CS 130
Exam I

Fall 2015

Name	
Student ID	
Signature	

You may not ask any questions during the test. If you believe that there is something wrong with a question, write down what you think the question is trying to ask and answer that.

Question	**Points**	**Score**
True/False		
1	2	
2	2	
3	2	
4	2	
5	2	
6	2	
7	2	
8	2	
9	2	
10	2	
Multiple Choice		
11	4	
12	4	
13	4	
14	4	
15	4	
16	4	
17	4	
18	4	
19	4	
20	4	
Written		
21	15	
22	10	
23	15	
Total	100	

1 True/False

For each question, indicate whether the statement is true or false by circling T or F, respectively.

1. (T/$\boxed{\text{F}}$) Rasterization occurs before vertex transformation in the graphics pipeline.

2. ($\boxed{\text{T}}$/F) Rasterization may generate multiple fragments per pixel.

3. ($\boxed{\text{T}}$/F) Matrix multiplication is associative but not commutative.

4. (T/$\boxed{\text{F}}$) Perspective transformation is a linear transformation.

5. ($\boxed{\text{T}}$/F) Applying a perspective transformation in the graphics pipeline to a vertex involves dividing by its 'z' coordinate.

6. (T/$\boxed{\text{F}}$) Clipping geometry against the view volume can be done independently on each vertex, make the graphics pipeline extremely fast. *When clipping triangles we need to use connectivity of vertices.*

7. (T/$\boxed{\text{F}}$) The directional light source idealization is appropriate for a light that is very close to the scene. *It is appropriate for a distant light source, e.g.,the sun.*

8. ($\boxed{\text{T}}$/F) When using the Phong Reflectance Model, we calculate the red, green, and blue color channels independently.

9. (T/$\boxed{\text{F}}$) When using the Phong Reflectance Model with smooth shading for a triangulated surface, we will calculate the Phong Reflectance Model once per triangle. *In smooth shading, the shading calculation is done once per vertex.*

10. ($\boxed{\text{T}}$/F) Modern day GPUs allow the user to supply custom vertex and pixel shaders.

2 Multiple Choice

For each question, circle exactly one of (a)-(e), unless otherwise stated.

11. Consider the use of homogeneous coordinates $(x, y, z, w)^T$ in the graphics pipeline.

    I. $(x, y, z, w)^T$ can be used to represent either a 3D point or a 3D vector.
    II. $w = 0$ for a 3D vector.
    III. Nonzero values of w are used to effect translation and perspective transformation.

    (a) I only
    (b) I and II only
    (c) II and III only
    (d) I and III only
    (e) $\boxed{\text{I, II and III}}$

12. Consider the OpenGL graphics pipeline. Which statements are true?

    I. Pipelining increases throughput and decreases latency.

    II. OpenGL sorts triangles to determine visibility.

    III. In modern OpenGL, the user may supply shaders which will execute on the GPU.

    (a) I only

    (b) II only

    (c) $\boxed{\text{III only}}$

    (d) I and II only

    (e) I and III only

13. Which of the following are true?

    I. Backface culling refers to eliminating geometry with backfacing normals.

    II. The Painter's algorithm attempts to sort fragments and draw them back to front.

    III. Using z-buffering, shading calculations will be done for fragments that may never appear on screen.

    (a) I only

    (b) I and II only

    (c) II and III only

    (d) $\boxed{\text{I and III only}}$

    (e) I, II and III

14. Concerning flat, smooth, and Phong shading,

    I. in flat shading the shading calculation is done once per triangle, while in Phong shading the shading calculation is done once per fragment.

    II. flat shading does not require any normals.

    III. smooth shading requires interpolation of normals to vertices.

    (a) I only

    (b) I and II only

    (c) $\boxed{\text{I and III only}}$

    (d) II and III only

    (e) I, II and III

15. Match the type of transformation in the left column with the example transformation matrix in the right by drawing lines between the matching boxes.

translation	$\begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
perspective	$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -3 & -2 \\ 0 & 0 & 1 & 1 \end{pmatrix}$
nonuniform scale	$\begin{pmatrix} 5 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
identity	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

16. Perspective transformations

    I. are nonlinear transformations.

    II. preserve the z ordering of vertices between the near and far planes.

    III. can change the sign of the z coordinate for vertices behind the eye.

    (a) I only

    (b) I and II only

    (c) I and III only

    (d) II and III only

    (e) $\boxed{\text{I, II and III}}$

17. Consider the Midpoint algorithm given here:

```
 (1)  y = y0
 (2)  d = f(x0+1,y0+1/2)
 (3)  for x = x0 to x1
 (4)  do
 (5)    draw(x,y)
 (6)    if (d<0)
 (7)    then
 (8)      y = y+1
 (9)      d = d+(y0-y1)+(x1-x0)
(10)    else
(11)      d = d+(y0-y1)
(12)    end
(13)  end
```

Which statements are true?

