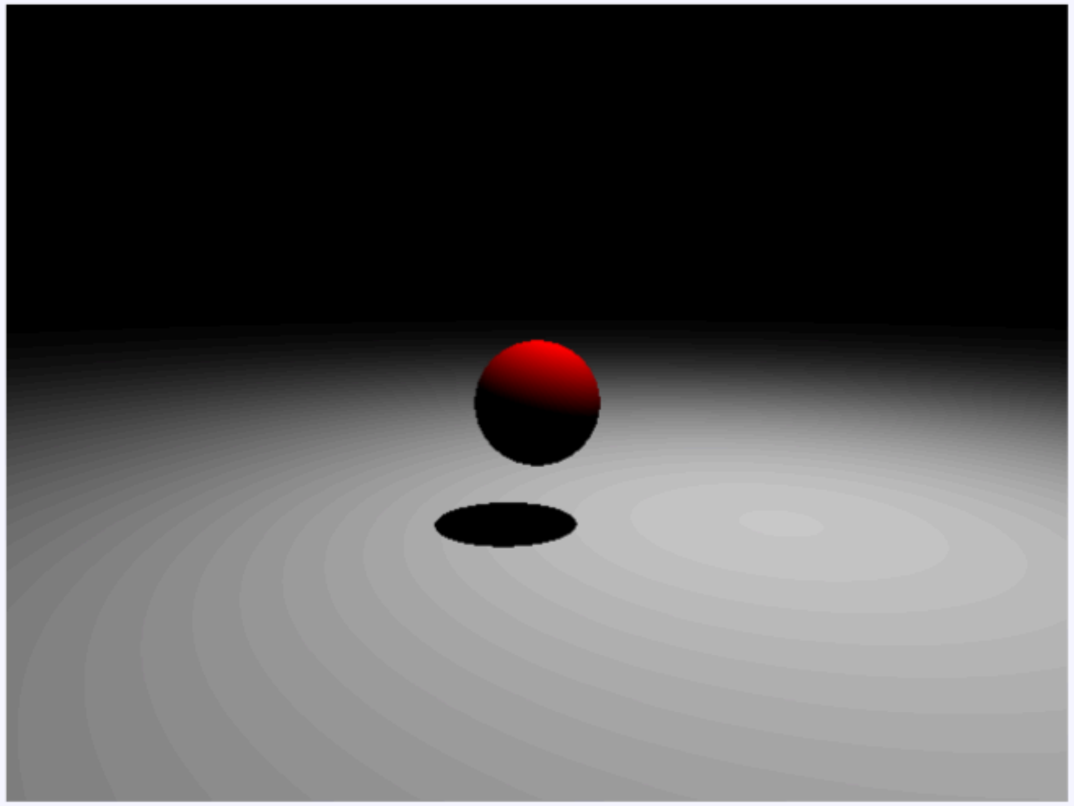
- Achieve uniform light level
- No shadows
- Same light level everywhere



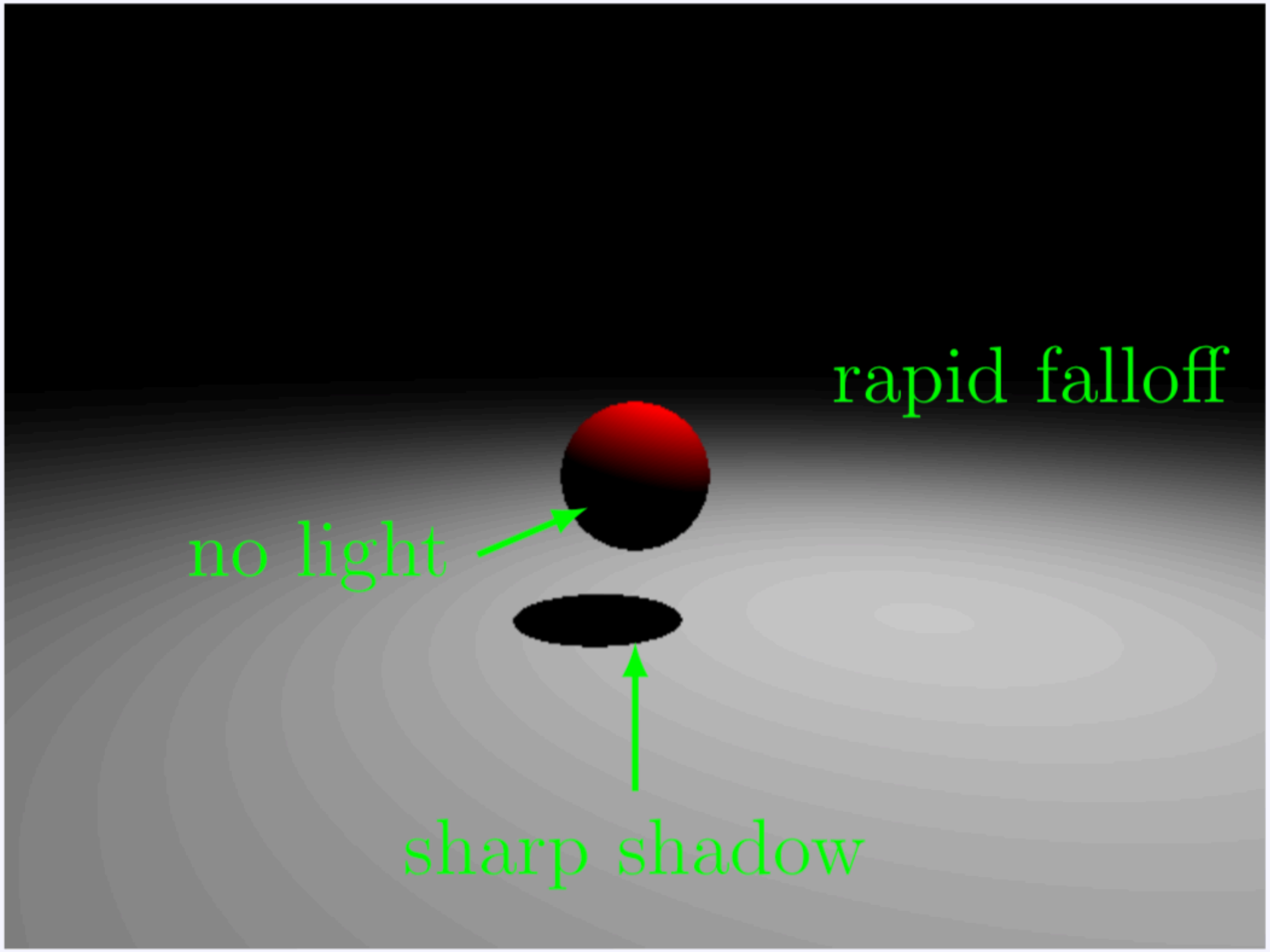
Point light

- Light emitted from a point $\mathbf{p}$
- Uniform in all directions
- Falls off with distance: $\ell(\mathbf{x}) = \frac{1}{\|\mathbf{x} - \mathbf{p}\|^2} L$

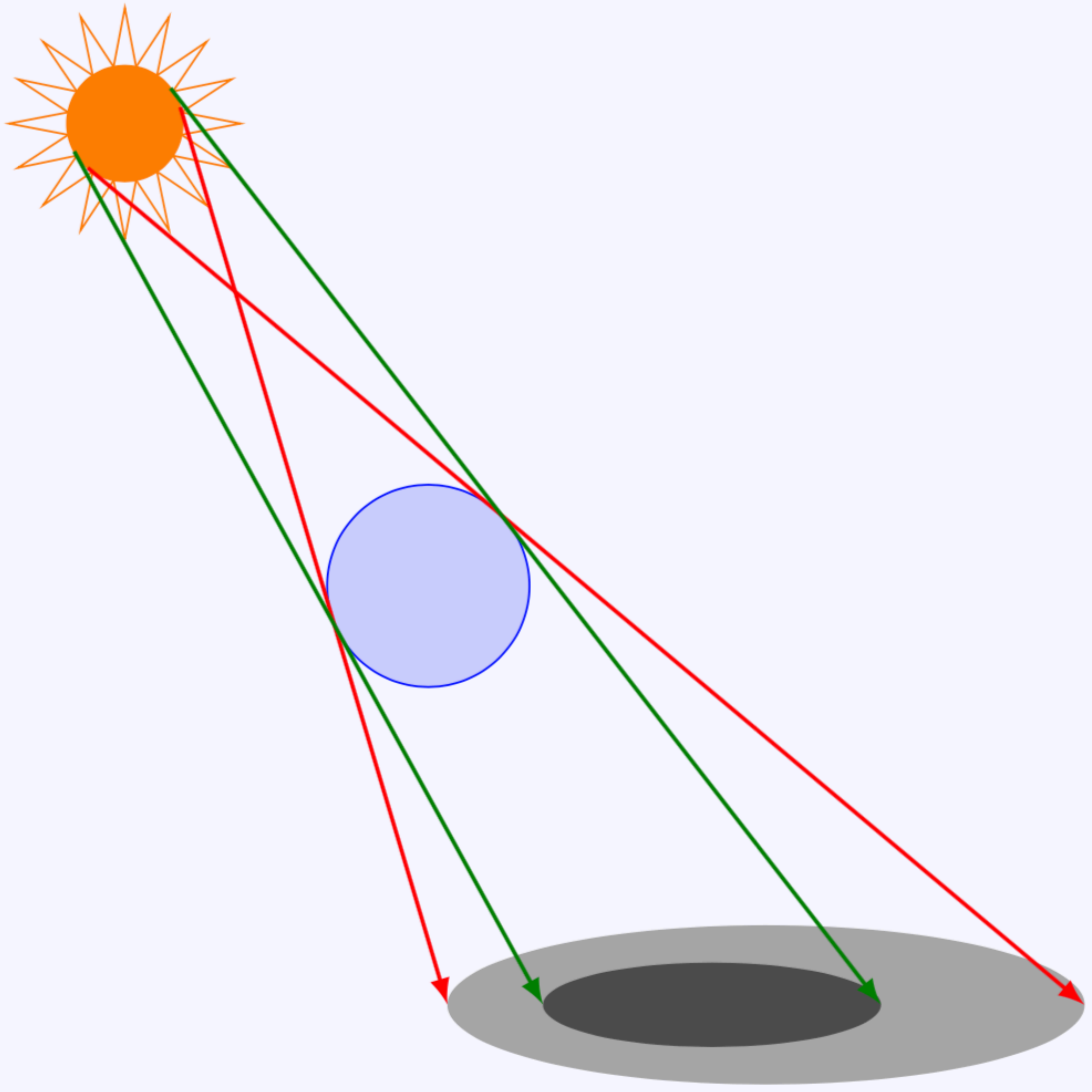




Point light - limitations

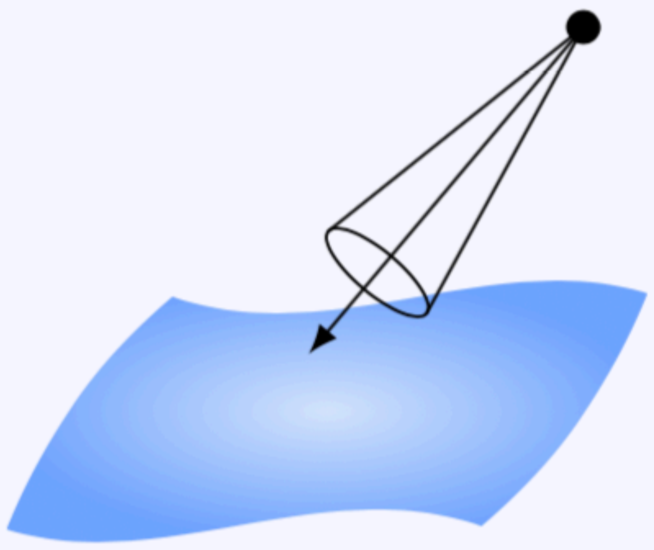


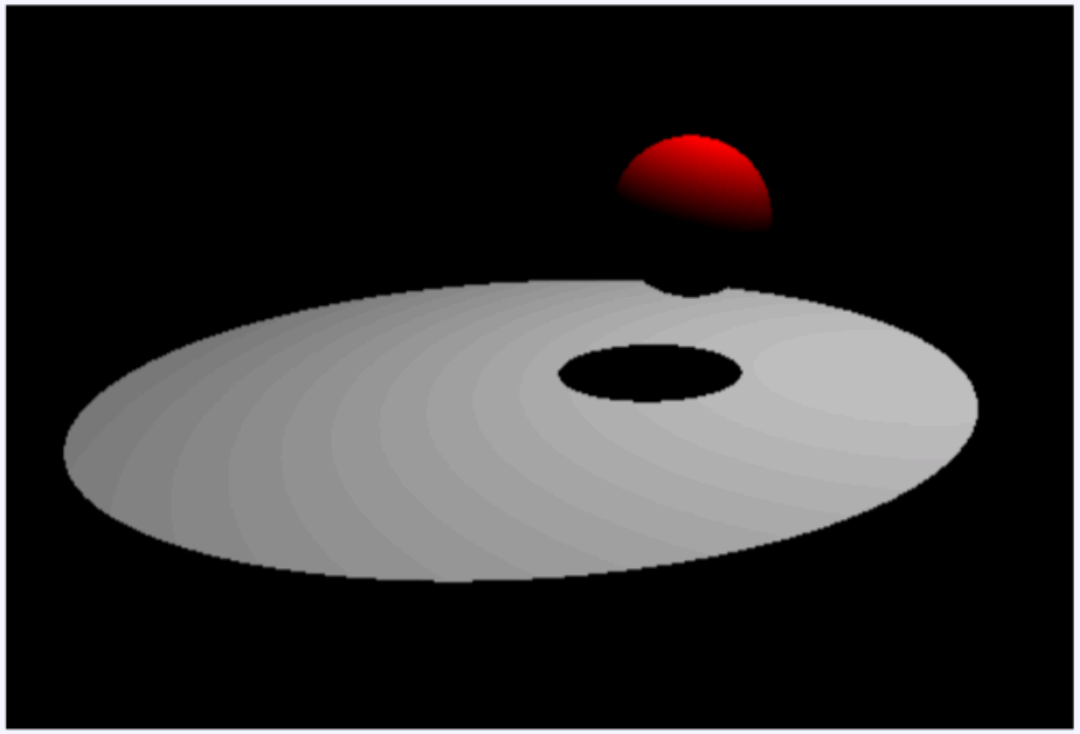
Soft shadows



Spotlight

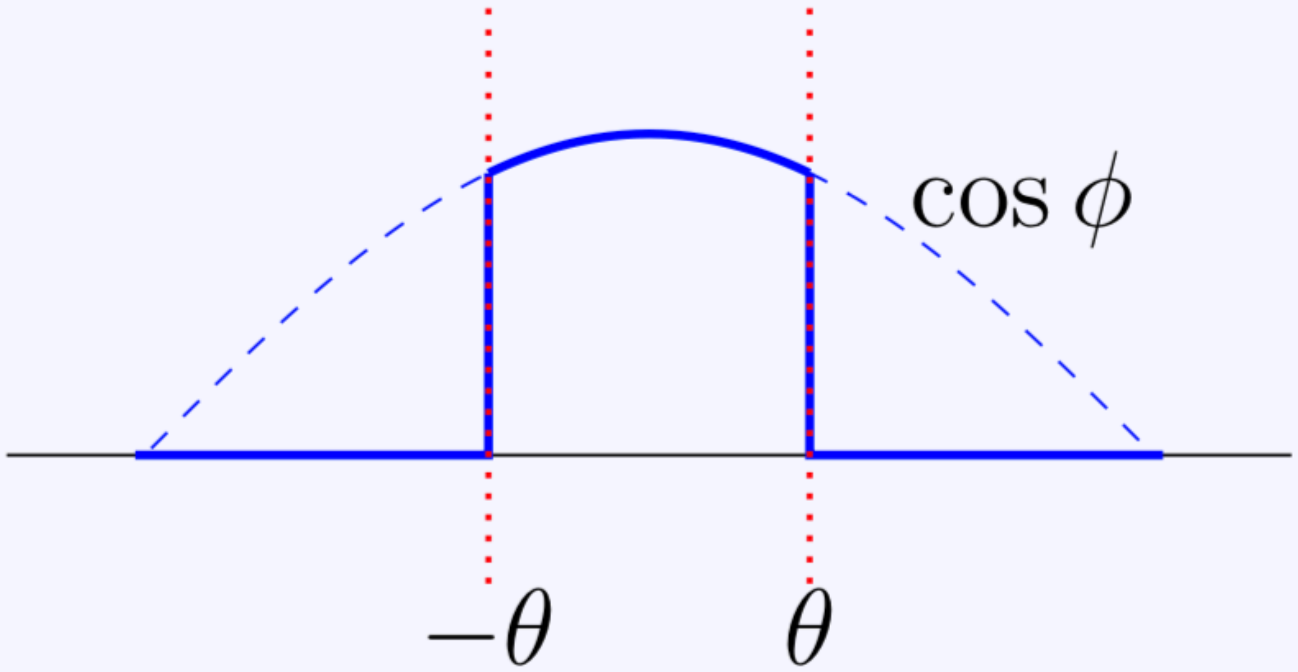
- Light emitted from a point $\mathbf{p}$
- Emited in a cone
- Brightest in middle of cone
- Falls off with distance

