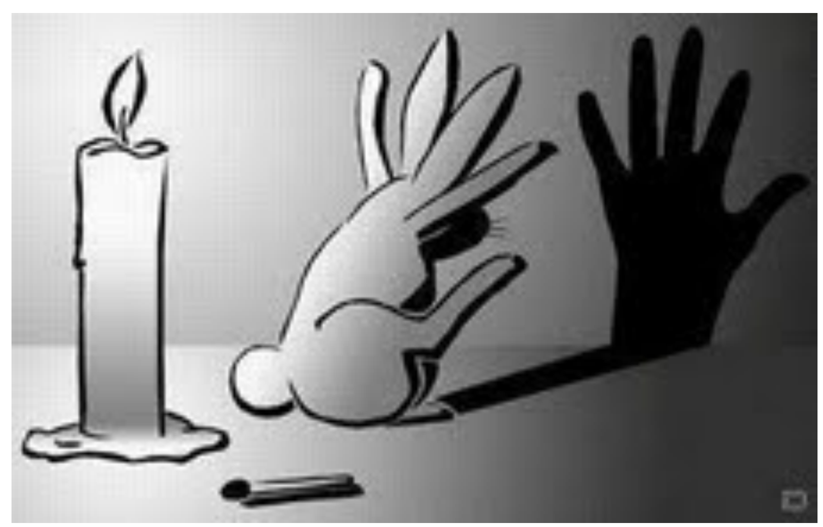
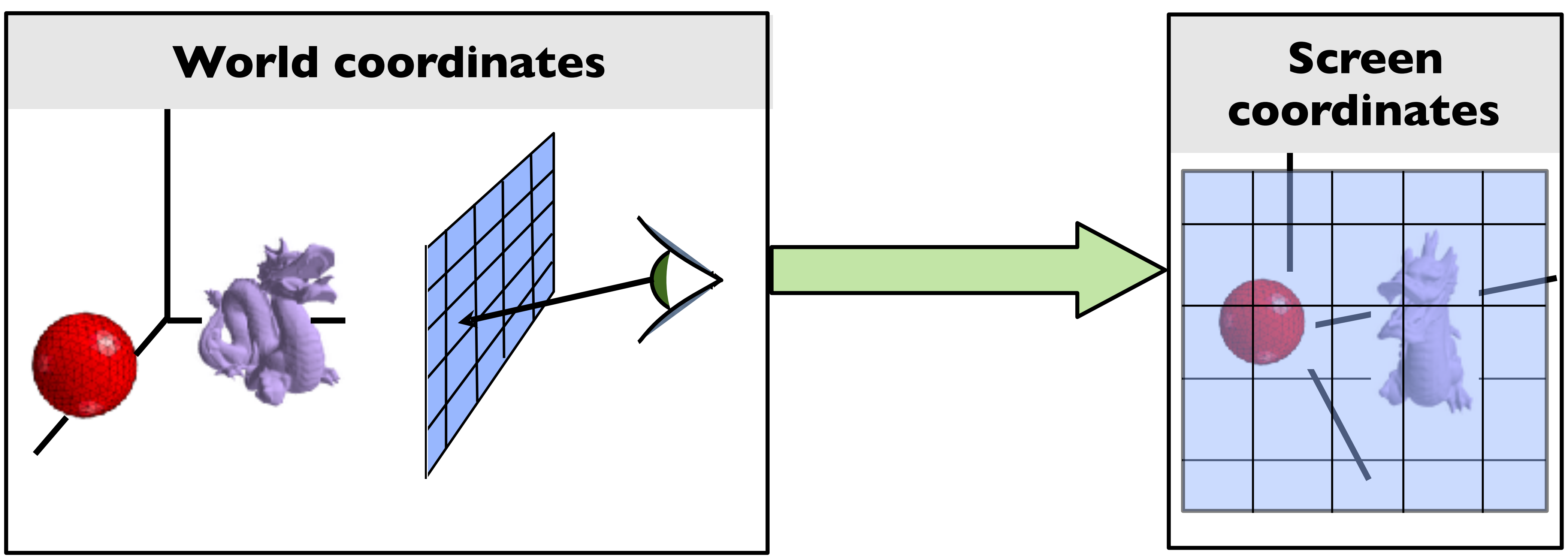
Viewing Transformations

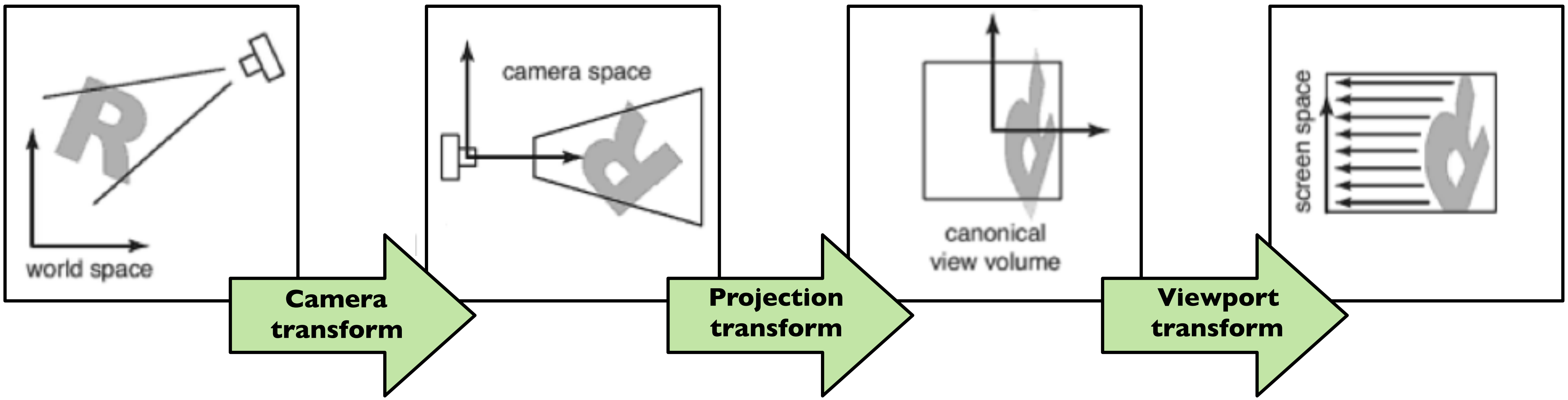


Viewing transformations

- Transform **vertices** from world coordinate descriptions to screen coordinate description



Decomposition of viewing transforms



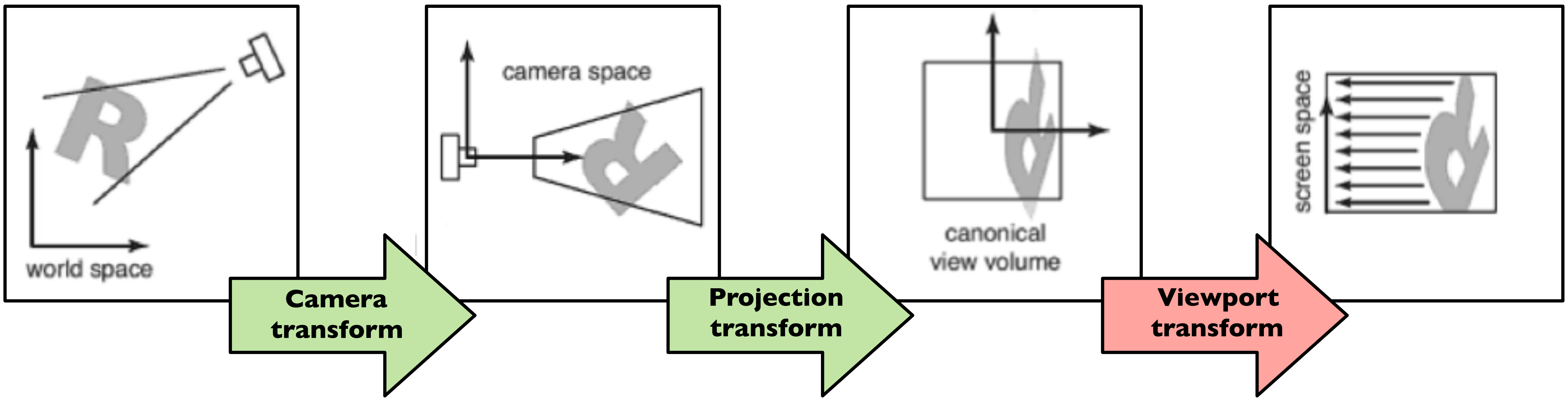
- rigid body transformation
- transform camera to origin

- x, y, z in [-1,1]
- depends on type of projection

- map to pixel coordinates

Viewing transforms depend on: camera position and orientation, type of projection, field of view, image resolution

