

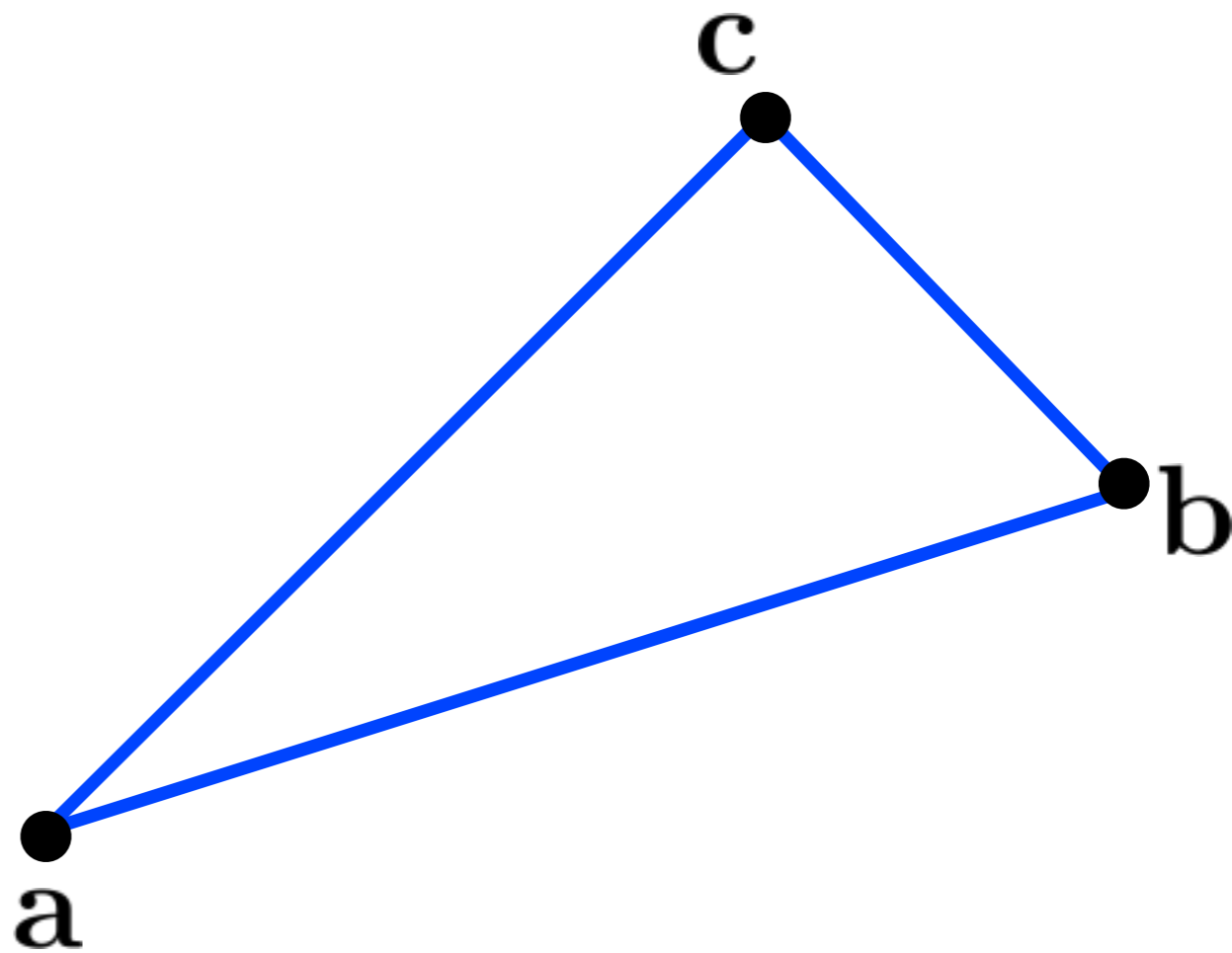
CS 130 : Computer Graphics

Rasterizing Triangles
and Graphics Pipeline (cont.)

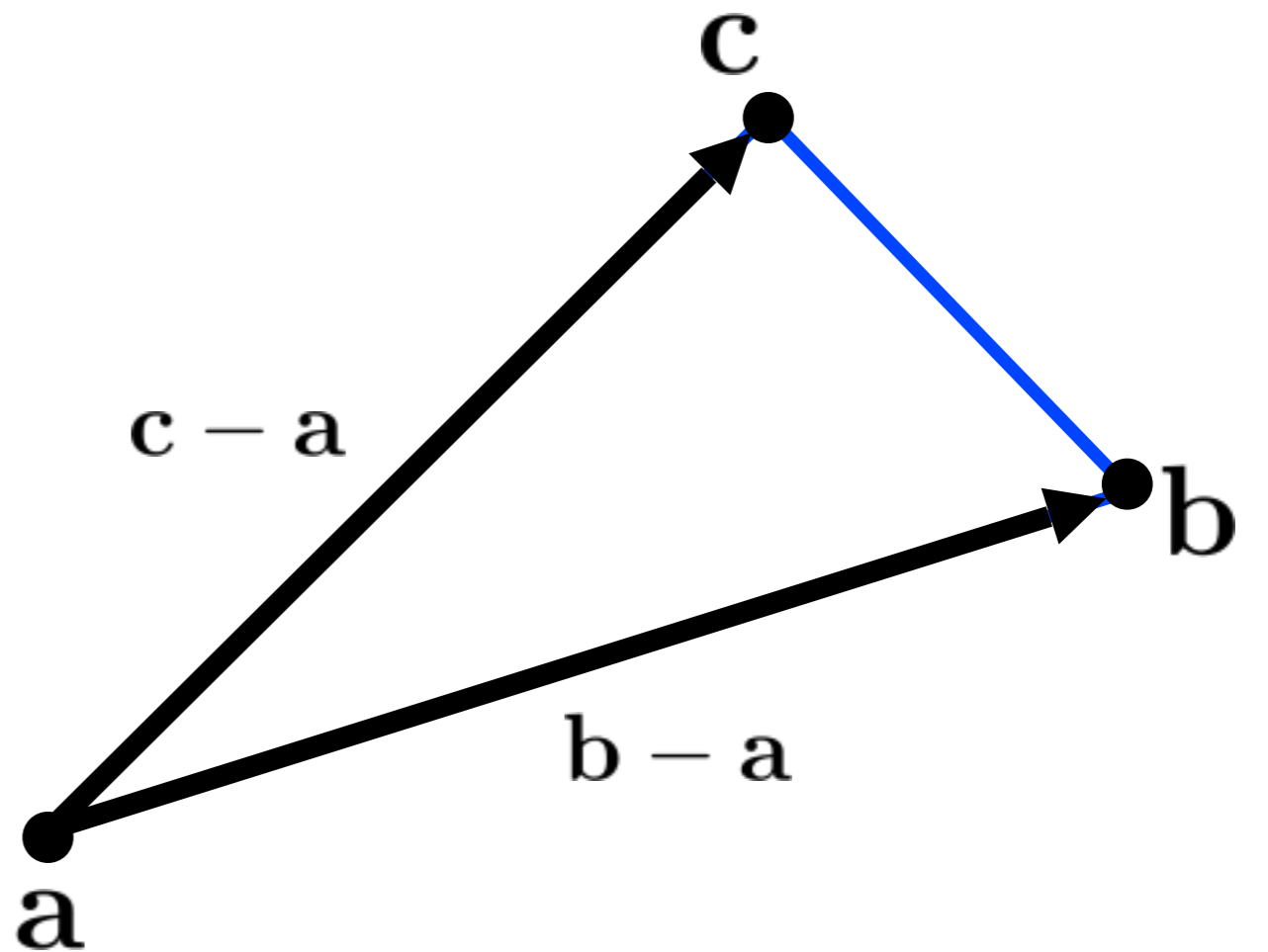
Tamar Shinar
Computer Science & Engineering
UC Riverside