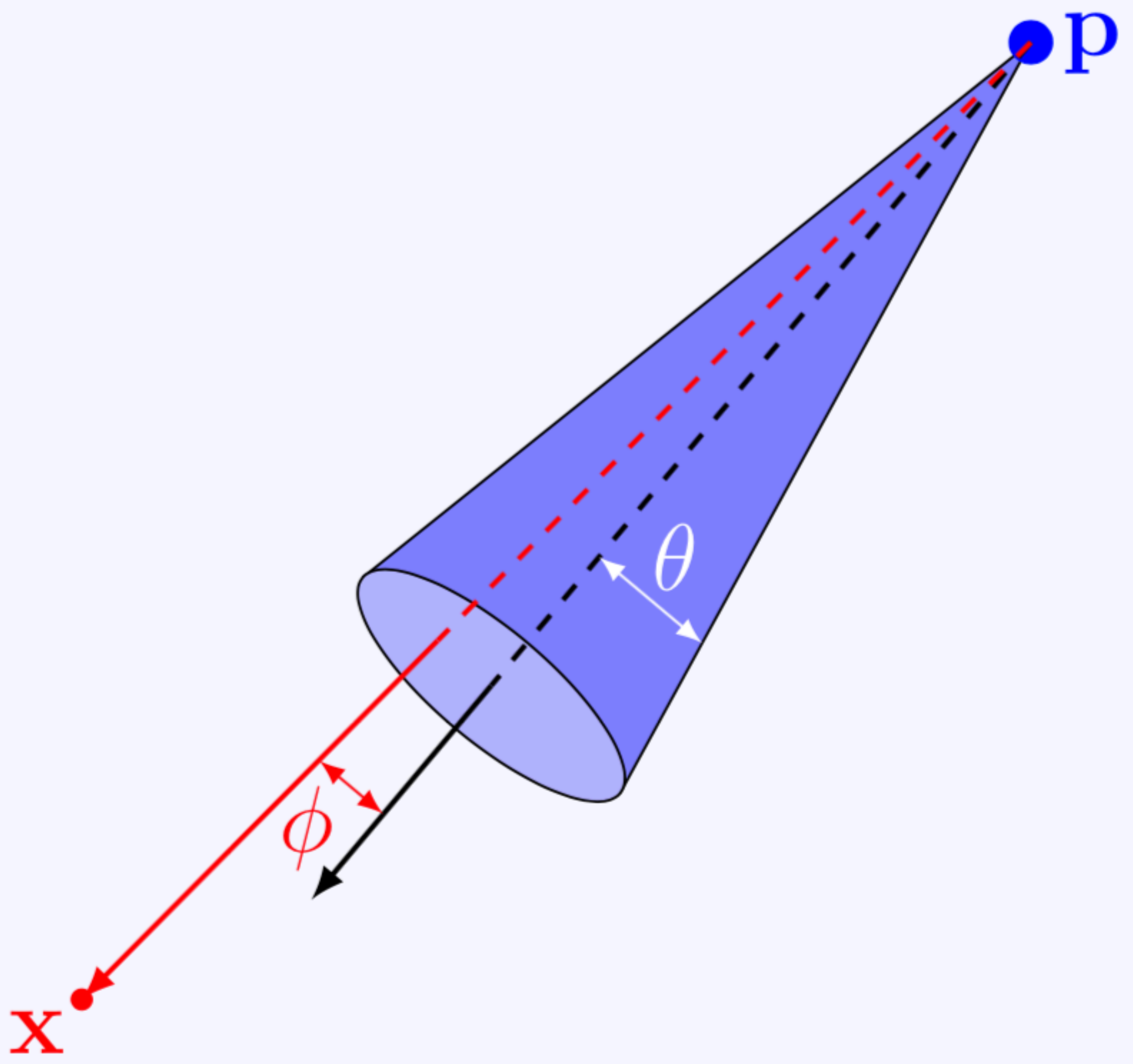




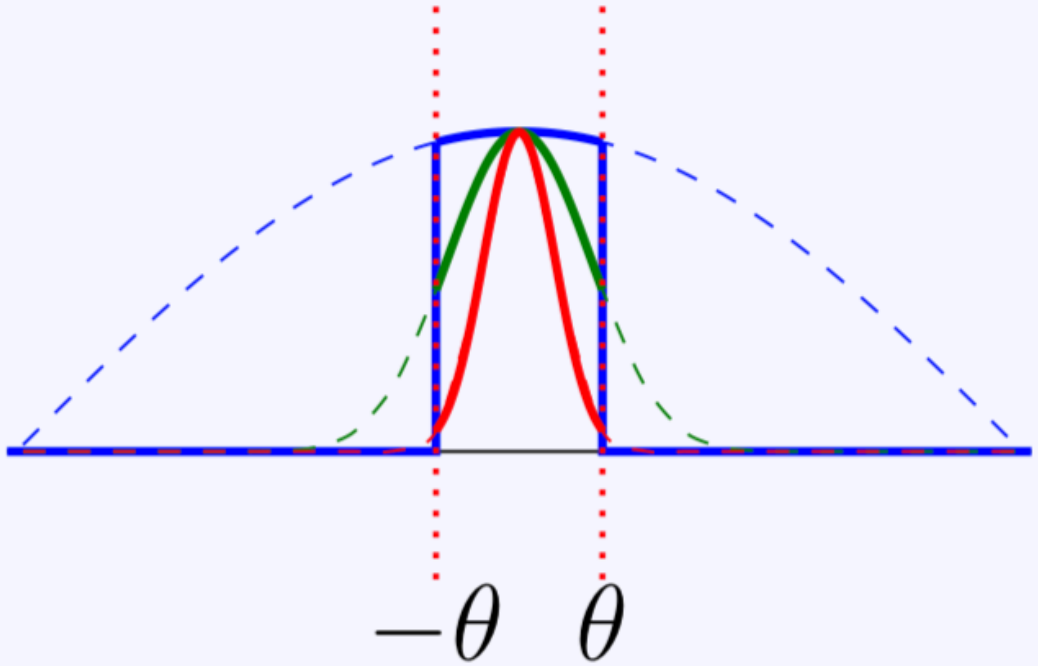
Spotlight

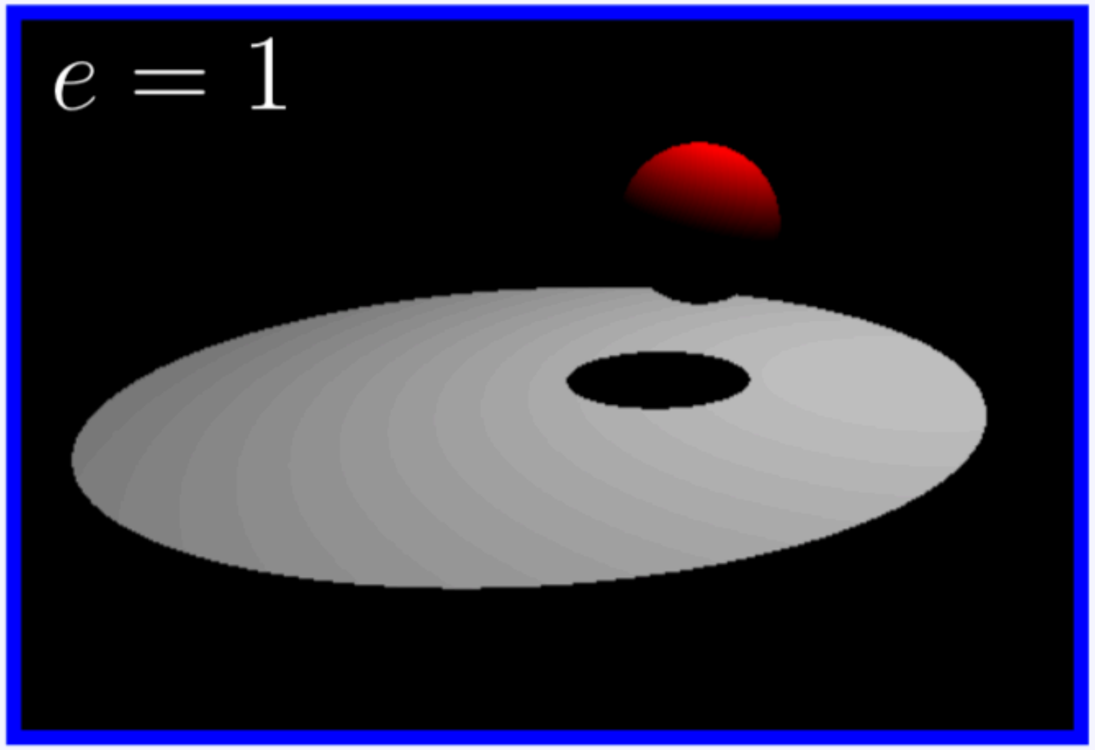
$$\ell(\mathbf{x}) = \frac{\cos^e \phi}{\|\mathbf{x} - \mathbf{p}\|^2} L$$

