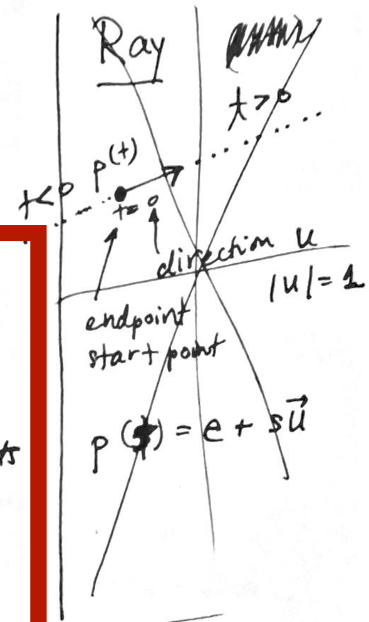
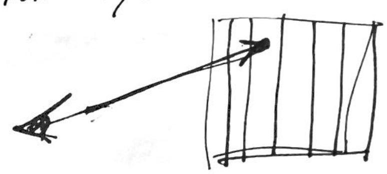
Types of Rays

1. Eye/pixel rays
2. Illumination/shadow rays.
3. Reflection rays
   - specular highlight
   - reflect other objects
4. Transmission/transparency rays.



$\boxed{1}$ Eye rays

* automatically get a perspective view when trace from eye.



Q. How would you get an ~~orthogonal~~ orthographic view?

Illumination/Shadow Rays.

2. Shadow rays

[Shirley, 4.7]

ray = ray(e, u, 0, ∞)

Cast-Ray (ray, parent-ray)

if (object = ... closest-intersection (ray))

then

$p = \left(e + \text{ray.t\_max}\, u \right) = \text{ray.Point(ray.t\_max)}$

color = $L_a R_a$

for each light source. → if (! closest-intersection (ray(p, l, ε, ∞)) then

Shadow ray.

avoid specks

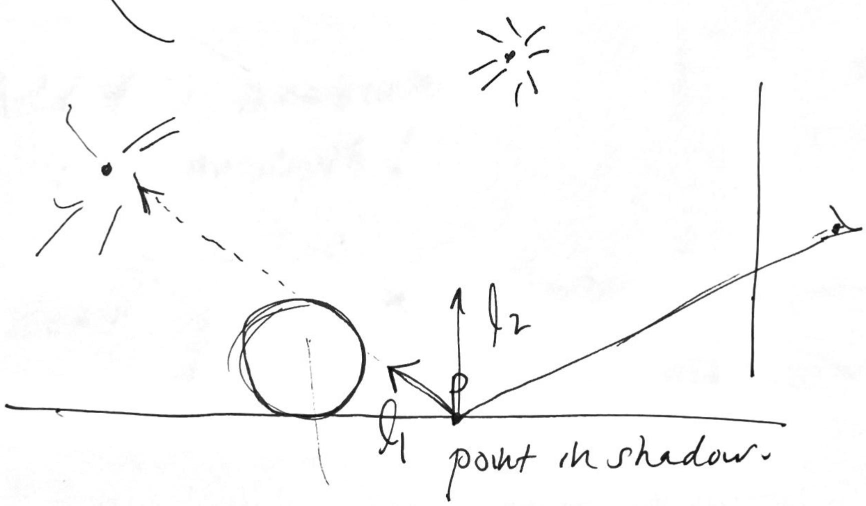
semi-inf = true

$h = \ \dots \ \dfrac{l + V}{\|l + V\|}$

$c \mathrel{+}= L_d R_d \cdot \max(0, n \cdot l) + L_s R_s (n \cdot h)^e$

else

return background color.

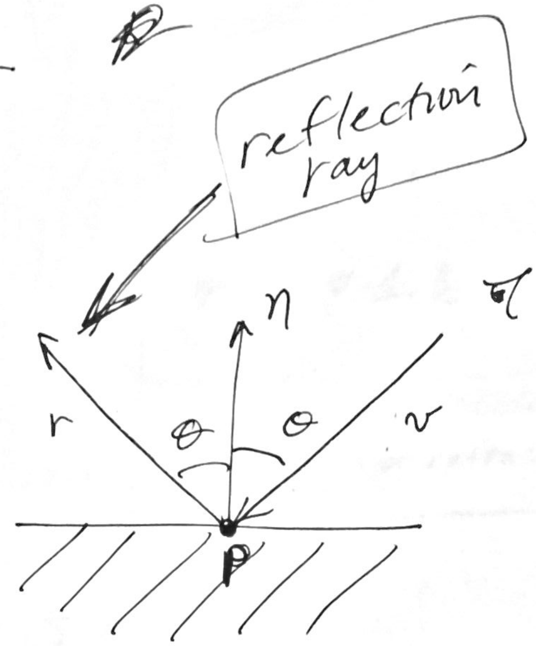


Ideal Specular Reflection

[Shirley 4.8]

mirror reflection

3. Reflection rays



$$r = v - 2(v \cdot n)n$$

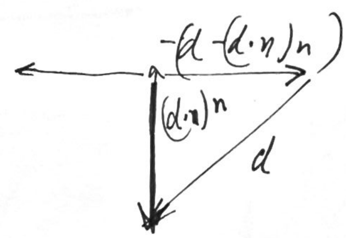
$(1-k_m)$ — in my solution

$$\text{Color } c = (1-k_m)\,c + k_m \,\text{Ray\_Color}(\text{ray}(p, r, \varepsilon, \infty))$$

important to avoid intersecting w/ self

- Add a maximum recursion depth!

- don't generate a reflection if $k_m == 0$ (no reflection).