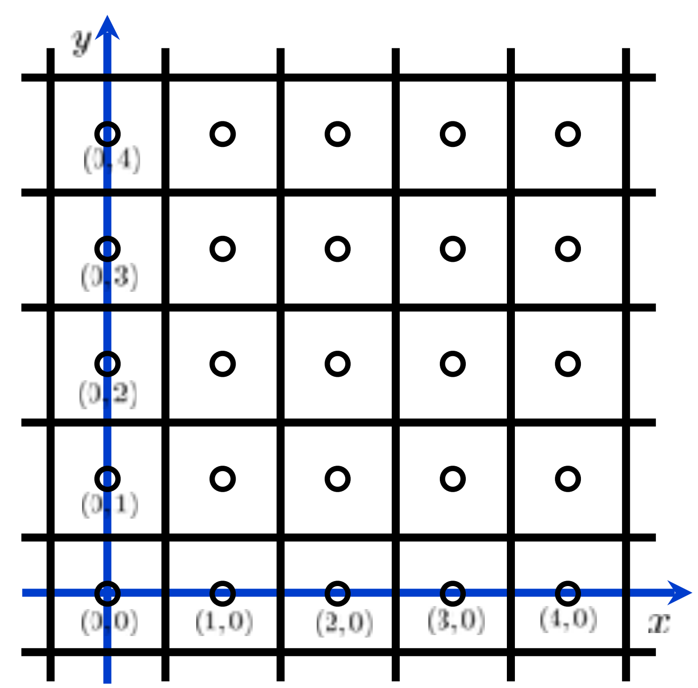
Viewport transform



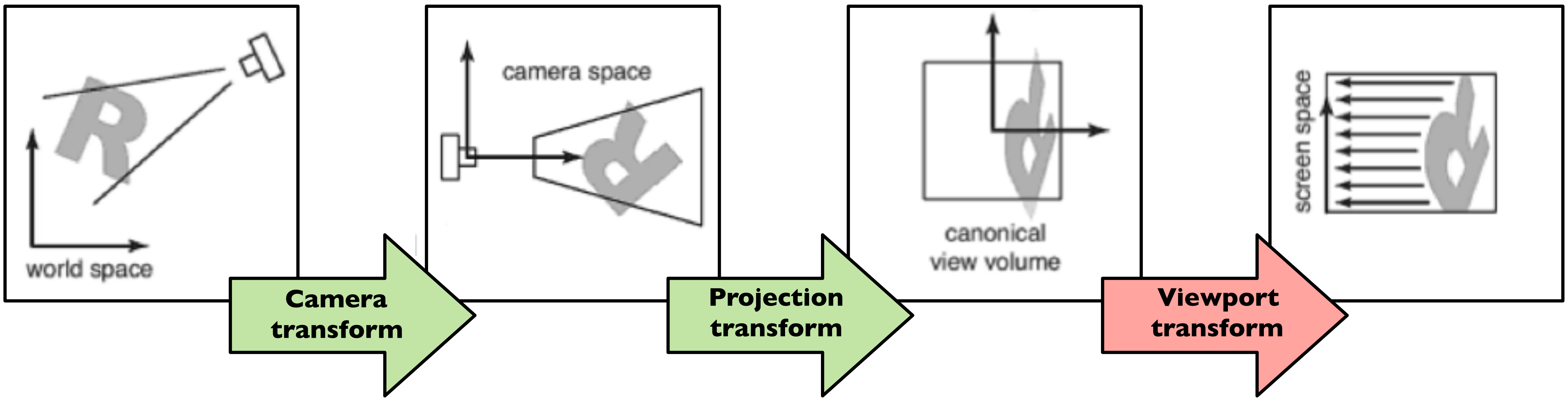
$$(x, y, z) \rightarrow (x', y', z')$$

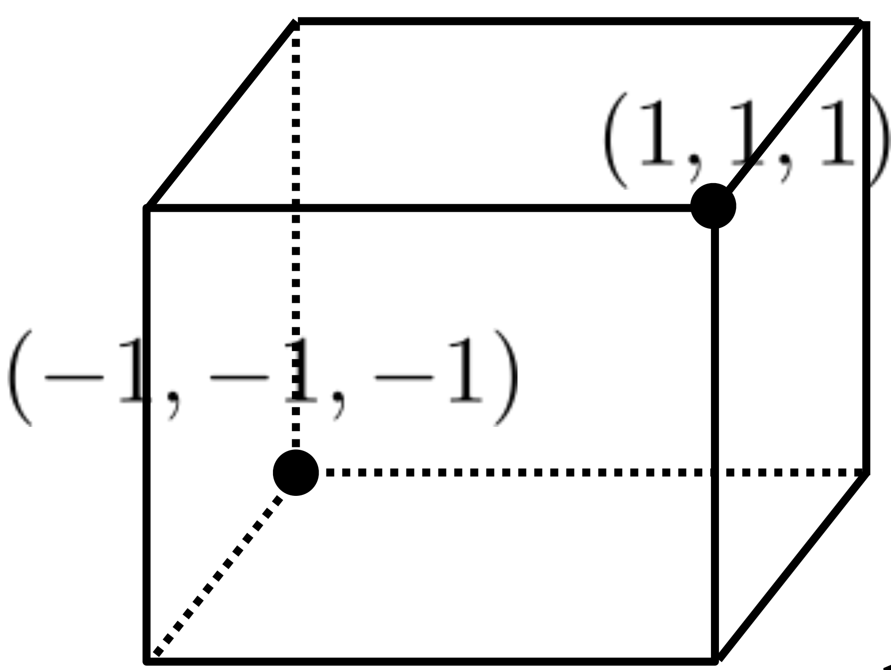
$$(x, y, z) \in [-1, 1]^3$$

$$x' \in [-.5, n_x - .5]$$
$$y' \in [-.5, n_y - .5]$$



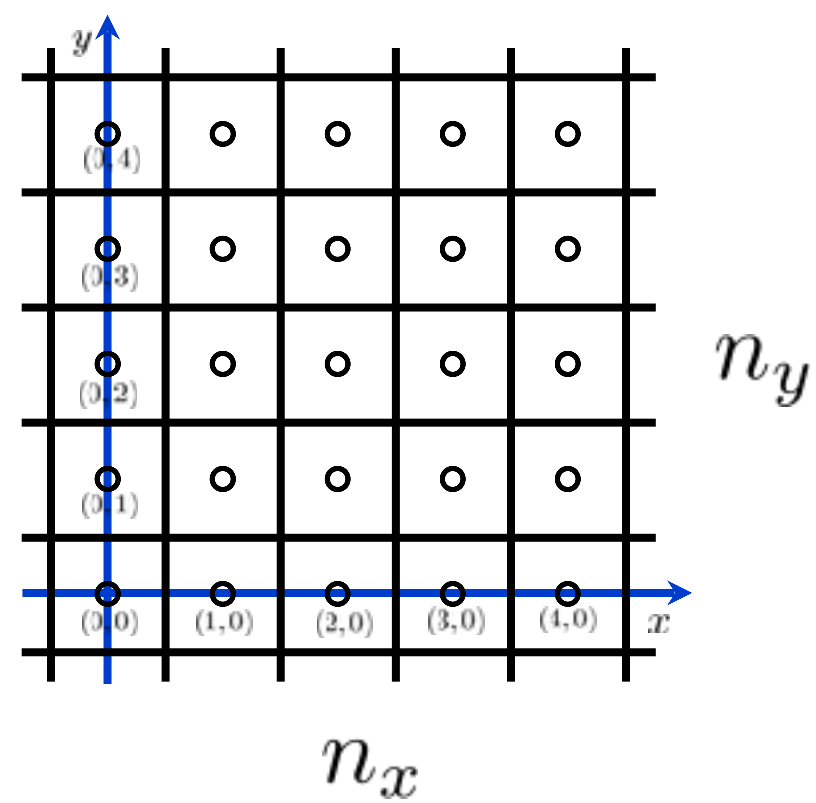
Viewport transform



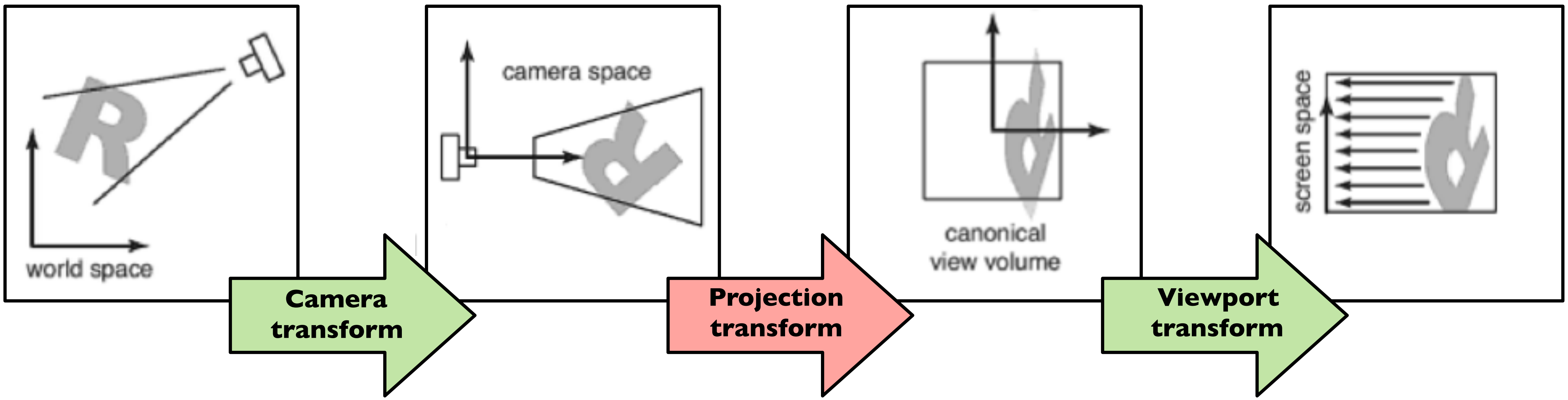


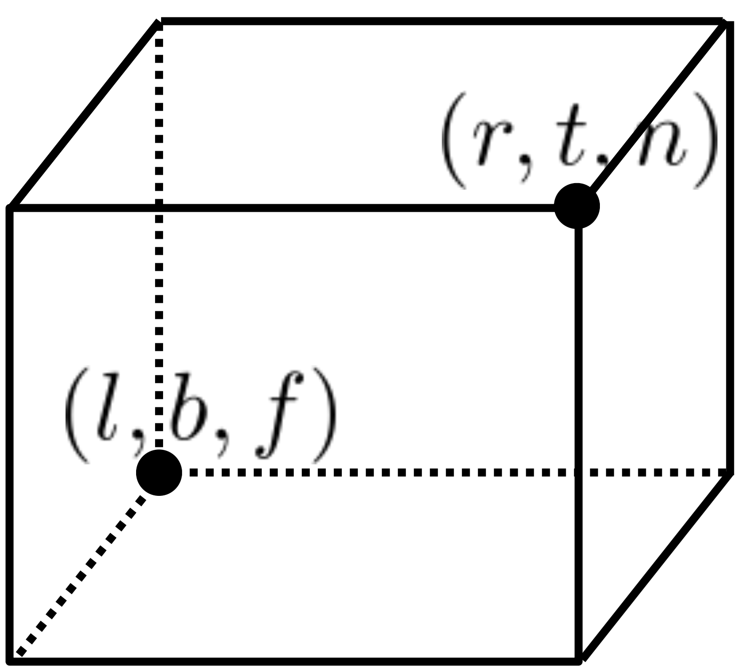
M_{vp}

<whiteboard>

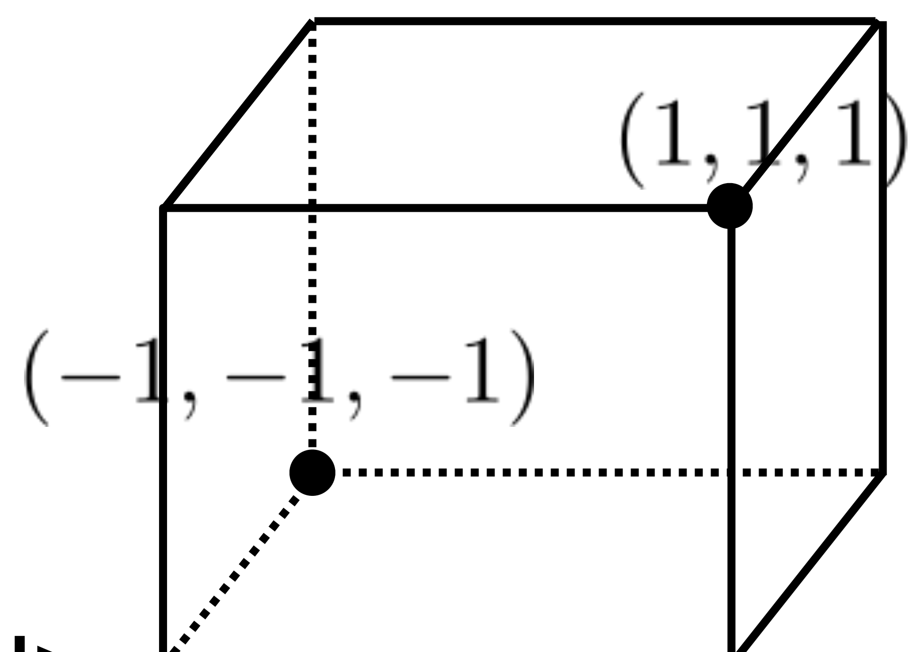


Orthographic Projection Transform