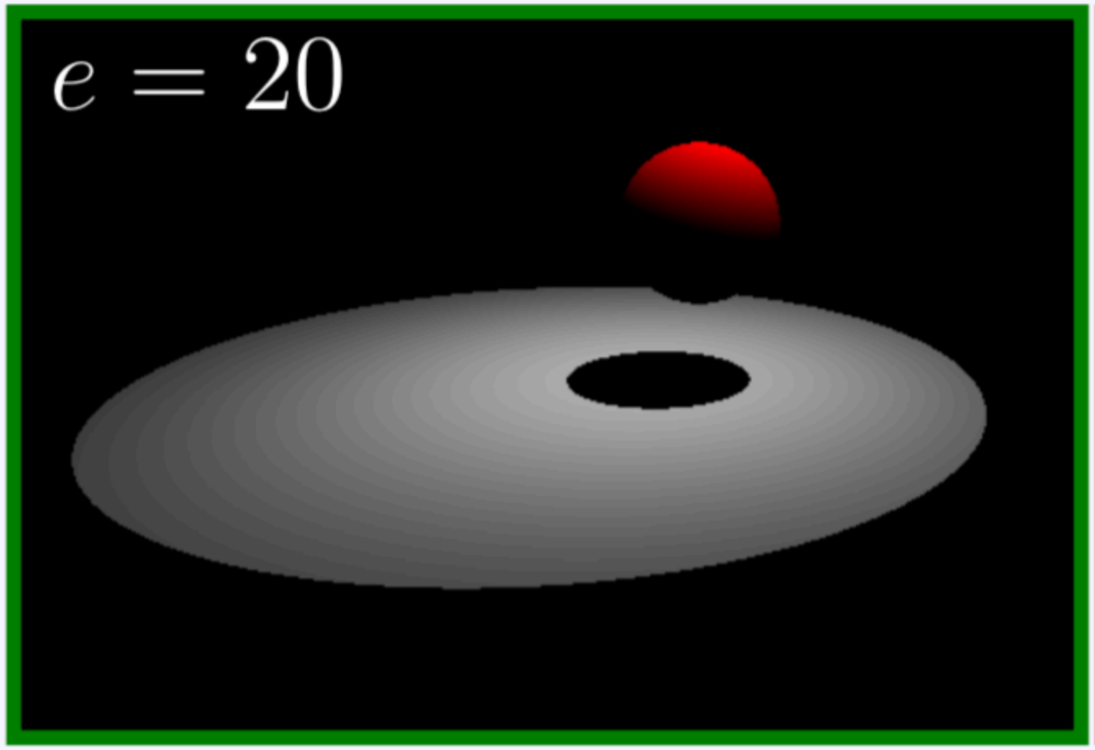


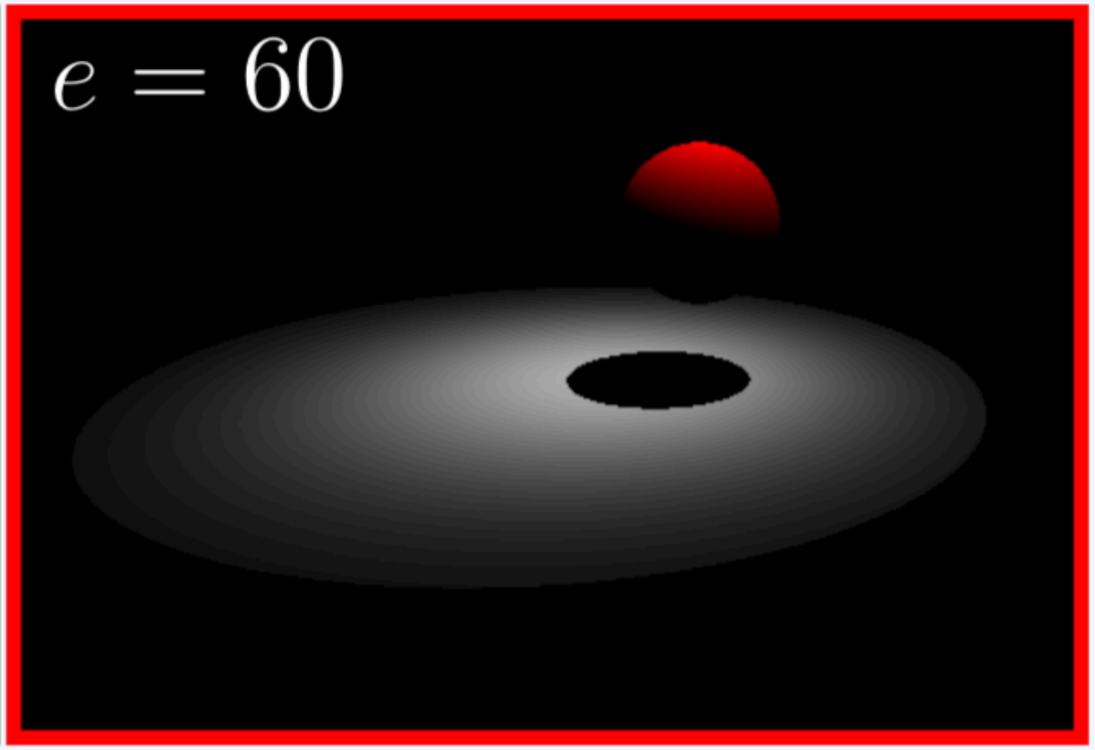


Spotlight - exploring e



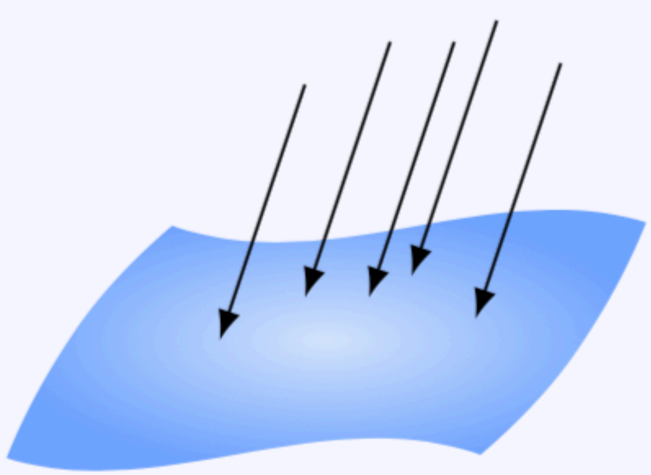


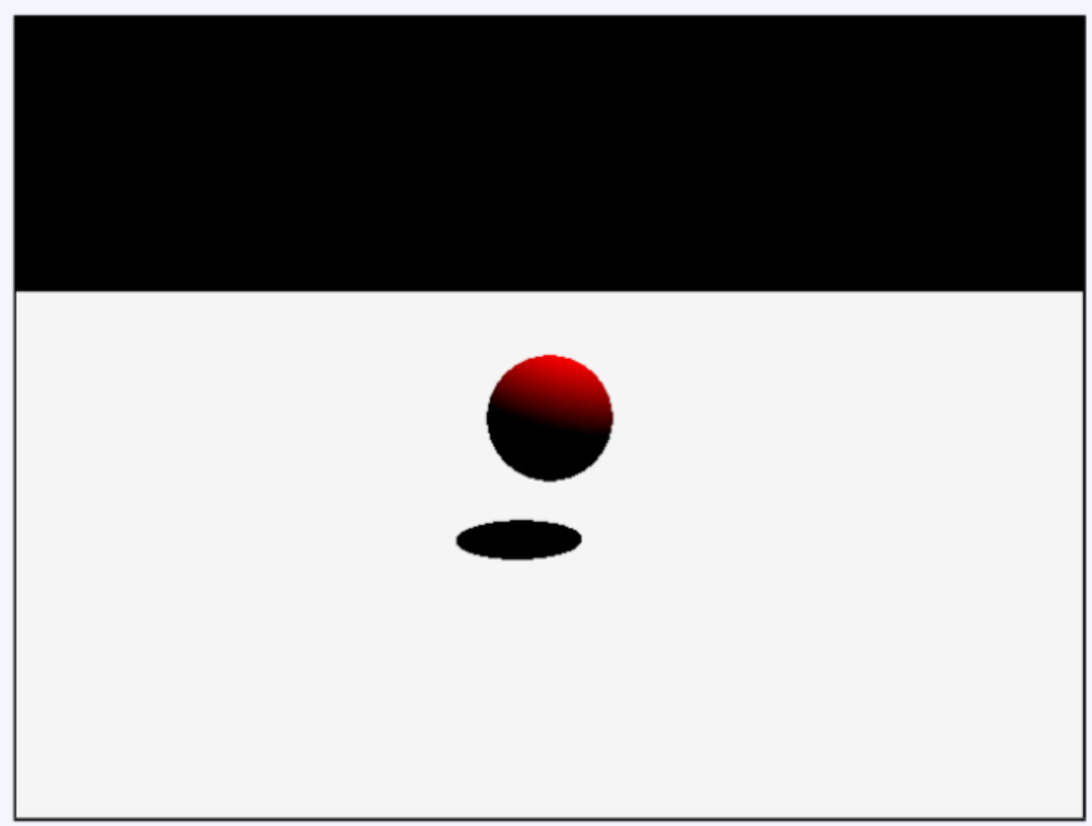


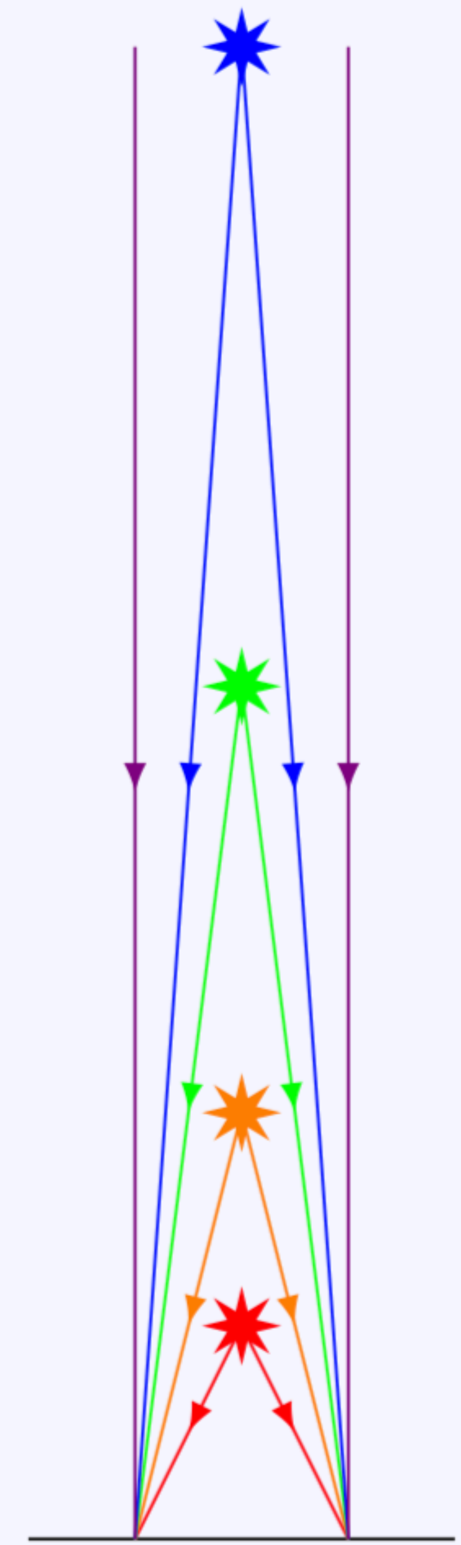


Directional light

- Light source at infinity
- Rays come in parallel
- No falloff
- Characterized by direction







Shading

Lambertian reflection model

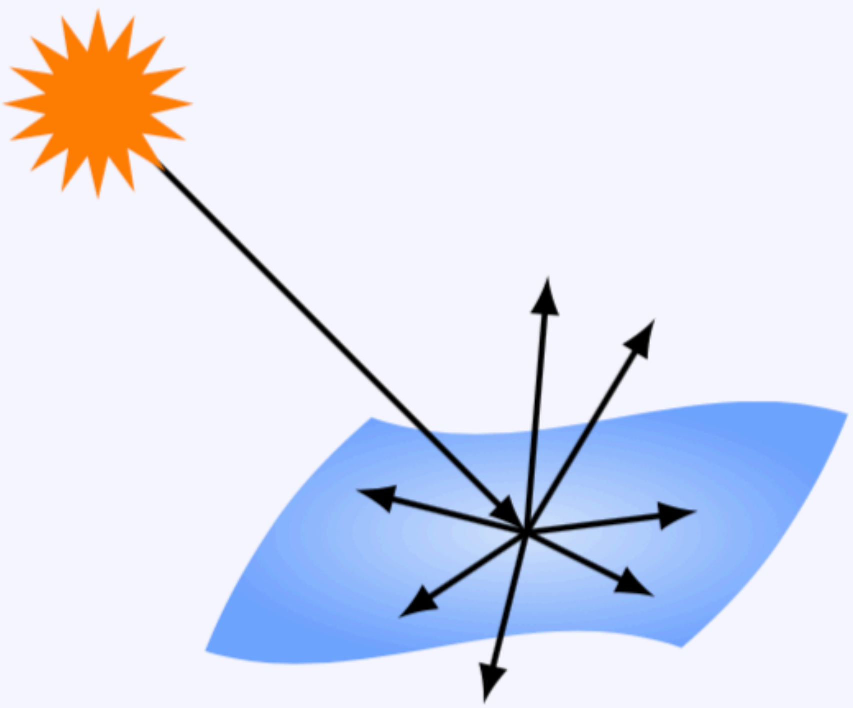
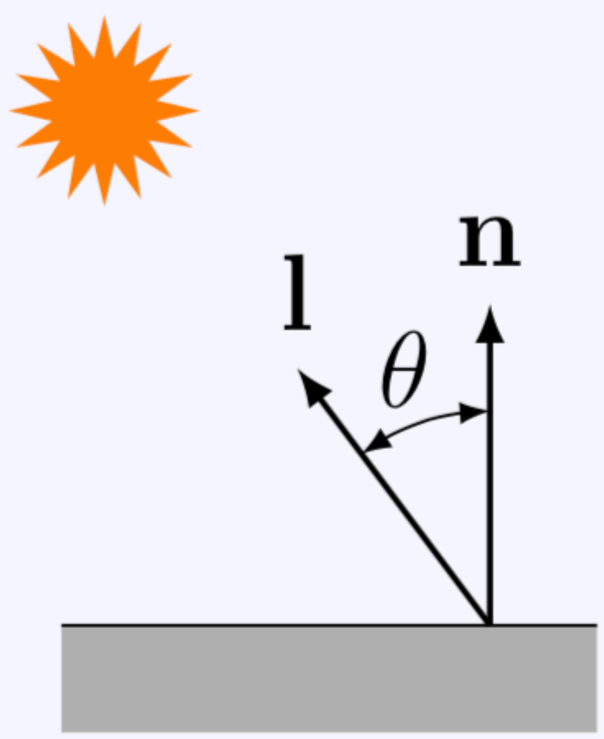
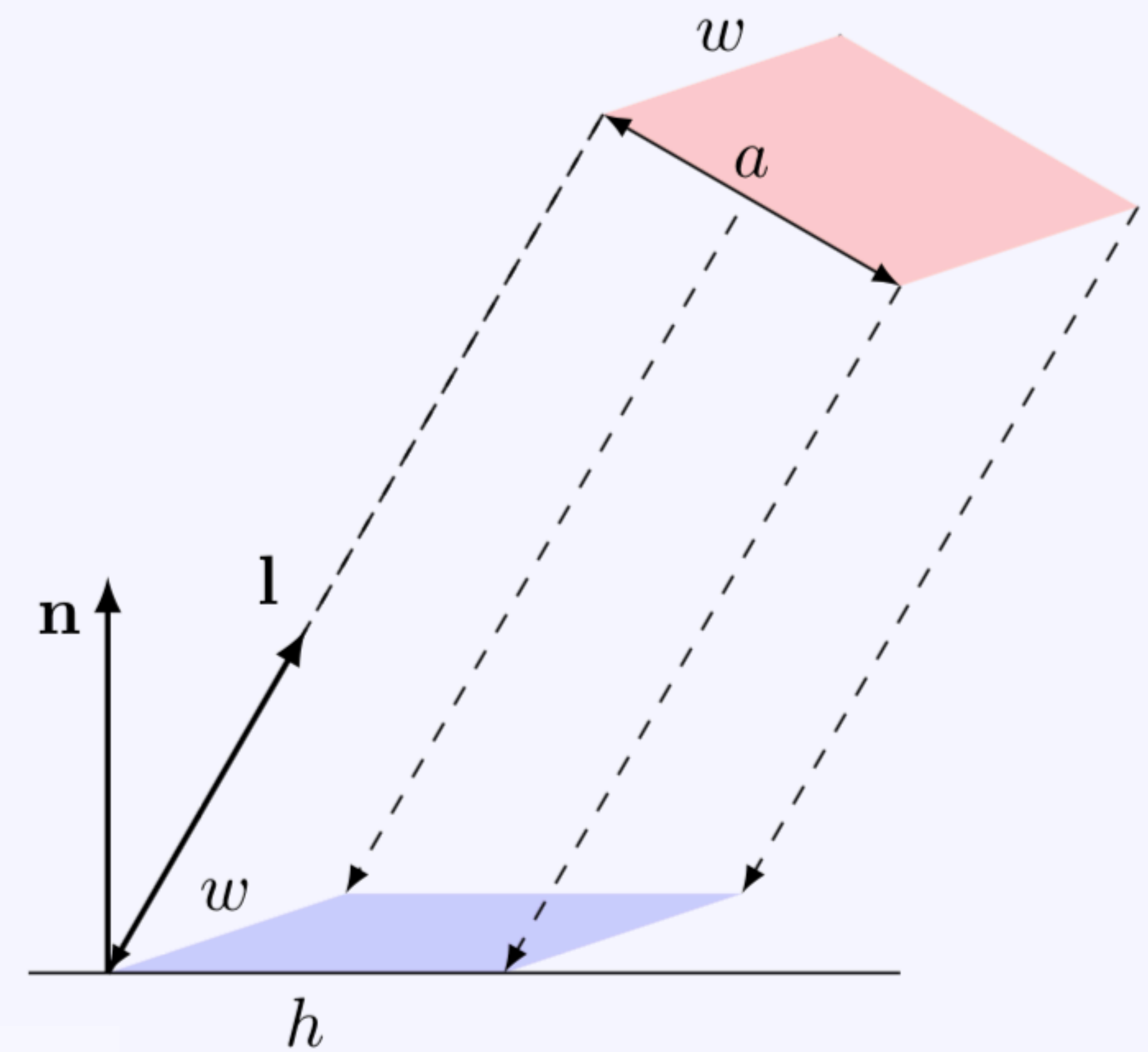


Image source: [1]

