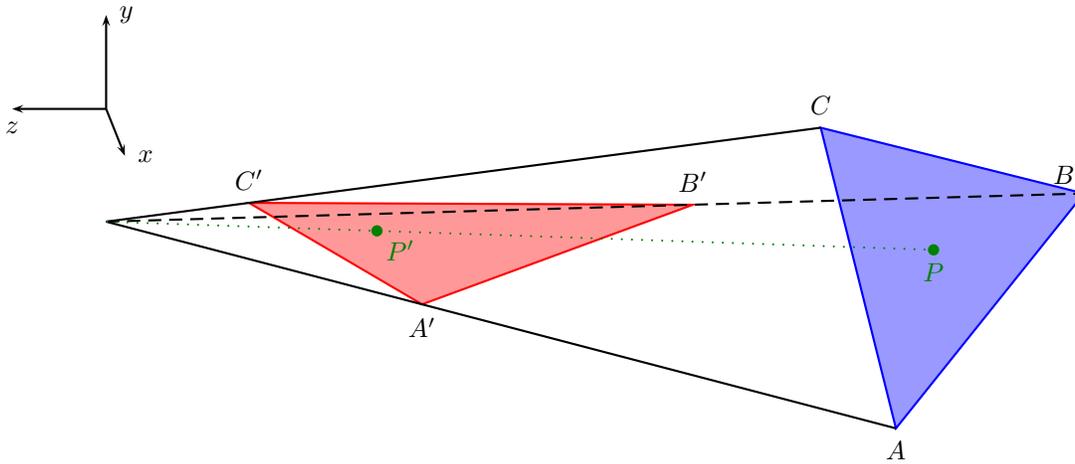Craig Schroeder

Perspective Correct Interpolation

CS 130

1. Viewing frustum in camera space. Camera is at the origin.

2. Transform from A, B, C, P to A', B', C', P' by homogeneous matrix $\mathbf{M}$.

$$\begin{pmatrix} A'w_a \\ w_a \end{pmatrix} = \mathbf{M} \begin{pmatrix} A \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} B'w_b \\ w_b \end{pmatrix} = \mathbf{M} \begin{pmatrix} B \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} C'w_c \\ w_c \end{pmatrix} = \mathbf{M} \begin{pmatrix} C \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} P'w_p \\ w_p \end{pmatrix} = \mathbf{M} \begin{pmatrix} P \\ 1 \end{pmatrix}$$

3. The real barycentric weights are α, β, γ. Because of the projection, they appear to be α', β', γ'.

$$P = \alpha A + \beta B + \gamma C$$
$$P' = \alpha' A' + \beta' B' + \gamma' C'$$

4. While rasterizing, we can compute α', β', γ' directly, but we will need the real weights α, β, γ to correctly interpolate color.

5. Noting $\alpha + \beta + \gamma = 1$,

$$\begin{pmatrix} P \\ 1 \end{pmatrix} = \alpha \begin{pmatrix} A \\ 1 \end{pmatrix} + \beta \begin{pmatrix} B \\ 1 \end{pmatrix} + \gamma \begin{pmatrix} C \\ 1 \end{pmatrix}$$

$$\mathbf{M} \begin{pmatrix} P \\ 1 \end{pmatrix} = \alpha \mathbf{M} \begin{pmatrix} A \\ 1 \end{pmatrix} + \beta \mathbf{M} \begin{pmatrix} B \\ 1 \end{pmatrix} + \gamma \mathbf{M} \begin{pmatrix} C \\ 1 \end{pmatrix}$$

$$P' w_p = \alpha A' w_a + \beta B' w_b + \gamma C' w_c$$

$$w_p = \alpha w_a + \beta w_b + \gamma w_c$$

$$P' = \frac{\alpha A' w_a + \beta B' w_b + \gamma C' w_c}{\alpha w_a + \beta w_b + \gamma w_c}$$

$$P' = \frac{\alpha w_a}{\alpha w_a + \beta w_b + \gamma w_c} A' + \frac{\beta w_b}{\alpha w_a + \beta w_b + \gamma w_c} B' + \frac{\gamma w_c}{\alpha w_a + \beta w_b + \gamma w_c} C'$$

$$\alpha' = \frac{\alpha w_a}{\alpha w_a + \beta w_b + \gamma w_c}$$

$$\beta' = \frac{\beta w_b}{\alpha w_a + \beta w_b + \gamma w_c}$$

$$\gamma' = \frac{\gamma w_c}{\alpha w_a + \beta w_b + \gamma w_c}$$

6. This is the wrong way around. We have α' but need α.

$$k = \frac{k}{\alpha w_a + \beta w_b + \gamma w_c}$$

$$\alpha' = \alpha w_a k$$

$$\beta' = \beta w_b k$$

$$\gamma' = \gamma w_c k$$

$$\alpha = \frac{\alpha'}{w_a k}$$

$$\beta = \frac{\beta'}{w_b k}$$

$$\gamma = \frac{\gamma'}{w_c k}$$

$$1 = \alpha + \beta + \gamma = \frac{\alpha'}{w_a k} + \frac{\beta'}{w_b k} + \frac{\gamma'}{w_c k}$$

$$k = \frac{\alpha'}{w_a} + \frac{\beta'}{w_b} + \frac{\gamma'}{w_c}$$

$$\alpha = \frac{\frac{\alpha'}{w_a}}{\frac{\alpha'}{w_a} + \frac{\beta'}{w_b} + \frac{\gamma'}{w_c}}$$

$$\beta = \frac{\frac{\beta'}{w_b}}{\frac{\alpha'}{w_a} + \frac{\beta'}{w_b} + \frac{\gamma'}{w_c}}$$

$$\gamma = \frac{\frac{\gamma'}{w_c}}{\frac{\alpha'}{w_a} + \frac{\beta'}{w_b} + \frac{\gamma'}{w_c}}$$

7. Can now use α, β, γ to interpolate colors.