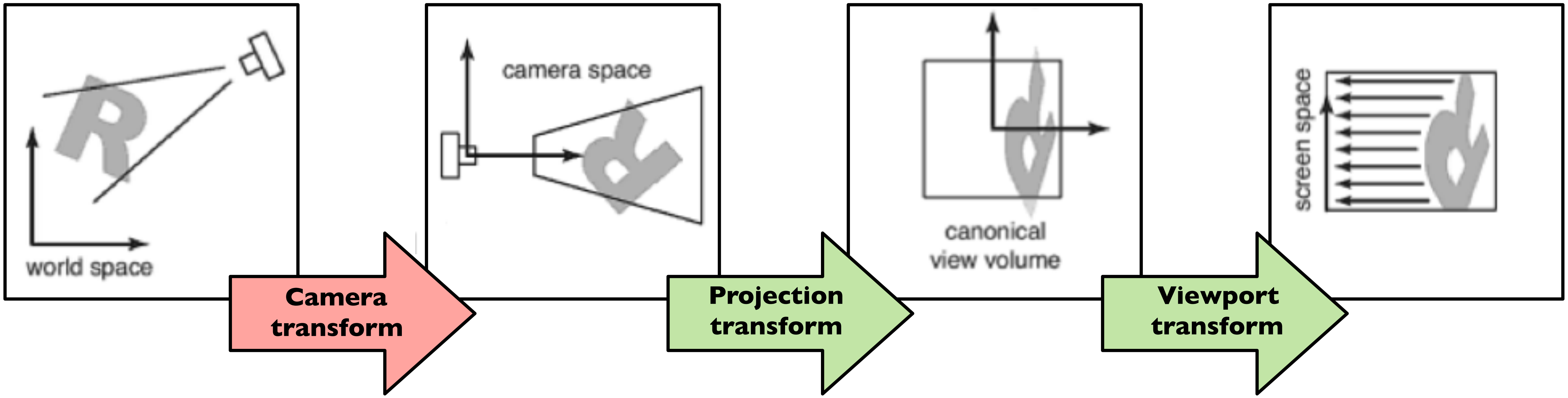


$$M_{orth}$$



<whiteboard>

Camera Transform

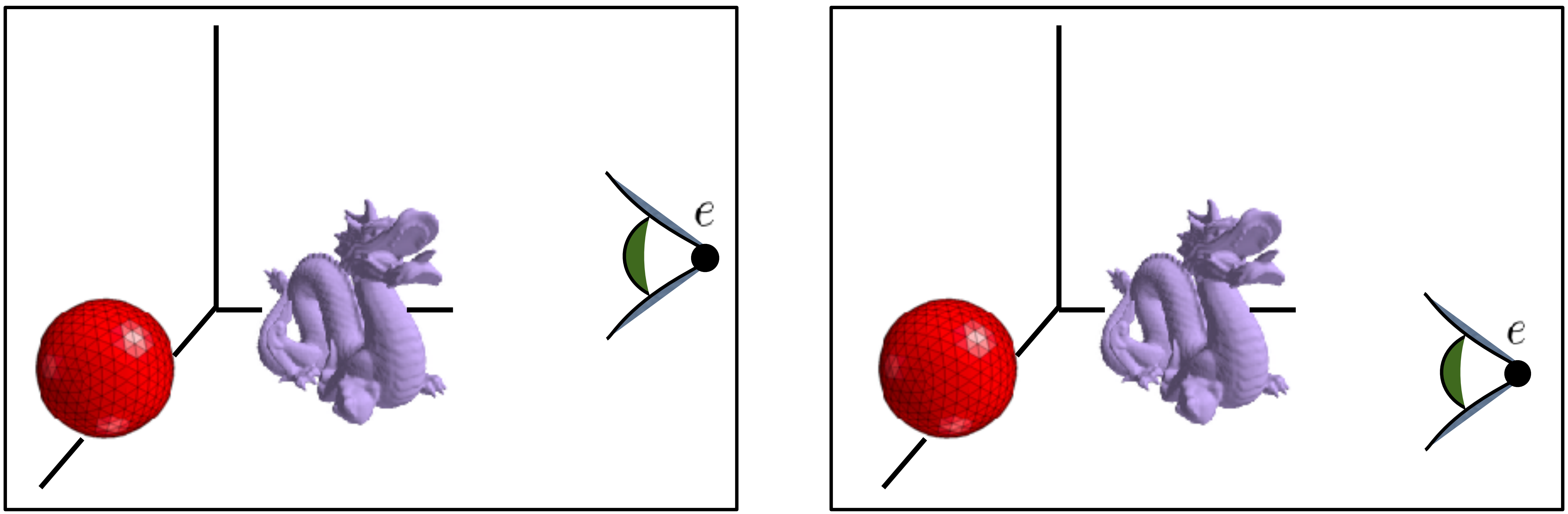


Camera Transform

How do we specify the camera configuration?

Camera Transform

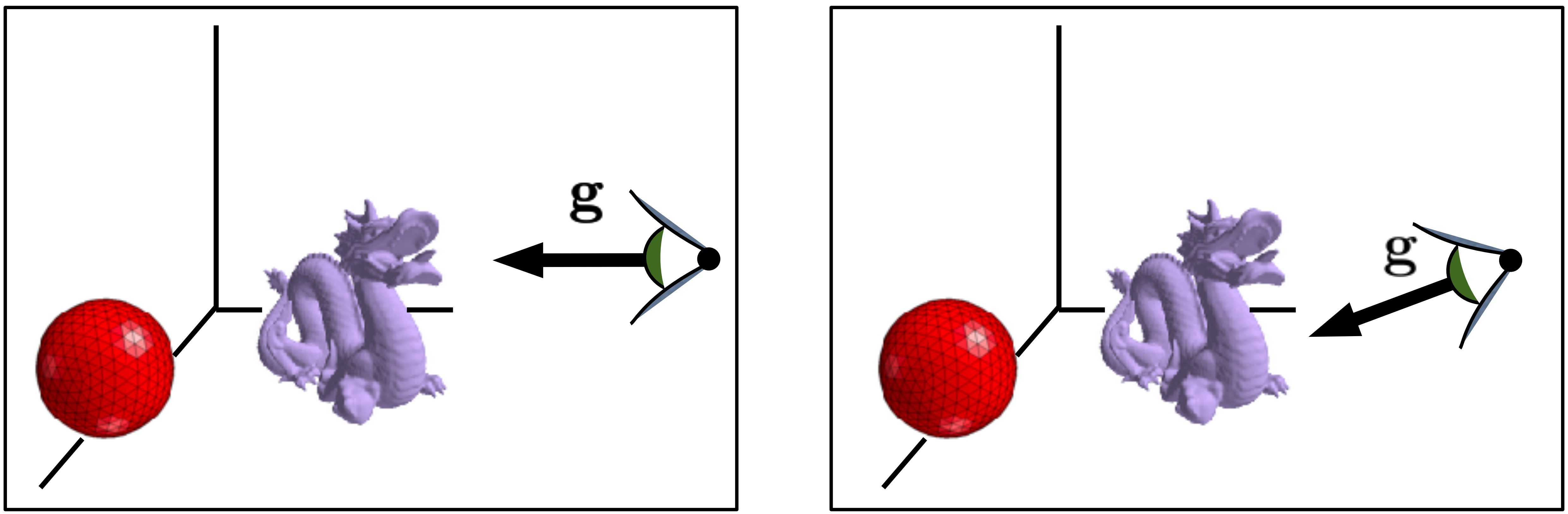
How do we specify the camera configuration? **eye position**



Camera Transform

How do we specify the camera configuration?

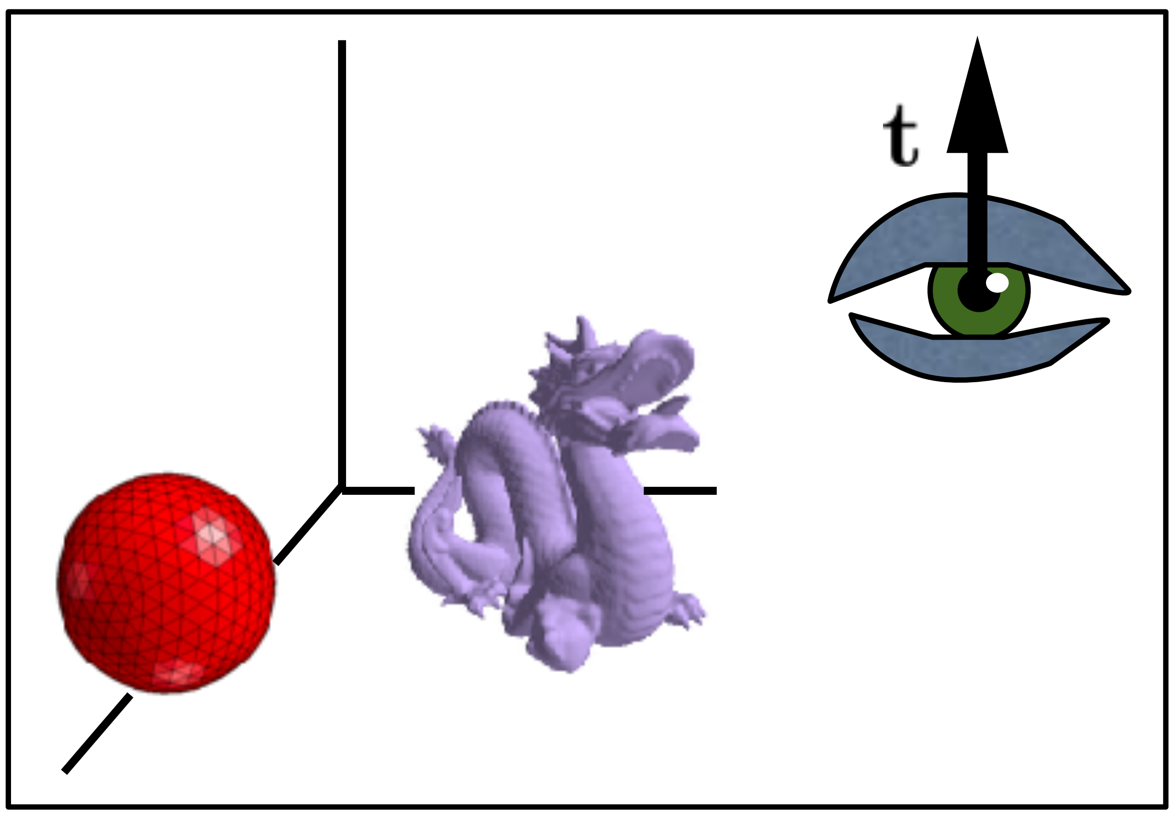
gaze direction

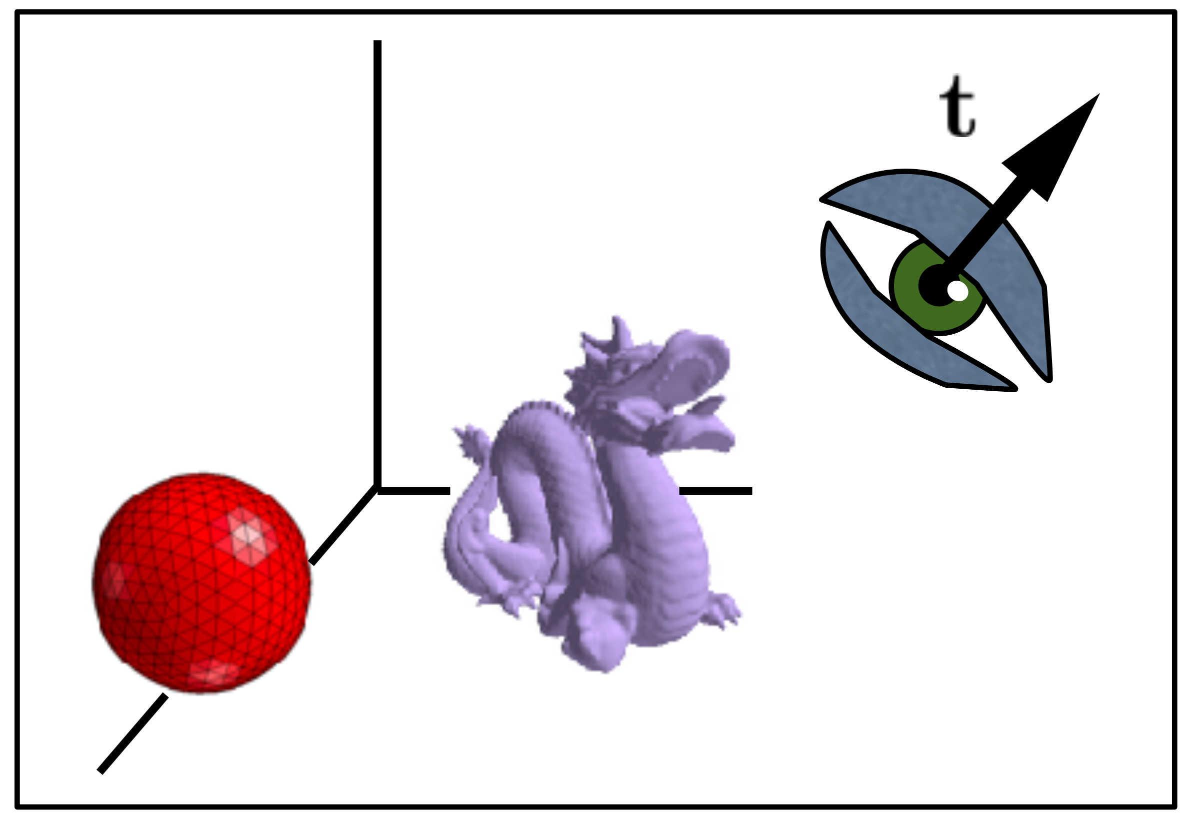


Camera Transform

How do we specify the camera configuration?