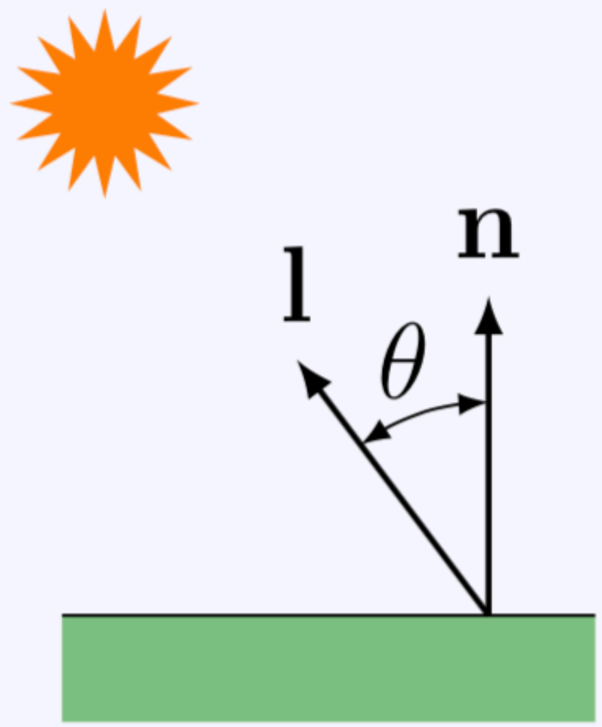
Lambertian reflection model

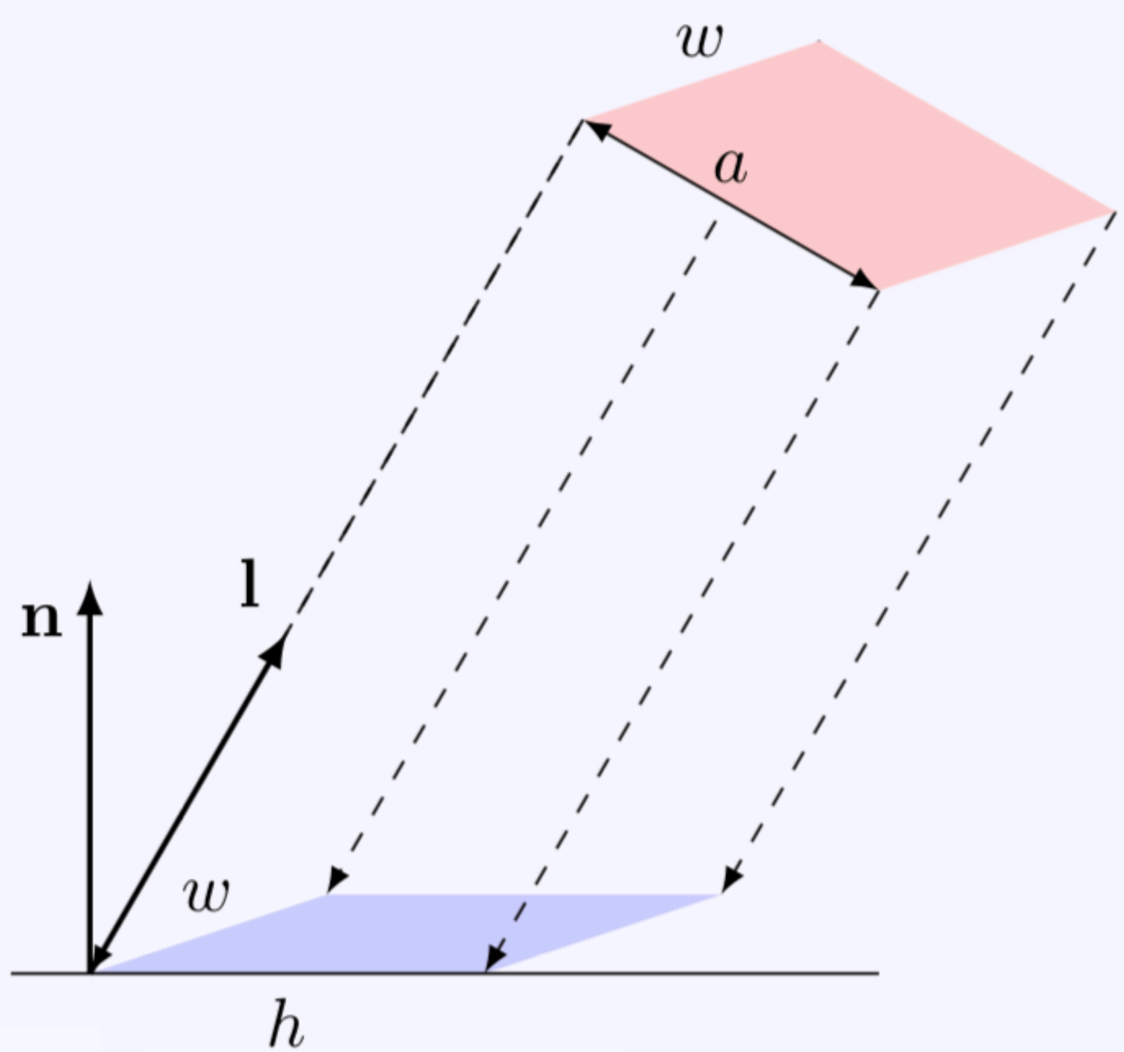




	Light	Incident
Intensity	L	L'
Energy	$E = Lwa$	$E = L'wh$

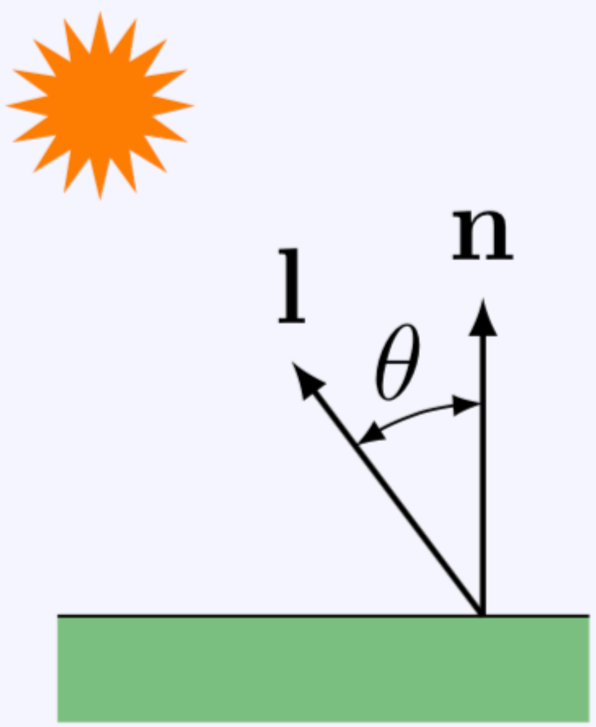
Lambertian reflection model

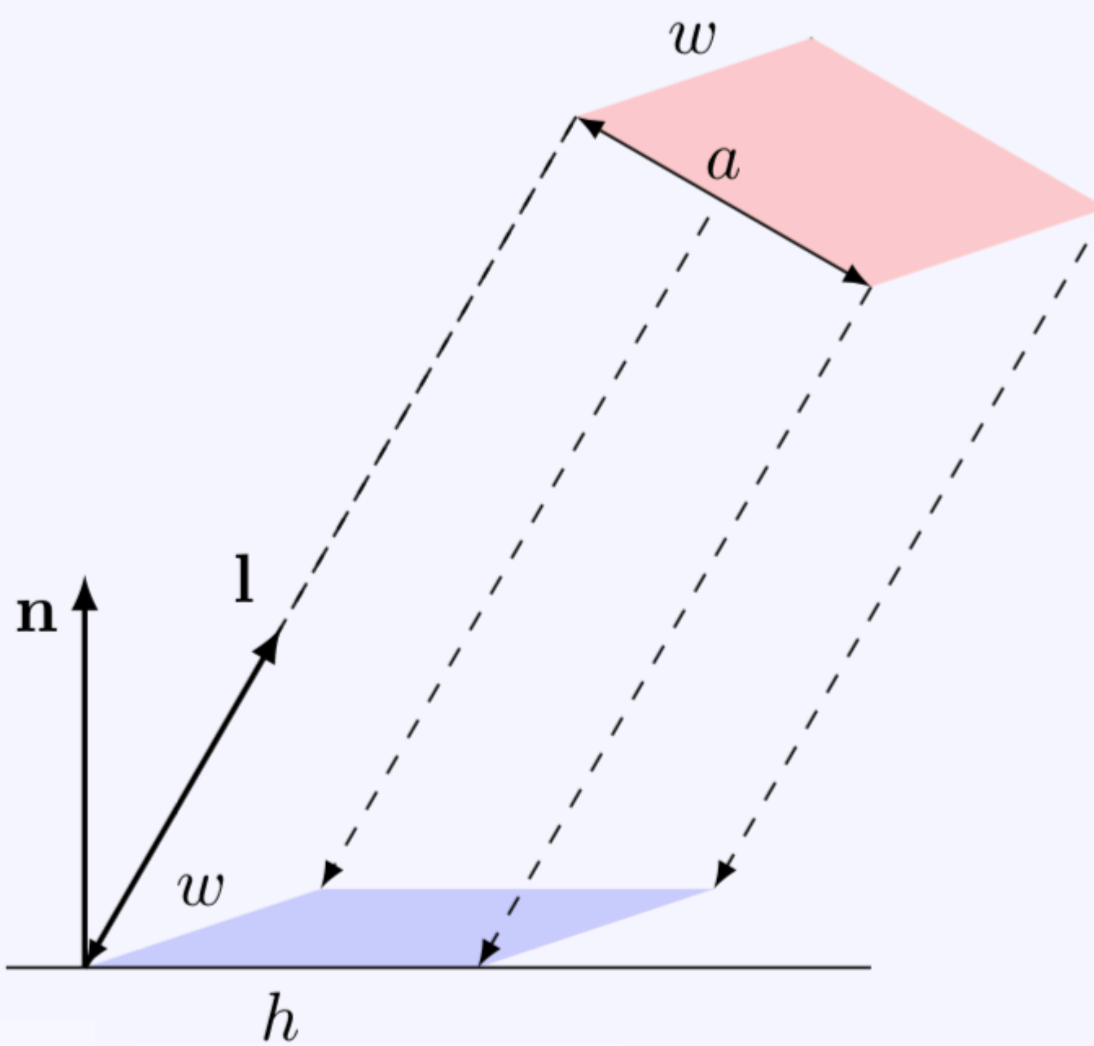




	Light	Incident	Reflected
Intensity	L	L'	$I = RL'$
Energy	$E = Lwa$	$E = L'wh$	RE

Lambertian reflection model

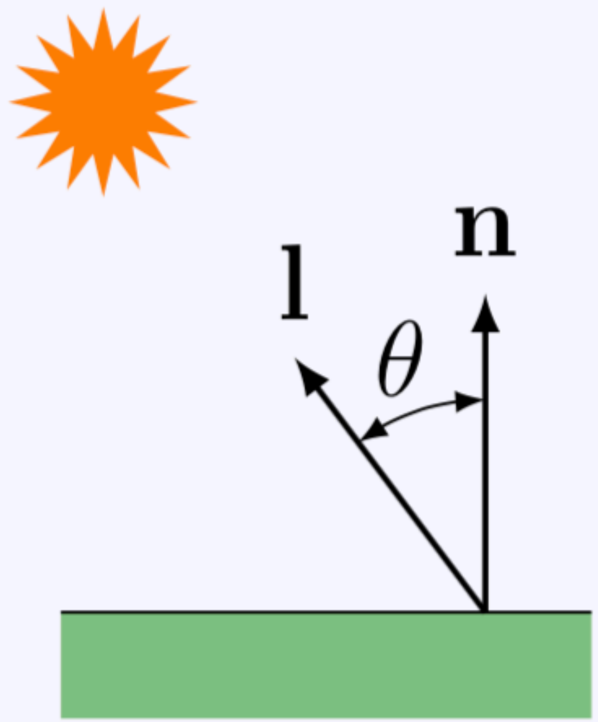


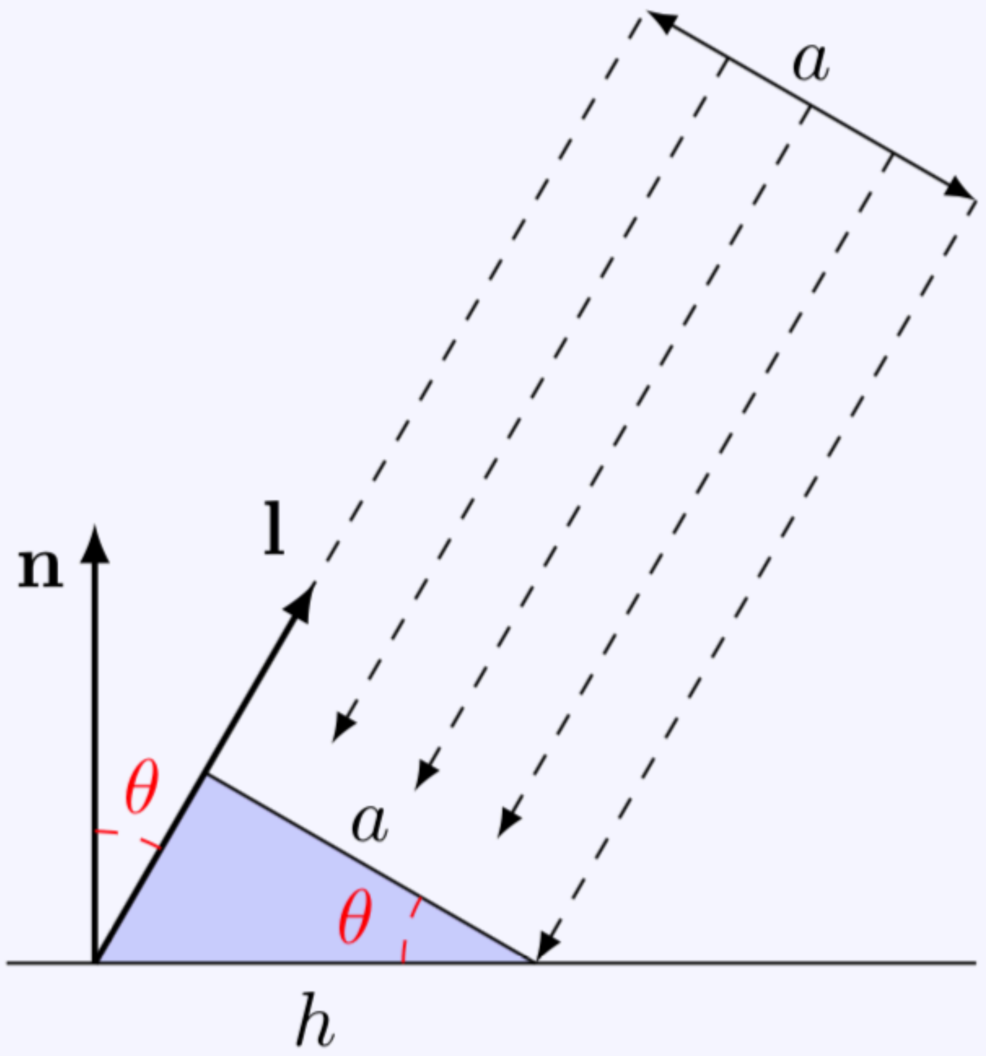


	Light	Incident	Reflected
Intensity	L	L'	$I = RL'$
Energy	$E = Lwa$	$E = L'wh$	RE

$$\boxed{I = LR\frac{a}{h}}$$

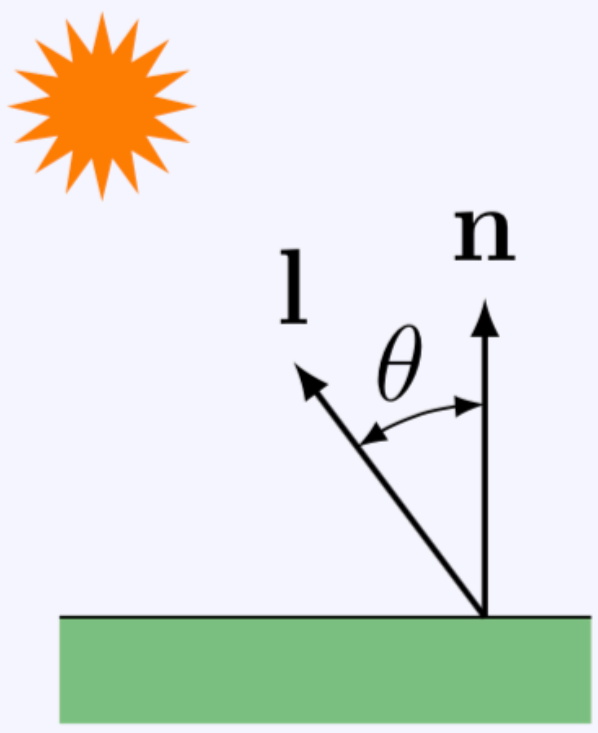
Lambertian reflection model

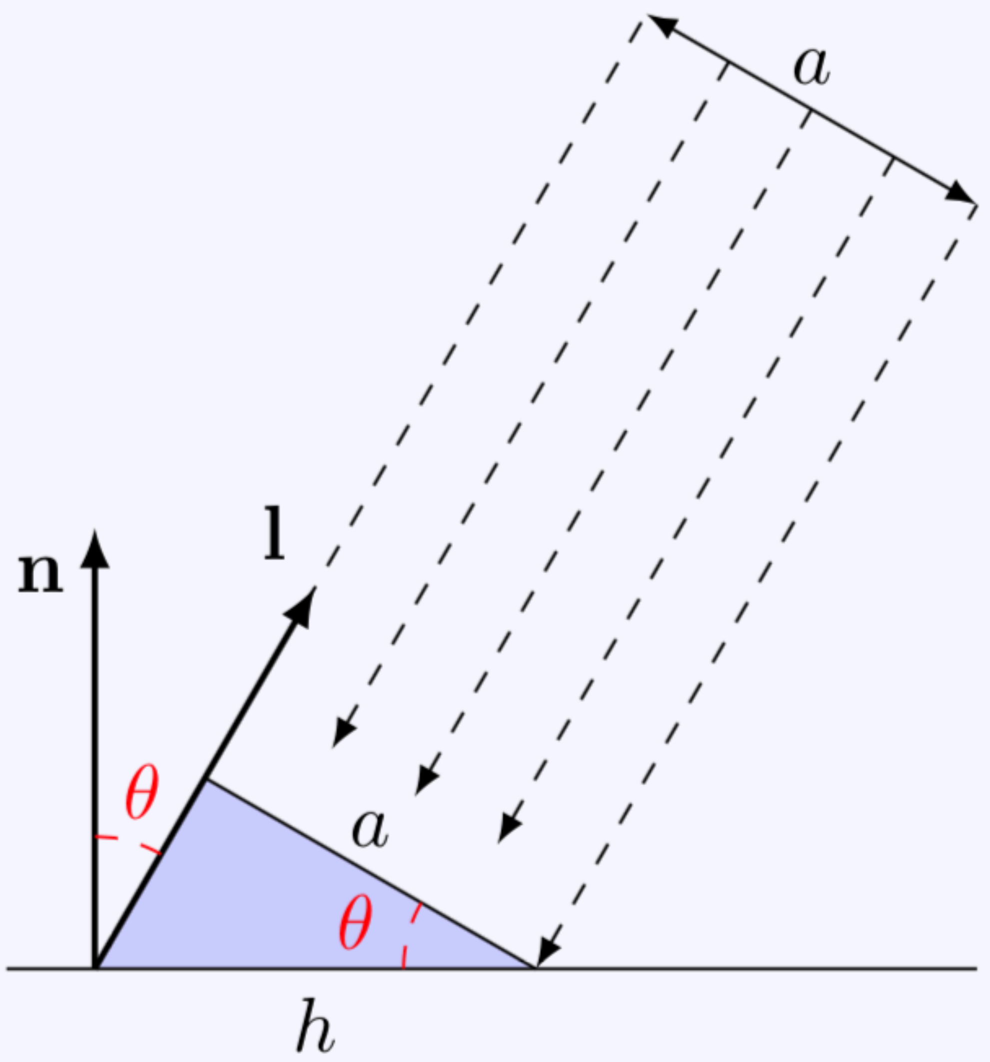




$$I = LR\frac{a}{h}$$