Triangles

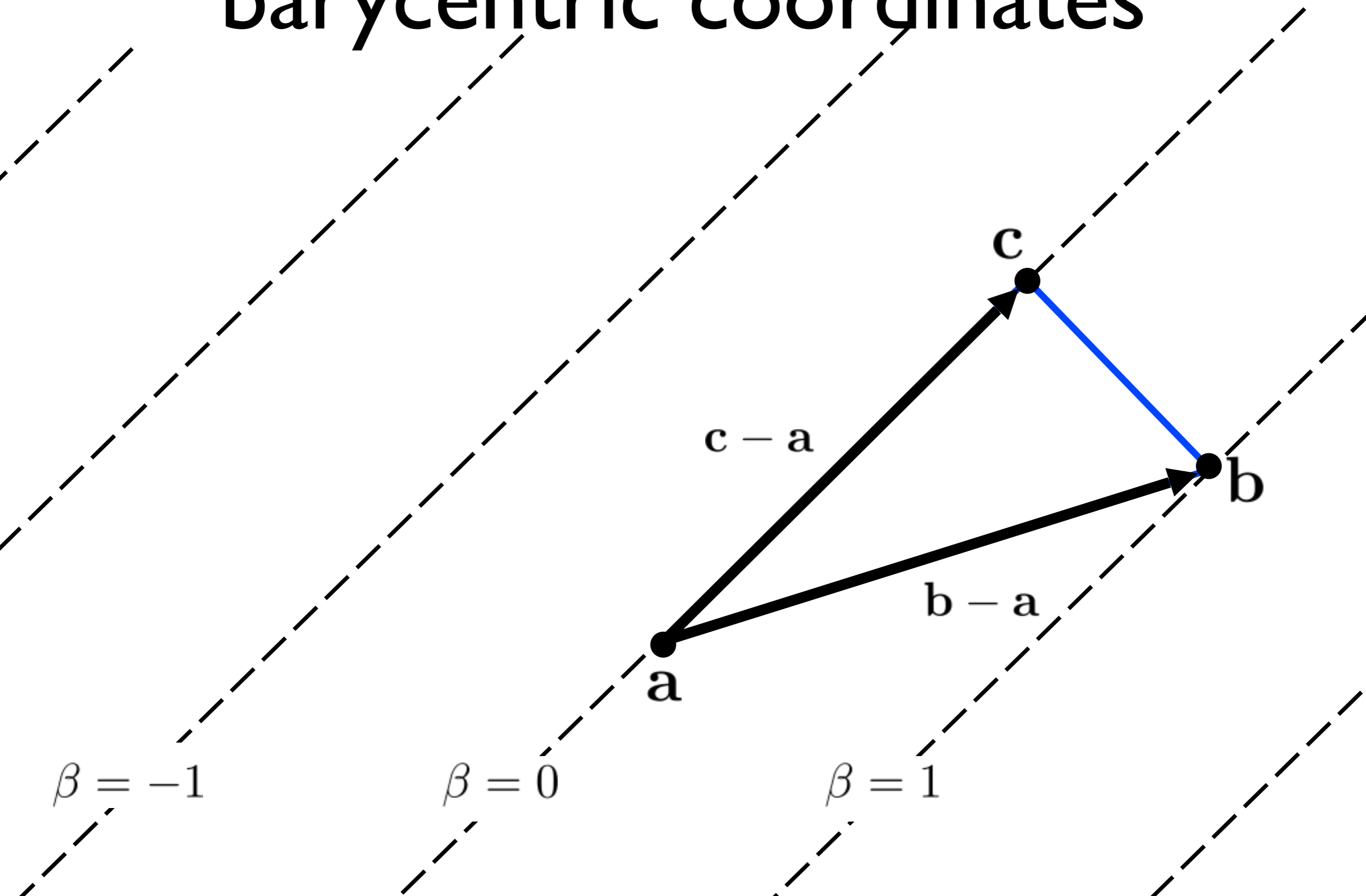
barycentric coordinates



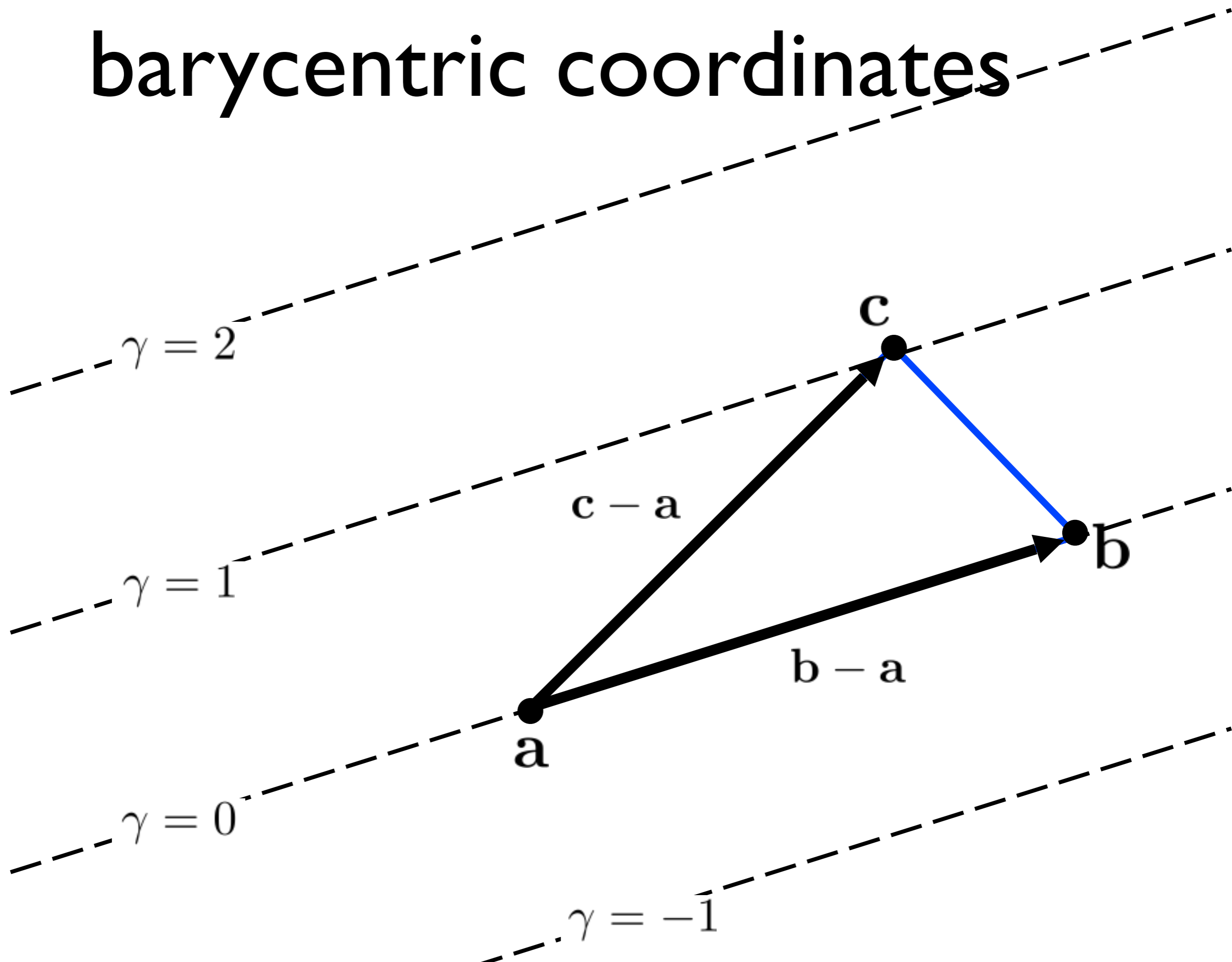
barycentric coordinates



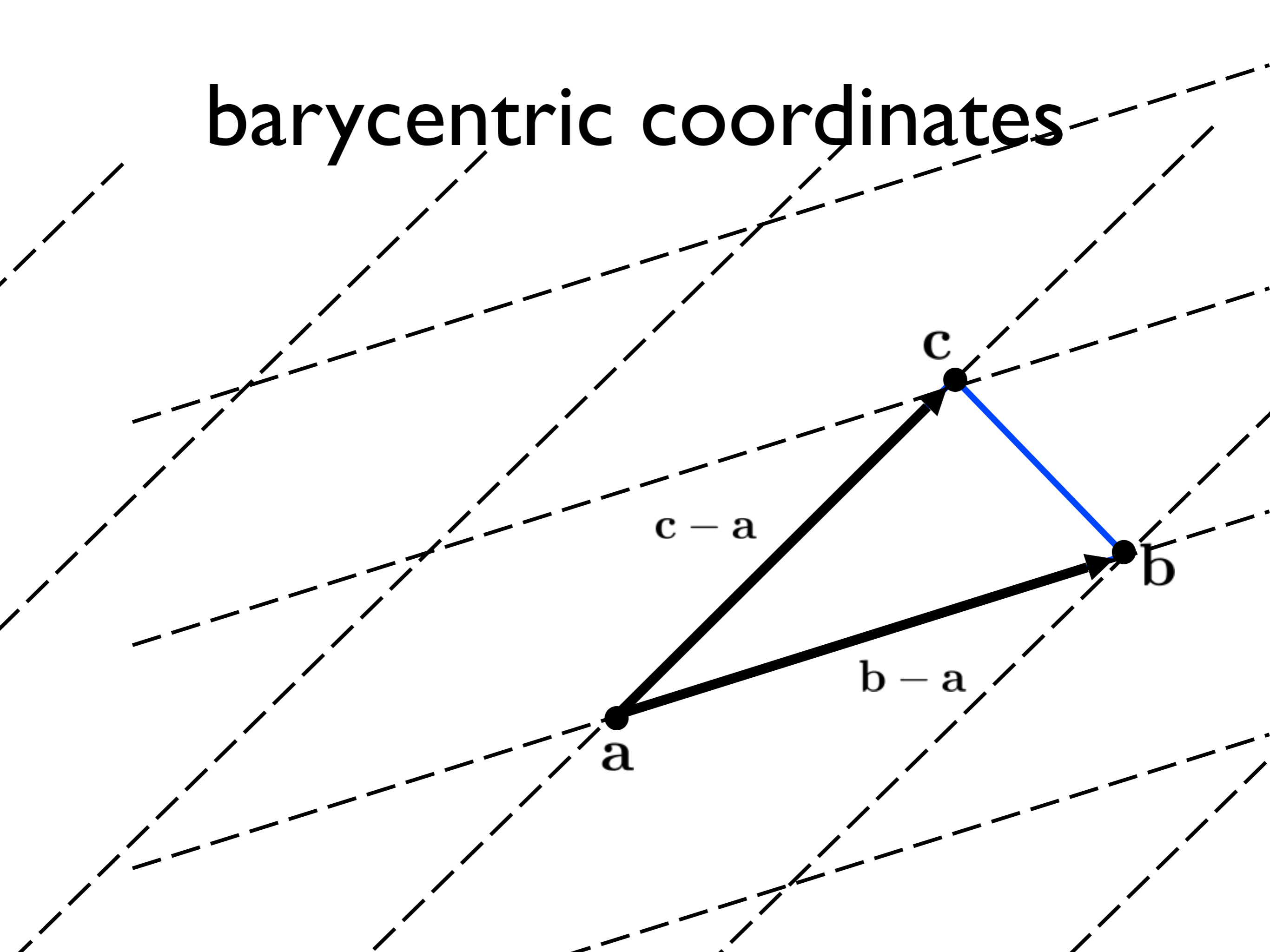
barycentric coordinates



barycentric coordinates



barycentric coordinates

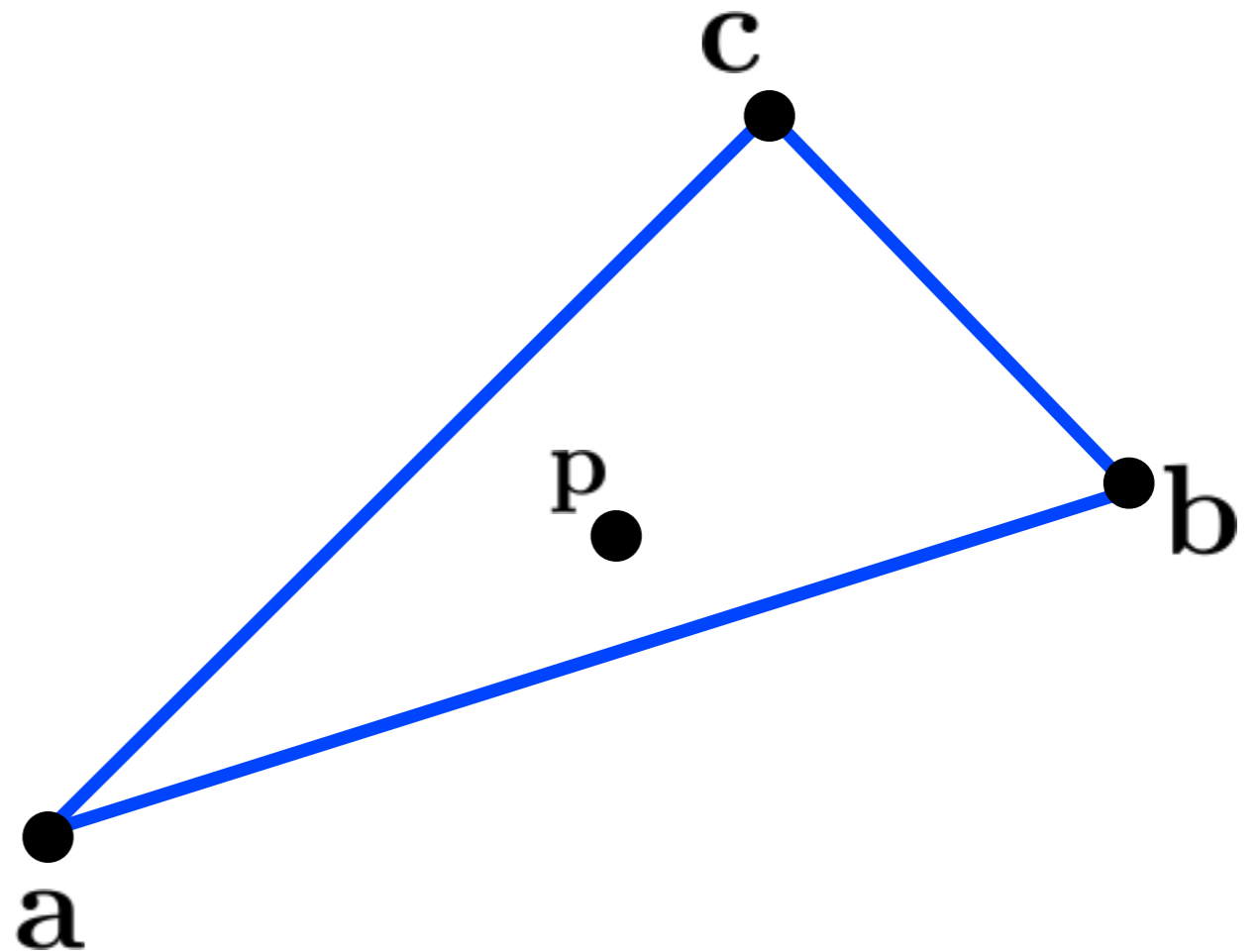


barycentric coordinates

$$\mathbf{p} = \alpha \mathbf{a} + \beta \mathbf{b} + \gamma \mathbf{c}$$

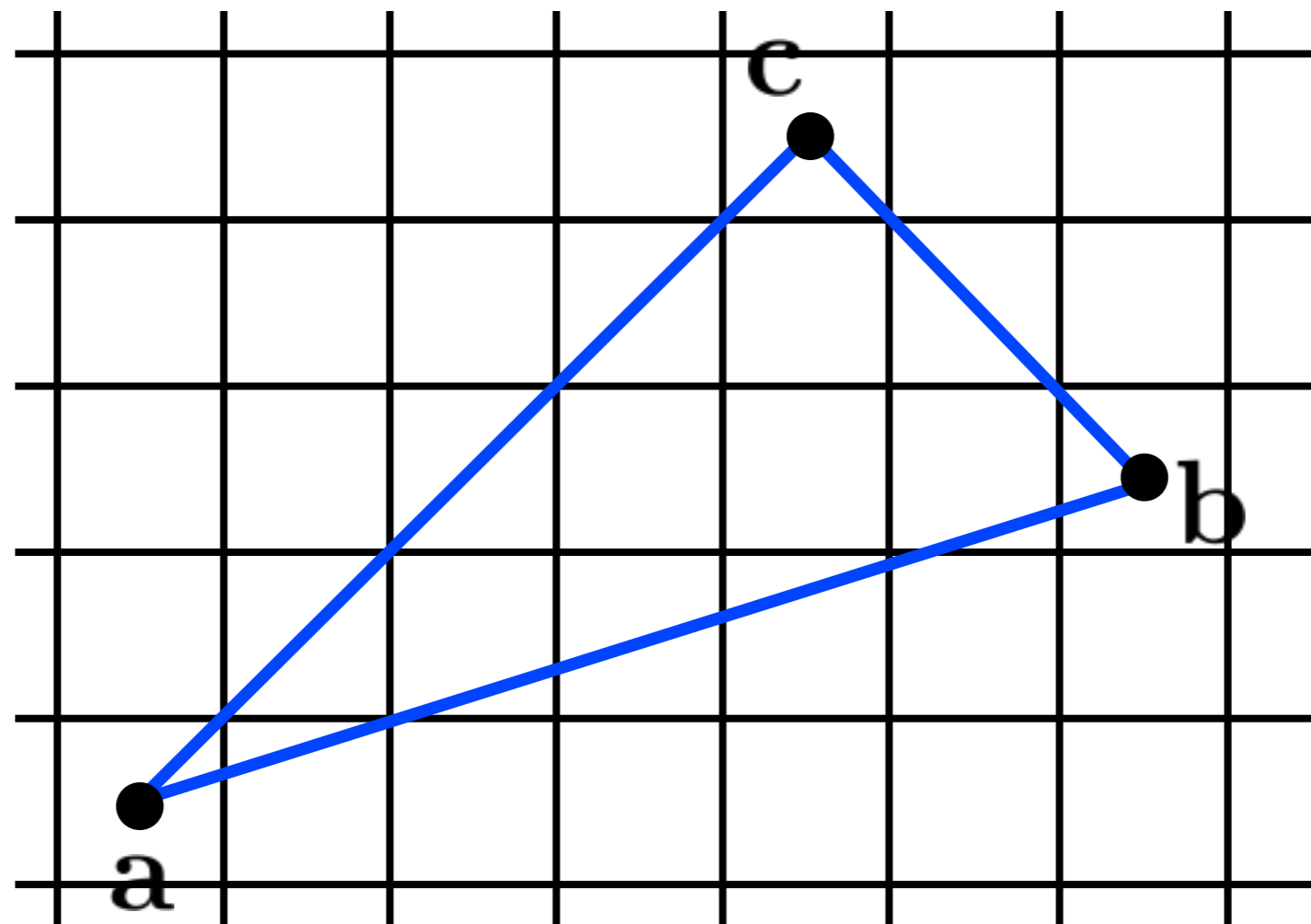
What are (α, β, γ) ?

<whiteboard>

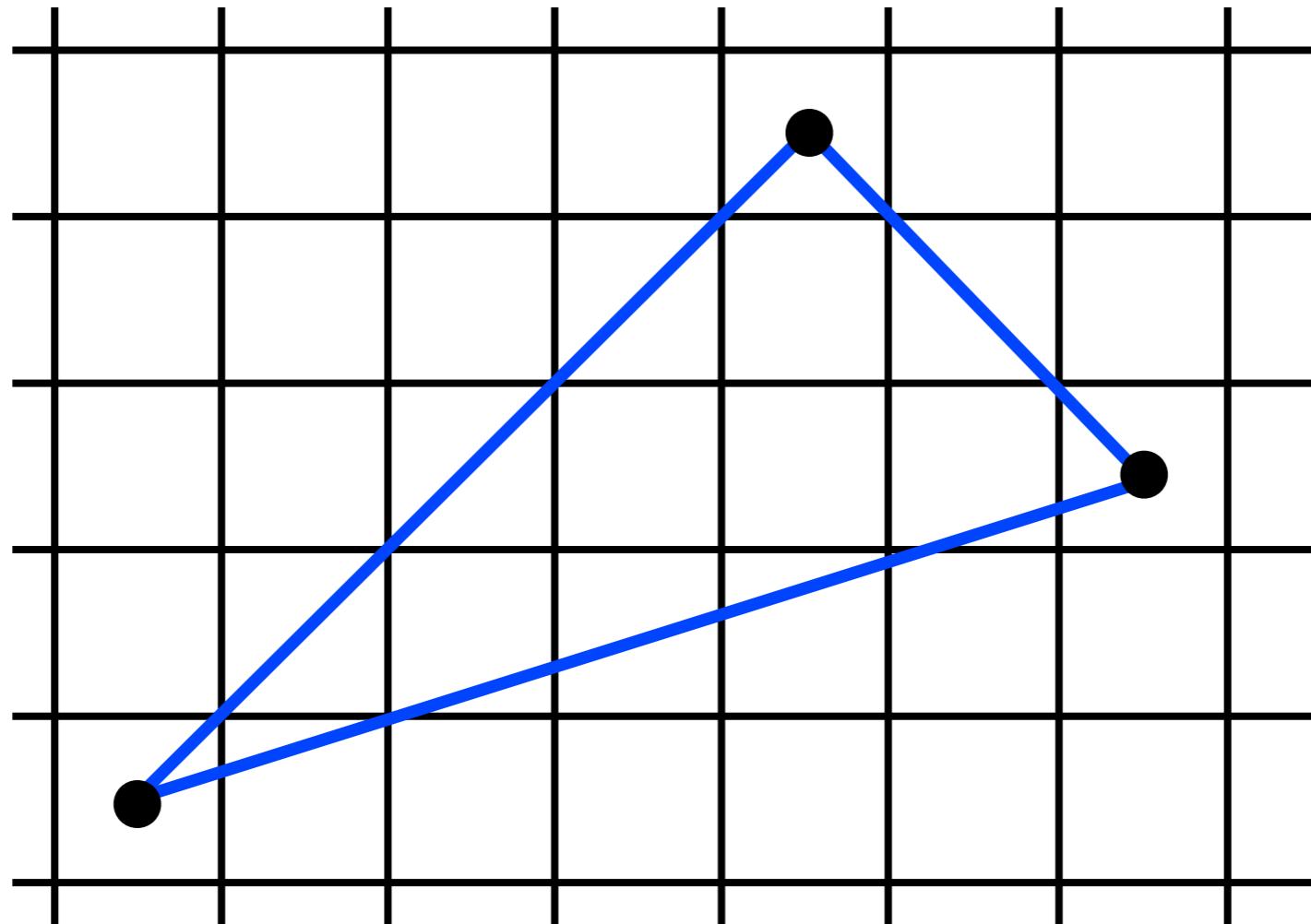


Triangle rasterization

Which pixels should be used to approximate a triangle?

