True/False

For each question, indicate whether the statement is true or false by circling T or F, respectively.

1. (T/F) Rasterization occurs before vertex transformation in the graphics pipeline.
2. (T/F) Clipping is performed after perspective division in the graphics pipeline.
3. (T/F) Given any matrices M_1, M_2, and M_3, $(M_1M_2)M_3 = M_1(M_2M_3)$.
4. (T/F) Given any matrices M_1, M_2, and M_3, $M_3M_2M_1 = M_1M_2M_3$.
5. (T/F) OpenGL supports z-buffering.
6. (T/F) In describing the orientation of a body, Euler angles are angles specified relative to a coordinate system fixed to the body.
7. (T/F) The perspective transformation is nonlinear in z.
8. (T/F) The viewport transformation maps from normalized device coordinates to screen space.
9. (T/F) Applying a perspective transformation in the graphics pipeline to a vertex involves dividing by its 'z' coordinate.
10. (T/F) This matrix is a rigid body transformation

$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 & 2 \\ \sin\theta & \cos\theta & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

11. (T/F) This matrix reflects about the x-axis.

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

12. (T/F) We can translate the vector

$$\begin{pmatrix} 3 \\ 2 \\ 1 \\ 0 \end{pmatrix}$$

by multiplying it by the matrix

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

13. (T/F) OpenGL sorts triangles to determine visibility.
14. (T/F) Gouraud shading requires more computation than Phong shading.
15. (T/F) Bezier curves are curves that interpolate all of their control points.

16. (T/F) An n^{th} order polynomial is uniquely determined by $n+1$ distinct control points.

17. (T/F) Piecewise polynomial curves are preferable to high order polynomials because interpolating a large number of points with a single high order polynomial can create a very oscillatory curve.

18. (T/F) Blending functions provide a convenient basis for expressing curves in terms of the control data.

19. (T/F) A cubic Bezier curve has 4 control points.

20. (T/F) A quadratic Bezier curve has degree two and three control points.

Multiple Choice

For each question, circle exactly one of (a)-(e), unless otherwise stated.

1. Consider the use of homogeneous coordinates $(x, y, z, w)^T$ in the graphics pipeline.

   I. $(x, y, z, w)^T$ can be used to represent either a 3D point or a 3D vector.

   II. $w = 0$ for a 3D vector.

   III. Nonzero values of w are used to effect translation and perspective transformation.

   (a) I only
   (b) I and II only
   (c) II and III only
   (d) I and III only
   (e) I, II and III

2. Match the type of transformation in the left column with the example transformation matrix in the right by drawing lines between the matching boxes.

translation	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
perspective	$\begin{pmatrix} 5 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
nonuniform scale	$\begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
identity	$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -3 & -2 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

3. Perspective transformations

    I. are nonlinear transformations.
    II. preserve the z ordering of vertices between the near and far planes.
    III. can change the sign of the z coordinate for vertices behind the eye.

    (a) I only
    (b) I and II only
    (c) I and III only
    (d) II and III only
    (e) I, II and III

4. Perspective transformations A) keep parallel lines parallel B) are affine transformations C) all of the above D) none of the above

5. Orthographic transformations A) keep parallel lines parallel B) are affine transformations C) all of the above D) none of the above

6. Which statements about the z-buffer approach to rendering are true?

    I. selects which fragment to draw based on its depth.
    II. orders triangles from back to front.
    III. orders triangles based on the average z-values of their vertices

    (a) I only
    (b) I and II only
    (c) I and III only
    (d) I, II and III
    (e) None

7. Which of the following statements about rotations are true?

    I. The vector component of the quaternion encodes the rotation axis.
    II. Gimbal locks remove a degree of freedom of rotation.
    III. Interpolation using Euler angles does not always yield geodesic (shortest) paths.

    (a) I only
    (b) II only
    (c) I and III only
    (d) II and III only
    (e) I, II and III

8. Which of the following statements about rotations are true?

   I. Any rotation in 3D space can be described using an angle and an axis.

   II. The inverse of a rotation matrix R is R^T.

   III. This rotation matrix will rotate the object pictured about its center.

$$\begin{pmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

   (a) II only
   (b) I and II only
   (c) I and III only
   (d) II and III only
   (e) I, II and III

9. Compared to flat shading, ____________improves the appearance of the objects silhouette.

   (a) Gouraud shading
   (b) Phong shading
   (c) none of the above

10. Concerning flat, smooth, and Phong shading,

    I. in flat shading the shading calculation is done once per triangle, while in Phong shading the shading calculation is done once per fragment.