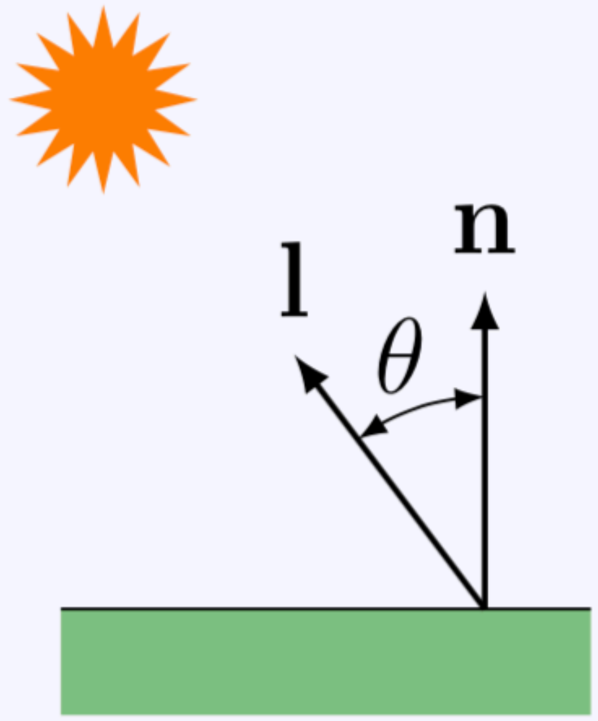
Lambertian reflection model

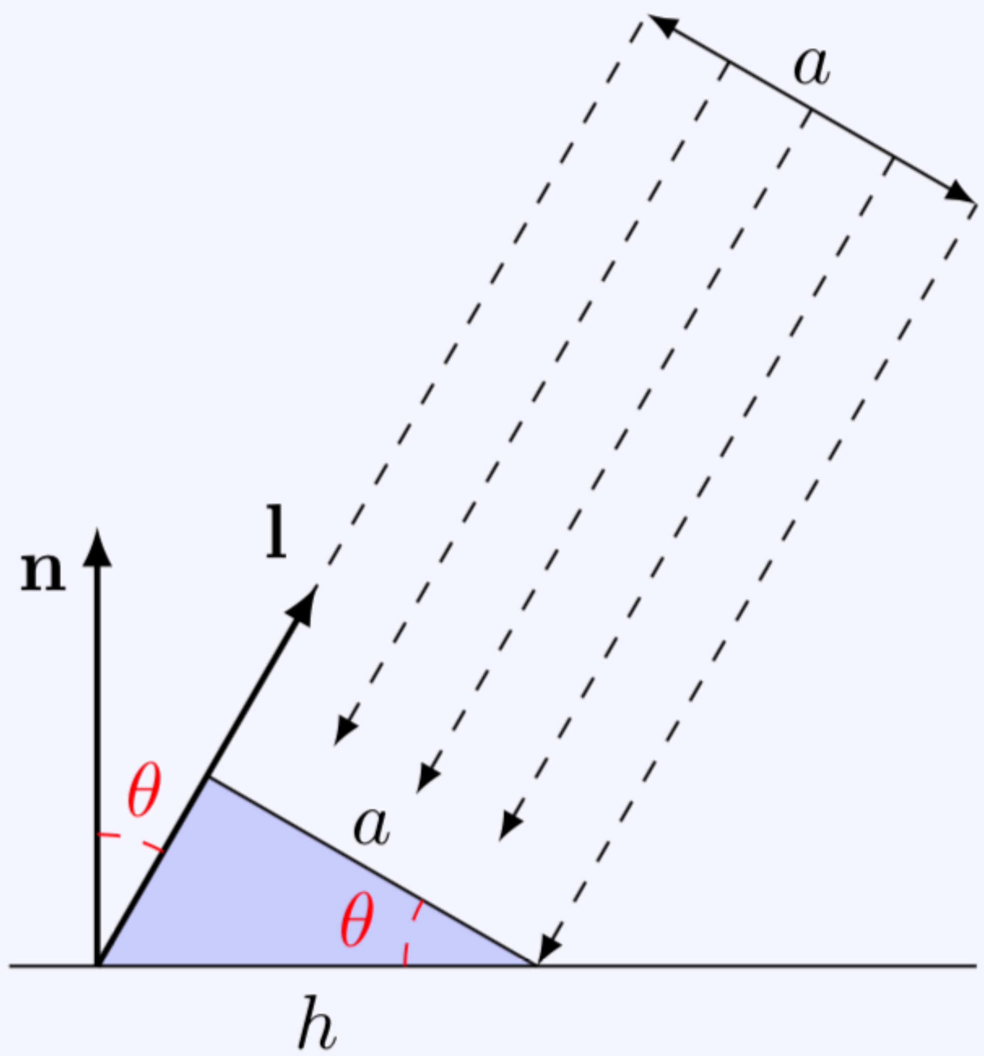




$$I = LR\frac{a}{h} = LR\cos\theta$$

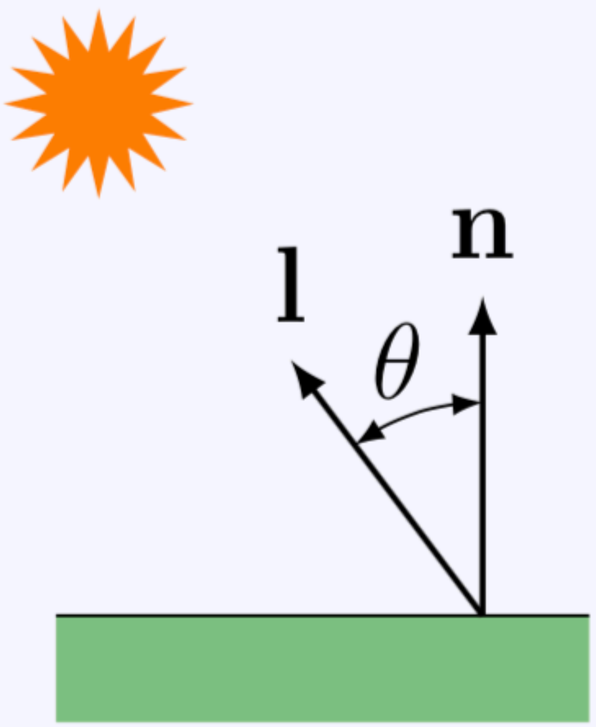
Lambertian reflection model

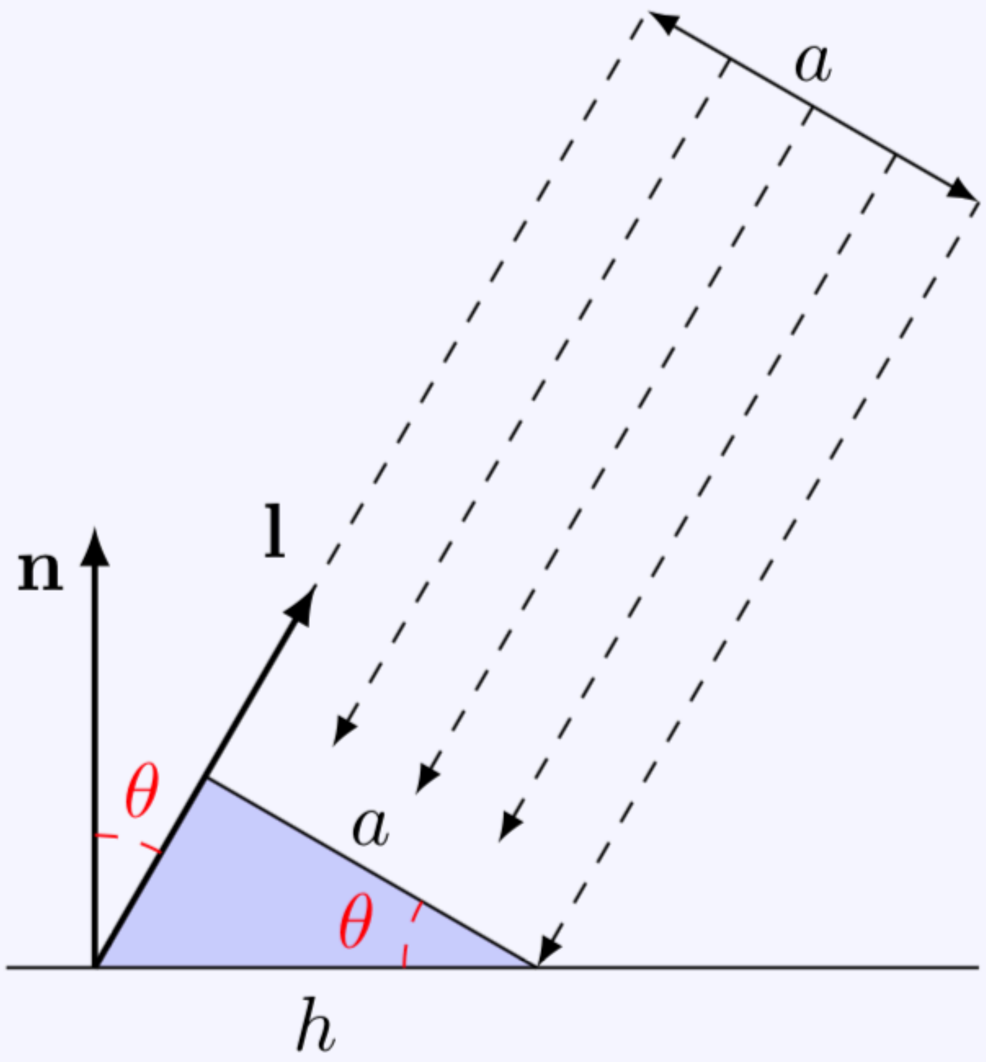




$$I = LR\frac{a}{h} = LR\cos\theta = LR\mathbf{n}\cdot\mathbf{l}$$

Lambertian reflection model





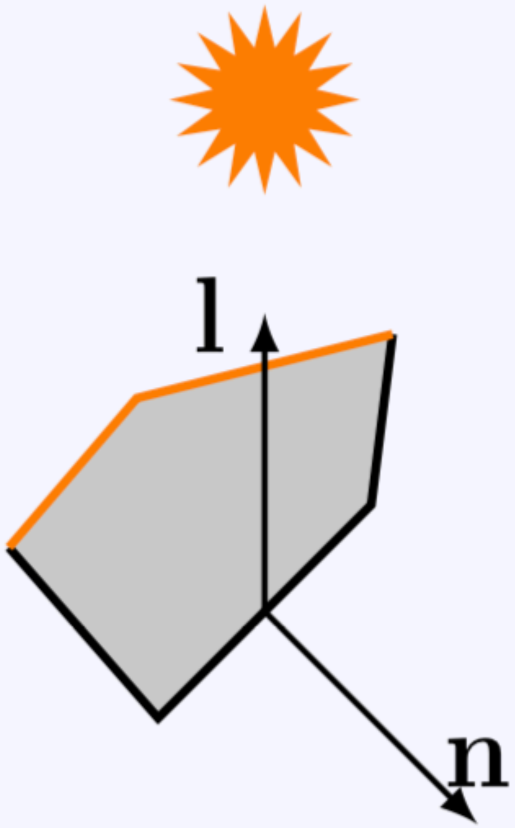
$$I = LR\frac{a}{h} = LR\cos\theta = LR\mathbf{n}\cdot\mathbf{l}$$

Avoid bug: $I = LR\max(\mathbf{n}\cdot\mathbf{l}, 0)$

Ambient reflection

$$I = {\color{orange}L}{\color{green}R}\max(\mathbf{n}\cdot\mathbf{l}, 0)$$

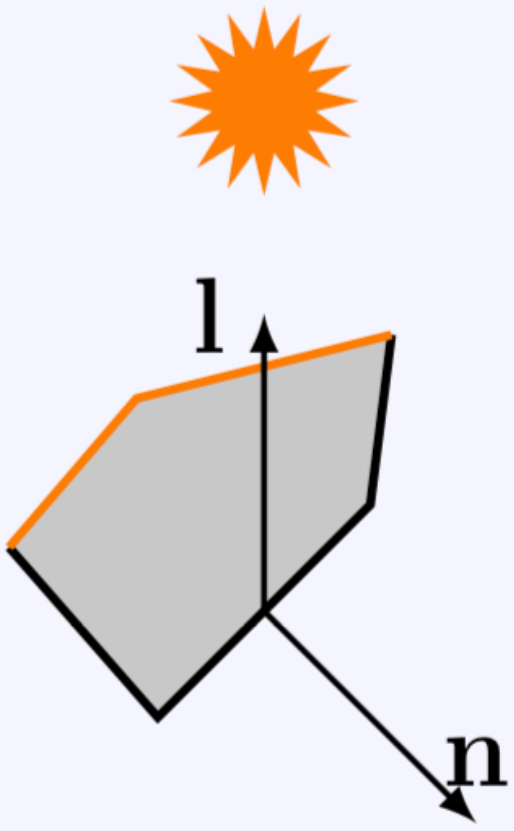
Surfaces facing away from the light will be totally **black**