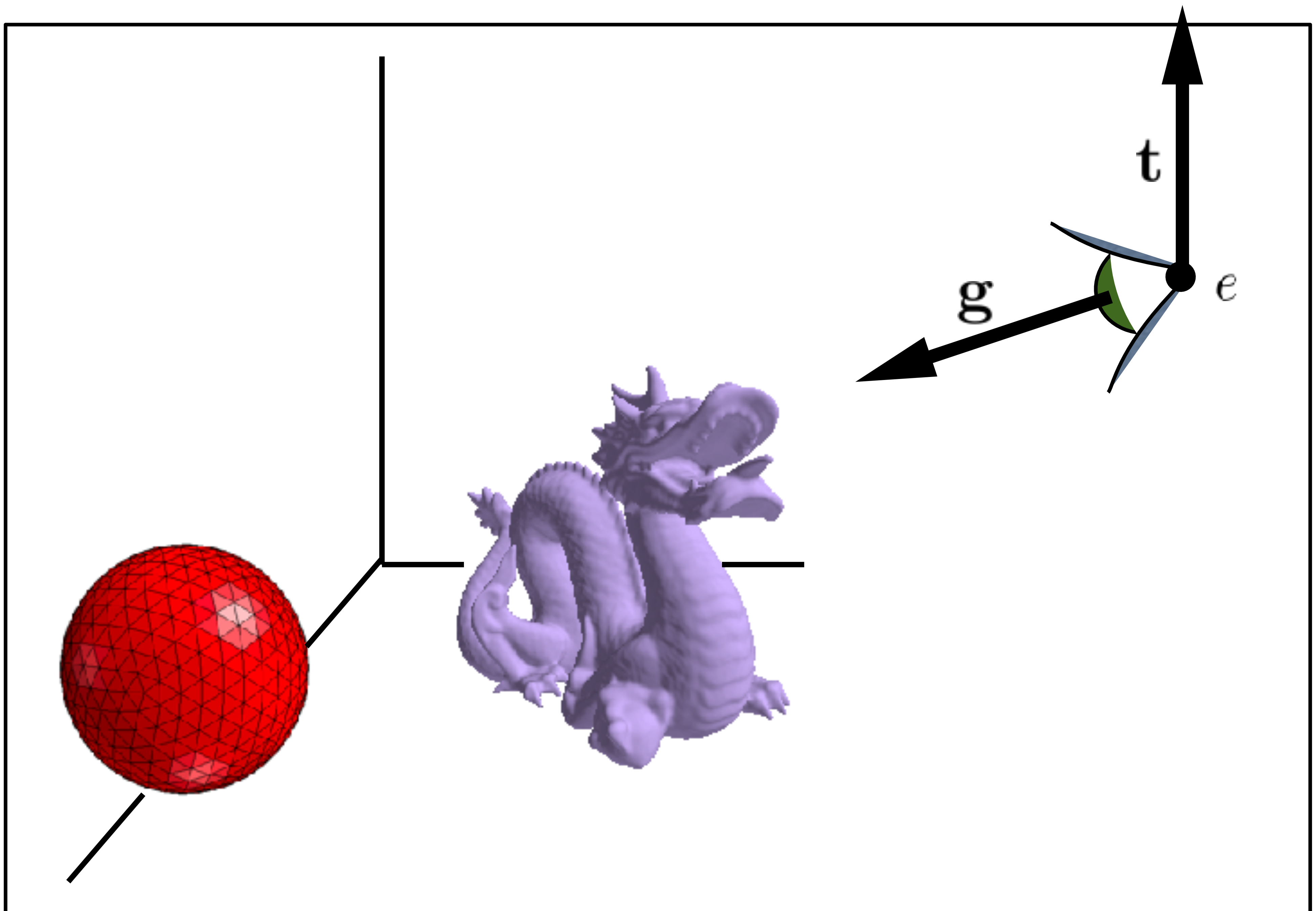
up vector



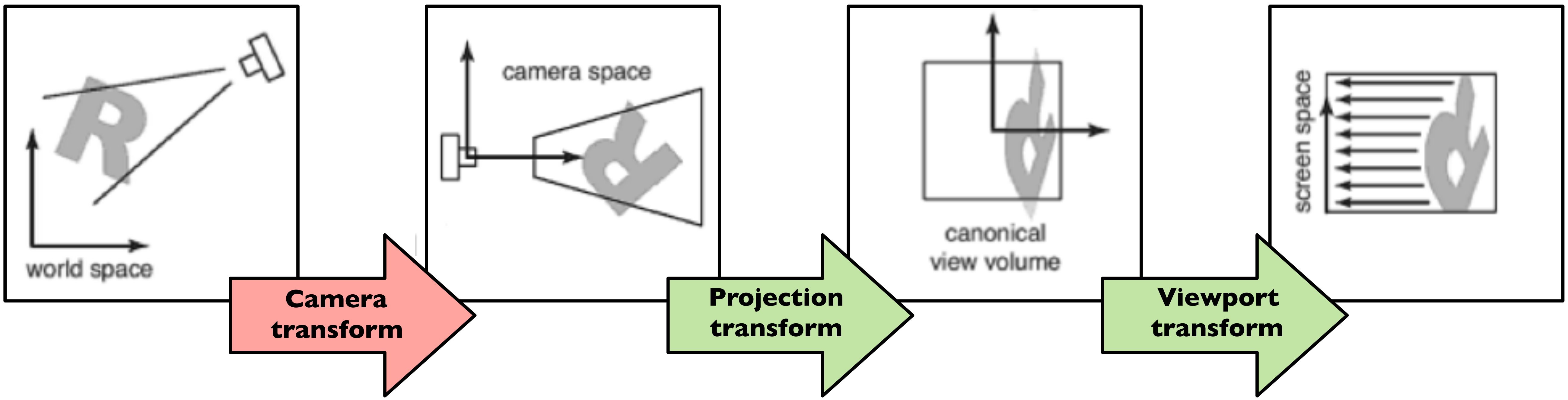


Camera Transform

How do we specify the camera configuration?



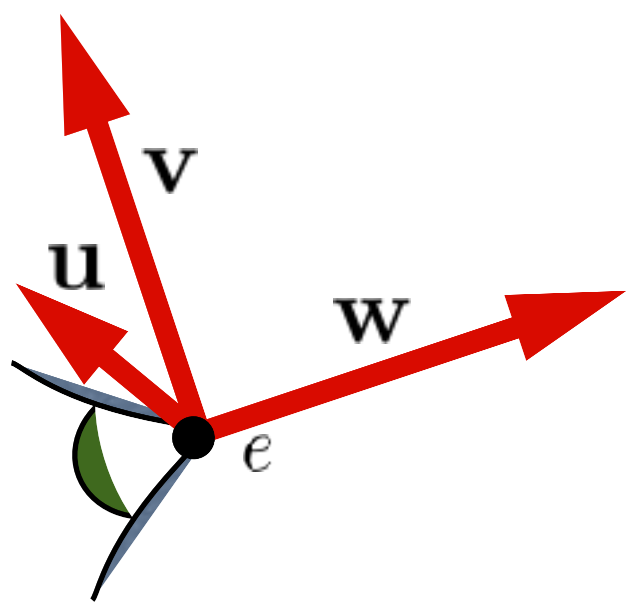
Camera Transform

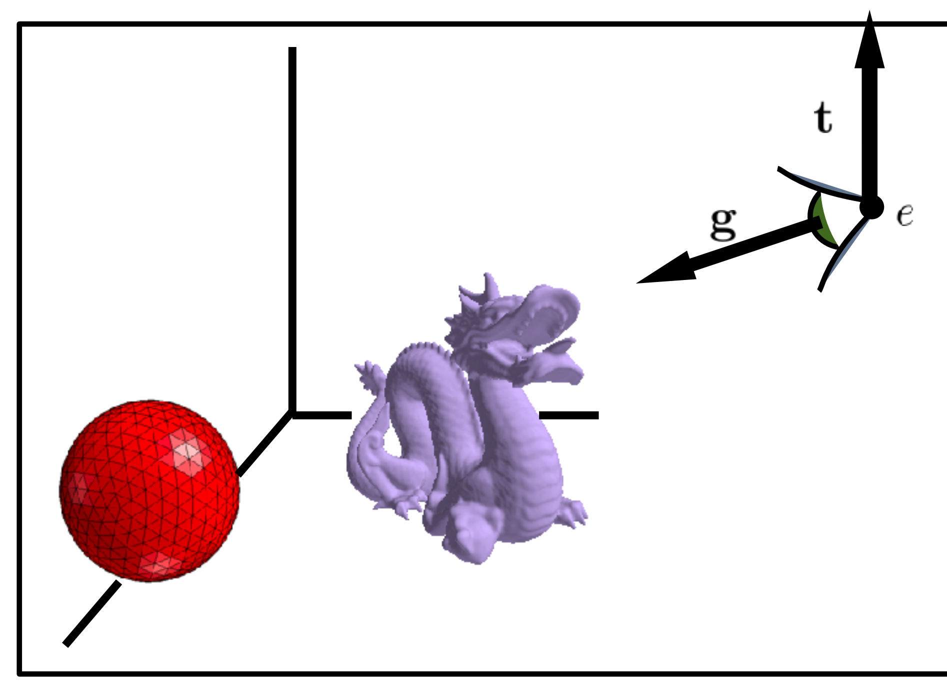


$$\mathbf{w} = -\frac{\mathbf{g}}{||\mathbf{g}||}$$

$$\mathbf{u} = \frac{\mathbf{t} \times \mathbf{w}}{||\mathbf{t} \times \mathbf{w}||}$$

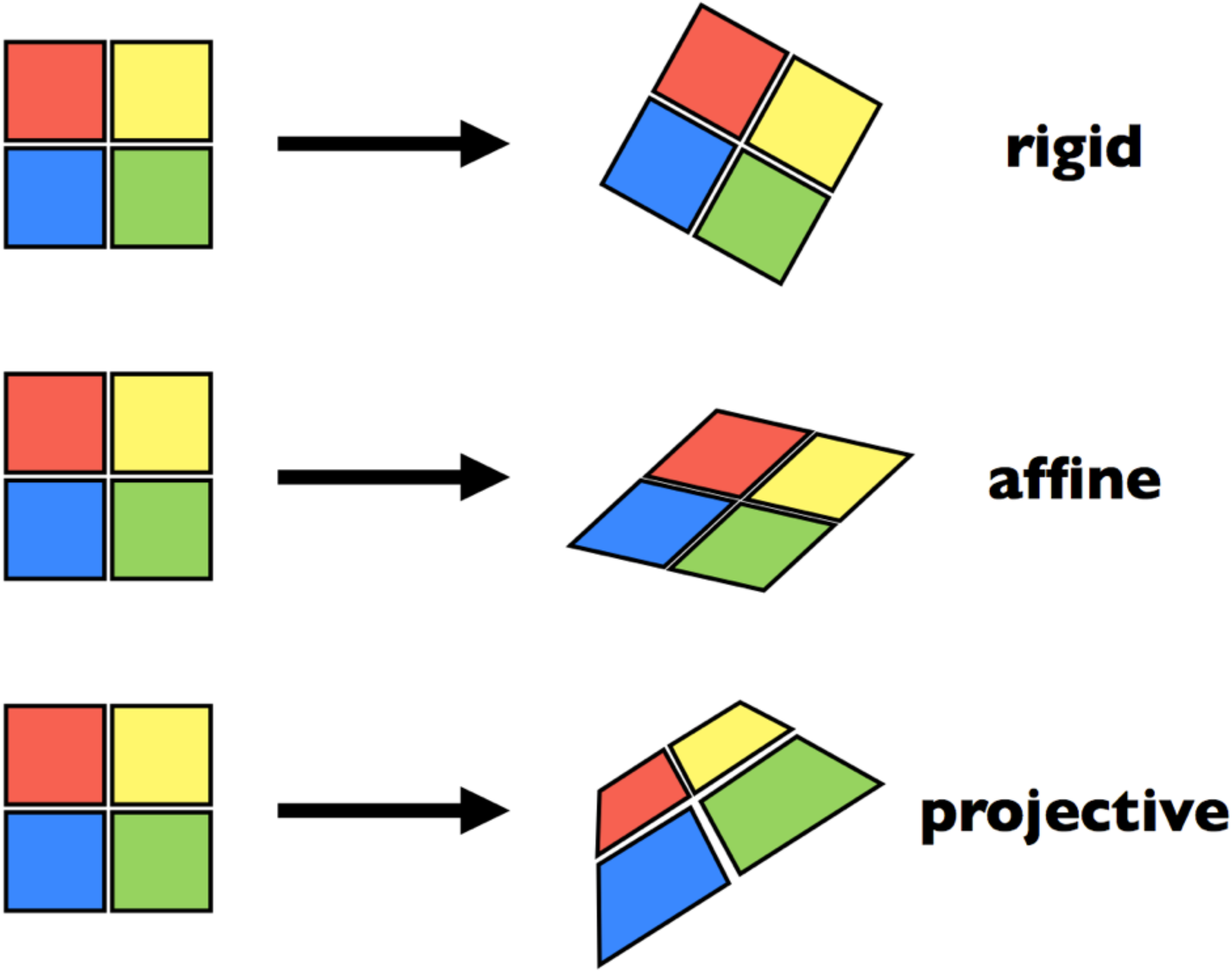
$$\mathbf{v} = \mathbf{w} \times \mathbf{u}$$