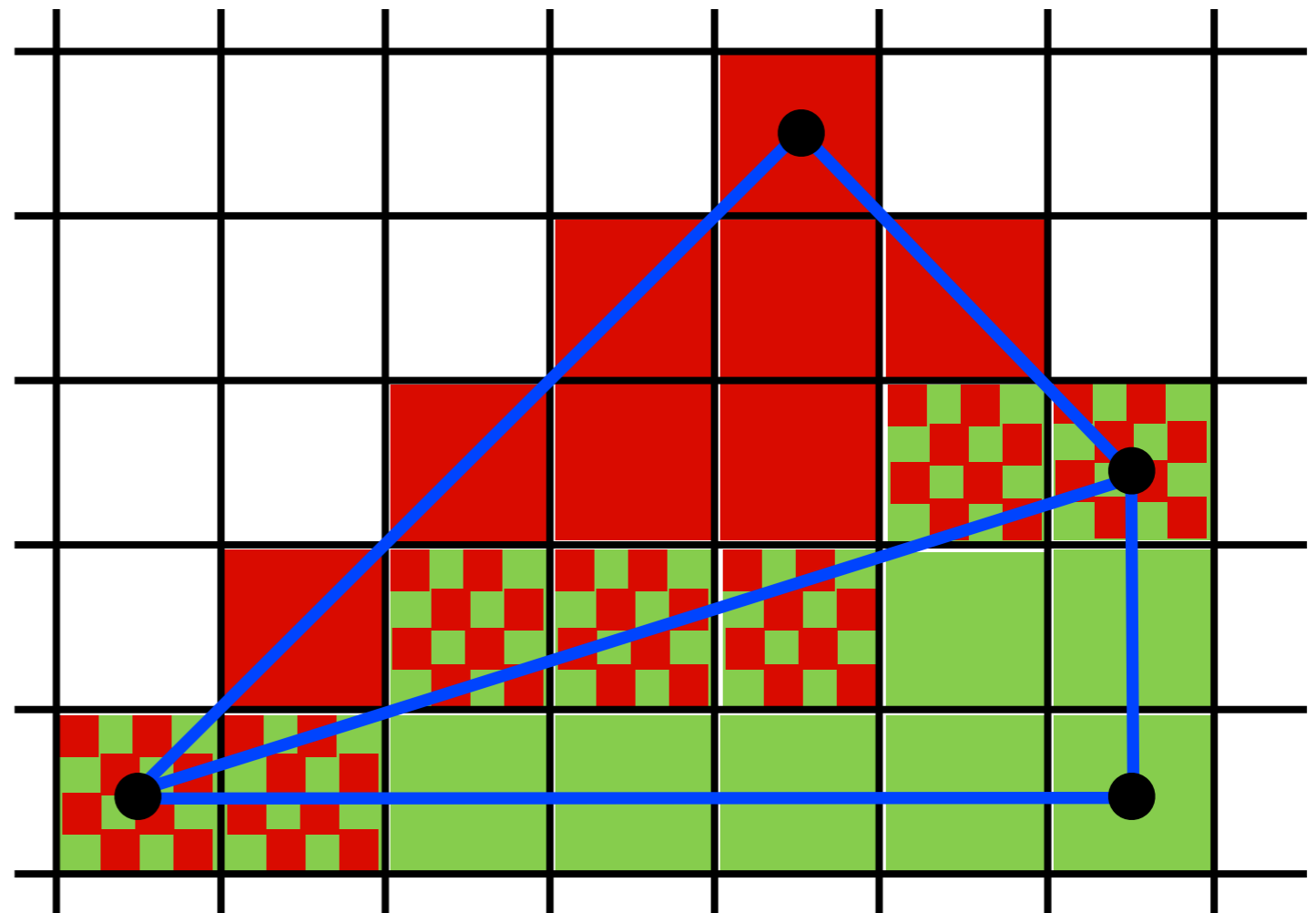


Triangle rasterization issues



Which pixels should be used to approximate a triangle?

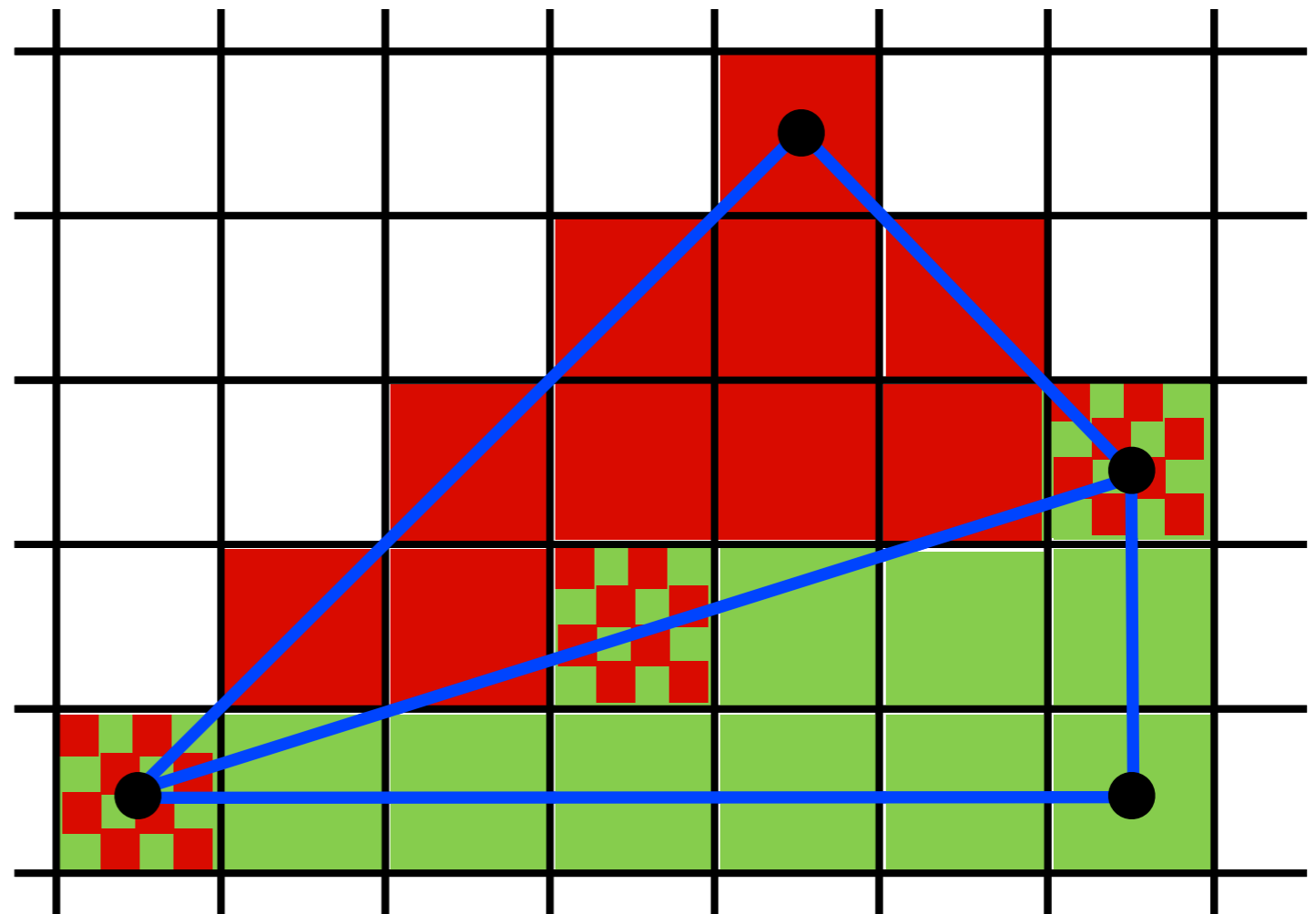
Which should fill in shared edge?



Which pixels should be used to approximate a triangle?

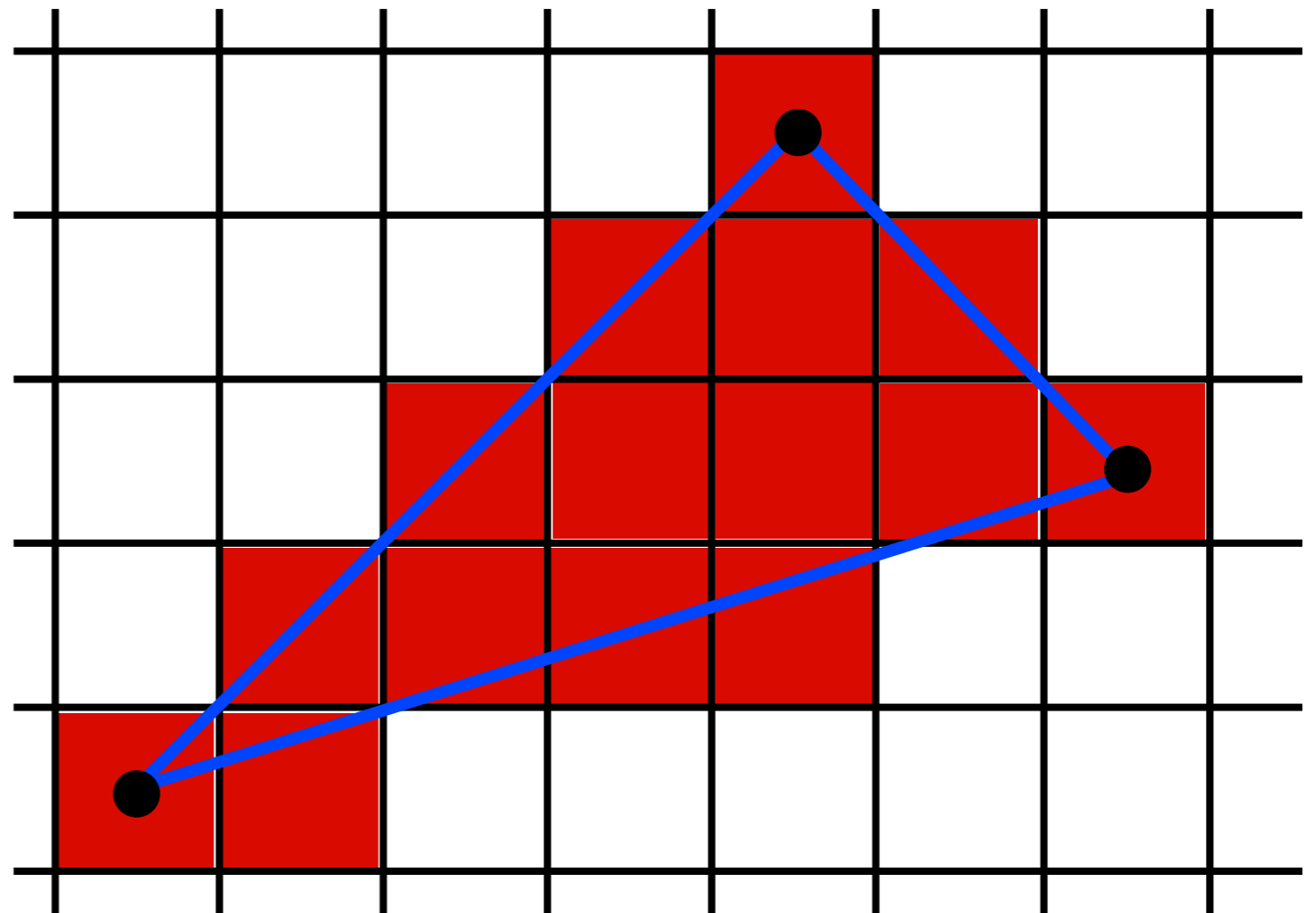
Which should fill in shared edge?

- triangle that contains pixel center
- still have some ties!
- neither? both?
- want a unique assignment



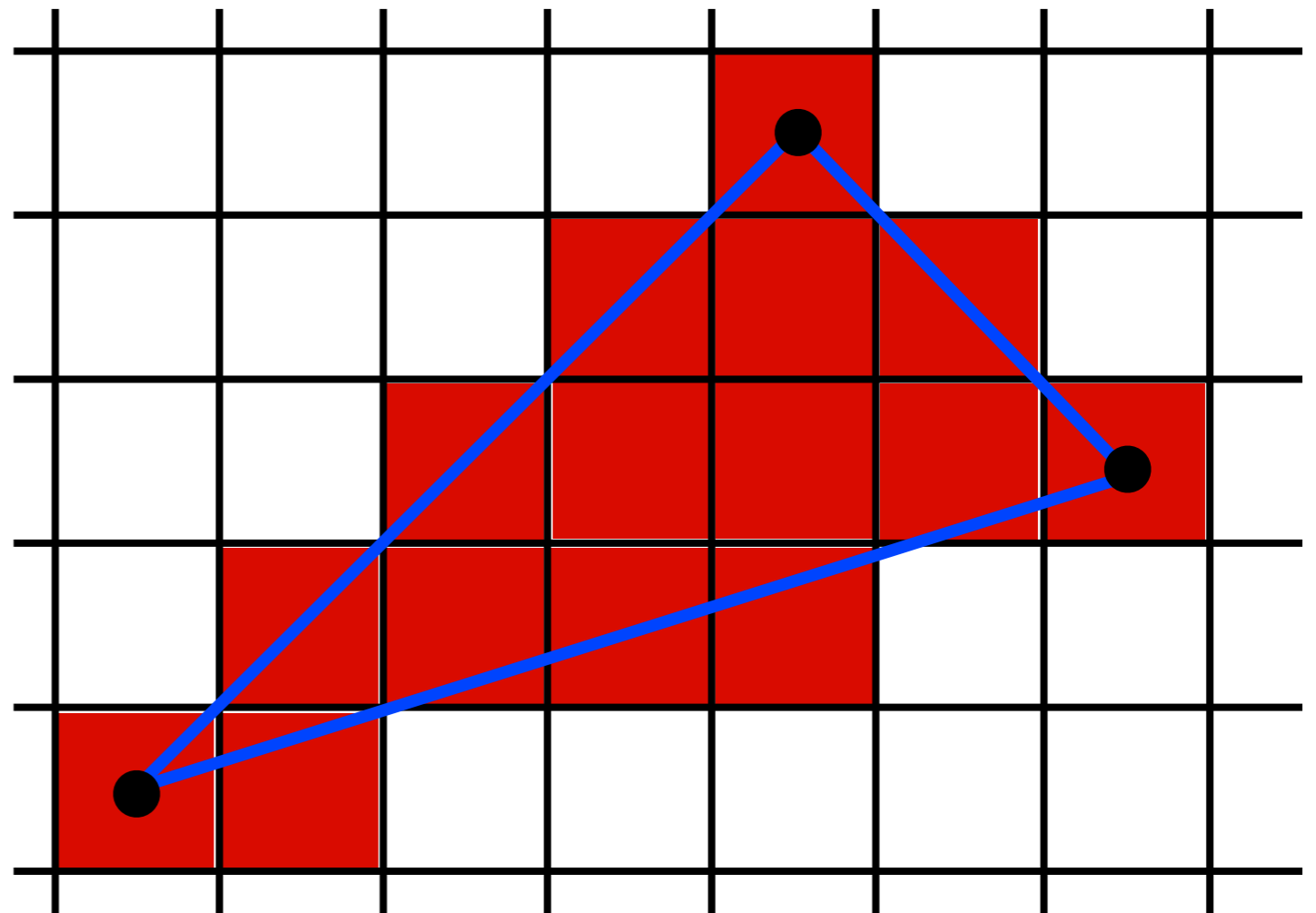
Which pixels should be used to approximate a triangle?

