general rotations

Rotation about x, y, z axes

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

X axis

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Y axis

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Z axis

Rotation about an arbitrary axis $\mathbf{w}$ by θ radians

- **complete orthonormal basis**

$$\mathbf{u}, \mathbf{v}, \mathbf{w}$$

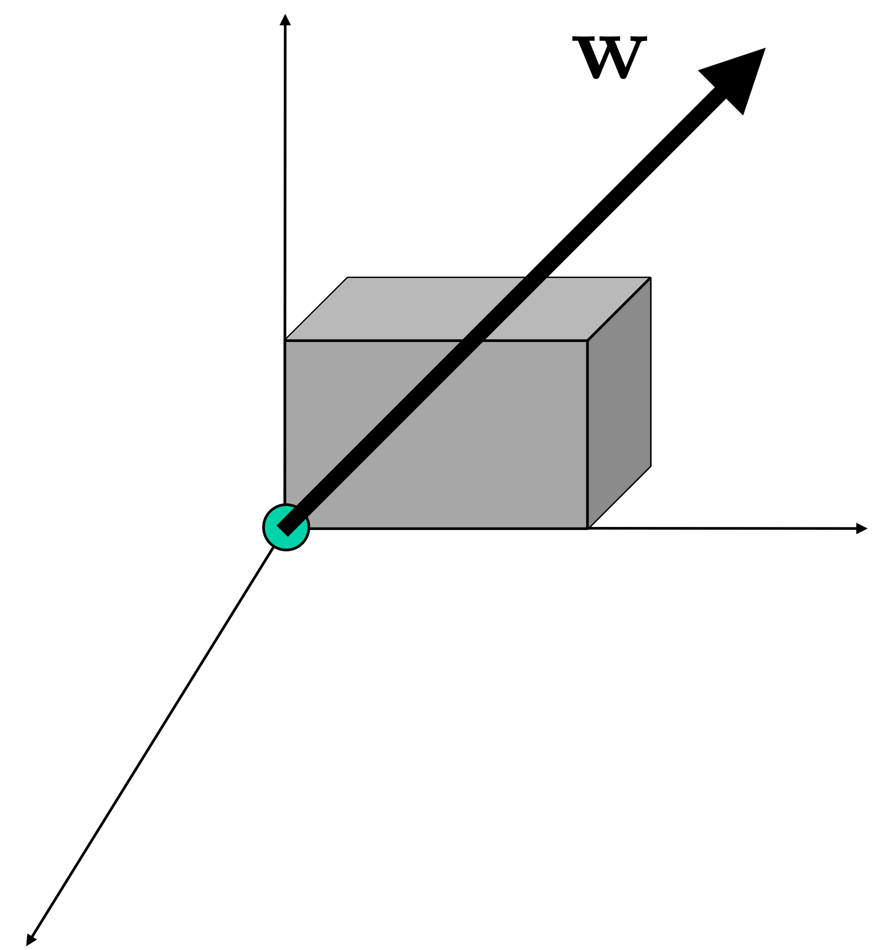
- **rotation matrix (w -> z-axis)**

$$R^T = \begin{pmatrix} \mathbf{u}^T \\ \mathbf{v}^T \\ \mathbf{w}^T \end{pmatrix}$$