I. For a line with slope $m > 1$, we should change the outer loop in line (3) to be over y.

II. Lines (9) and (11) update the decision variable d through an incremental evaluation of the line equation f.

III. This algorithm fails if d is ever 0.

(a) I only

(b) $\boxed{\text{I and II only}}$

(c) I and III only

(d) II and III only

(e) I, II and III

18. Which of the following statements about barycentric coordinates (α, β, γ) for triangles are true?

I. If $s = \alpha + \beta + \gamma$, then $s < 1$ for points inside the triangle, $s > 1$ for points outside the triangle, and $s = 1$ for points on the triangle.

II. At least one of α, β, and γ will be 0 for a point on the triangle.

III. α, β, and γ can be used to interpolate vertex attributes across the face of the triangle.

(a) I only

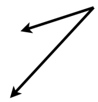
(b) I and II only

(c) I and III only

(d) $\boxed{\text{II and III only}}$

(e) I, II and III

19. Consider the 3D vectors, $\mathbf{x}$, $\mathbf{y}$, illustrated below, and dot product $\cdot$ and cross product $\times$. Which statements are true?



I. $\mathbf{x} \cdot \mathbf{y} > 0$.

II. $\mathbf{x} \times \mathbf{y} = 0$, because $\mathbf{x}$ and $\mathbf{y}$ lie in the same plane.

III. $\mathbf{x} \cdot \mathbf{x} = \mathbf{y} \cdot \mathbf{y}$.

(a) $\boxed{\text{I only}}$
(b) II only
(c) I and II only
(d) I and III only
(e) None

20. Consider the following equation from the Lambertian reflectance model, where R_a, R_d, L_a, and L_d are the ambient and diffuse reflectance of the object, and the ambient and diffuse components of the light, respectively, $\mathbf{l}$ is the light vector, and $\mathbf{n}$ is the object normal vector.

$$I = R_a L_a + R_d L_d \max(0, \mathbf{l} \cdot \mathbf{n})$$

I. Polygons facing away from the light will necessarily have I = 0.

II. This formula can capture specular highlights.

III. Generally $\mathbf{n}$ will vary over the surface of the object but $\mathbf{l}$ will be constant.

(a) I only
(b) II only
(c) I and II only
(d) I and III only
(e) $\boxed{\text{None}}$

3 Written Response

21. Consider a line in the plane going through the points $(1, 2)$ and $(2, 4)$.

    (a) Write down the explicit equation for the line, with x the independent variable and $y = f(x)$ the dependent variable.

    Answer The explicit equation is of the form $y = f(x) = mx+b$. In this case $m = \frac{4-2}{2-1} = 2,\ b = 0$, so

    $$y = f(x) = 2x.$$

    (b) Write down an implicit equation for the line. Identify a normal to the line.

    Answer

    $$f(x, y) = -2x + y = 0.$$

    This can also be expressed as

    $$f(x, y) = (-2, 1) \cdot (x, y) = 0,$$

    so the vector $(-2, 1)^T$ is normal to the line.

    (c) Write down a parametric equation for the line *segment* going through the two points, in terms of a single parameter $t \in [0, 1]$.

    Answer The parametric equation is of the form $\mathbf{p}(t) = \mathbf{a} + t(\mathbf{b} - \mathbf{a})$ for segment endpoints $\mathbf{a}$ and $\mathbf{b}$. In this case,

    $$\mathbf{p}(t) = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad 0 \le t \le 1.$$

22. Come up with a series of matrices as well as an order of multiplication (you don't need to actually perform the multiplication) to transform the triangle (0,0), (1,0), (0,3) to (-1,0), (-3,0),(-1,-6). Sketch the triangle at every step of the transformation.