Use Midpoint Algorithm for edges and fill in?



Which pixels should be used to approximate a triangle?

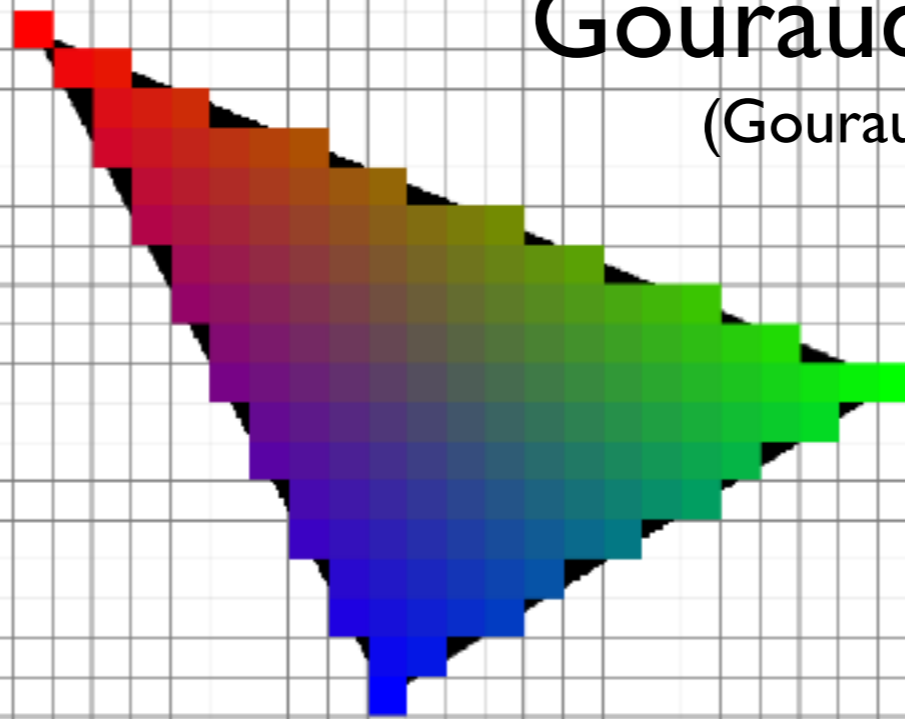
Use an approach based on **barycentric coordinates**



Advantage: we can easily interpolate attributes using barycentric coordinates

$$\mathbf{c} = \alpha \mathbf{c}_0 + \beta \mathbf{c}_1 + \gamma \mathbf{c}_2$$

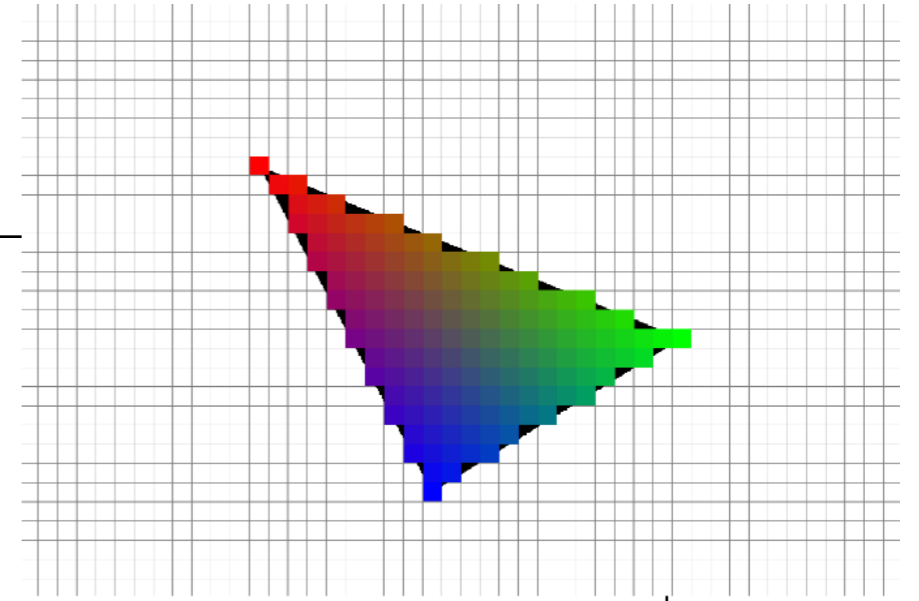
Gouraud shading
(Gouraud, 1971)



<http://jtibble.dyndns.org/graphics/eecs487/eecs487.html>

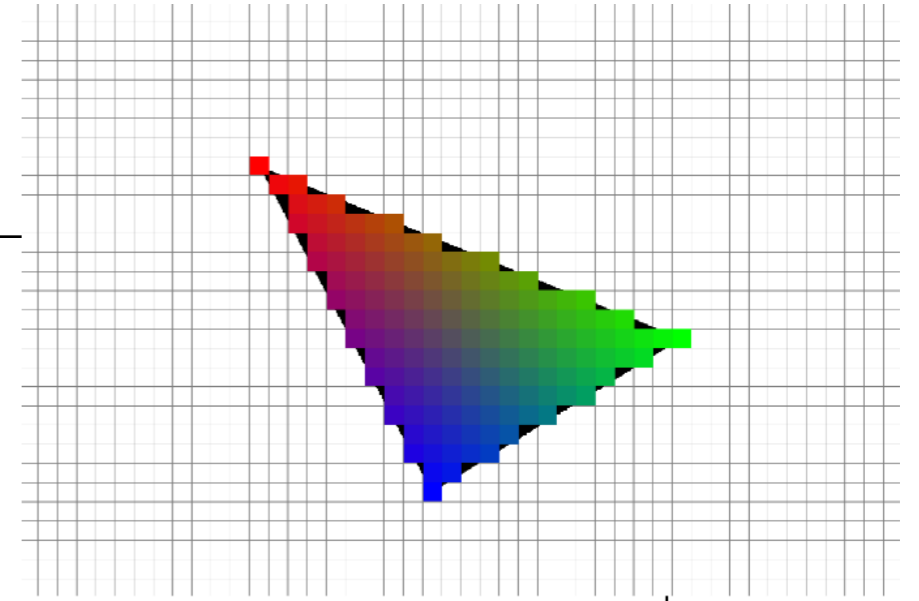
Triangle rasterization algorithm

```
for all x do
  for all y do
    compute  $(\alpha, \beta, \gamma)$  for  $(x, y)$ 
    if  $(\alpha \in [0, 1]$  and  $\beta \in [0, 1]$  and  $\gamma \in [0, 1])$  then
       $\mathbf{c} = \alpha \mathbf{c}_0 + \beta \mathbf{c}_1 + \gamma \mathbf{c}_2$ 
      drawpixel $(x, y)$  with color  $\mathbf{c}$ 
```



Triangle rasterization algorithm

```
for all x do
  for all y do
    compute  $(\alpha, \beta, \gamma)$  for  $(x, y)$ 
    if  $(\alpha \in [0, 1]$  and  $\beta \in [0, 1]$  and  $\gamma \in [0, 1])$  then
       $\mathbf{c} = \alpha\mathbf{c}_0 + \beta\mathbf{c}_1 + \gamma\mathbf{c}_2$ 
      drawpixel $(x, y)$  with color  $\mathbf{c}$ 
```

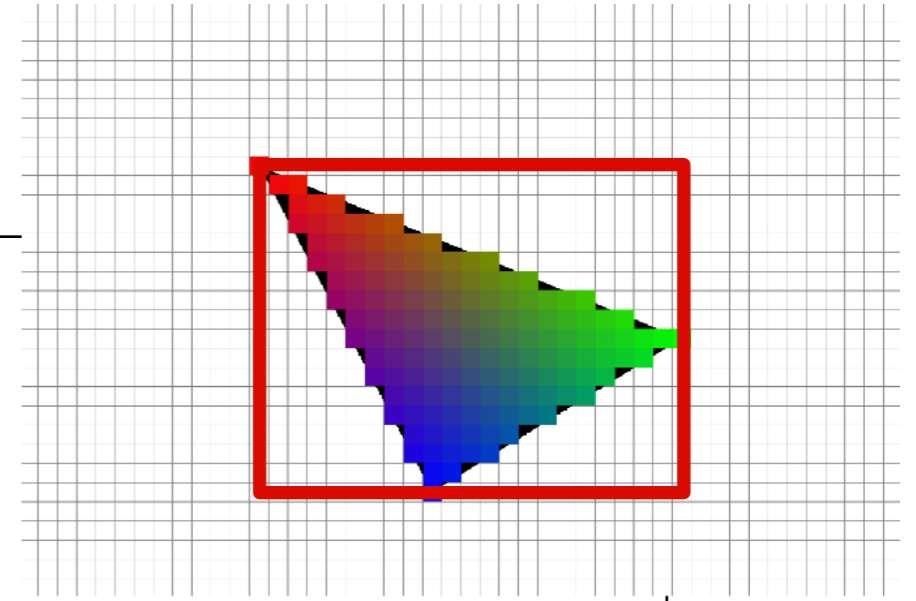


the rest of the algorithm is to make the steps in **red** more **efficient**

Triangle rasterization algorithm

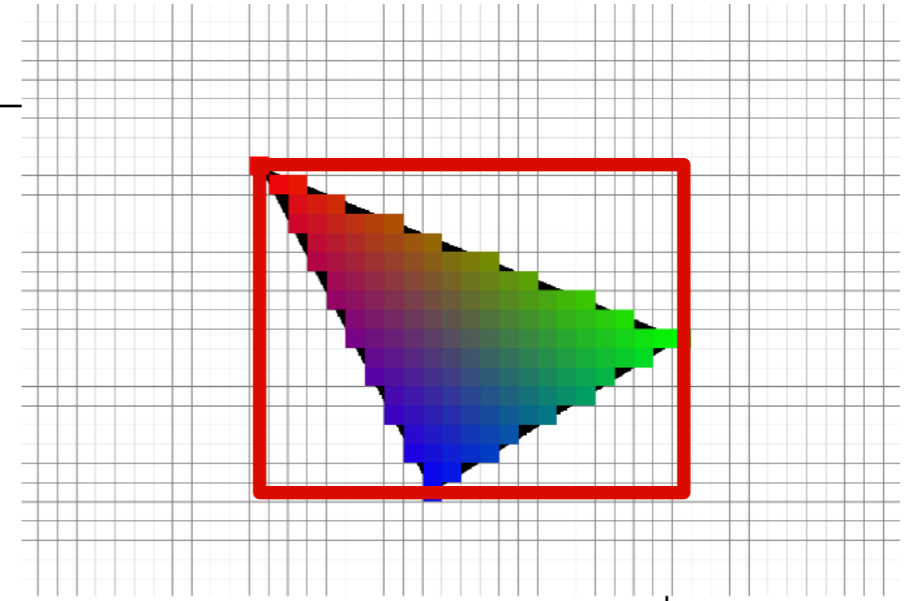
use a bounding rectangle

```
for x in [x_min, x_max]
  for y in [y_min, y_max]
    compute  $(\alpha, \beta, \gamma)$  for  $(x, y)$ 
    if  $(\alpha \in [0, 1]$  and  $\beta \in [0, 1]$  and  $\gamma \in [0, 1])$  then
       $c = \alpha c_0 + \beta c_1 + \gamma c_2$ 
      drawpixel(x, y) with color c
```



Triangle rasterization algorithm

```
for x in [x_min, x_max]
  for y in [y_min, y_max]
     $\alpha = f_{bc}(x, y) / f_{bc}(x_a, y_a)$ 
     $\beta = f_{ca}(x, y) / f_{ca}(x_b, y_b)$ 
     $\gamma = f_{ab}(x, y) / f_{ab}(x_c, y_c)$ 
    if ( $\alpha \in [0, 1]$  and  $\beta \in [0, 1]$  and  $\gamma \in [0, 1]$ ) then
       $\mathbf{c} = \alpha \mathbf{c}_0 + \beta \mathbf{c}_1 + \gamma \mathbf{c}_2$ 
      drawpixel(x,y) with color c
```

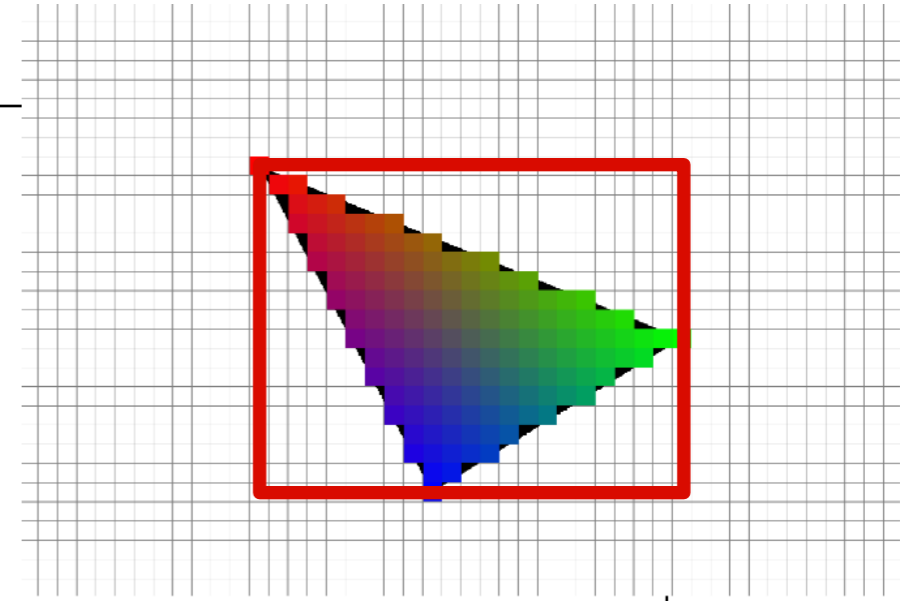


<whiteboard>

Triangle rasterization algorithm

Optimizations?