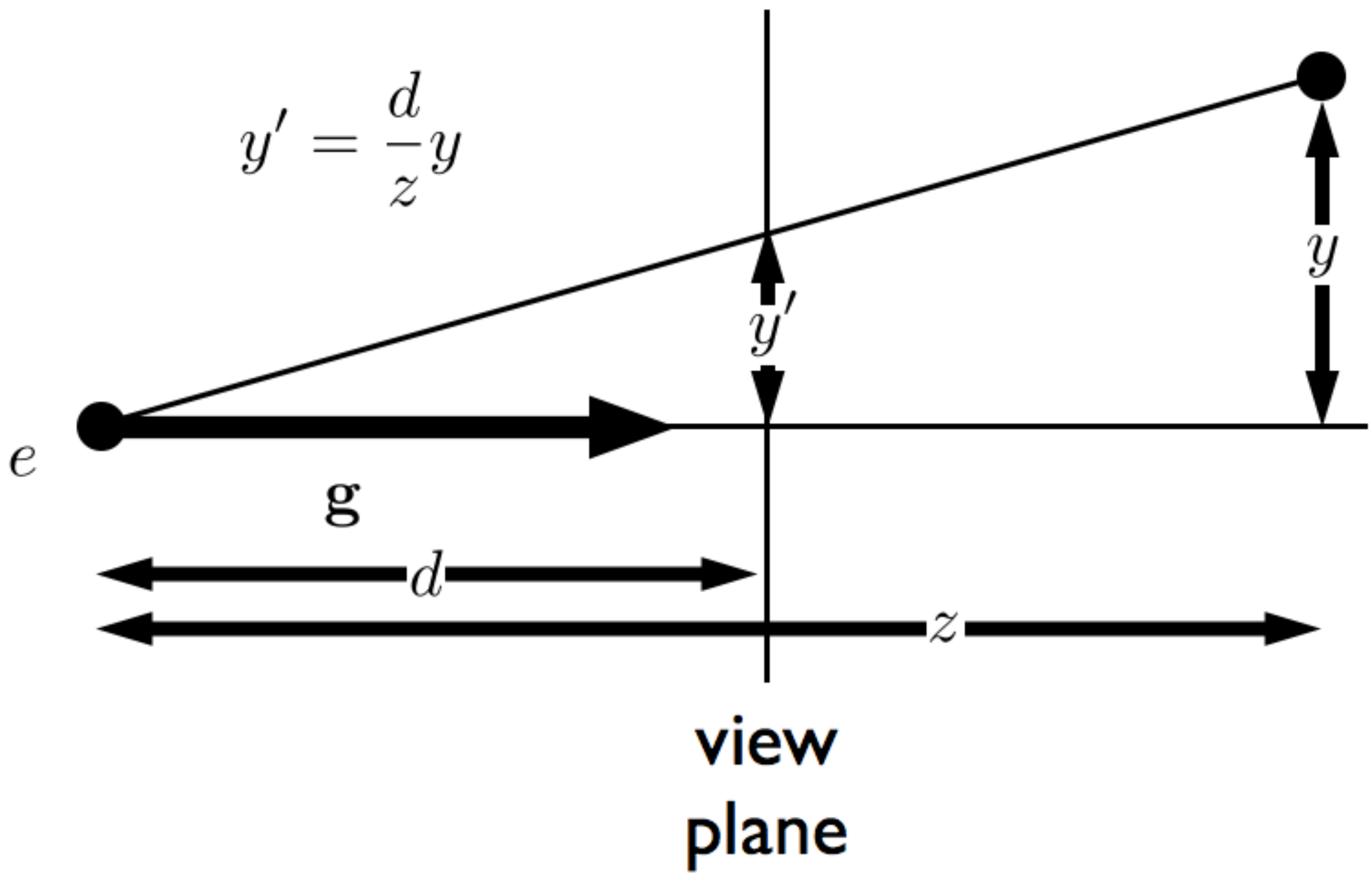


M_{cam} <whiteboard>

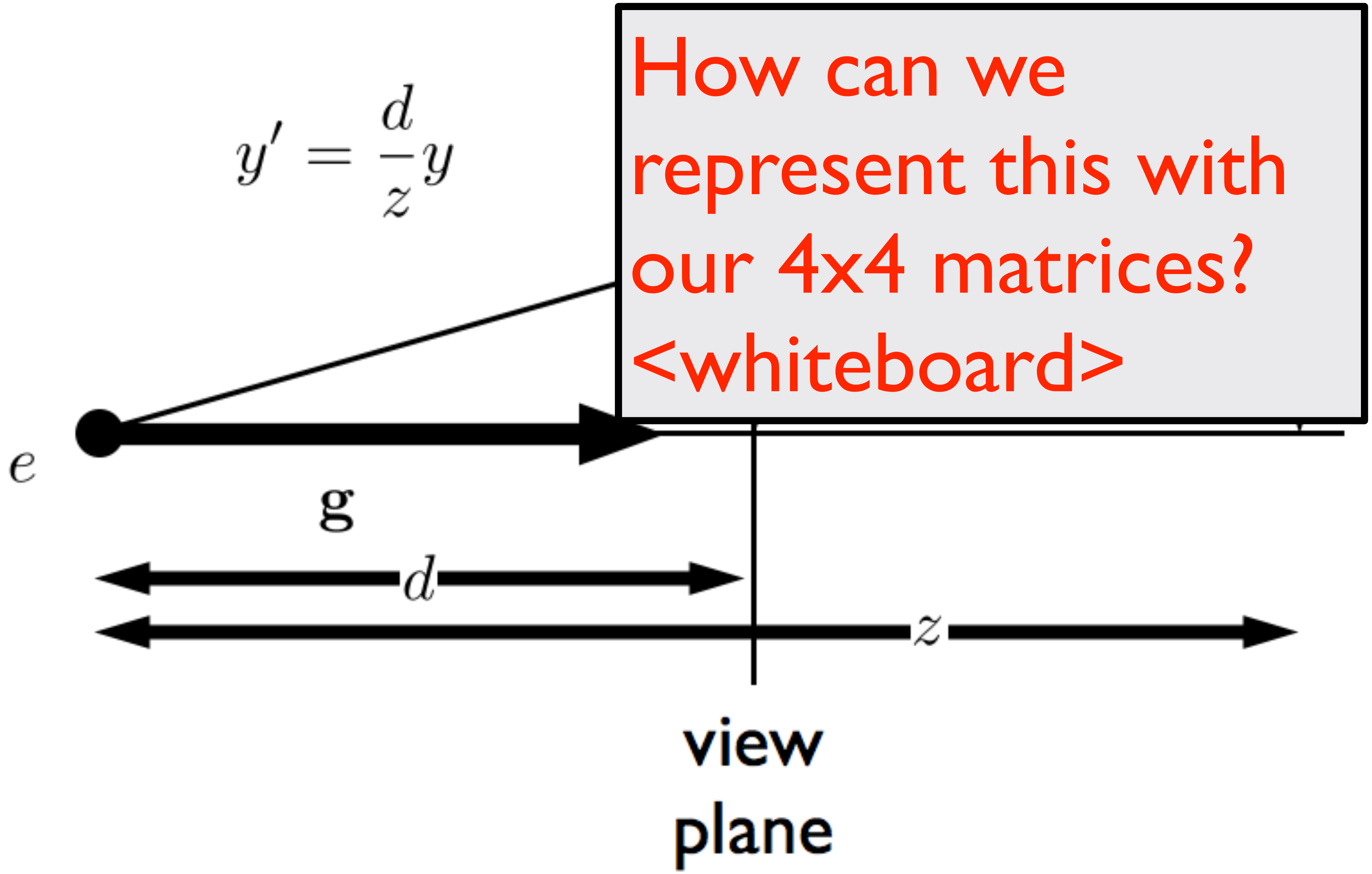
Perspective Viewing


rigid
affine
projective

Projective Transformations



Projective Transformations

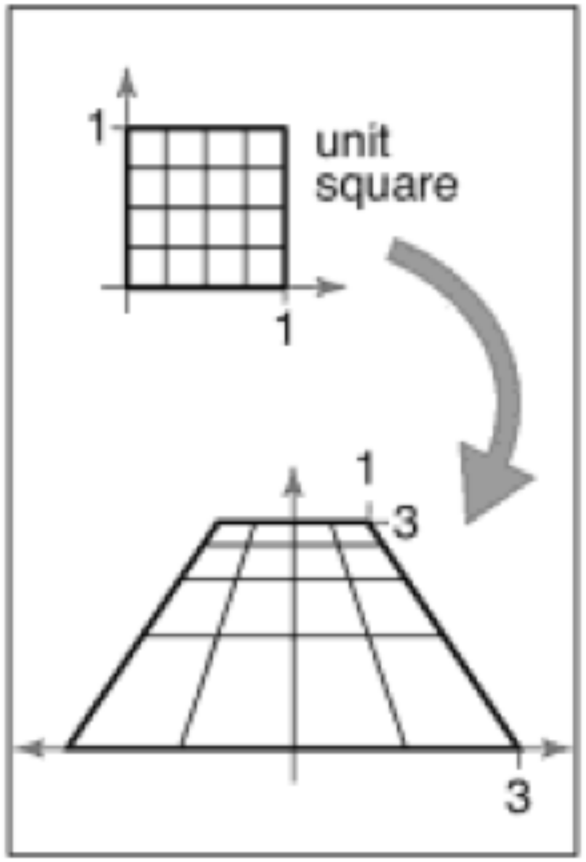


Projective Transformations

$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \\ w \end{pmatrix} \rightarrow \begin{array}{l} x = \dfrac{\tilde{x}}{w} \\ y = \dfrac{\tilde{y}}{w} \\ z = \dfrac{\tilde{z}}{w} \end{array}$$

Example:

$$M = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 3 & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} \end{pmatrix}$$



[Shirley, Marschner]

<whiteboard>

Projective Transformations

$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \\ w \end{pmatrix} \rightarrow \begin{array}{l} x = \dfrac{\tilde{x}}{w} \\ y = \dfrac{\tilde{y}}{w} \\ z = \dfrac{\tilde{z}}{w} \end{array}$$

