CS 130
Exam II

Fall 2015

Name	
Student ID	
Signature	

You may not ask any questions during the test. If you believe that there is something wrong with a question, write down what you think the question is trying to ask and answer that.

Question	**Points**	**Score**
True/False		
1	2	
2	2	
3	2	
4	2	
5	2	
6	2	
7	2	
8	2	
9	2	
10	2	
Multiple Choice		
11	4	
12	4	
13	4	
14	4	
15	4	
16	4	
17	4	
Written		
18	15	
19	10	
Total	73	

True/False

For each question, indicate whether the statement is true or false by circling T or F, respectively.

1. (T/$\boxed{\text{F}}$) One way to represent a rotation of θ degrees about an axis $\mathbf{u}$ (with $||\mathbf{u}|| = 1$) is with the quaternion $q = \cos\theta + \sin\theta\mathbf{u}$.
2. ($\boxed{\text{T}}$/F) In describing the orientation of a body, Euler angles are angles specified relative to a coordinate system fixed to the body.
3. ($\boxed{\text{T}}$/F) Texture coordinates are typically assigned at vertices and interpolated to the interior of a triangle.
4. (T/$\boxed{\text{F}}$) Textures are applied in the vertex processing stage of the graphics pipeline.
5. ($\boxed{\text{T}}$/F) Mipmapping involves generating and utilizing a hierarchy of textures to mitigate minification artifacts.
6. (T/$\boxed{\text{F}}$) Bezier curves are curves that interpolate all of their control points.
7. ($\boxed{\text{T}}$/F) Blending functions provide a convenient basis for expressing curves in terms of the control points.
8. ($\boxed{\text{T}}$/F) A cubic Bezier curve has 4 control points.
9. ($\boxed{\text{T}}$/F) The direction of a ray transmitted through a dielectric material can be computed using Snell's law.
10. ($\boxed{\text{T}}$/F) The initial ray cast in a ray tracing algorithm is the view ray, which goes from the eye in the direction of the pixel.

Multiple Choice

For each question, circle exactly one of (a)-(e), unless otherwise stated.

11. Textures

    (a) may be 2D images or 3D solid textures.
    (b) can also be used to implement light maps, shadow maps, environment maps, and bump maps.
    (c) can appear distored if perspective correct interpolation is not employed.
    (d) $\boxed{\text{all of the above}}$
    (e) none of the above

12. Which statements regarding texture mapping are true?

    I. Texture mapping adds realism without increasing polygon count.
    II. OpenGL supports applying multiple textures to objects.
    III. Textures typically change object silhouettes.

    (a) I only
    (b) II only
    (c) $\boxed{\text{I and II only}}$

(d) I and III only
(e) I, II and III

13. Which of the following statements regarding ray tracing are true?

I. Using a regular pixel grid can alleviate aliasing artifacts.
II. Depth of field can be implemented by perturbing the starting point of view rays.
III. A bounding volume hierarchy can be used to accelerate ray tracing.

(a) I only
(b) II only
(c) I and III only
(d) $\boxed{\text{II and III only}}$
(e) I, II and III

14. In ray tracing,

(a) flat shading uses diffuse lighting to determine the color of an object.
(b) point light sources lead to softer shadows than area light sources.
(c) $\boxed{\text{testing for ray-sphere intersection requires solving a quadratic equation.}}$
(d) reflected rays originate at an intersection point, and bounce in the negative direction of the incident ray.
(e) rays may reflect up to a maximum of two times.

15. A cubic Bezier curve

(a) is a way to implicitly represent a cubic.
(b) $\boxed{\text{interpolates the first and last of its 4 control points.}}$
(c) has degree 2.
(d) may extend outside the convex hull of its control points.
(e) is seldom used in practice in computer graphics due to difficulty in evaluation of points on the curve.

16. Which of the following statements regarding curves are true?

I. There is a unique n degree polynomial that interpolations $n + 1$ distinct data points.
II. A monomial basis for curves up to order 3 is set $1, u, u^2, u^3$.
III. When using piecewise polynomial curves to interpolate a set of data points, care must be taken at join points to ensure desired level of continuity.

(a) II only
(b) I and II only
(c) I and III only
(d) II and III only
(e) $\boxed{\text{I, II and III}}$

17. Which of the following statements regarding rotations are true?

   I. The product of several rotation matrices is itself a rotation.

   II. Quaternions are four-dimensional vectors that can be used to specify rotations.

   III. Quaternion representations of rotations suffer from a problem known as gimbal lock.

   (a) I only

   (b) II only

   (c) $\boxed{\text{I and II only}}$

   (d) II and III only

   (e) I, II and III

Written Response

18. Consider a ray with endpoint $\mathbf{e}$ and direction $\mathbf{d}$, given by the ray equation

$$\mathbf{p}(t) = \mathbf{e} + t\mathbf{d}, \tag{1}$$

and a triangle with vertices $\mathbf{a}, \mathbf{b}, \mathbf{c}$.