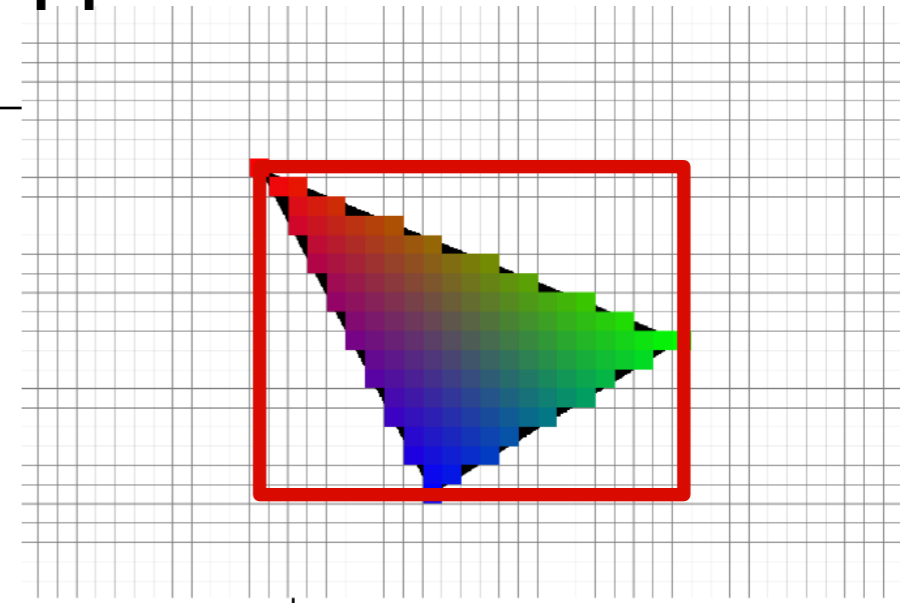
```
for x in [x_min, x_max]
  for y in [y_min, y_max]
     $\alpha = f_{bc}(x, y) / f_{bc}(x_a, y_a)$ 
     $\beta = f_{ca}(x, y) / f_{ca}(x_b, y_b)$ 
     $\gamma = f_{ab}(x, y) / f_{ab}(x_c, y_c)$ 
    if ( $\alpha \in [0, 1]$  and  $\beta \in [0, 1]$  and  $\gamma \in [0, 1]$ ) then
       $\mathbf{c} = \alpha \mathbf{c}_0 + \beta \mathbf{c}_1 + \gamma \mathbf{c}_2$ 
      drawpixel(x,y) with color c
```



Triangle rasterization algorithm

Optimizations? don't need to check upper bound

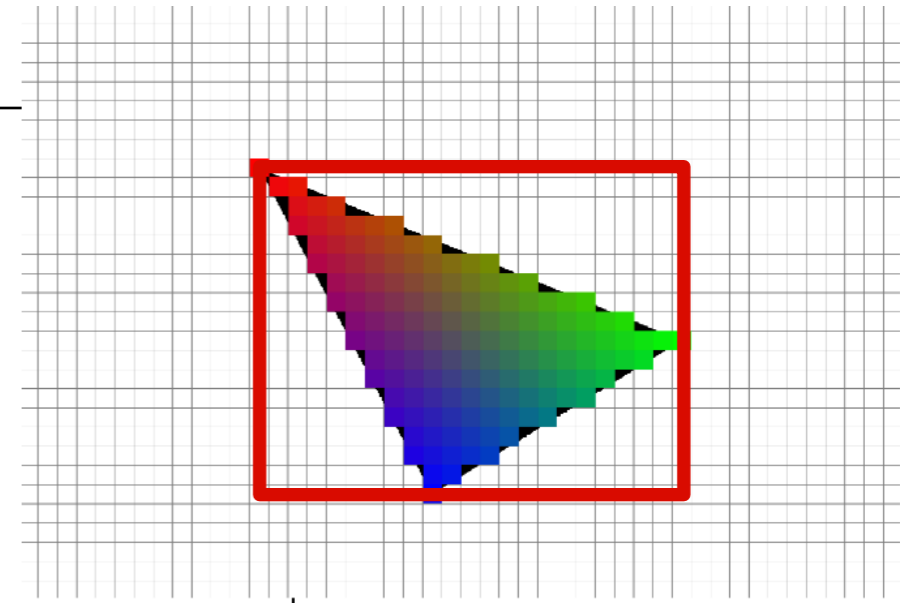
```
for x in [x_min, x_max]
  for y in [y_min, y_max]
     $\alpha = f_{bc}(x, y) / f_{bc}(x_a, y_a)$ 
     $\beta = f_{ca}(x, y) / f_{ca}(x_b, y_b)$ 
     $\gamma = f_{ab}(x, y) / f_{ab}(x_c, y_c)$ 
    if ( $\alpha \geq 0$  and  $\beta \geq 0$  and  $\gamma \geq 0$ ) then
       $\mathbf{c} = \alpha \mathbf{c}_0 + \beta \mathbf{c}_1 + \gamma \mathbf{c}_2$ 
      drawpixel(x,y) with color c
```



Triangle rasterization algorithm

Optimizations? compute bary. coord. and colors incrementally

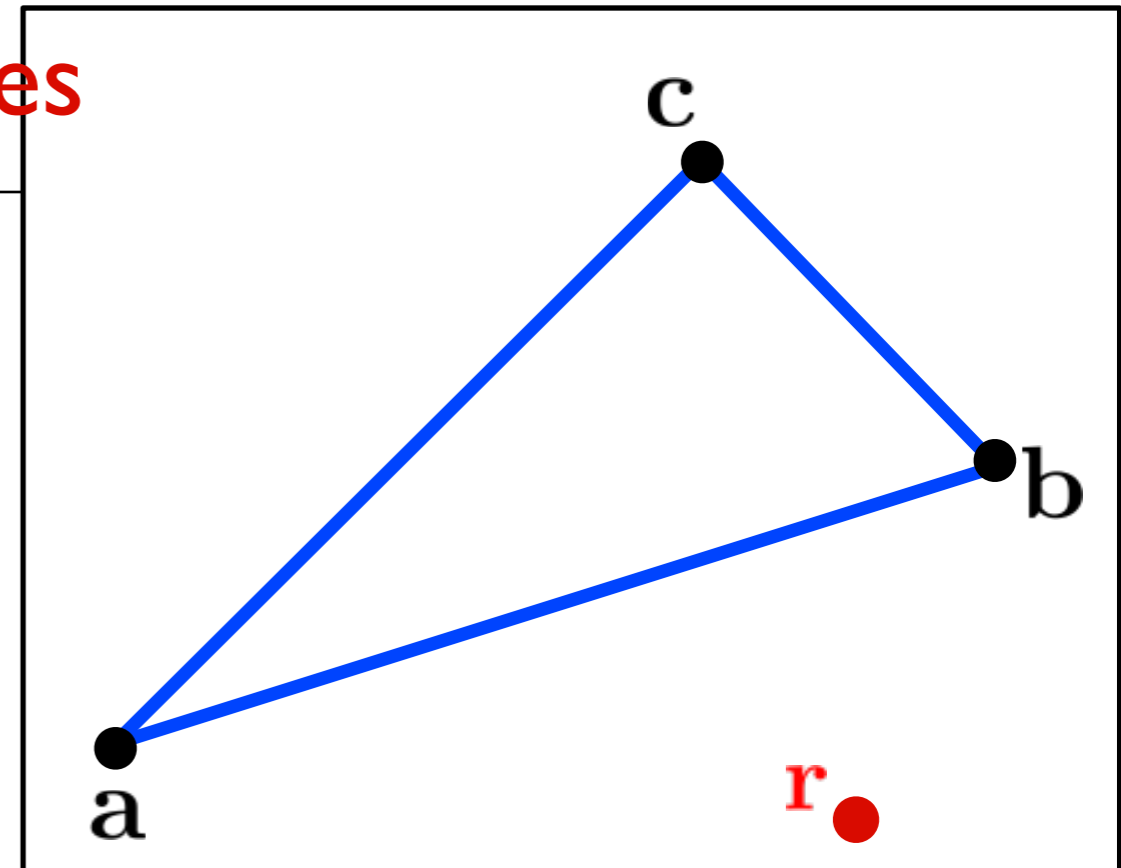
```
for x in [x_min, x_max]
  for y in [y_min, y_max]
     $\alpha = f_{bc}(x, y) / f_{bc}(x_a, y_a)$ 
     $\beta = f_{ca}(x, y) / f_{ca}(x_b, y_b)$ 
     $\gamma = f_{ab}(x, y) / f_{ab}(x_c, y_c)$ 
    if ( $\alpha \geq 0$  and  $\beta \geq 0$  and  $\gamma \geq 0$ ) then
       $\mathbf{c} = \alpha \mathbf{c}_0 + \beta \mathbf{c}_1 + \gamma \mathbf{c}_2$ 
      drawpixel(x,y) with color c
```



Triangle rasterization algorithm

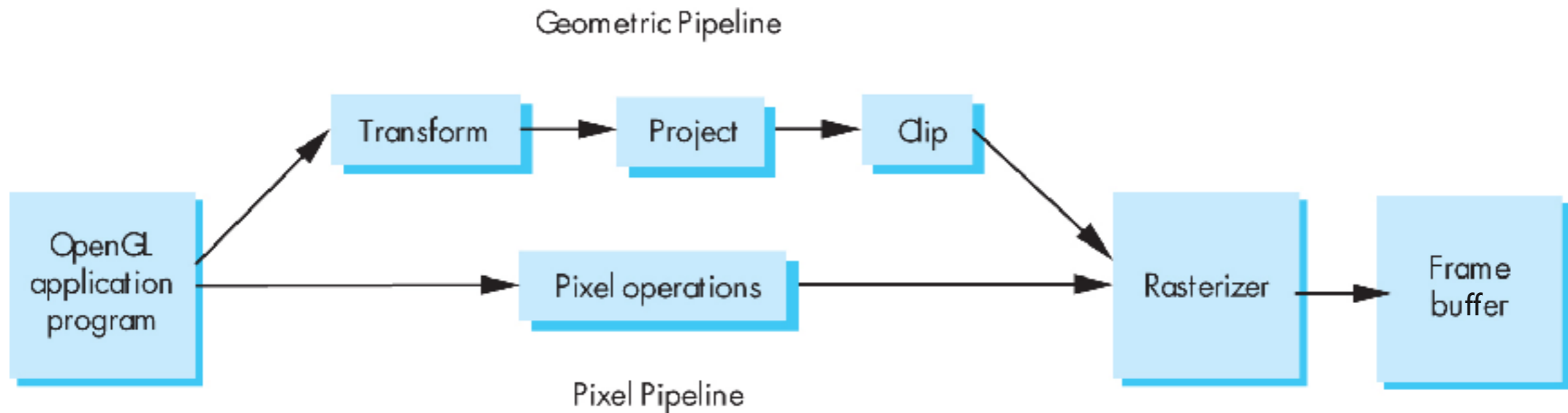
dealing with shared triangle edges

```
for x in [x_min, x_max]
  for y in [y_min, y_max]
     $\alpha = f_{bc}(x, y) / f_{bc}(x_a, y_a)$ 
     $\beta = f_{ac}(x, y) / f_{ac}(x_b, y_b)$ 
     $\gamma = f_{ab}(x, y) / f_{ab}(x_c, y_c)$ 
    if ( $\alpha \geq 0$  and  $\beta \geq 0$  and  $\gamma \geq 0$ ) then
      if ( $\alpha > 0$  or  $f_{bc}(\mathbf{a})f_{bc}(\mathbf{r}) > 0$ ) and
         ( $\beta > 0$  or  $f_{ca}(\mathbf{b})f_{ca}(\mathbf{r}) > 0$ ) and
         ( $\gamma > 0$  or  $f_{ab}(\mathbf{c})f_{ab}(\mathbf{r}) > 0$ )
         $\mathbf{c} = \alpha\mathbf{c}_0 + \beta\mathbf{c}_1 + \gamma\mathbf{c}_2$ 
        drawpixel(x,y) with color c
```

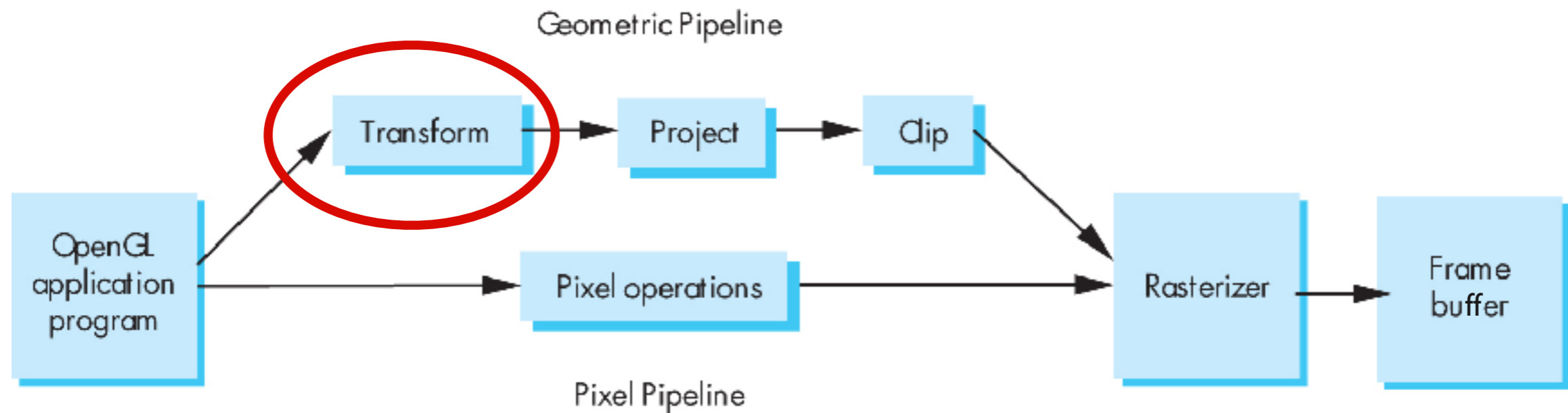


Graphics Pipeline (cont.)