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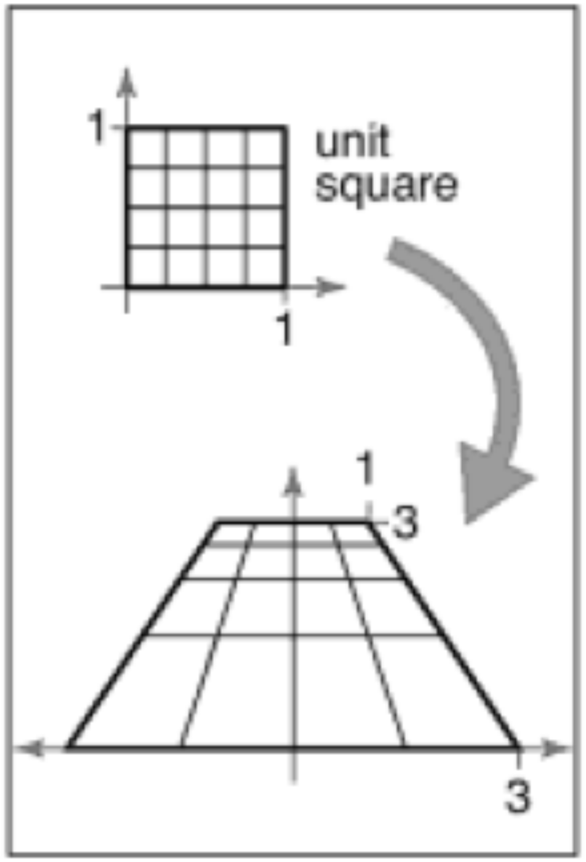
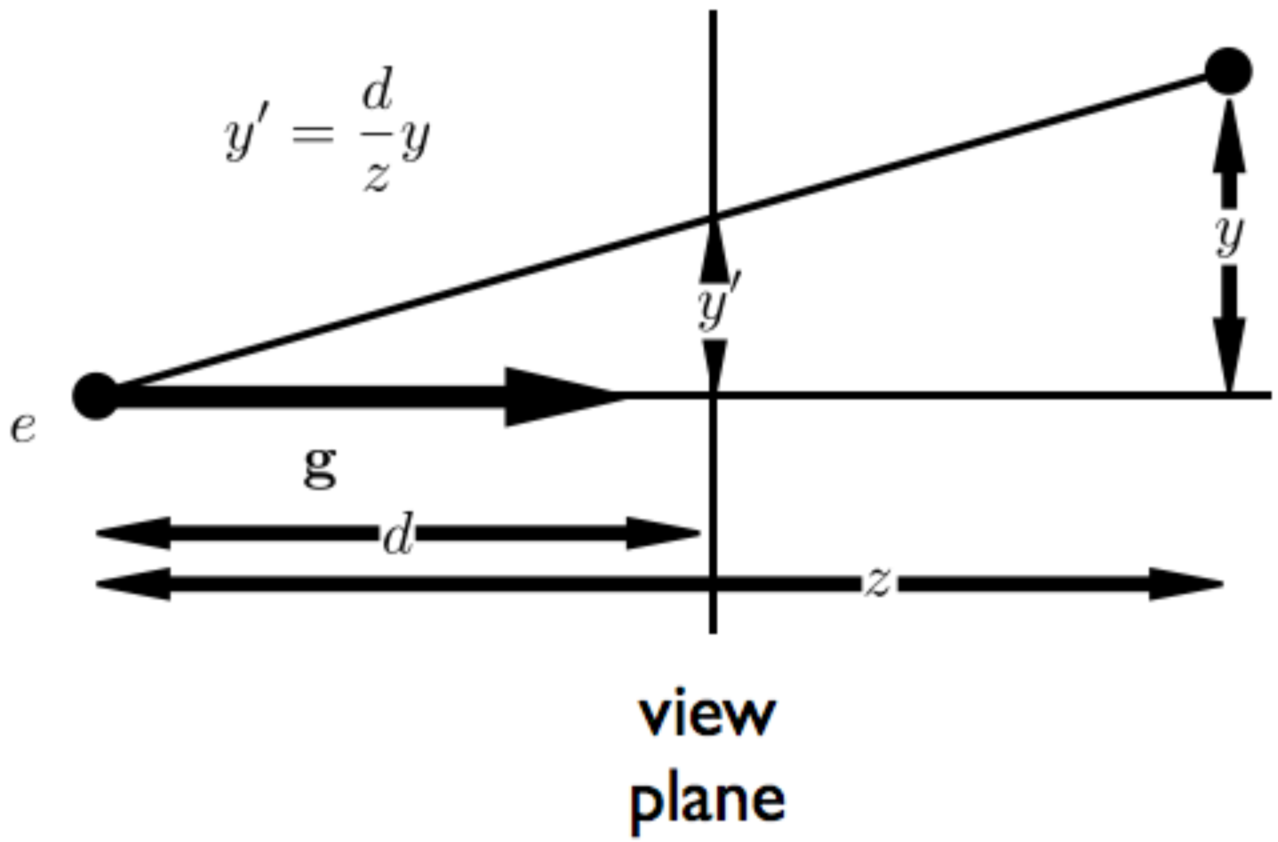
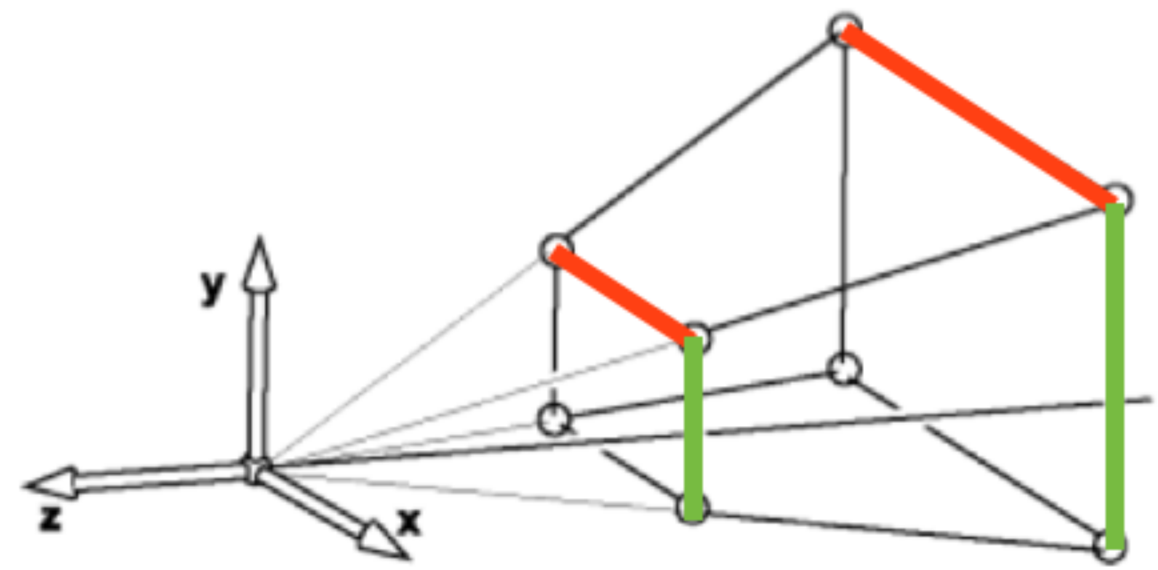


We can now implement perspective projection!

[Shirley, Marschner]

Perspective Projection





both x and y get multiplied by d/z

[Shirley, Marschner]

Simple perspective projection

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \\ z/d \end{pmatrix} \Rightarrow \begin{cases} x' = \frac{d}{z}x \\ y' = \frac{d}{z}y \\ z' = \frac{d}{z}z = d \end{cases}$$

This achieves a simple perspective projection onto the view plane z = d

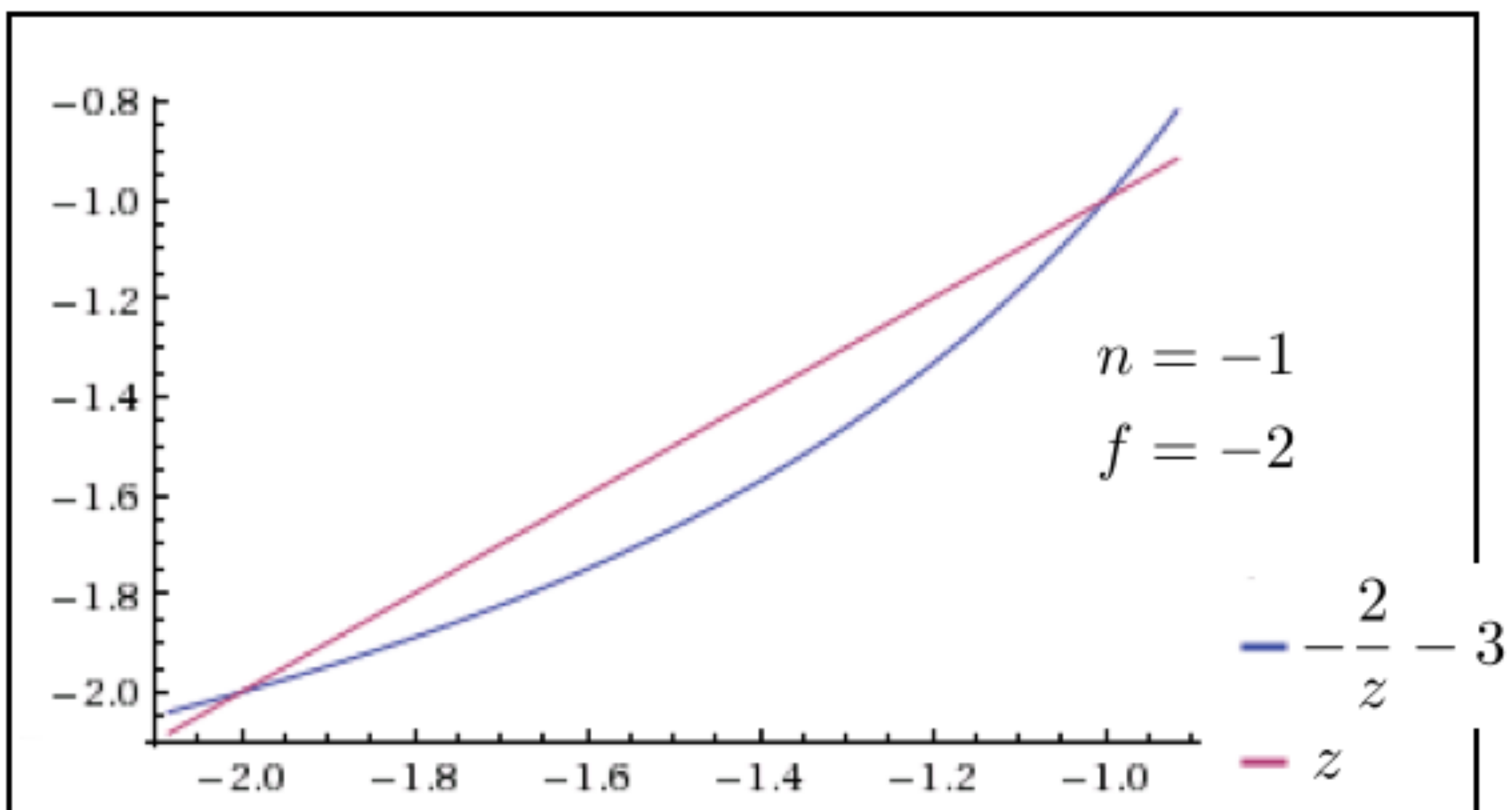
but we've lost all information about z!

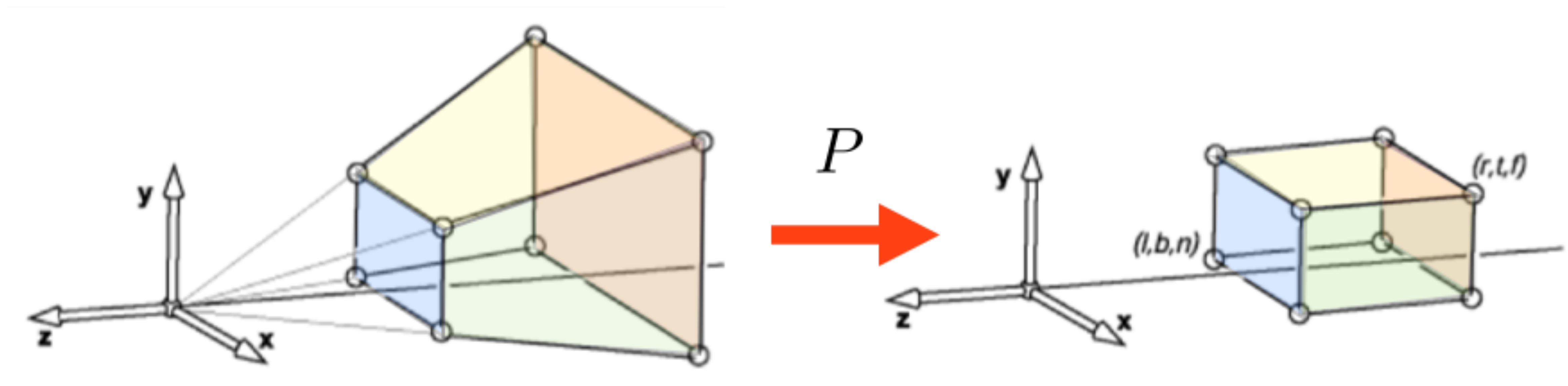
<whiteboard>

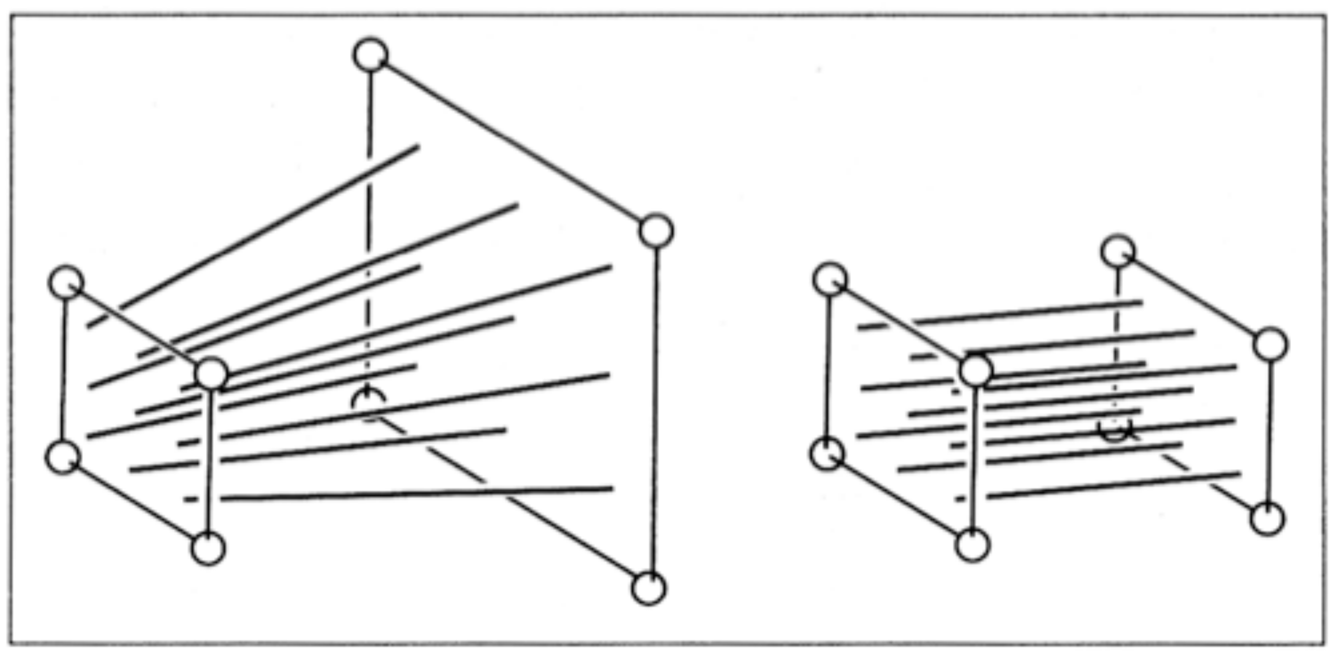
Perspective Projection

$$P = \begin{pmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad z' = (n+f) - \frac{nf}{z}$$

