CS130 - LAB 1 - Section:

Name: **SID:**

Part 1: vectors, dot and cross product

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1. Solve the given vector equation for x:
3 * [1, 2, -1] + 4 * [2, 0, x] = [11, 6, 17]

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2. Solve the given vector equation for the scalar x. Is there a solution?
x * [1, 2, -1] + 4 * [3, 4, 2] = [-1, 0, 4]

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3. Calculate the cosine of the angle between the vectors
[2, 4, 4] and [4, 3, 0].

4. Given 2 vectors, a and b, explain the geometrical relationship between a and b for the following cases:

a) $a \cdot b = 0$:

b) $a \cdot b > 0$:

c) $a \cdot b < 0$:

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5. Calculate the cross product: [1, 2, 3] x [4, 5, 6].

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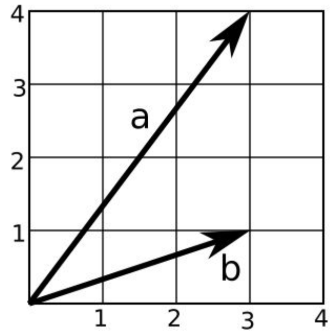
6. Given the triangle with vertices [0, 2, -1], [2, 0, -1] and [1, 0, 0], calculate the normal of the plane that contains the triangle.



7. Calculate the vector that bisects the angle between the vectors a and b in the figure above.

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8. Calculate a vector in the form ∝a + βb = u where ∝ and β are scalars and u is a vector orthogonal to b. Suppose ∝ is 1, draw the three vectors ∝a, βb and u in the figure above (draw ∝a from βb to u).

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Part 2: Matrices

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 3 & 7 & 19 \end{bmatrix}, B = \begin{bmatrix} 5 & 2 \\ 1 & -3 \\ -1 & 1 \end{bmatrix}$$

You don't have to do the divisions, just keep the values in division format.

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1. Calculate:

a) $A + B^T$

b) AB

c) $(AB)^{-1}$

2. Solve (AB)x = c, where c is the vector [1, 2] and x is [x_1, x_2].
Show the following steps:

a) Isolate x in the left-hand side of the equation.

b) Compute $(AB)^{-1}c$ to find the values of x.

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Part 3: Ray/plane intersection

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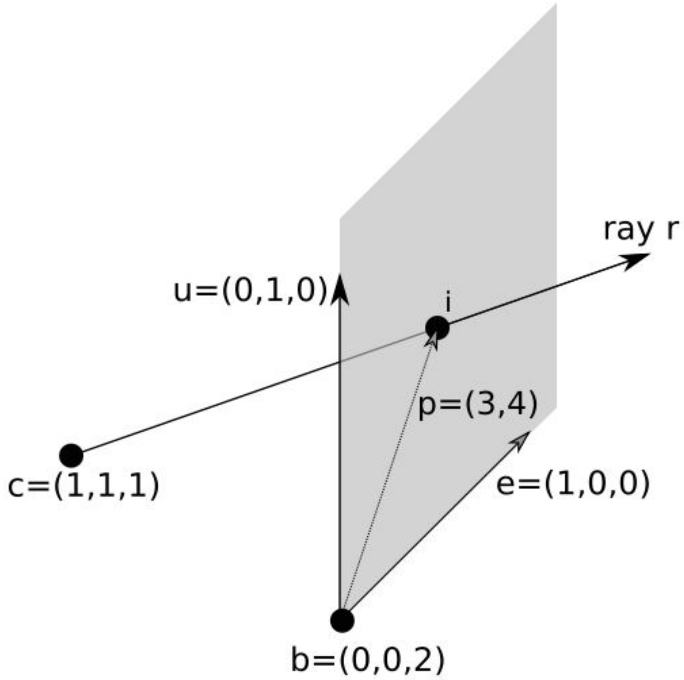
1. Calculate the endpoint and direction of the ray r in the figure below.
   - u and e are unitary vectors that define the shaded plane.
   - u and e originate at b.
   - The ray pass through the plane at the intersection point i.
   - The 2D vector p is on the plane. p goes from b (origin) to i.



ray endpoint:

ray direction (don't forget to normalize):

2. Consider a ray with endpoint e and (unitary) direction u. Consider a plane with (unitary) normal vector n and with a point x0 located anywhere on the plane. The equations for the ray and the plane are:

- ray: R(t) = e + ut
- plane: P(x) = (x - x0) • n = 0

Any point r on the ray can be found using the real value t >= 0.
Any point x that satisfies the plane equation is on the plane.

a) In the ray equation, what does t represent geometrically?

b) Follow the steps below and calculate t* such that the point in the ray intersects the plane.

Step 1: Combine the ray and plane equations to find t* that solves both equations by expanding P(R(t*)) = 0.

Step 2: Group terms with t* together.

Step 3: Fill out below by leaving t* alone on the left side.

t* =

c) What are the cases that will make t* undefined (e.g. division by 0), and what does it mean geometrically?

d) Explain the geometric meaning of two cases:

t* > 0:

t* < 0:

e) Write a code in C++ that receives e, u, n and x0, and returns *true* if the ray intersects the plane. Assume all vectors have the same size.

```
// you can use vec_f as a shortcut for a vector of floats
typedef vector<float> vec_f;

bool intersects(vec_f &e, vec_f &u, vec_f &n, vec_f &x0){

}

// computes dot product between the vectors a and b
float dot(vec_f &a, vec_f &b) {
      float d = 0;

      return d;
}

// compute the difference between vectors a and b
vec_t sub(vec_f &a, vec_f &b) {
      vec_t result(a.size());

      return result;
}
```

Home Exercise (Optional. Try to solve before next lab)

Consider a ray with endpoint e and (unitary) direction u. Consider a sphere with center c and radius r. The equations for the ray and the sphere are:

- ray: $R(t) = e + ut$
- sphere: $S(x) = (x - c) \cdot (x - c) = r^2$

Any point r on the ray can be found using the real value t >= 0.
Any point x that satisfies the sphere equation is on the sphere.

Calculate t*(s) such that the point in the ray intersects the sphere.

Hint: You can use steps and the reasoning that we used for ray-plane intersections.