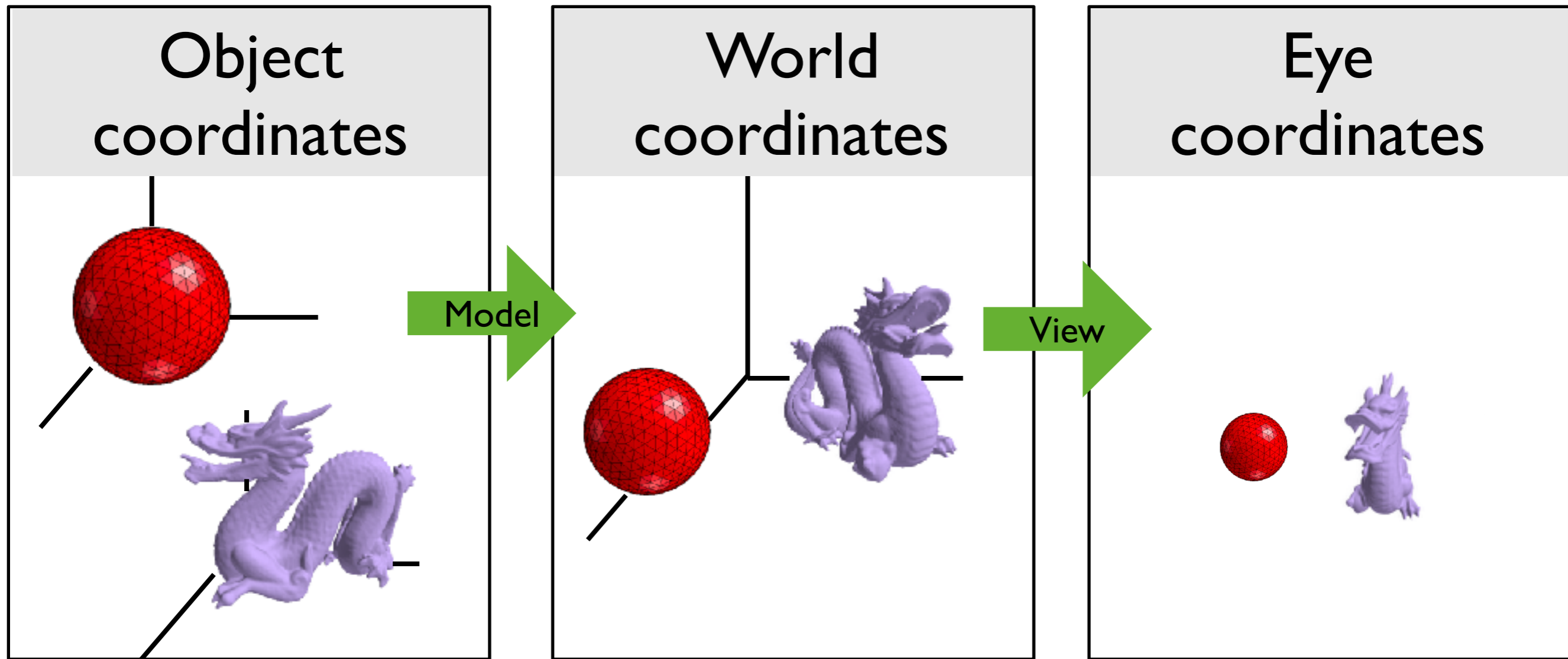
Graphics Pipeline



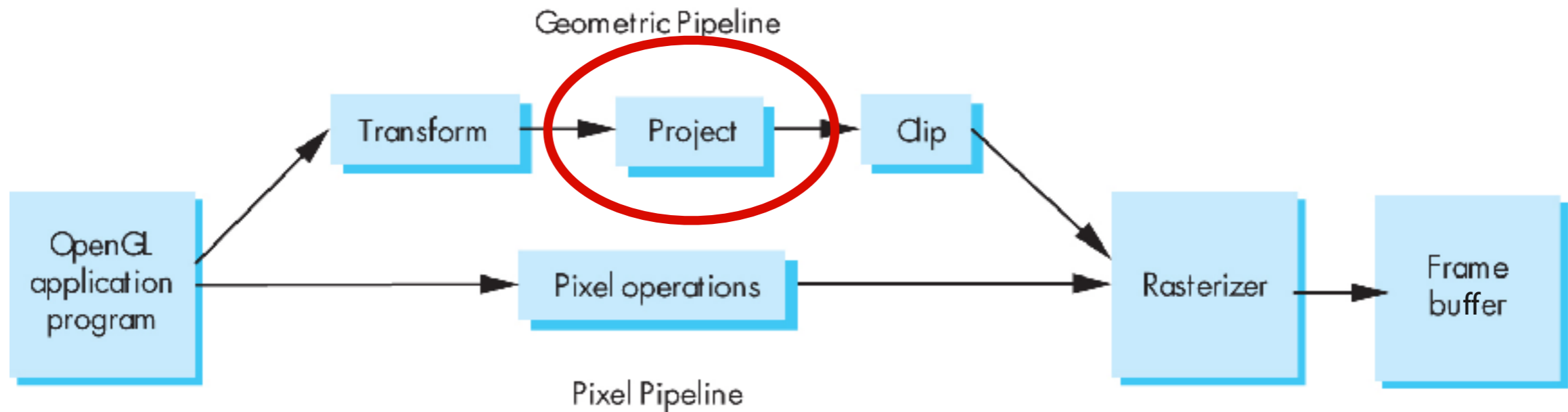
Transform



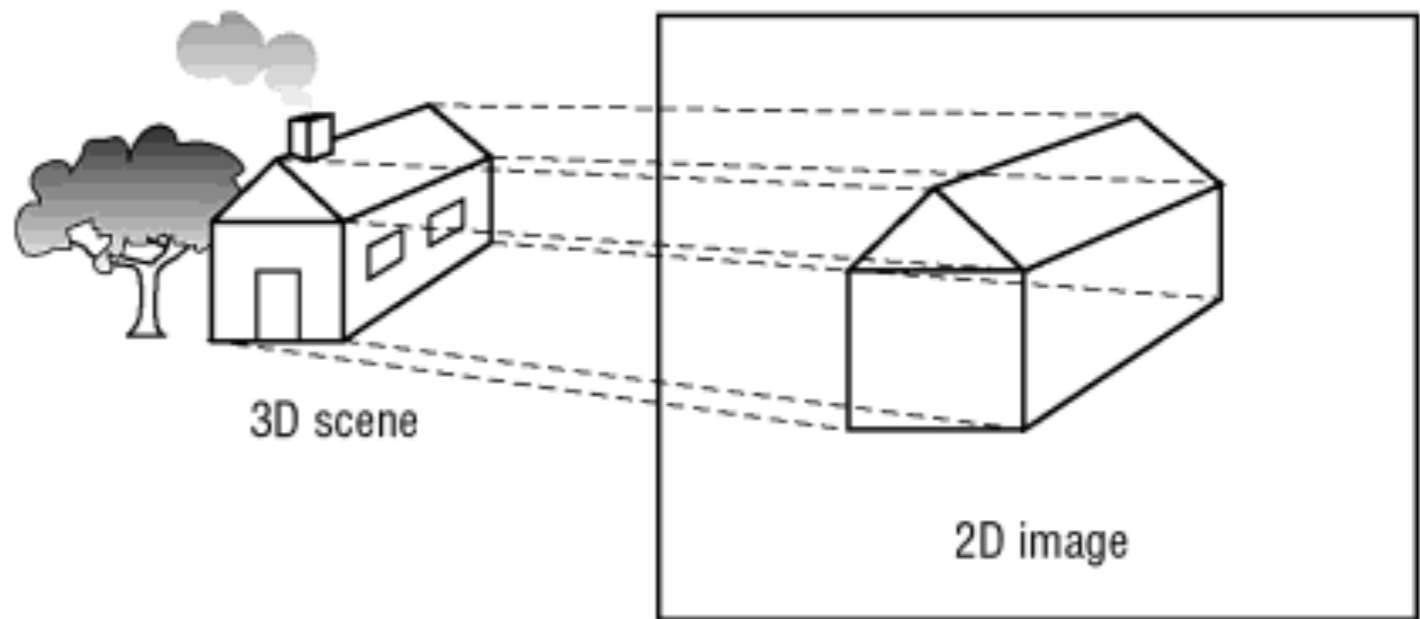
“Modelview” Transformation



Project

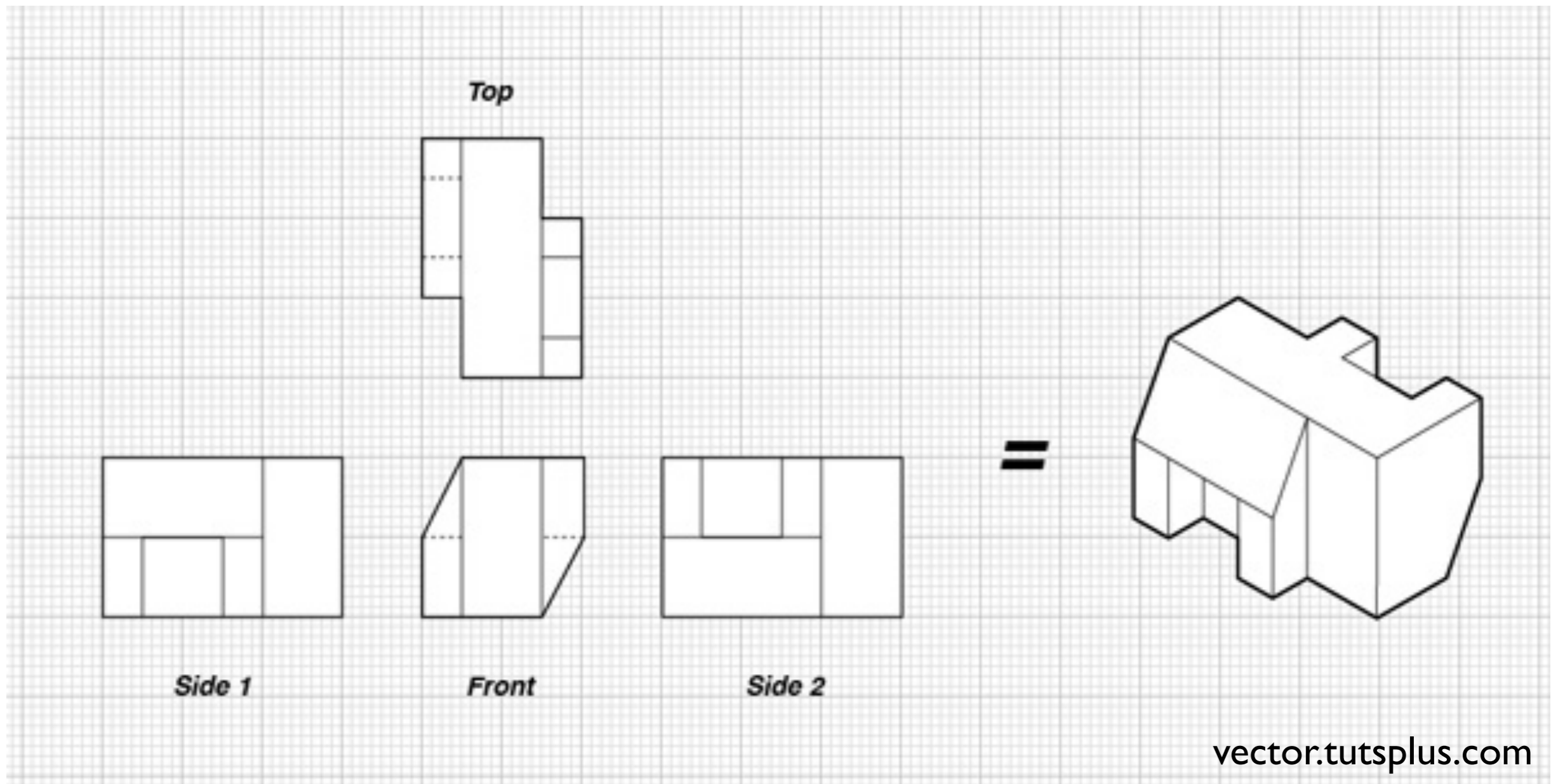


Projection: map
3D scene to
2D image



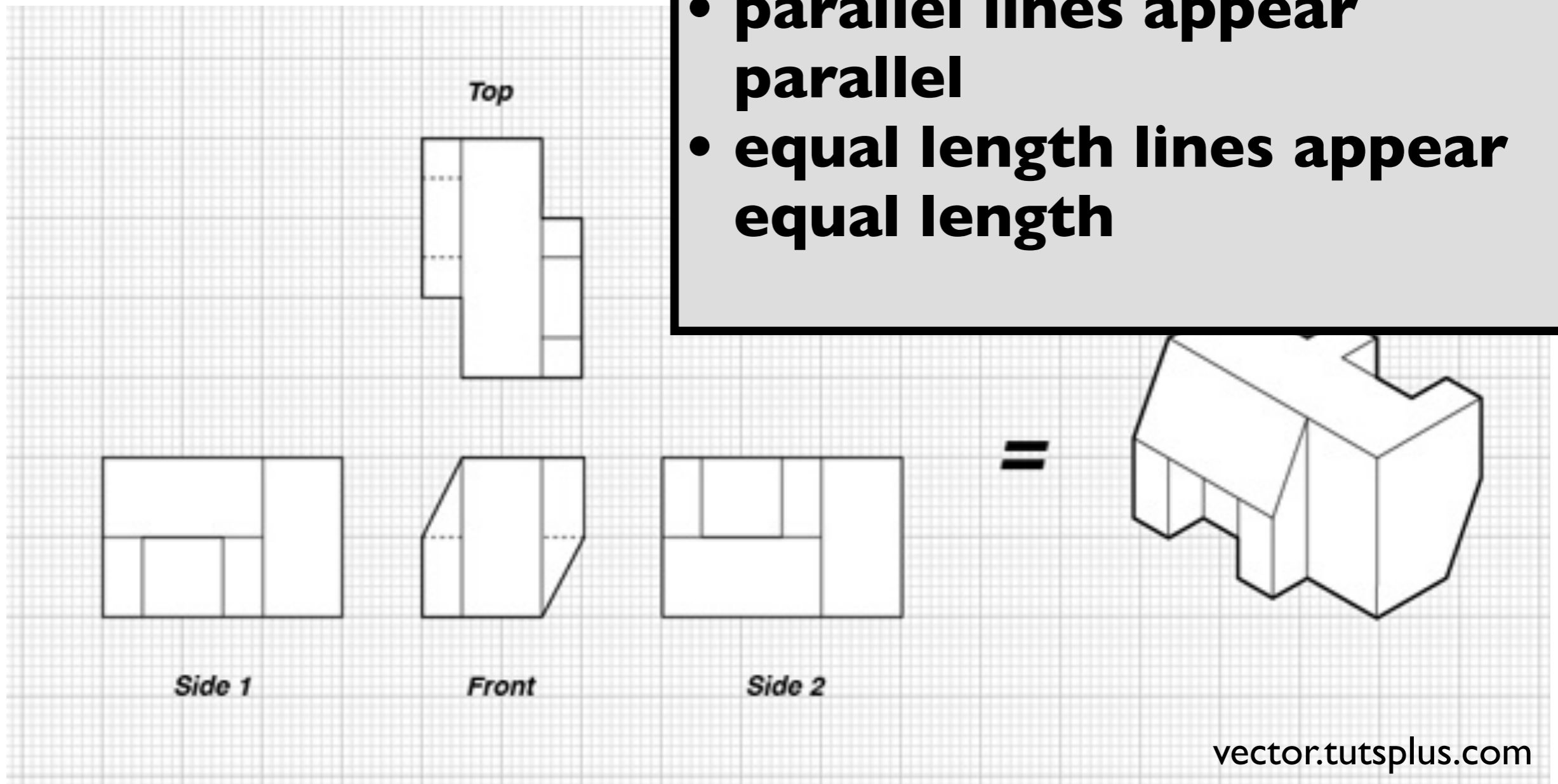
OpenGL Super Bible, 5th Ed.