Ambient reflection

$$I = L_a R_a + L_d R_d \max(\mathbf{n} \cdot \mathbf{l}, 0)$$

All surfaces get the same amount of ambient light



Phong reflection model

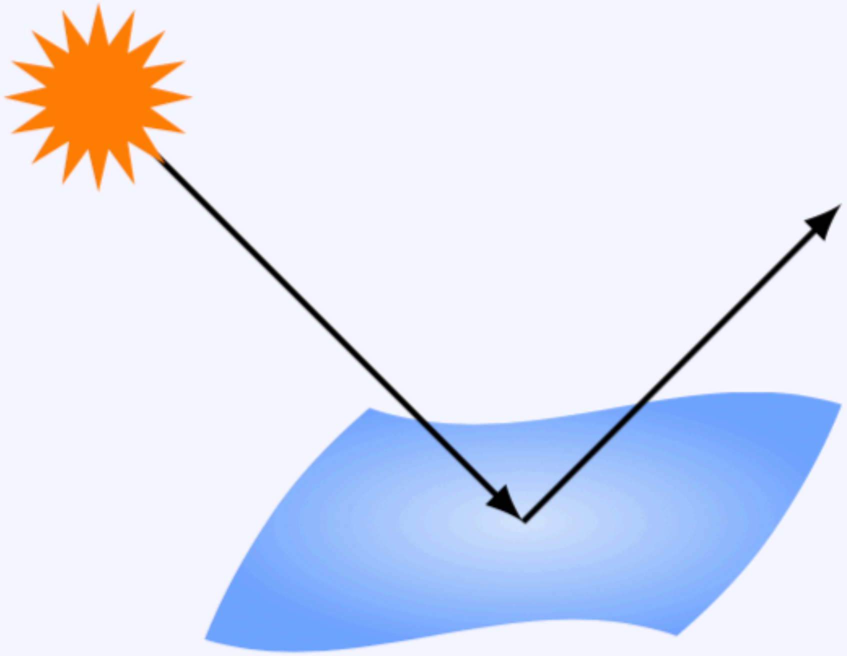
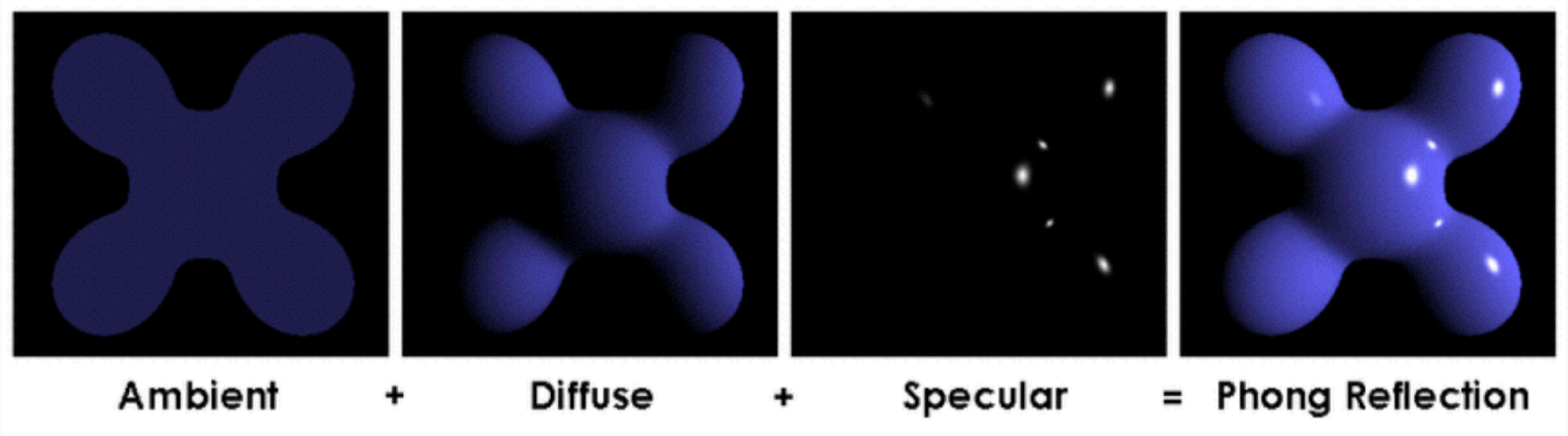
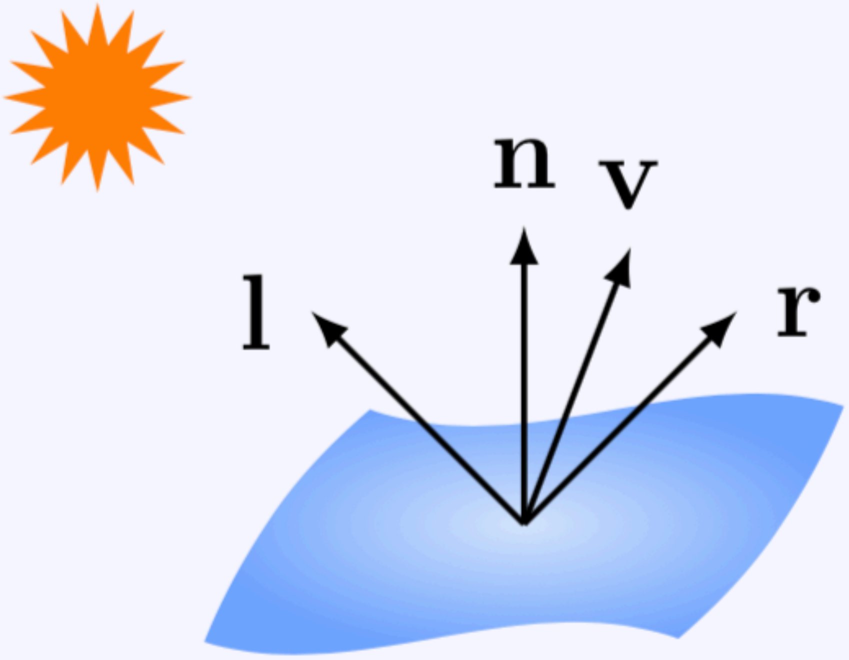


Image source: [1]

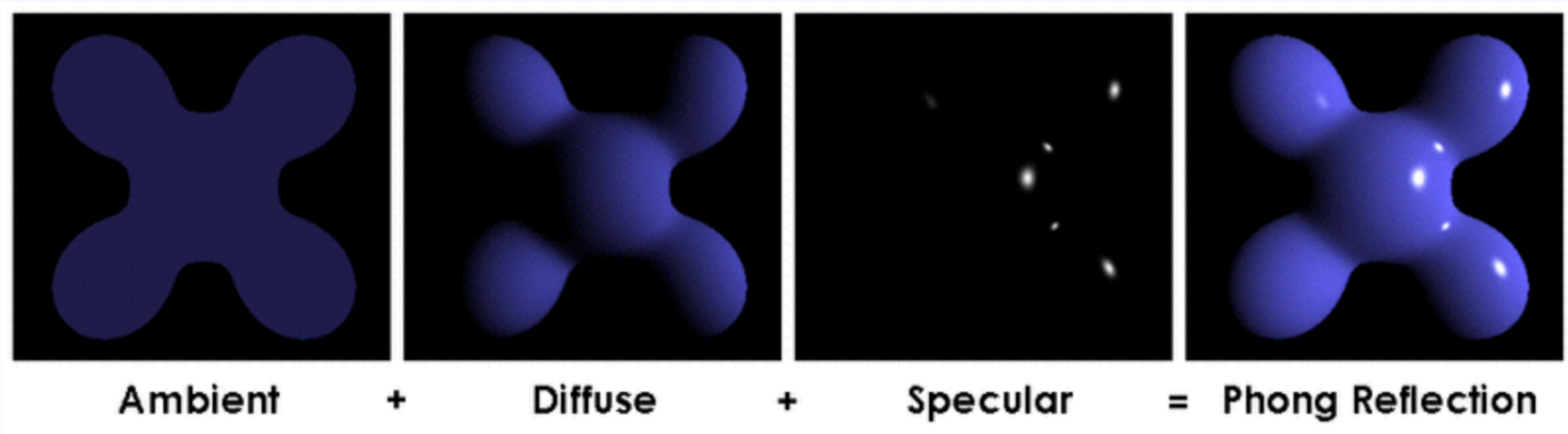
Phong reflection model



- Efficient
- Reasonably realistic
- 3 components
- 4 vectors

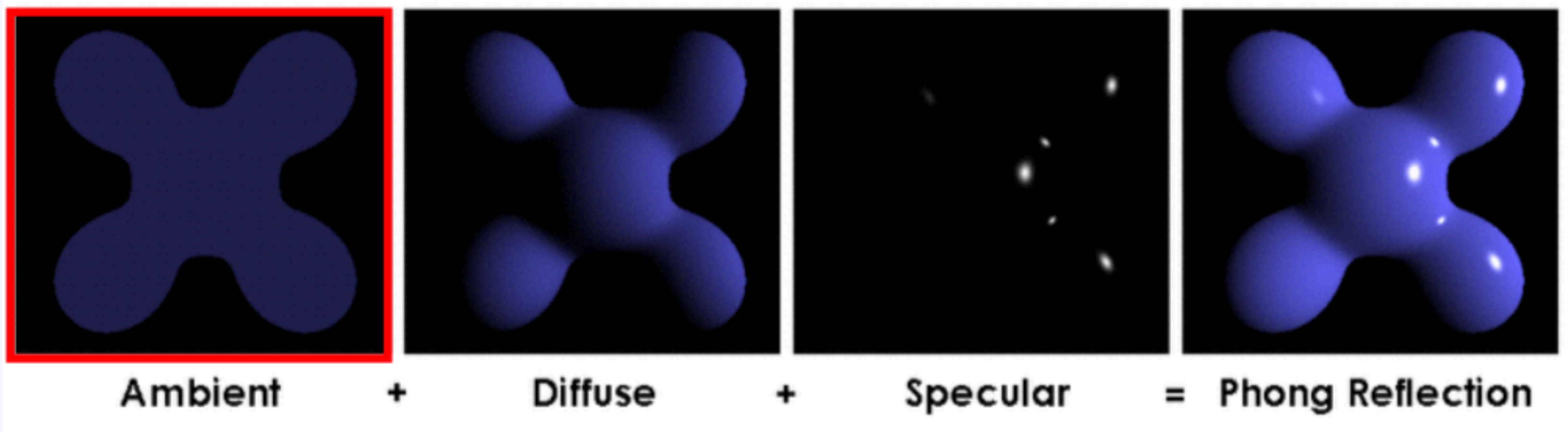


Phong reflection model



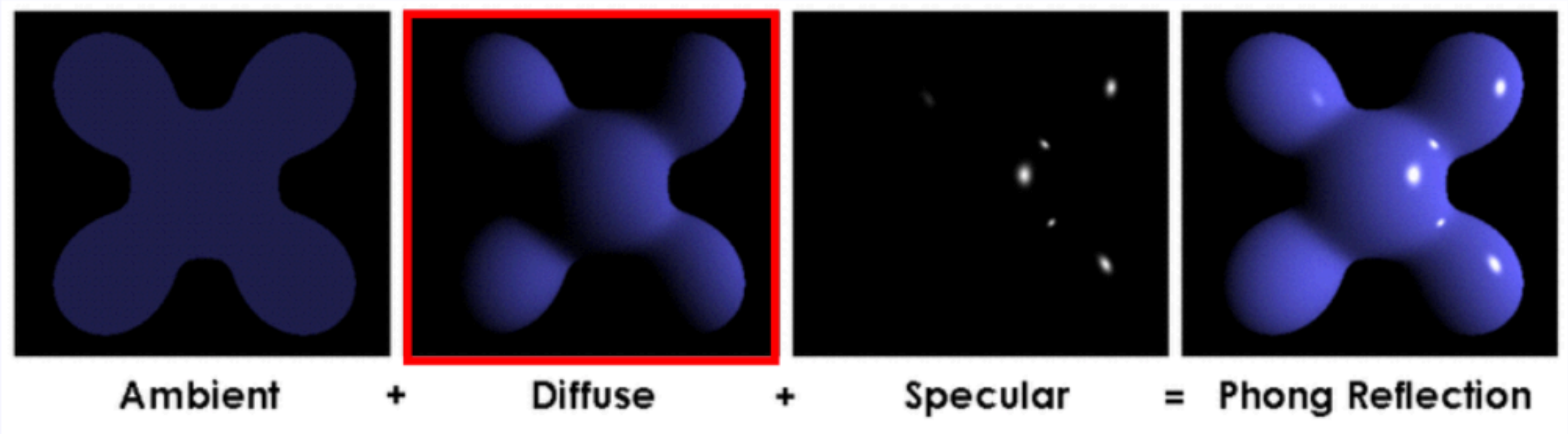
$$
\begin{aligned}
I &= I_a + I_d + I_s \\
&= R_a L_a + R_d L_d \max(\mathbf{n} \cdot \mathbf{l}, 0) + R_s L_s \max(\cos\phi, 0)^{\alpha}
\end{aligned}
$$

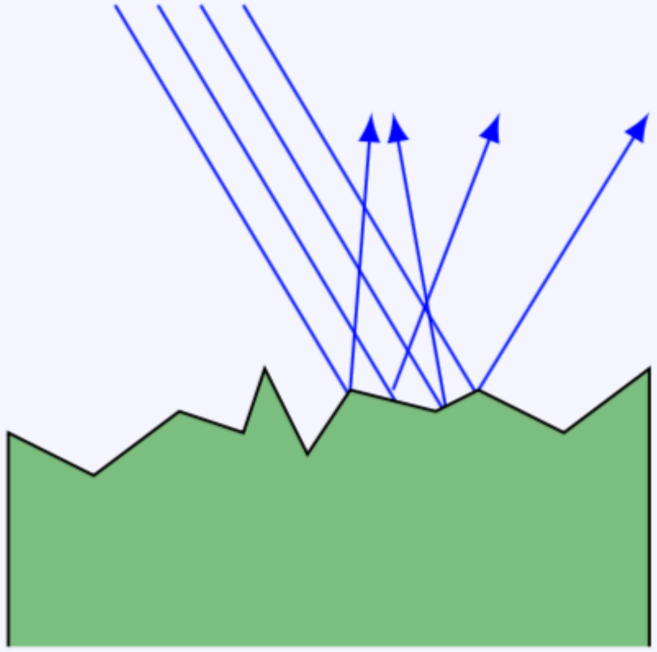
Ambient reflection



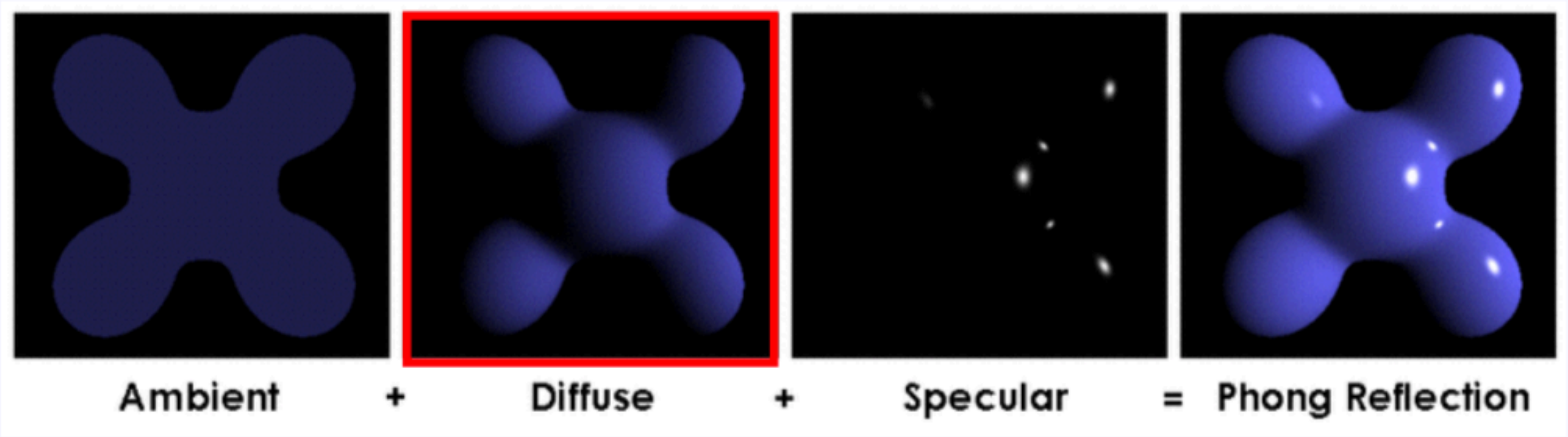
$$I_a = R_a L_a \qquad\qquad 0 \leq R_a \leq 1$$