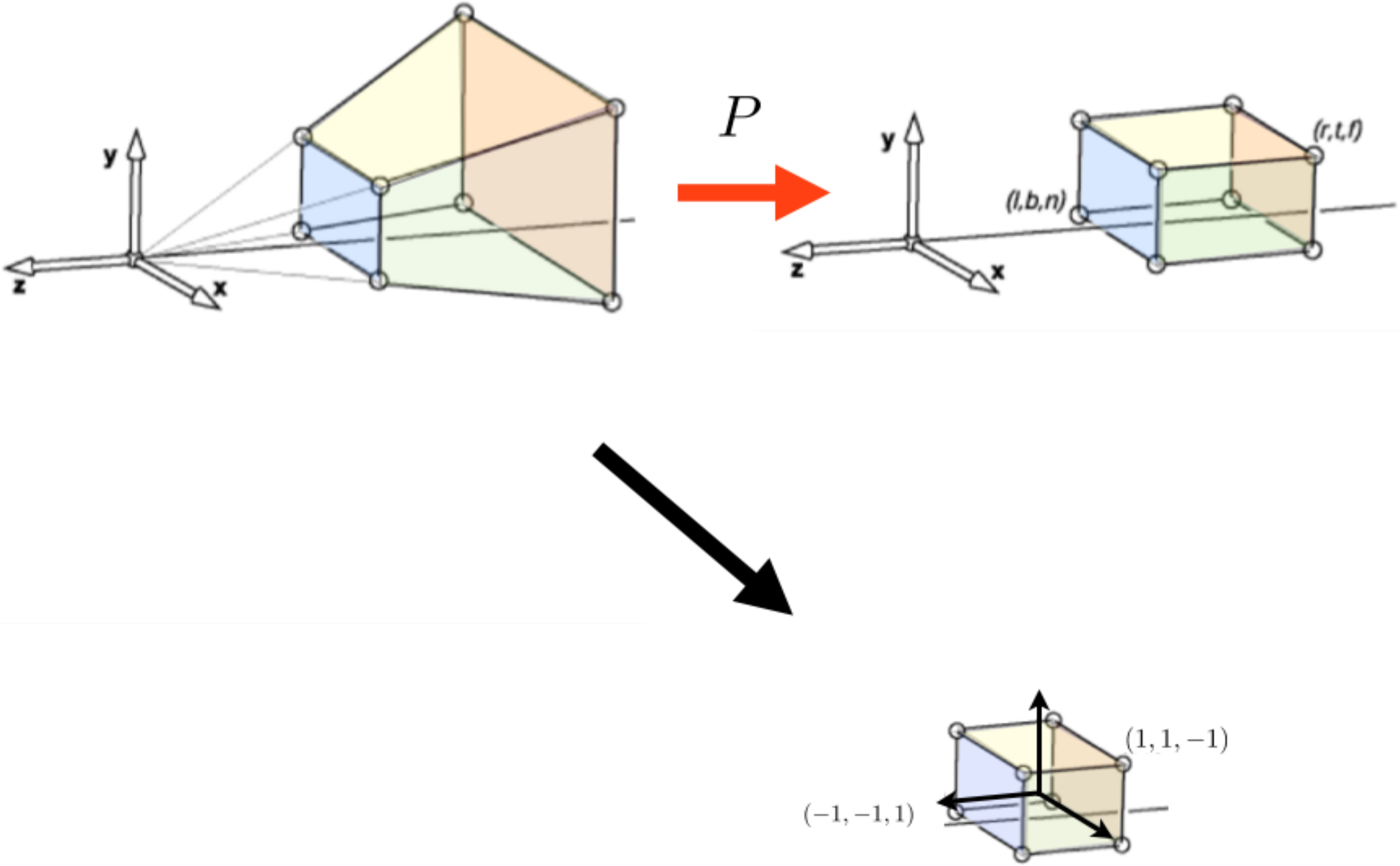
Example:

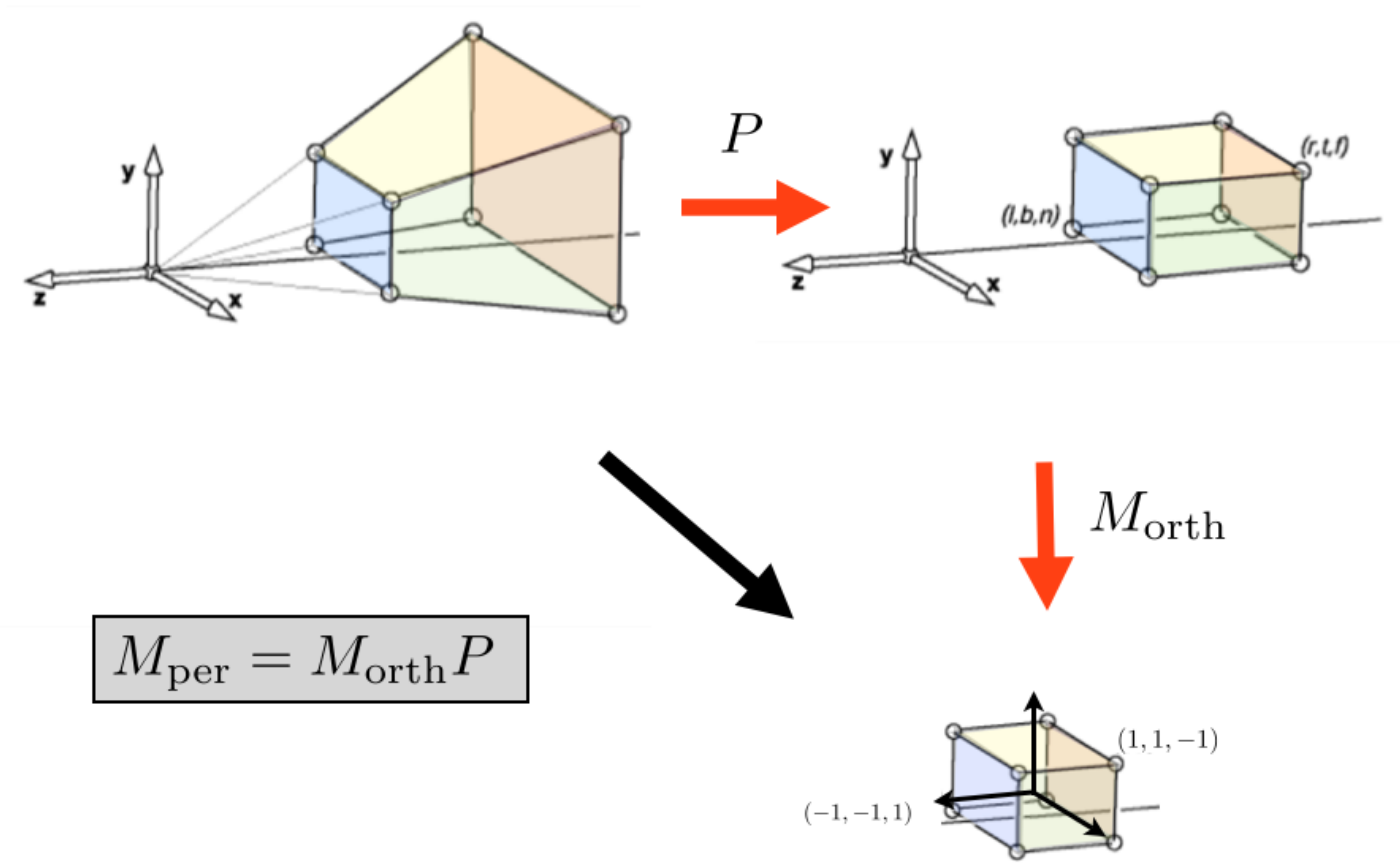






[Shirley, Marschner]


y
z
x
P
y
z
x
(l,b,n)
(r,t,f)
(1, 1, −1)
(−1, −1, 1)