Orthographic projection



Orthographic projection

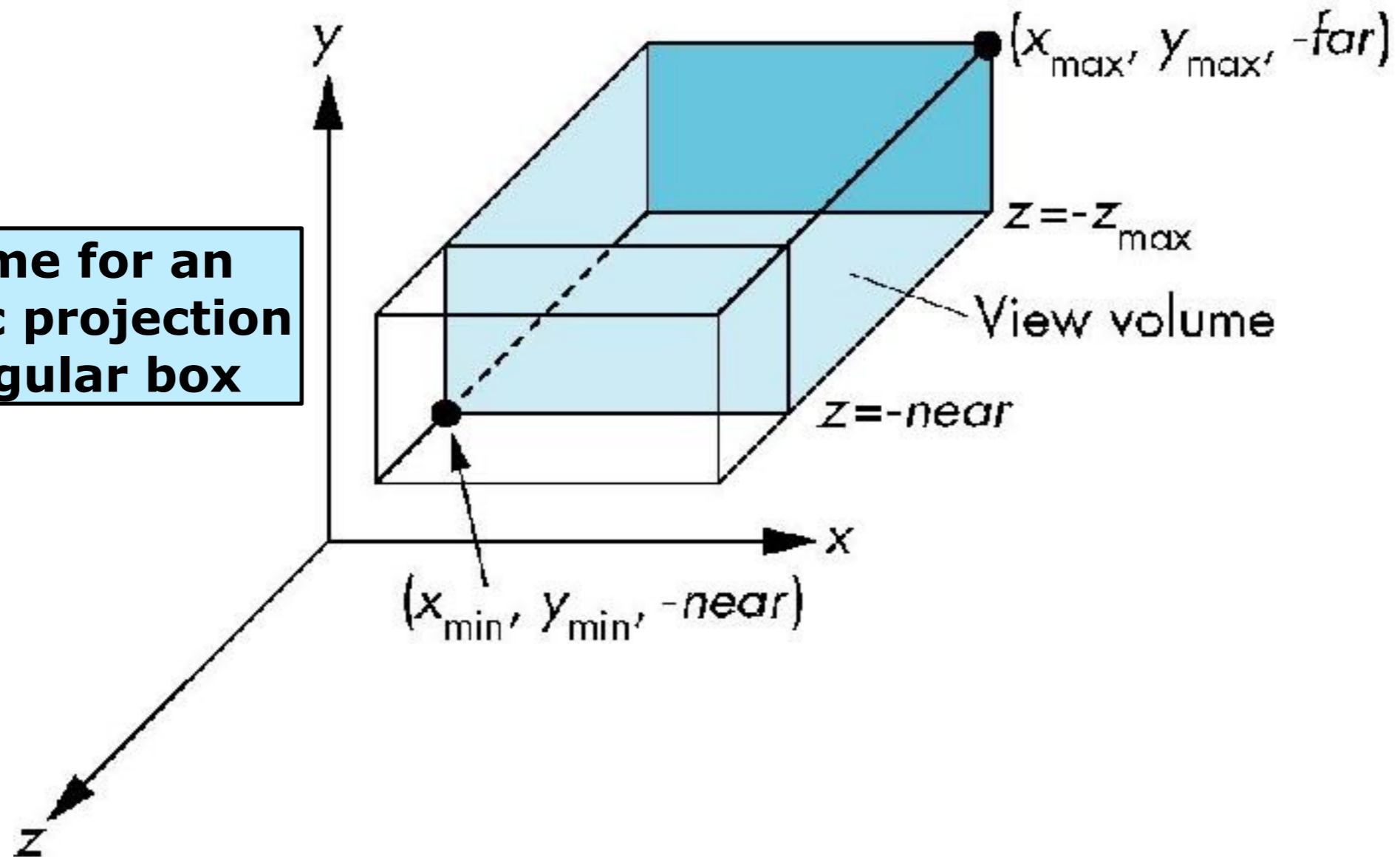
- **parallel lines appear parallel**
- **equal length lines appear equal length**