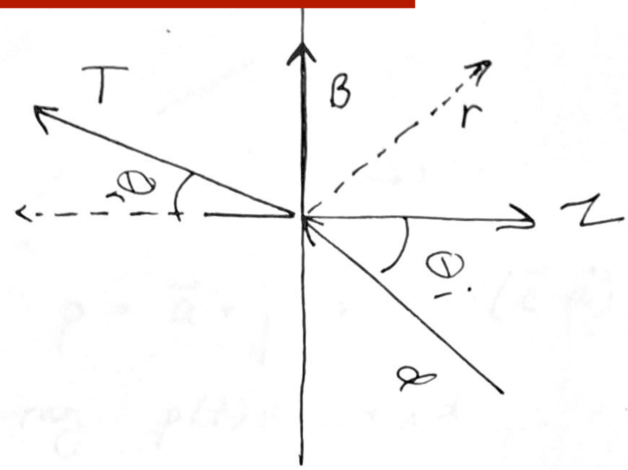


~~d + r =~~

$$r = -d + 2(d - (d \cdot n)n)$$
$$= +d - 2(d \cdot n)n$$

Transparency and Refraction

4. Transmission rays



η = index of refraction

$\eta_{air} \cong 1$

$\eta_{water} \cong 1.33$

θ_i = angle of incidence

θ_r = angle of refraction

What is the transmitted ray T?

First, find θ_r using Snell's law:

$$\eta_r \sin\theta_r = \eta_i \sin\theta_i$$

using cosine's, this is

$$\eta_r^2 (1-\cos^2\theta_r) = \eta_i^2 (1-\cos^2\theta_i)$$

$$\Rightarrow \cos^2\theta_r = -\frac{\eta_i^2}{\eta_r^2}(1-\cos^2\theta_i) + 1$$

$$\Rightarrow \cos\theta_r = \Big[\underbrace{1 - \frac{\eta_i^2}{\eta_r^2}(1-\cos^2\theta_i)}_{\circledast}\Big]^{1/2}$$

if $\circledast < 0$, no refracted ray "total internal reflection"

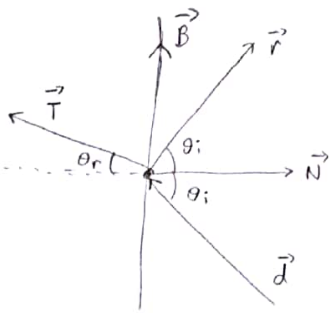
$$-\vec{d} = \cos\theta_i \vec{N} - \sin\theta_i \vec{B} \quad \Rightarrow \quad \vec{B} = \frac{+d + \cos\theta_i \vec{N}}{+\sin\theta_i}$$

$$\vec{T} = -\cos\theta_r \vec{N} + \sin\theta_r \vec{B}$$

$$\vec{T} = -\cos\theta_r \vec{N} + \frac{\sin\theta_r}{\sin\theta_i}\left[\vec{d} + \cos\theta_i \vec{N}\right]$$

$$= -\cos\theta_r \vec{N} + \frac{\eta_i}{\eta_r}\vec{d} + \frac{\eta_i}{\eta_r}\cos\theta_i \vec{N} = -\cos\theta_r \vec{N} + \frac{\eta_i}{\eta_r}\left[\vec{d} - (N \cdot d)N\right]$$

$$\vec{T} = \frac{\eta_i}{\eta_r}\left[d - (N \cdot d)N\right] - \cos\theta_r N$$



$\vec{T}$: transmited ray

$\vec{B}$: tangent to the surface

First let's find θ_r using Snell's Law:

$$n_r \sin\theta_r = n_i \sin\theta_i$$

n_r is refraction index of the object's material (water, for instance, will have $n_r = 1.33 - 1.34$)

n_i is refraction index oside the object (air, for instance, will have $n_i = 1$)

-> take the square of both sides

$$n_r^2 \sin^2\theta_r = n_i^2 \sin^2\theta_i$$ "replace sin by cos: $\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta$

$$n_r^2 (1 - \cos^2\theta_r) = n_i^2 (1 - \cos^2\theta_i)$$ "isolate $\cos^2\theta_r$

$$-n_r^2 \cos^2\theta_r = n_i^2 (1 - \cos^2\theta_i) - n_r^2 \Rightarrow \cos^2\theta_r = 1 - \frac{n_i^2}{n_r^2}(1 - \cos^2\theta_i)$$

$$\Rightarrow \boxed{\cos\theta_r = \Big[\underbrace{1 - \left(\frac{n_i}{n_r}\right)^2 (1 - \cos^2\theta_i)}\Big]^{1/2}}$$

what happens if this is negative? total internal reflection

$$-\vec{d} = \cos\theta_i \vec{N} - \sin\theta_i \vec{B} \Rightarrow \vec{B} = \frac{\vec{d} + \cos\theta_i \vec{N}}{\sin\theta_i}$$

$$\vec{T} = -\cos\theta_r \vec{N} + \sin\theta_r \vec{B}$$ "replacing $\vec{B}$"

$$= -\cos\theta_r \vec{N} + \frac{\sin\theta_r}{\sin\theta_i}\left(\vec{d} + \cos\theta_i \vec{N}\right)$$ " $\frac{\sin\theta_r}{\sin\theta_i} = \frac{n_i}{n_r}$ "

$$= -\cos\theta_r \vec{N} + \frac{n_i}{n_r}\left(\vec{d} + \cos\theta_i \vec{N}\right)$$ " $\cos\theta_i = (-\vec{d}) \cdot N \quad \|\vec{d}\| = 1$ "

$$\boxed{= \frac{n_i}{n_r}\left(\vec{d} - (\vec{d} \cdot \vec{N}) \cdot \vec{N}\right) - \cos\theta_r \vec{N}}$$ "dot product is not associative!"