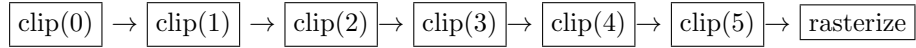


# 1 Clipping Pipeline



Triangle clipping is done against one plane at a time. The planes are represented by the arguments 0-5, and signify, e.g., the right, left, top, bottom, far, and near planes, respectively. Clipping a triangle against a single plane results in 0, 1, or 2 triangles being passed on to the next stage in the clipping pipeline. Once the end of the clipping pipeline is reached (argument 6), the triangle can simply be passed on to the rasterization step.

## 2 Clipping against a plane in clip space

Clipping must be done **before** the perspective divide, as the perspective divide can cause with negative  $z$  to flip sign, creating invalid clipped triangles. Therefore, clipping is carried out in what is called “clip space”, which differs from normalized device coordinates only in that the perspective divide has not been carried out.

### 2.1 Example: clipping against the right plane

The right plane is the plane that will be  $x = 1$  in normalized device coordinates. Therefore, in clip space, the right plane consists of all points satisfying  $x = w$ , i.e., points of the form  $(w, y, z, w)^T$ .

In clipping, we are looking for the intersection of triangle edges with planes. We first establish whether the vertices are inside or outside the clipping plane as follows:

$$\begin{aligned}x \leq w &\Rightarrow \text{inside} \\x > w &\Rightarrow \text{outside}\end{aligned}$$

If two vertices of the triangle  $\mathbf{a}$  and  $\mathbf{b}$  are determined to be on opposite sides of the clipping plane, we proceed to find the point of intersection of the edge with the plane. The points along the edge can be parameterized as

$$\mathbf{p}(\alpha) = \alpha \mathbf{a} + (1 - \alpha) \mathbf{b}, \tag{1}$$

where  $\mathbf{a} = (x_a, y_a, z_a, w_a)^T$  and  $\mathbf{b} = (x_b, y_b, z_b, w_b)^T$ .

We look for an  $\alpha$  that gives a point on the plane by setting

$$\alpha \begin{pmatrix} x_a \\ y_a \\ z_a \\ w_a \end{pmatrix} + (1 - \alpha) \begin{pmatrix} x_b \\ y_b \\ z_b \\ w_b \end{pmatrix} = \begin{pmatrix} w \\ y \\ z \\ w \end{pmatrix}.$$

The equations for  $\alpha$  and  $w$  can be solved independently of the other two:

$$\begin{aligned}\alpha x_a + (1 - \alpha)x_b &= w \\ \alpha w_a + (1 - \alpha)w_b &= w\end{aligned}$$

Subtracting the two equations and solving for  $\alpha$ , we get

$$\alpha = \frac{w_b - x_b}{x_a - w_a + w_b - x_b}.$$

This value of  $\alpha$  can be substituted into (1) to get the intersection point  $\mathbf{p}(\alpha)$ .

## 2.2 Example: clipping against the near plane

Clipping against any of the other planes proceeds in much the same way. To illustrate, we also do the clipping against the near plane. The near plane is the plane that will be  $z = -1$  in NDC, so in clip space is the set of points with  $z = -w$ , i.e., points of the form  $(x, y, -w, w)^T$ . If two vertices  $\mathbf{a}$  and  $\mathbf{b}$  are determined to be on either side of the  $z = -w$  plane, then we look for the point of intersection of the edge with the plane

$$\alpha \begin{pmatrix} x_a \\ y_a \\ z_a \\ w_a \end{pmatrix} + (1 - \alpha) \begin{pmatrix} x_b \\ y_b \\ z_b \\ w_b \end{pmatrix} = \begin{pmatrix} x \\ y \\ -w \\ w \end{pmatrix}.$$

The third and fourth equations,

$$\begin{aligned}\alpha z_a + (1 - \alpha)z_b &= -w \\ \alpha w_a + (1 - \alpha)w_b &= w\end{aligned}$$

can be solved simultaneously to get

$$\alpha = \frac{-w_b - z_b}{z_a + w_a - w_b - z_b}.$$

The other four cases proceed similar to the two cases shown above.