(a) Find an implicit equation for the plane containing the triangle, of the form

$$f(\mathbf{p}) = \mathbf{N} \cdot (\mathbf{p} - \mathbf{q}) = 0$$

where $\mathbf{N}$ is a normal to the plane and $\mathbf{q}$ is a point in the plane. Specify $\mathbf{N}$ and $\mathbf{q}$ in terms of the triangle vertices.
Answer
We can get the normal as

$$\mathbf{n} = (\mathbf{c} - \mathbf{a}) \times (\mathbf{b} - \mathbf{a})$$

$$\mathbf{N} = \frac{\mathbf{n}}{||\mathbf{n}||} \tag{2}$$

We can use any point of the triangle as a point in the plane. We choose $\mathbf{a}$. So a valid implicit equation for the plane is

$$f(\mathbf{p}) = \mathbf{N} \cdot (\mathbf{p} - \mathbf{a}) = 0 \tag{3}$$

with $\mathbf{N}$ as in (2).

(b) Find the intersection point of the ray with the plane, if any, or specify how to determine that there is no intersection point.
Answer
We plug the ray equation (1) into the plane equation (3)

$$f(\mathbf{p}(t)) = \mathbf{N} \cdot (\mathbf{e} + t\mathbf{d} - \mathbf{a}) = 0$$

and rearrange to get

$$\mathbf{N} \cdot (\mathbf{e} - \mathbf{a}) + t\mathbf{N} \cdot \mathbf{d} = 0$$

If $\mathbf{N} \cdot \mathbf{d} = 0$, then there are two possible cases. If $f(\mathbf{e}) = 0$, the ray lies in the plane and hence there are infinitely many intersections. If $f(\mathbf{e}) \neq 0$ are no intersections of the plane and ray. If $\mathbf{N} \cdot \mathbf{d} \neq 0$, we can divide by this quantity to solve for t as

$$t = \frac{\mathbf{N} \cdot (\mathbf{a} - \mathbf{e})}{\mathbf{N} \cdot \mathbf{d}}.$$

If $t \geq 0$, then the intersection point of the ray and plane is given by $\mathbf{p}(t)$.

(c) How would you determine whether the ray intersects the original triangle or not? You do not need to give all the mathematical details, but simply outline in words a procedure.
Answer
If there is an intersection point $\mathbf{p}(t)$ of the ray and plane, we can find the barycentric coordinates α, β, and γ of the point. If they satisfy $0 \leq \alpha \leq 1$, $0 \leq \beta \leq 1$, and $0 \leq \gamma \leq 1$, then the ray intersects the triangle. Otherwise it does not.

19. Consider a quadratic curve that interpolates three control points $\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2$. We wish to find a parametric representation of the curve of the form

$$\mathbf{f}(u) = \mathbf{a}_0 + \mathbf{a}_1 u + \mathbf{a}_2 u^2.$$

(a) Set up a linear system of equations relating the known control points $\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2$ to the unknown coefficients $\mathbf{a}_0, \mathbf{a}_1, \mathbf{a}_2$, by choosing $\mathbf{f}(0) = \mathbf{p}_0$, $\mathbf{f}(.5) = \mathbf{p}_1$, and $\mathbf{f}(1) = \mathbf{p}_2$.
Answer
We have

$$\begin{aligned}
\mathbf{f}(0) &= \mathbf{a}_0 = \mathbf{p}_0 \\
\mathbf{f}(.5) &= \mathbf{a}_0 + .5\mathbf{a}_1 + .25\mathbf{a}_2 = \mathbf{p}_1 \\
\mathbf{f}(1) &= \mathbf{a}_0 + \mathbf{a}_1 + \mathbf{a}_2 = \mathbf{p}_2
\end{aligned}$$

which can be written in matrix form as

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & .5 & .25 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{a}_0 \\ \mathbf{a}_1 \\ \mathbf{a}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{p}_0 \\ \mathbf{p}_1 \\ \mathbf{p}_2 \end{pmatrix}$$

(b) If your linear system in part (a) is given by $C\mathbf{a} = \mathbf{p}$, with

$$\mathbf{a} = \begin{pmatrix} \mathbf{a}_0 \\ \mathbf{a}_1 \\ \mathbf{a}_2 \end{pmatrix}, \quad \mathbf{p} = \begin{pmatrix} \mathbf{p}_0 \\ \mathbf{p}_1 \\ \mathbf{p}_2 \end{pmatrix}$$

and $\mathbf{f}(u) = \mathbf{u}^T \mathbf{a}$ with

$$\mathbf{u} = \begin{pmatrix} 1 \\ u \\ u^2 \end{pmatrix}$$

identify a set of blending functions that can be used to specify $\mathbf{f}$ directly in terms of the control points $\mathbf{p}_i$. You do not need to find the blending functions explicitly, but only identify how you would find them.
Answer
We have

$$\begin{aligned}
\mathbf{f}(u) &= \mathbf{u}^T \mathbf{a} \\
&= \mathbf{u}^T C^{-1} \mathbf{p}
\end{aligned}$$

$\mathbf{u}^T C^{-1}$ is a row vector whose elements are the blending functions, i.e.,

$$\mathbf{u}^T C^{-1} = \begin{pmatrix} b_0(u) & b_1(u) & b_2(u) \end{pmatrix}$$

such that

$$\mathbf{f}(u) = b_0(u)\mathbf{p}_0 + b_1(u)\mathbf{p}_1 + b_2(u)\mathbf{p}_2.$$