$$R = \begin{pmatrix} \mathbf{u} & \mathbf{v} & \mathbf{w} \end{pmatrix}$$

- **composite rotation**

$$R R_z(\theta) R^T$$



Inverse of a rotation matrix

- complete orthonormal basis

$$\mathbf{u}, \mathbf{v}, \mathbf{w}$$

- R, R^T are rotation matrices

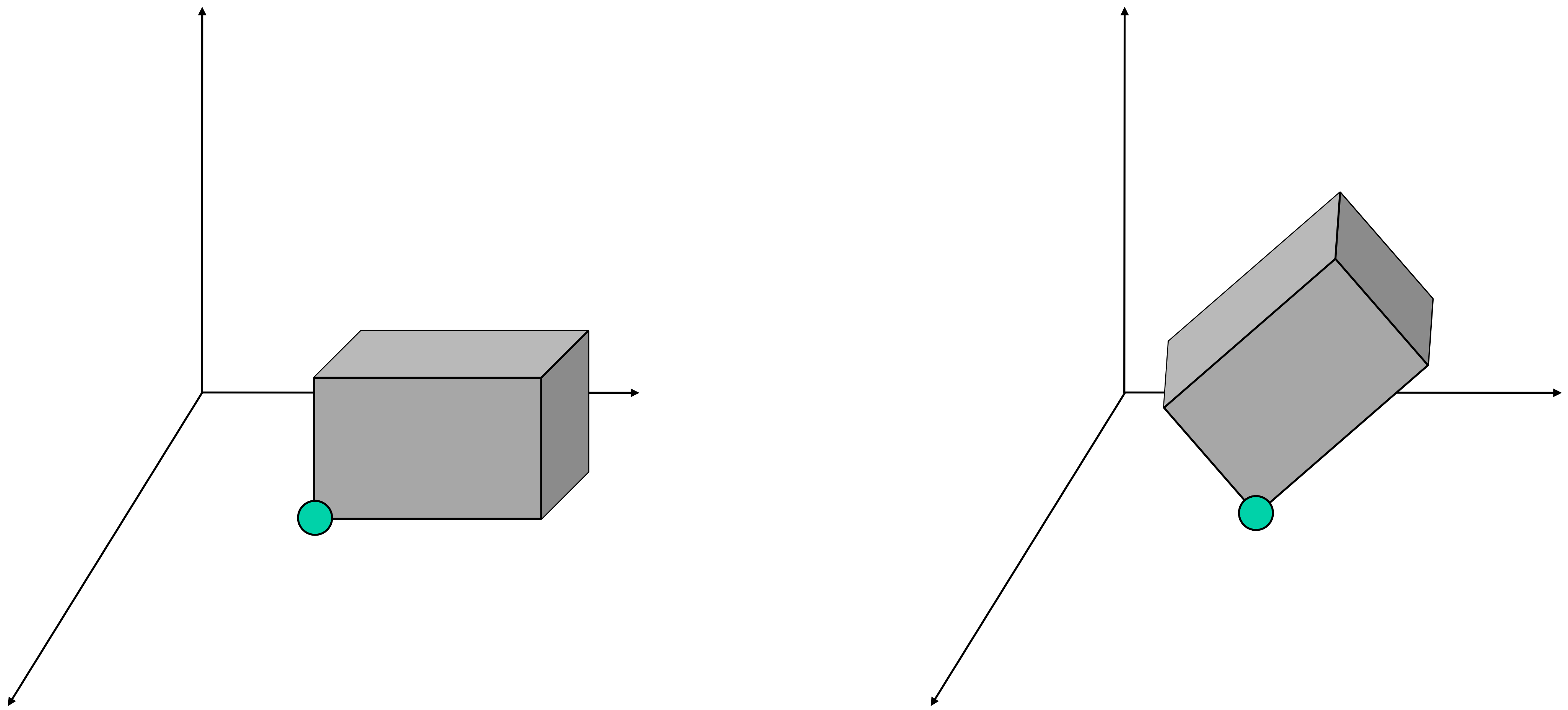
$$R = \begin{pmatrix} \mathbf{u} & \mathbf{v} & \mathbf{w} \end{pmatrix} \qquad R^T = \begin{pmatrix} \mathbf{u}^T \\ \mathbf{v}^T \\ \mathbf{w}^T \end{pmatrix}$$

- R, R^T are **orthogonal** matrices

$$RR^T = R^T R = I$$

Composite Transformations

- Rotating about a fixed point
  - **basic** rotation alone will rotate about origin but we want:



Composite Transformations

- Rotating about a fixed point
- Move fixed point (px,py,pz) to origin
- Rotate by desired amount
- Move fixed point back to original position

$$\mathbf{M} = \mathbf{T}(px, py, pz)\,\mathbf{R}\,\mathbf{T}(-px, -py, -pz)$$