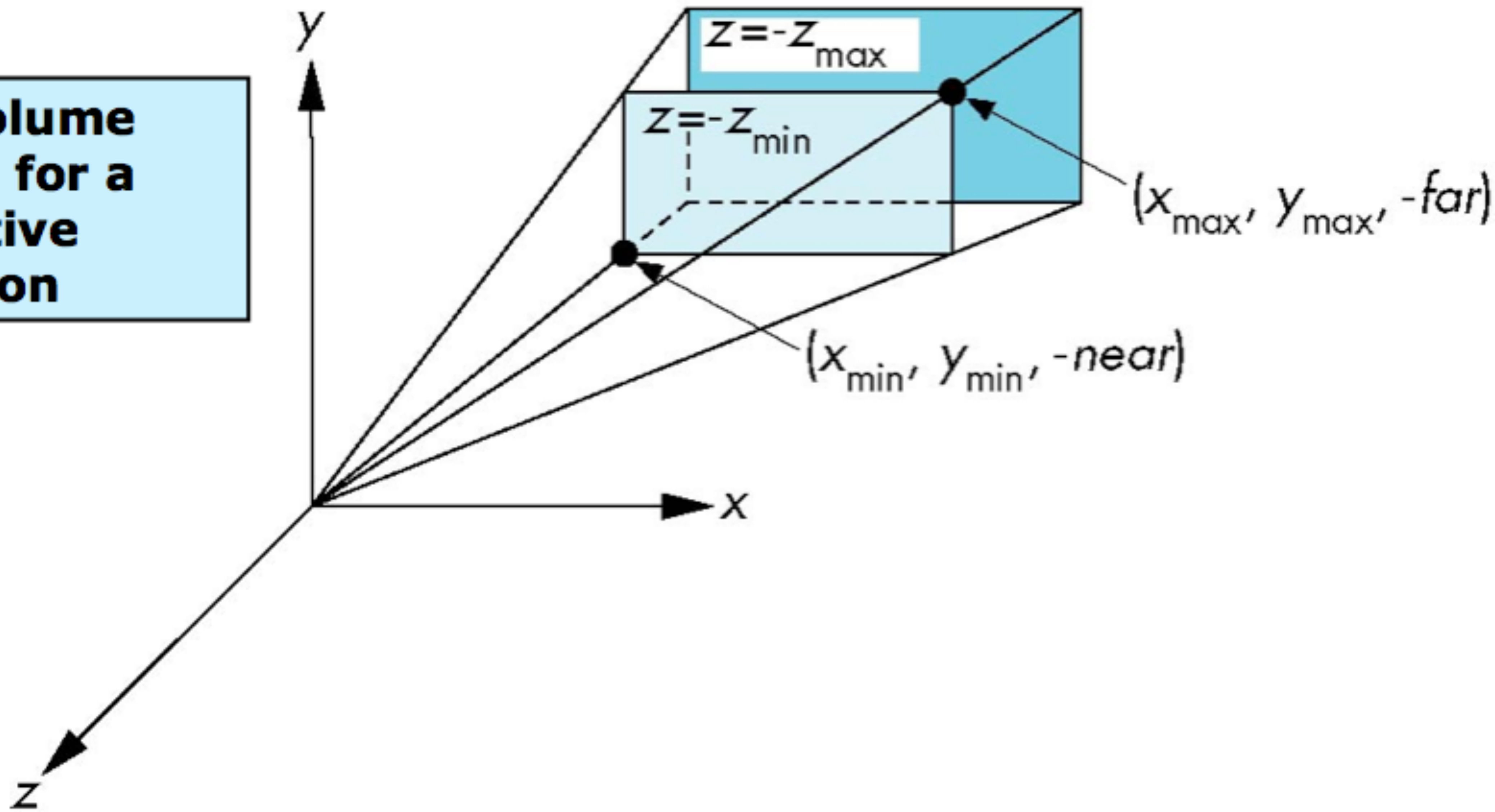


$$M_{\text{per}} = M_{\text{orth}} P$$

OpenGL Perspective Viewing

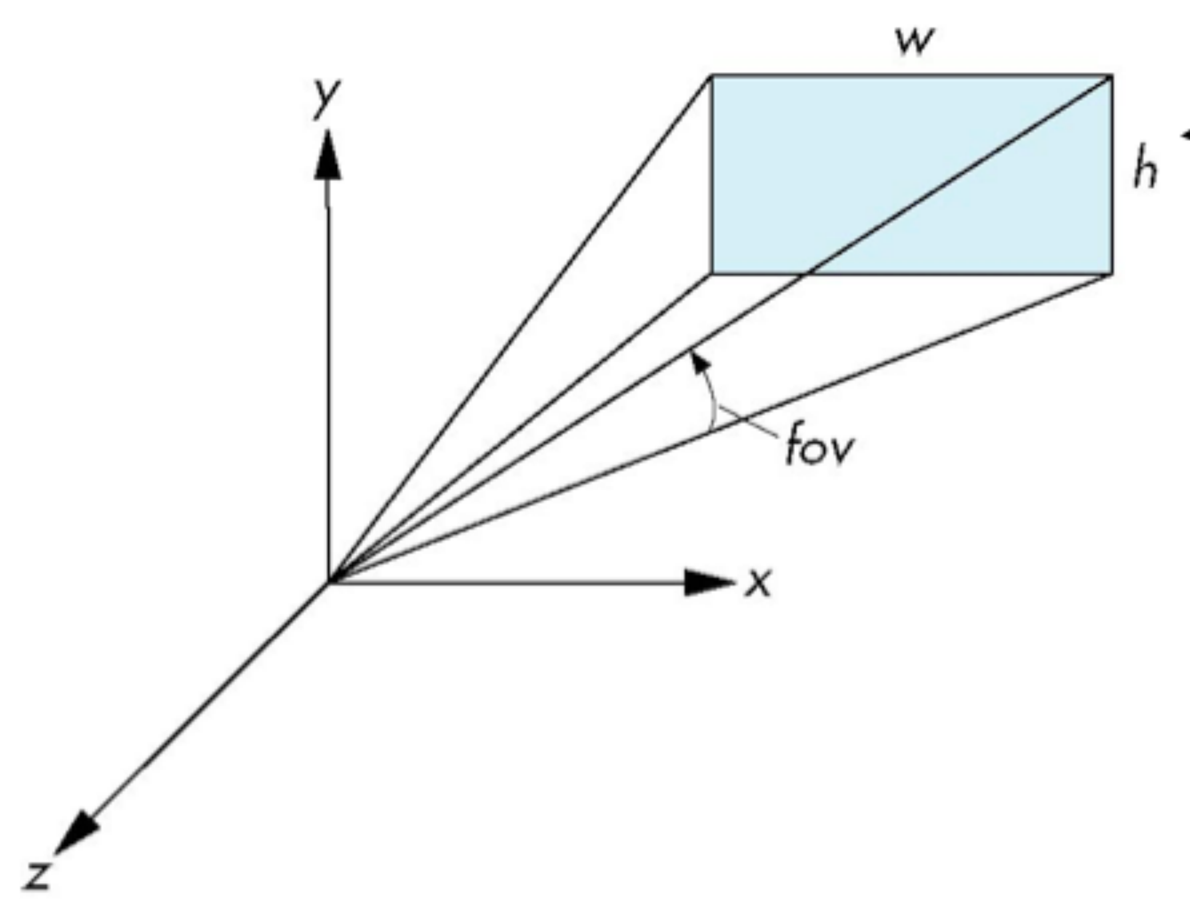
glFrustum(xmin,xmax,ymin,ymax,near,far)

Clipping volume (frustrum) for a perspective projection



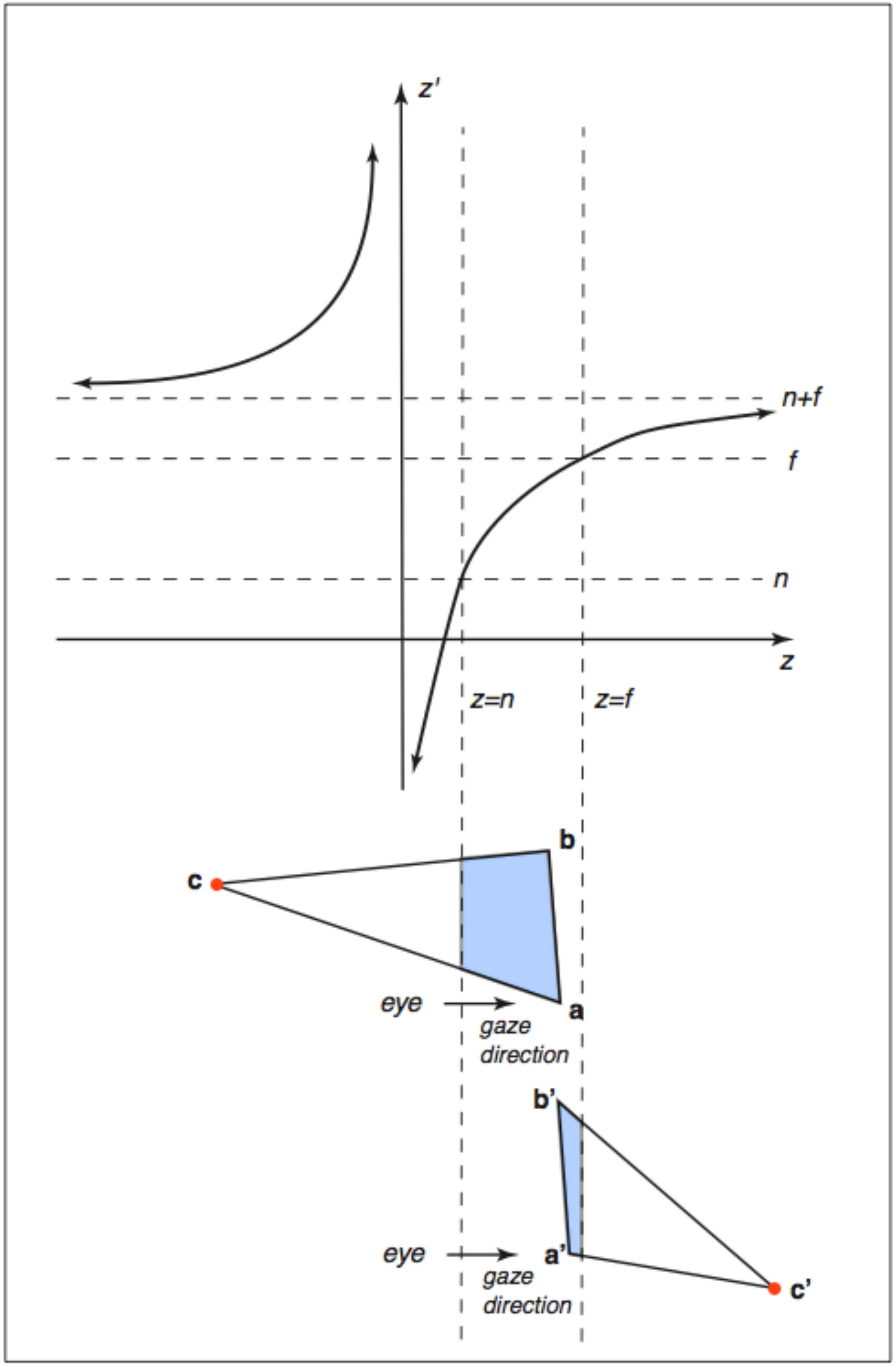
Using Field of View

With `glFrustum` it is often difficult to get the desired view `gluPerpective(fovy, aspect, near, far)` often provides a better interface



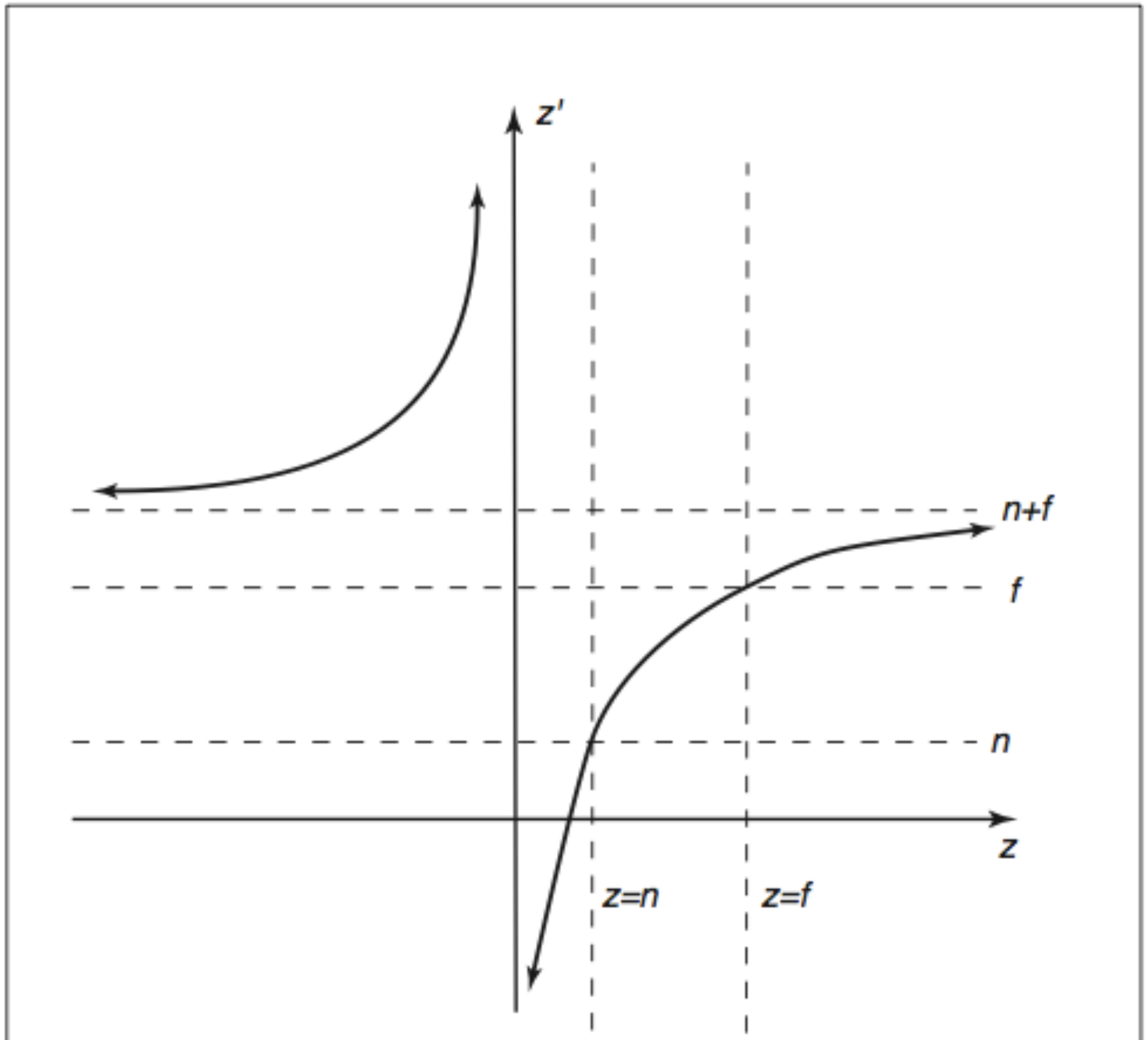
front plane

`aspect = w/h`



[Shirley, Marschner]

Clipping after the perspective transformation can cause problems



[Shirley, Marschner]

OpenGL clips **after** projection and **before** perspective division

$$-w \leq x \leq w$$

$$-w \leq y \leq w$$

$$-w \leq z \leq w$$