The transformations are:

1. Scale uniformly by 2:

$$S = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2. Rotate by π radians CCW:

$$R = \begin{pmatrix} \cos(\pi) & -\sin(\pi) & 0 \\ \sin(\pi) & \cos(\pi) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

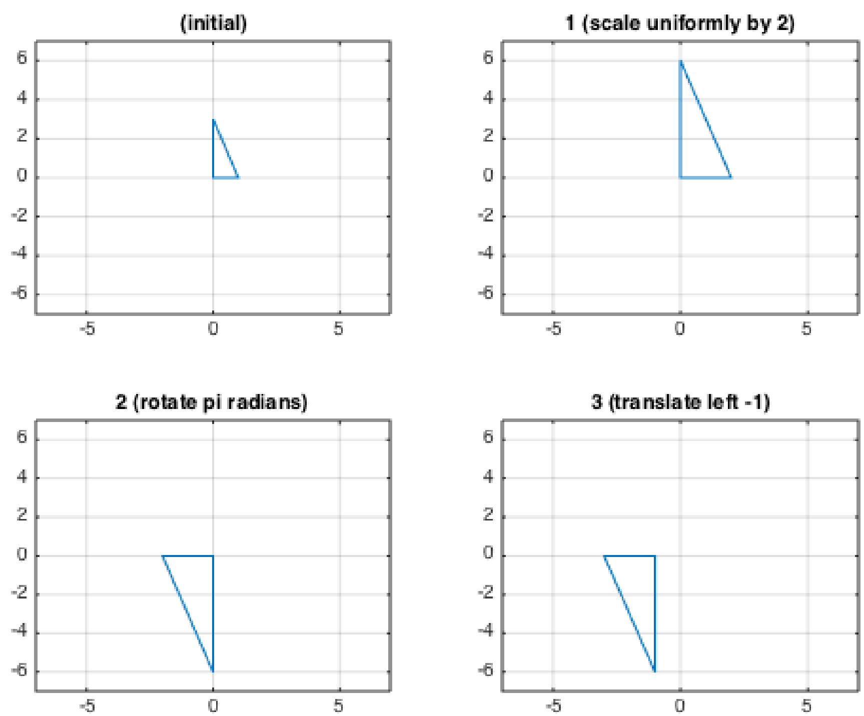
3. Translate left by -1:

$$T = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Thus each point in the triangle would be multiplied by the matrix

$$TRS.$$

The effect on the triangle is illustrated below



23. Consider a ray with endpoint $\mathbf{a}$ and a normalized direction $\mathbf{u}$,

$$\mathbf{p}(t) = \mathbf{a} + t\mathbf{u}, \quad t \geq 0, \tag{1}$$

and a sphere of radius r, centered at the origin. The implicit equation for the sphere is given as follows:

$$f(\mathbf{p}) = \mathbf{p} \cdot \mathbf{p} - r^2 = 0 \tag{2}$$

(a) Describe geometrically the ways in which the ray can intersect/not intersect with the sphere. I.e., when is there exactly one intersection, when are there two intersections, and when are there no intersections?

Answer

We enumerate the ways in which the line containing the ray can intersect the sphere, and for each case how the ray itself ($t \geq 0$) can intersect the sphere. These cases are illustrated below.

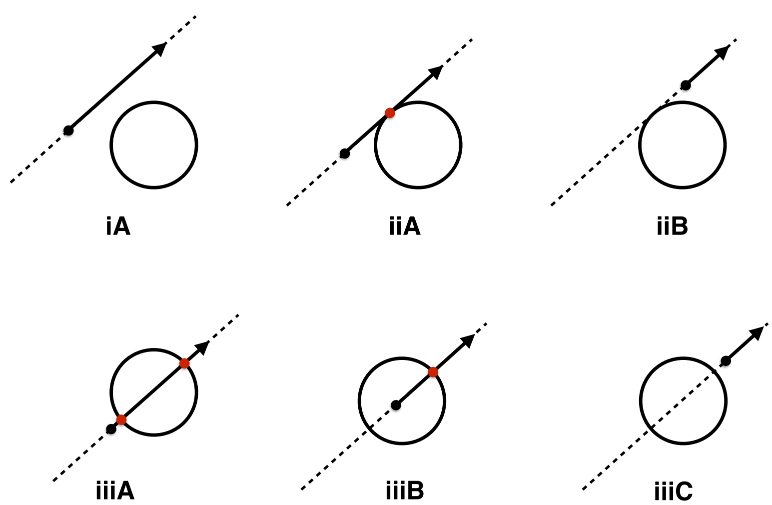
i. 0 intersection points of line and sphere
   - A. the ray also also does not intersect with the sphere

ii. 1 intersection point of line and sphere
   - A. the ray contains the intersection point
   - B. the ray does not contain the intersection point

iii. 2 intersection points of line and sphere
   - A. the ray contains point intersection points
   - B. the contains exactly one intersection point
   - C. the ray does not contain either intersection point