    II. flat shading does not require any normals.

    III. smooth shading requires interpolation of normals to vertices.

    (a) I only
    (b) I and II only
    (c) I and III only
    (d) II and III only
    (e) I, II and III

11. How many degrees of freedom does a rigid body have in <u>two</u> dimensions?

    (a) 1
    (b) 2
    (c) 3
    (d) 4
    (e) 6

12. What is the correct order of operations of the OpenGL graphics pipeline?
    (a) projection transformation, modelview transformation, divide by w, viewport transform
    (b) modelview transformation, divide by w, projection transformation, viewport transform
    (c) modelview transformation, viewport transform, divide by w, projection transformation
    (d) modelview transformation, projection transformation, divide by w, viewport transform
13. A cubic Bezier curve
    (a) is a way to implicitly represent a cubic.
    (b) interpolates the first and last of its 4 control points.
    (c) has degree 2.
    (d) may extend outside the convex hull of its control points.
    (e) is seldom used in practice in computer graphics due to difficulty in evaluation of points on the curve.
14. If a curve is C^0 continuous, then A) it can have sharp corners B) its tangent vectors are continuous C) A and B D) none of the above
15. If a curve is C^1 continuous, then A) it can have sharp corners B) its tangent vectors are continuous C) A and B D) none of the above
16. Which of the following statements regarding curves are true?
    I. There is a unique n degree polynomial that interpolations $n + 1$ distinct data points.
    II. A monomial basis for curves up to order 3 is set $1, u, u^2, u^3$.
    III. When using piecewise polynomial curves to interpolate a set of data points, care must be taken at join points to ensure desired level of continuity.

    (a) II only
    (b) I and II only
    (c) I and III only
    (d) II and III only
    (e) I, II and III
17. When doing physical simulation, the advantage of having a small timestep(h) is that it: A) reduces computation time B) reduces the effects of errors due to numerical integration in time C) prevents rigid bodies from non-physically deforming D) there is no advantage; any nonzero timestep will do.
18. When doing physical simulation, the advantage of having a large timestep(h) is that it: A) reduces computation time B) reduces the effects of errors due to numerical integration in time C) prevents rigid bodies from non-physically deforming D) there is no advantage; any nonzero timestep will do

Written Response

1. Come up with a series of matrices as well as an order of multiplication (you don't need to actually perform the multiplication) to transform the triangle (0,0), (1,0), (0,3) to (-1,0), (-3,0),(-1,-6). Sketch the triangle at every step of the transformation.

2. Homogeneous Transformations

   (a) Write a matrix to transform a point by first rotating it $\frac{\pi}{2}$ radians about the y-axis, and then translating it by $(1, 3, 0)$.

   (b) Write down a vector pointing in direction $(1, 1, 1)$ in homogeneous coordinates and apply the transformation matrix from part (a) to it.

   (c) Explain the difference between how the transformation matrix would transform the point and how it transformed the vector.

3. Implicit and Parametric Equations

   (a) Give an implicit equation for a 2D circle of radius R centered at (x_0, y_0).

   (b) Give a parametric equation for the same circle as in part (a), i.e. complete the following equations:

$$x(t) = ?$$
$$y(t) = ?$$

   (c) Given two points A and B, write down an equation for the line segment between them parameterized by $t \in [0, 1]$ (It should linearly interpolate between A and B such that $f(0) = A$ and $f(1) = B$).

   (d) Give an implicit equation of a square centered at the origin with side length $2S$. Hint: your equation can be piecewise.

4. Given a particle with mass m, with state $\mathbf{x}$,$\mathbf{v}$ (position, velocity), and forces $\mathbf{F}$ on the particle, describe an algorithm for advancing the particle state to the next time step(the step size is h).

5. Consider a quadratic curve that interpolates three control points $\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2$. We wish to find a parametric representation of the curve of the form

$$\mathbf{f}(u) = \mathbf{a}_0 + \mathbf{a}_1 u + \mathbf{a}_2 u^2.$$

   (a) Set up a linear system of equations relating the known control points $\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2$ to the unknown coefficients $\mathbf{a}_0, \mathbf{a}_1, \mathbf{a}_2$, by choosing $\mathbf{f}(0) = \mathbf{p}_0$, $\mathbf{f}(.5) = \mathbf{p}_1$, and $\mathbf{f}(1) = \mathbf{p}_2$.

   (b) If your linear system in part (a) is given by $C\mathbf{a} = \mathbf{p}$, with

$$\mathbf{a} = \begin{pmatrix} \mathbf{a}_0 \\ \mathbf{a}_1 \\ \mathbf{a}_2 \end{pmatrix}, \quad \mathbf{p} = \begin{pmatrix} \mathbf{p}_0 \\ \mathbf{p}_1 \\ \mathbf{p}_2 \end{pmatrix}$$

   and $\mathbf{f}(u) = \mathbf{u}^T\mathbf{a}$ with

$$\mathbf{u} = \begin{pmatrix} 1 \\ u \\ u^2 \end{pmatrix}$$

   identify a set of blending functions that can be used to specify $\mathbf{f}$ directly in terms of the control points $\mathbf{p}_i$. You do not need to find the blending functions explicitly, but only identify how you would find them.