Diffuse reflection

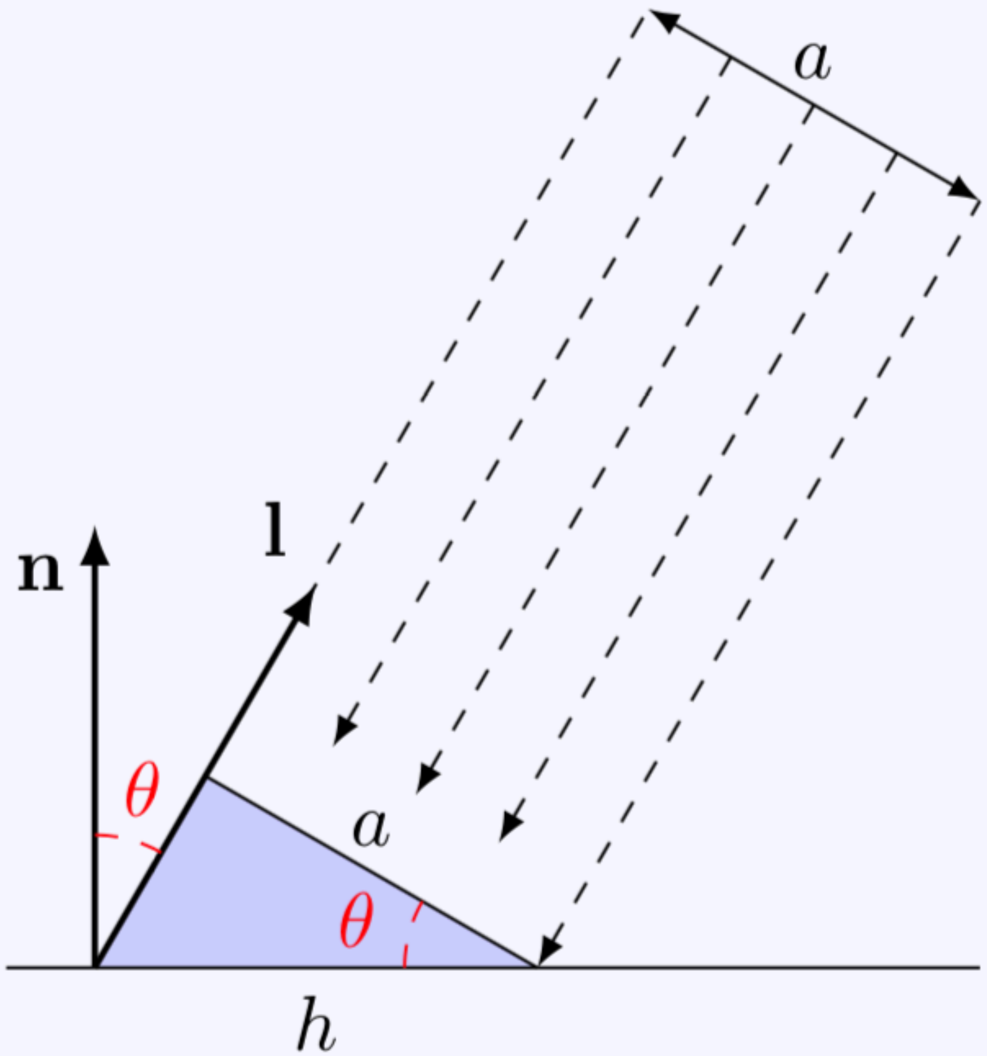




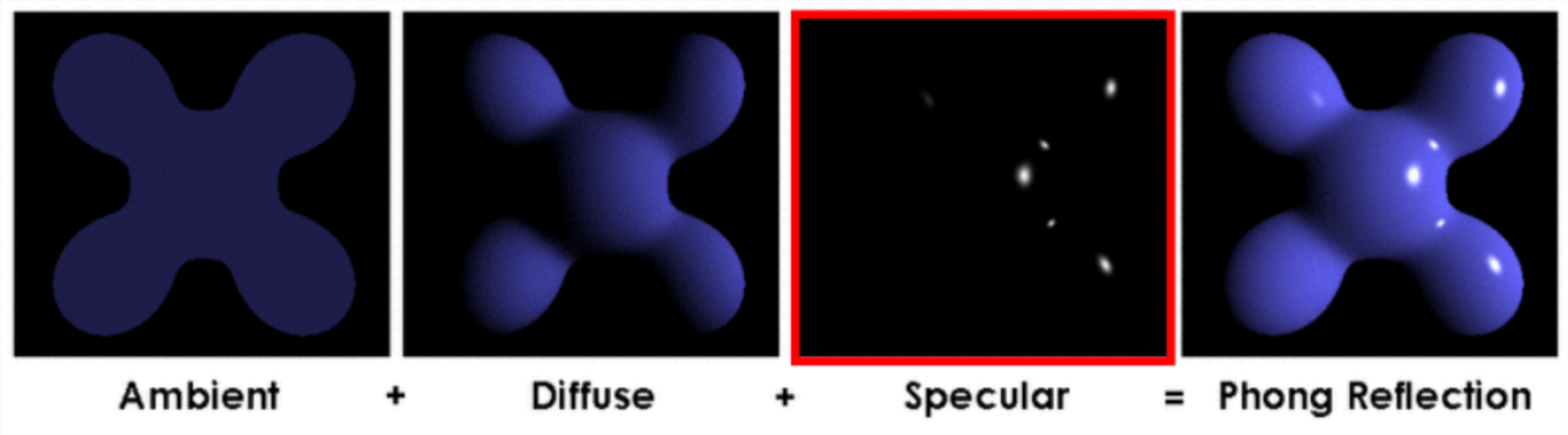
Diffuse reflection

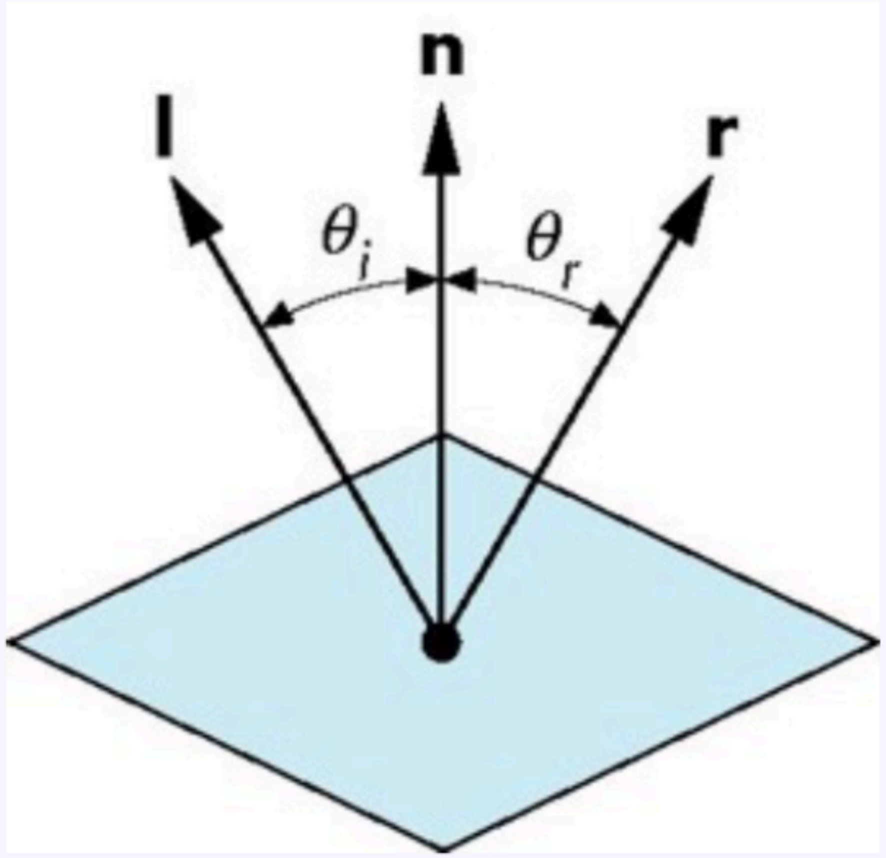


$$I_d = R_d L_d \max(\mathbf{n} \cdot \mathbf{l}, 0)$$



Specular reflection



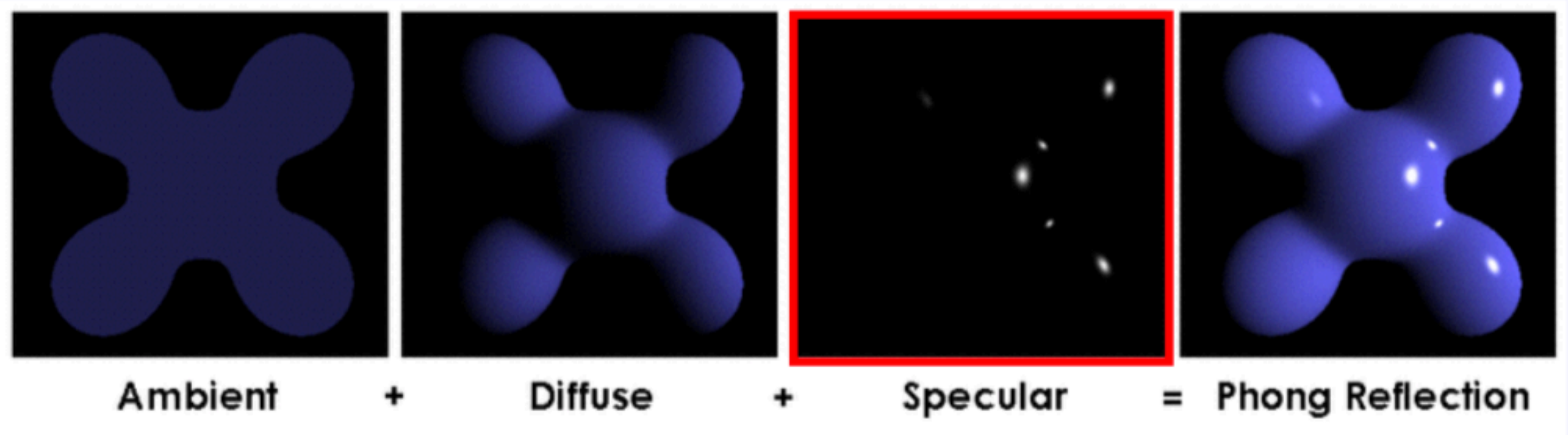


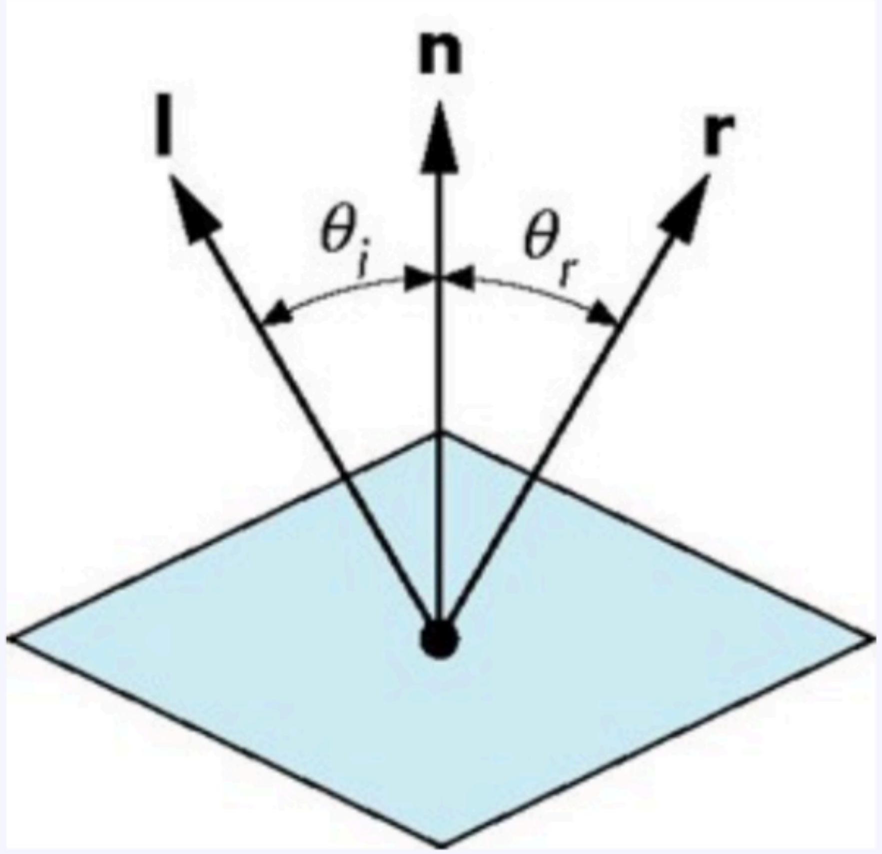
Ideal reflector

$$\theta_i = \theta_r$$

r is the mirror reflection direction

Specular reflection

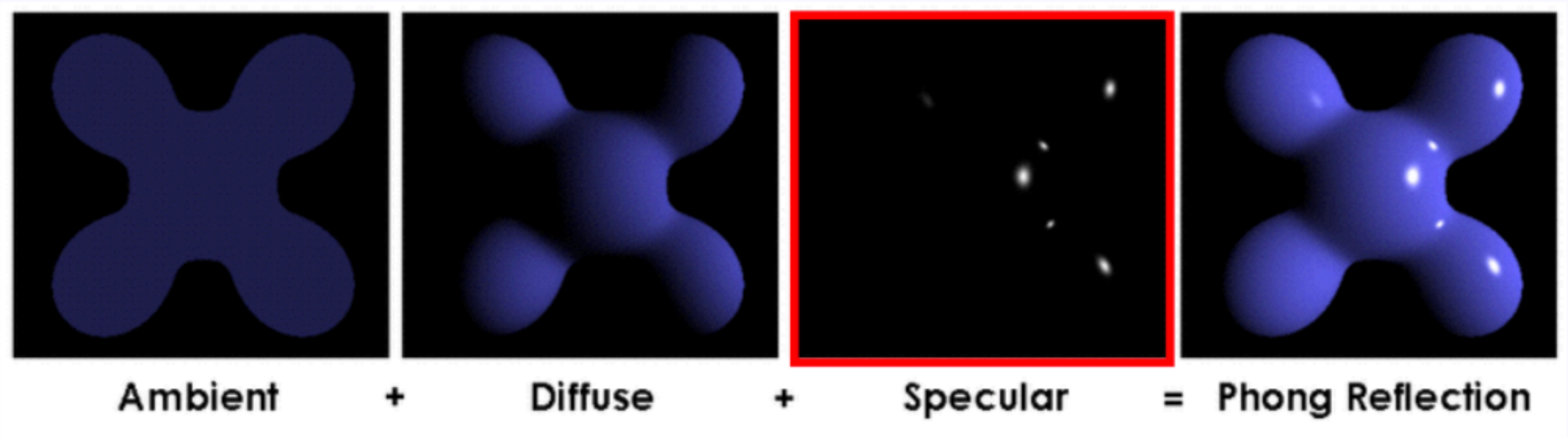


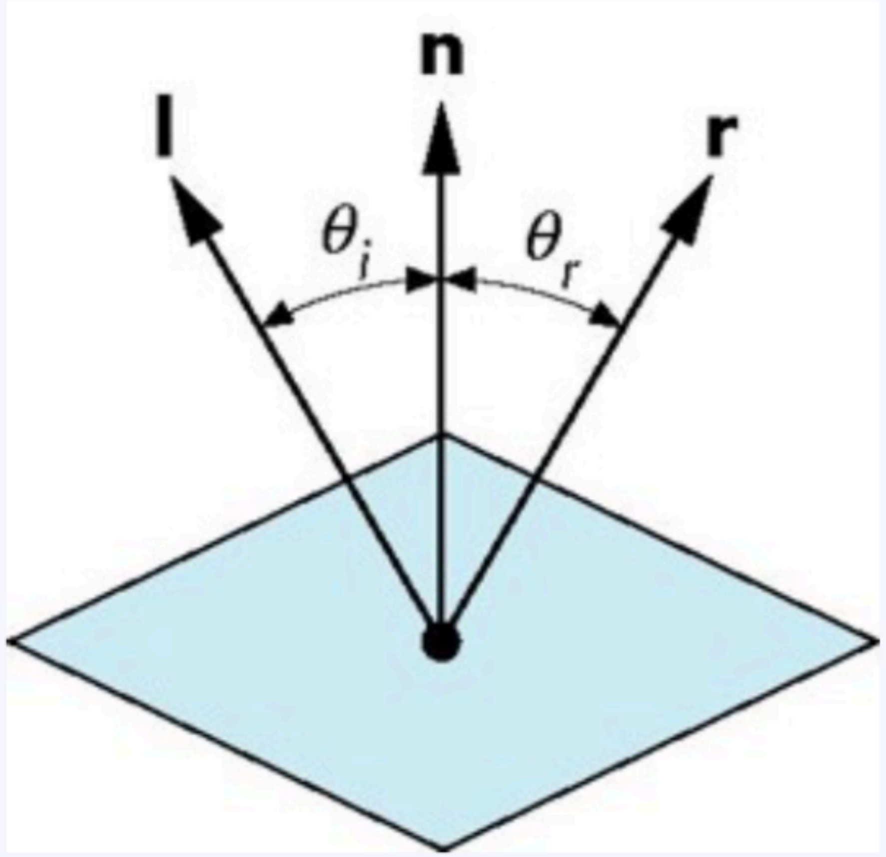


Specular surface

specular reflection is strongest in reflection direction

Specular reflection

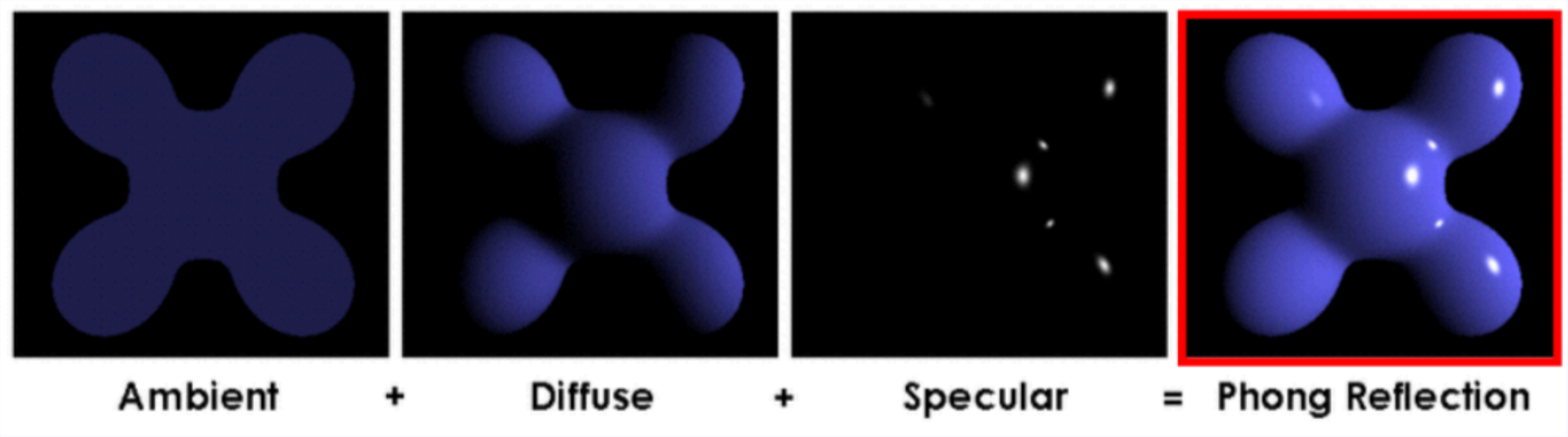




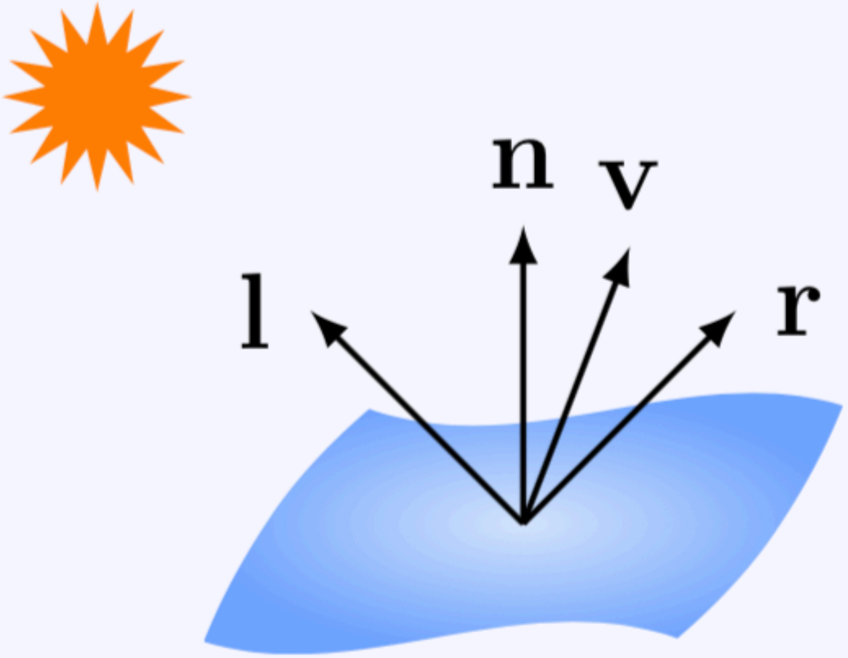
$$I_s = R_s L_s \max(\cos\phi, 0)^\alpha$$

specular reflection drops off with increasing ϕ

Phong reflection model



$$
\begin{aligned}
I &= I_a + I_d + I_s \\
&= R_a L_a + R_d L_d \max(\mathbf{n}\cdot\mathbf{l}, 0) + R_s L_s \max(\mathbf{v}\cdot\mathbf{r}, 0)^{\alpha}
\end{aligned}
$$



Attribution

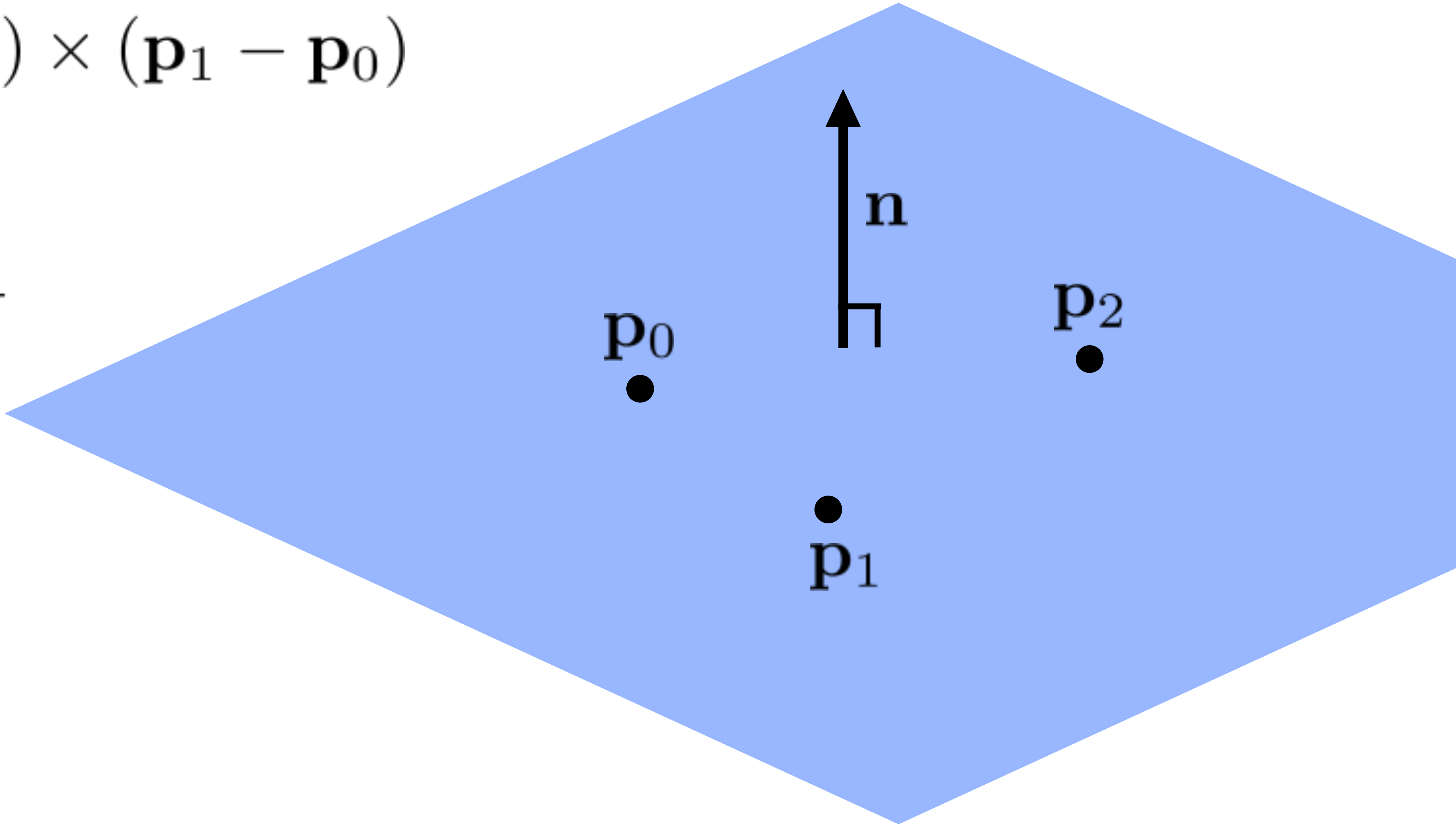