In summary,

- 0 intersection points: iA, iiB, and iiiC
- 1 intersection point: iiA and iiiB
- 2 intersection points: iiiA



(b) Find an expression for t where the intersection occurs by plugging eq. (1) into eq. (2) and solving for t. How can this expression to be used to distinguish the three cases described in part (a)? *Hint (Quadratic formula): Solutions to* $ax^2 + bx + c = 0$ *are* $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Answer

Plugging eq. (1) into eq. (2), we get

$$f(\mathbf{p}(t)) = (\mathbf{a} + t\mathbf{u}) \cdot (\mathbf{a} + t\mathbf{u}) - r^2 = 0.$$

This gives

$$\begin{aligned}(\mathbf{a} + t\mathbf{u}) \cdot (\mathbf{a} + t\mathbf{u}) - r^2 &= 0\\ \mathbf{a} \cdot \mathbf{a} + 2t\mathbf{a} \cdot \mathbf{u} + t^2 \mathbf{u} \cdot \mathbf{u} - r^2 &= 0\\ (\mathbf{u} \cdot \mathbf{u})\, t^2 + (2\mathbf{a} \cdot \mathbf{u})\, t + \left(\mathbf{a} \cdot \mathbf{a} - r^2\right) &= 0.\end{aligned}$$

We know $\mathbf{u} \cdot \mathbf{u} = 1$ since the direction vector $\mathbf{u}$ of the ray is normalized. So this simplifies to

$$t^2 + (2\mathbf{a} \cdot \mathbf{u})\, t + \left(\mathbf{a} \cdot \mathbf{a} - r^2\right) = 0.$$

Using the quadratic formula given in the hint, we solve for t:

$$t = \frac{-2\mathbf{a} \cdot \mathbf{u} \pm \sqrt{4(\mathbf{a} \cdot \mathbf{u})^2 - 4(\mathbf{a} \cdot \mathbf{a} - r^2)}}{2}$$

Simplifying further, the expression for t is

$$\boxed{t = -\mathbf{a} \cdot \mathbf{u} \pm \sqrt{(\mathbf{a} \cdot \mathbf{u})^2 - (\mathbf{a} \cdot \mathbf{a} - r^2)}}$$

We will label the two roots as

$$\begin{aligned}t1 &= -\mathbf{a} \cdot \mathbf{u} - \sqrt{(\mathbf{a} \cdot \mathbf{u})^2 - (\mathbf{a} \cdot \mathbf{a} - r^2)},\\ t2 &= -\mathbf{a} \cdot \mathbf{u} + \sqrt{(\mathbf{a} \cdot \mathbf{u})^2 - (\mathbf{a} \cdot \mathbf{a} - r^2)}.\end{aligned}$$

The different intersection cases can be distinguished by the value of the discriminant

$$d = (\mathbf{a} \cdot \mathbf{u})^2 - (\mathbf{a} \cdot \mathbf{a} - r^2),$$

and of $t1$ and $t2$. These are summarized in this table, with l = number of line/sphere intersections, r = number of ray/sphere intersections

d	$t1$	$t2$	fig	l	r
$d < 0$	$t1$ complex	$t2$ complex	iA	0	0
$d = 0$	$t1 \geq 0$	$t2 \geq 0$	iiA	1	1
$d = 0$	$t1 \leq 0$	$t2 \leq 0$	iiB	1	0
$d > 0$	$t1 \geq 0$	$t2 \geq 0$	iiiA	2	2
$d > 0$	$t1 < 0$	$t2 \geq 0$	iiiB	2	1
$d > 0$	$t1 < 0$	$t2 < 0$	iiiC	2	0

(c) Write pseudocode for an algorithm for finding the intersection points or identifying that there is is no intersection.

Answer

```
// returns number of intersections found
// sets intersection1 to first intersection, if applicable
// sets intersection2 to second intersection, if applicable
FindIntersections(a,u,r)

d = sqr(dot(a,u)) - dot(a,a) + sqr(r)

if (d < 0)
  return 0 // no intersections
end

t1 = -dot(a,u) - sqrt(d)
t2 = -dot(a,u) + sqrt(d)

if (d == 0)
  if(t1 >= 0)
     intersection1 =  intersection2 = a+ t1 * u
     return 1
  end
end

if (d > 0)
  if (t2 < 0)
     return 0
  else if (t1 < 0)
     intersection2 = a + t2 * u
     return 1
  else
     intersection1 = a + t1 * u
     intersection2 = a + t2 *u
   end
end
```