[1] Andrea Fisher Fine Pottery. jody-folwell-jar05big.jpg.
https://www.eyesofthepot.com/santa-clara/jody_folwell.

Computing Normal Vectors

Plane Normals

$$\mathbf{v} = (\mathbf{p}_2 - \mathbf{p}_0) \times (\mathbf{p}_1 - \mathbf{p}_0)$$

$$\mathbf{n} = \frac{\mathbf{v}}{||\mathbf{v}||}$$



Implicit function normals

$$f(\mathbf{p}) = 0$$

$$\nabla f(\mathbf{p})$$

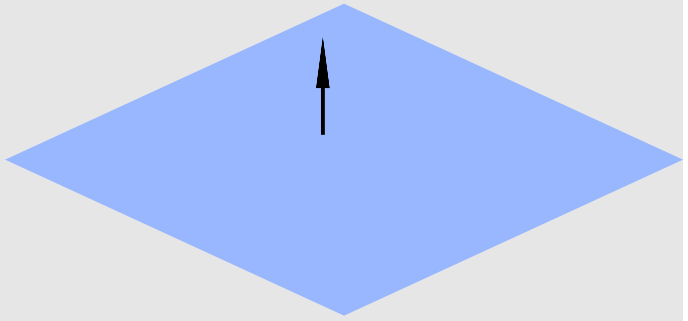
$$\nabla f = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{pmatrix}$$

sphere

$$\mathbf{p} \cdot \mathbf{p} - r^2 = 0$$

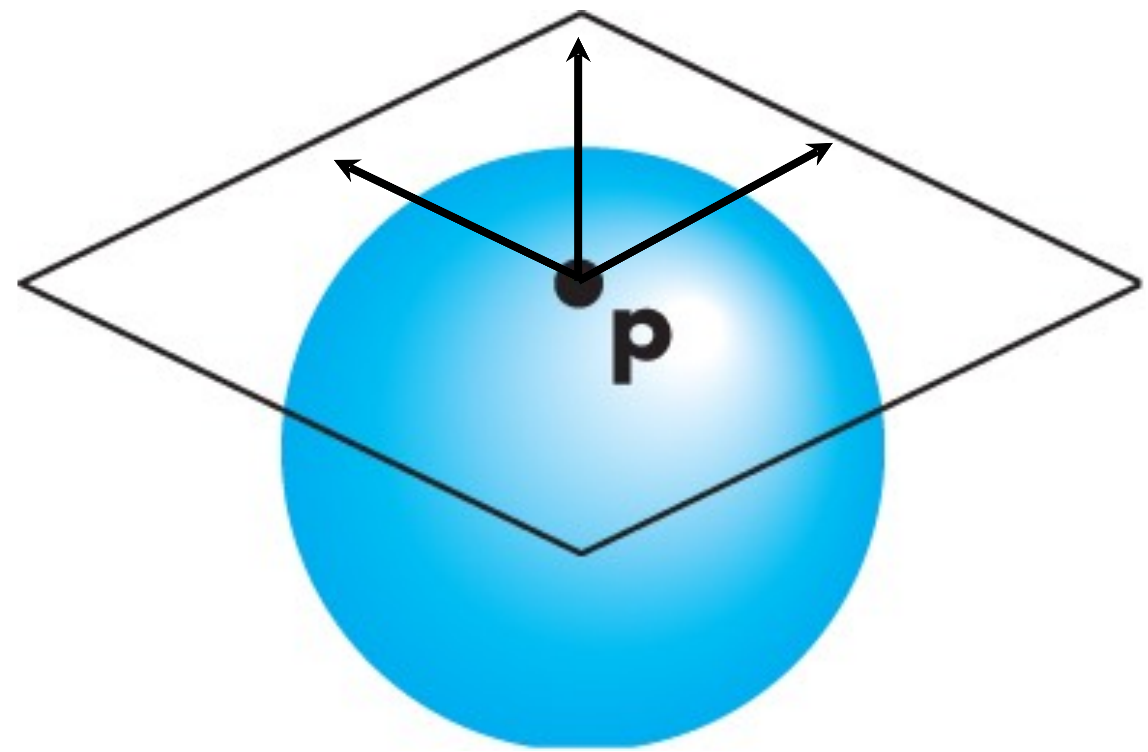
plane

$$\mathbf{n} \cdot (\mathbf{p} - \mathbf{p}_0) = 0$$



Parametric form

$$\mathbf{p}(u,v) = \begin{pmatrix} x(u,v) \\ y(u,v) \\ z(u,v) \end{pmatrix}$$



tangent vectors $\dfrac{\partial \mathbf{p}}{\partial u}$ $\dfrac{\partial \mathbf{p}}{\partial v}$

normal $\dfrac{\frac{\partial \mathbf{p}}{\partial u} \times \frac{\partial \mathbf{p}}{\partial v}}{||\frac{\partial \mathbf{p}}{\partial u} \times \frac{\partial \mathbf{p}}{\partial v}||}$