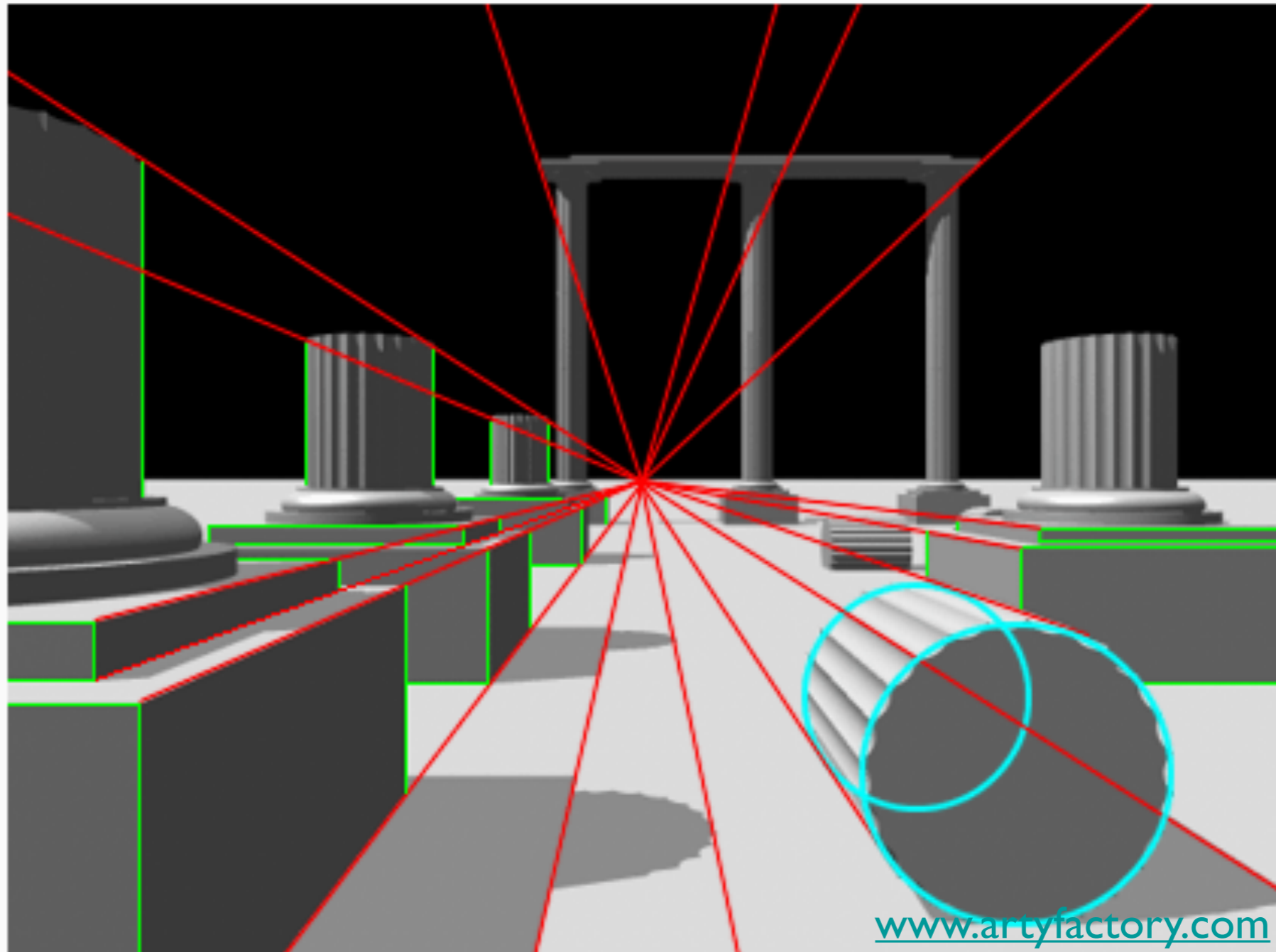
OpenGL Orthogonal Viewing

`glOrtho(left, right, bottom, top, near, far)`

View volume for an orthographic projection is a rectangular box



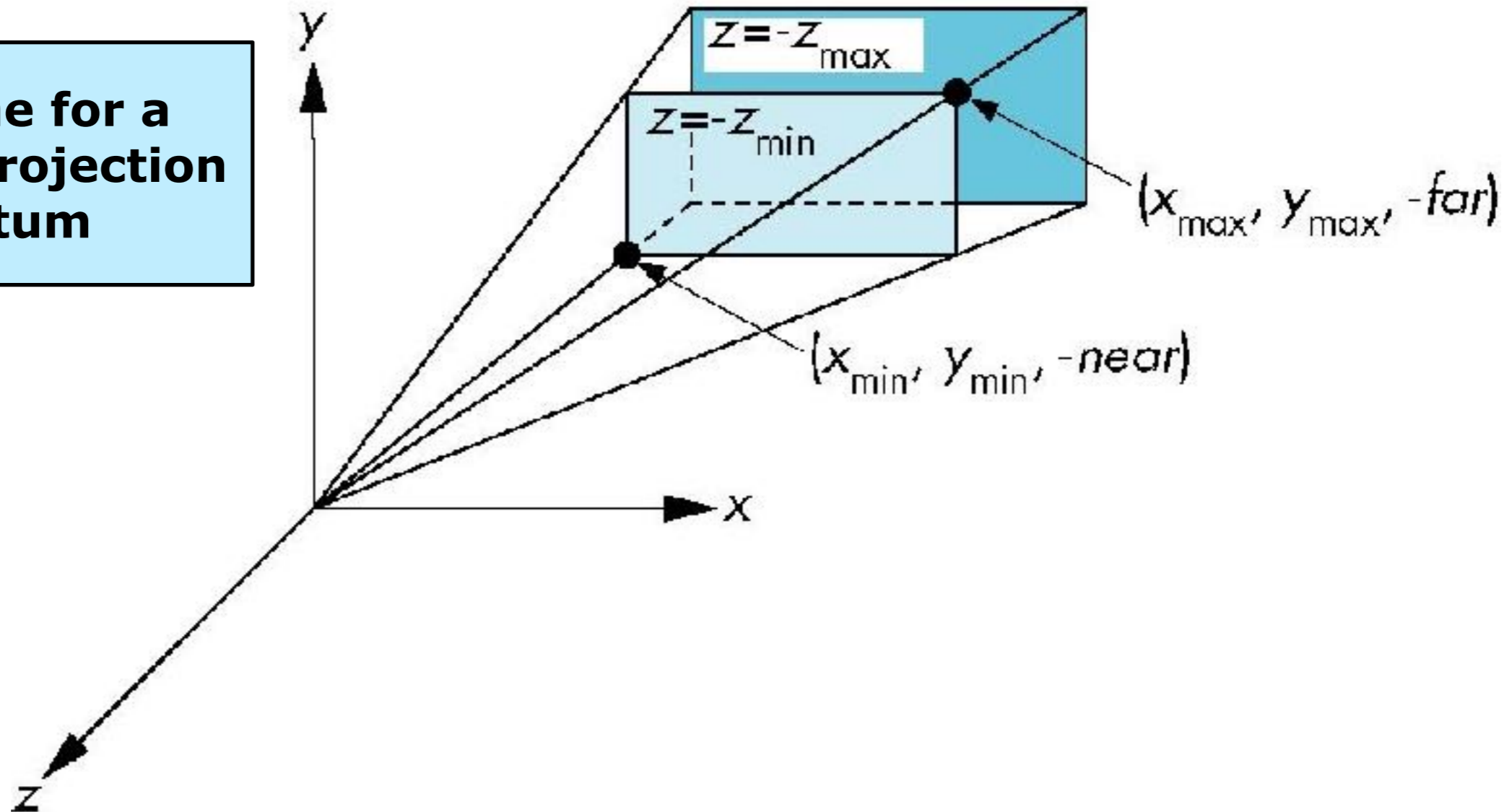
Perspective projection



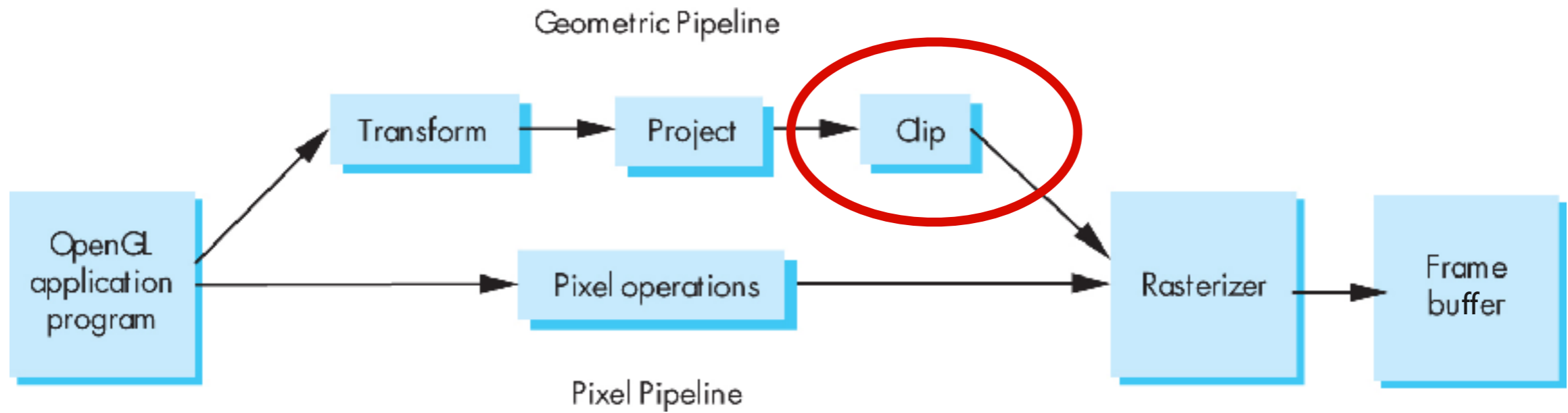
OpenGL Perspective Viewing

`glFrustum(xmin, xmax, ymin, ymax, near, far)`

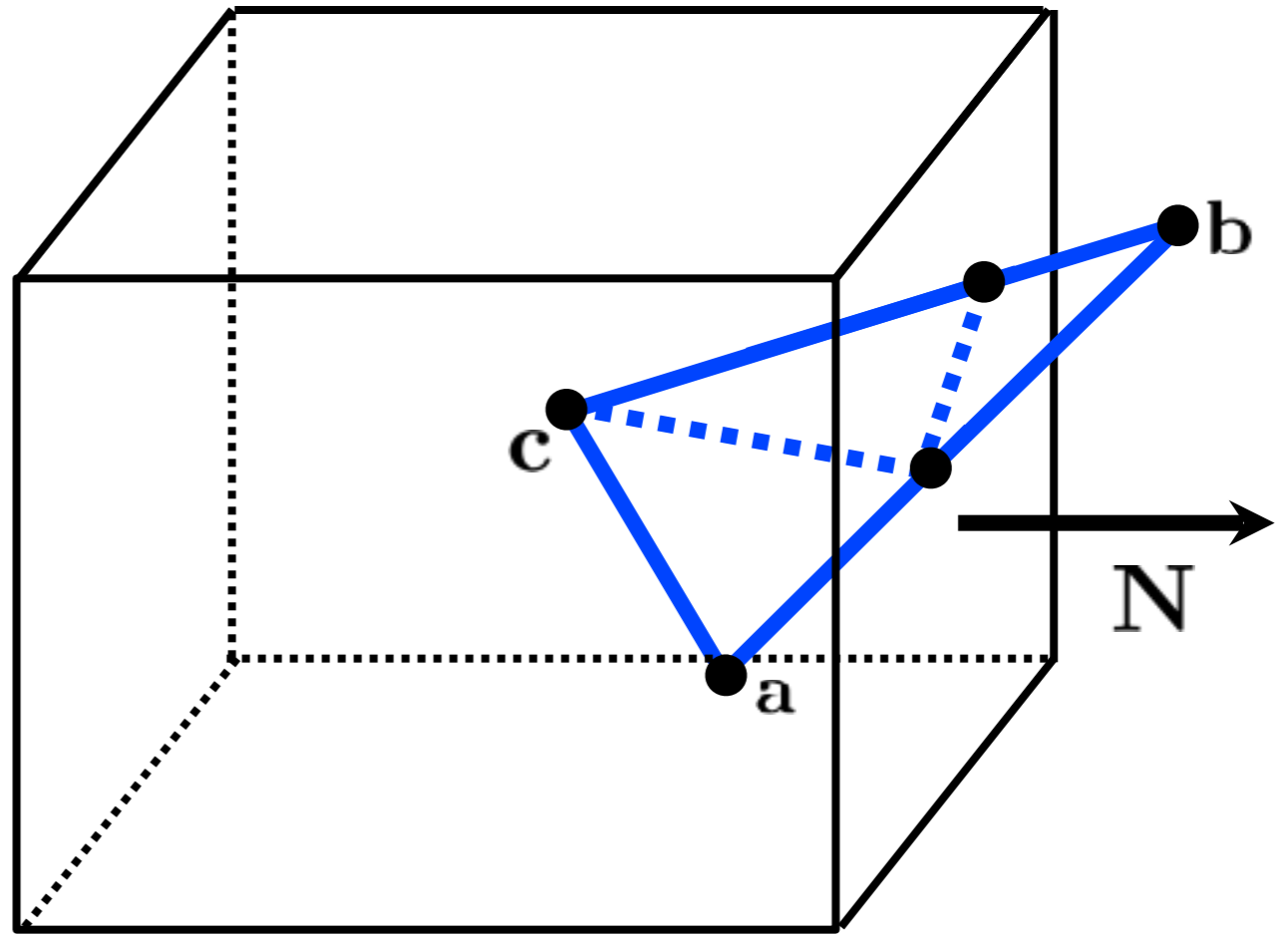
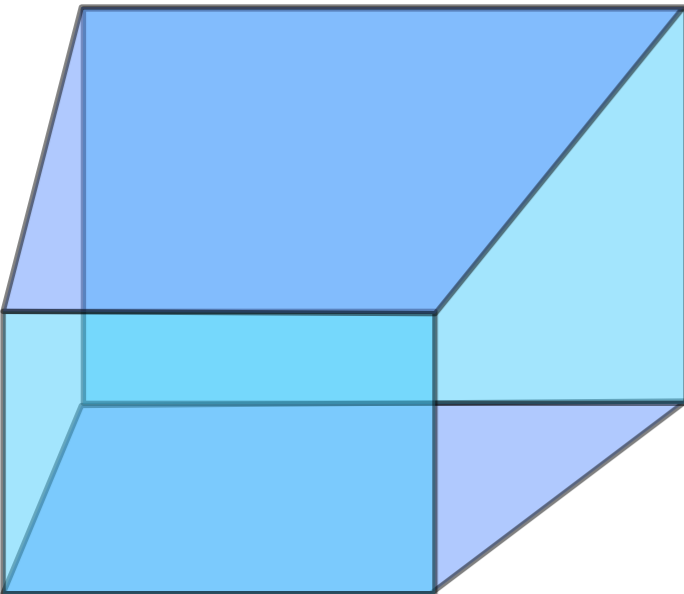
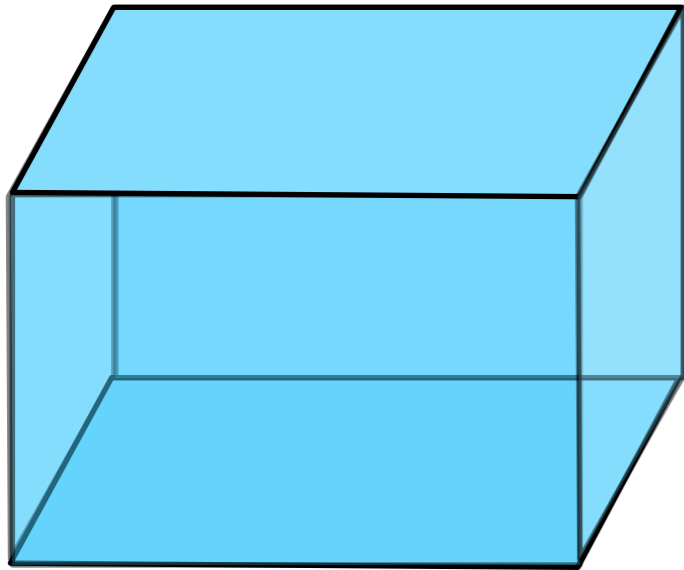
View volume for a perspective projection is a frustum



Clip

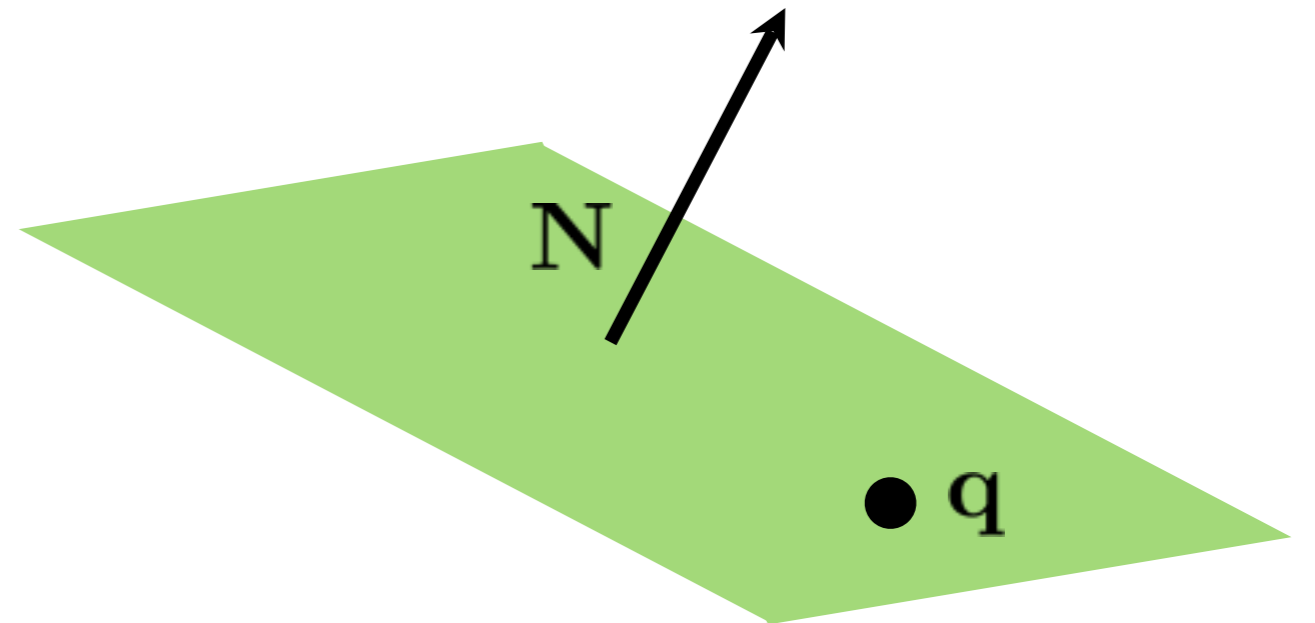


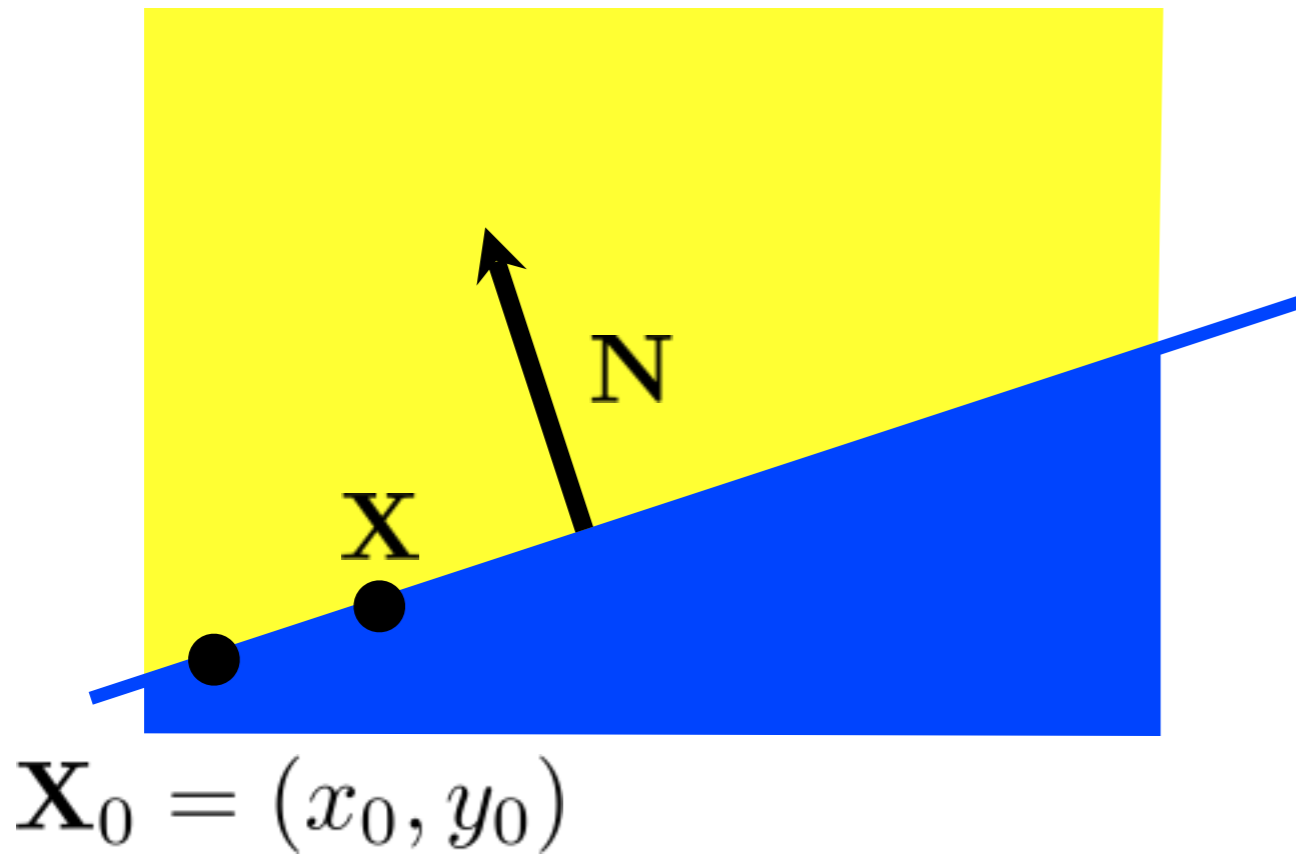
Clip against view volume



Clipping against a plane

What's the equation for
the plane through \mathbf{q}
with normal \mathbf{N} ?





implicit line equation:

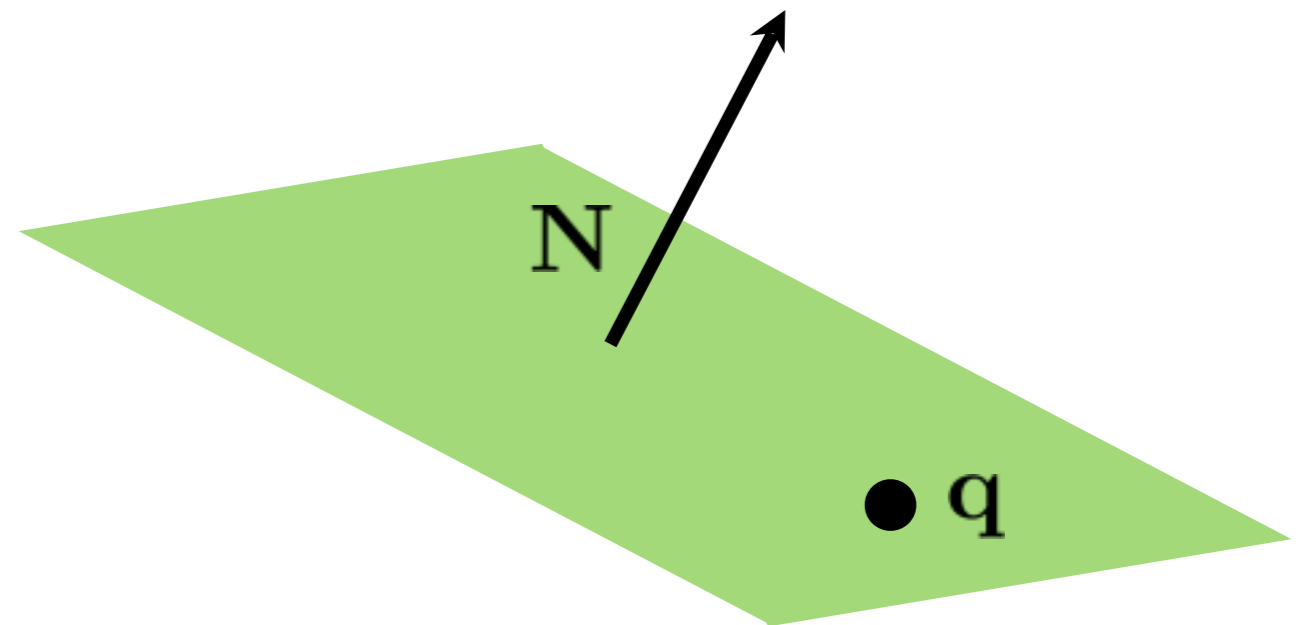
$$f(\mathbf{X}) = \mathbf{N} \cdot (\mathbf{X} - \mathbf{X}_0) = 0$$

Clipping against a plane

What's the equation for
the plane through \mathbf{q}
with normal \mathbf{N} ?

$$f(\mathbf{p}) = ? = 0$$

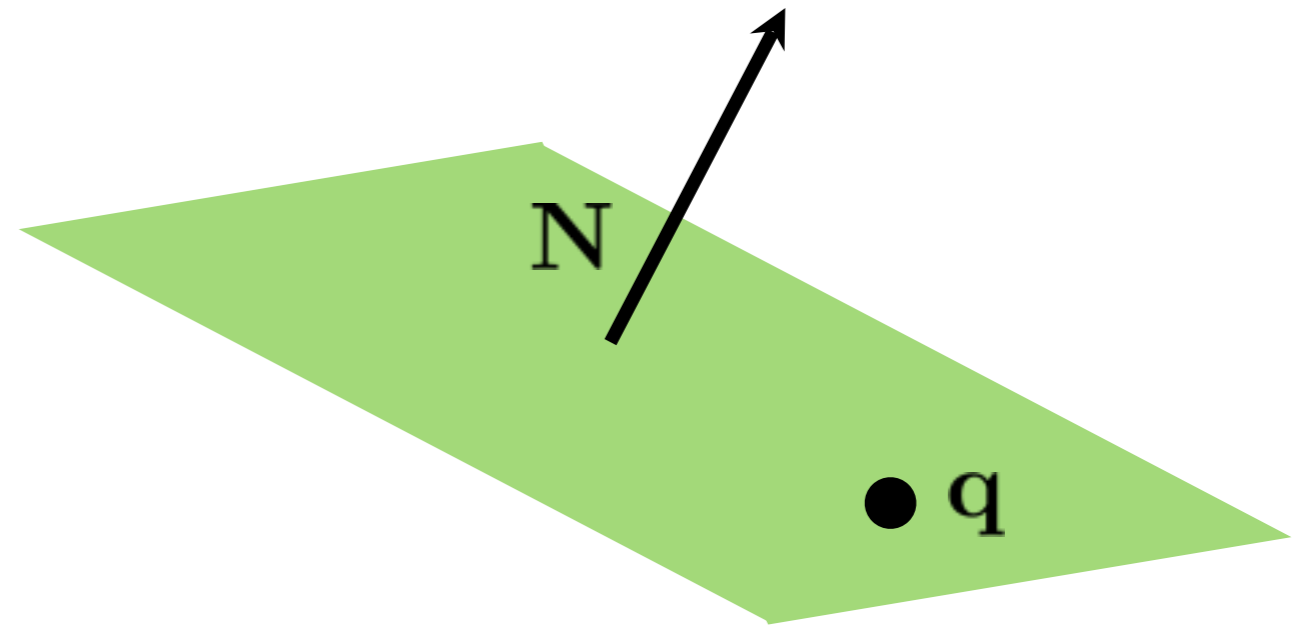
<whiteboard>



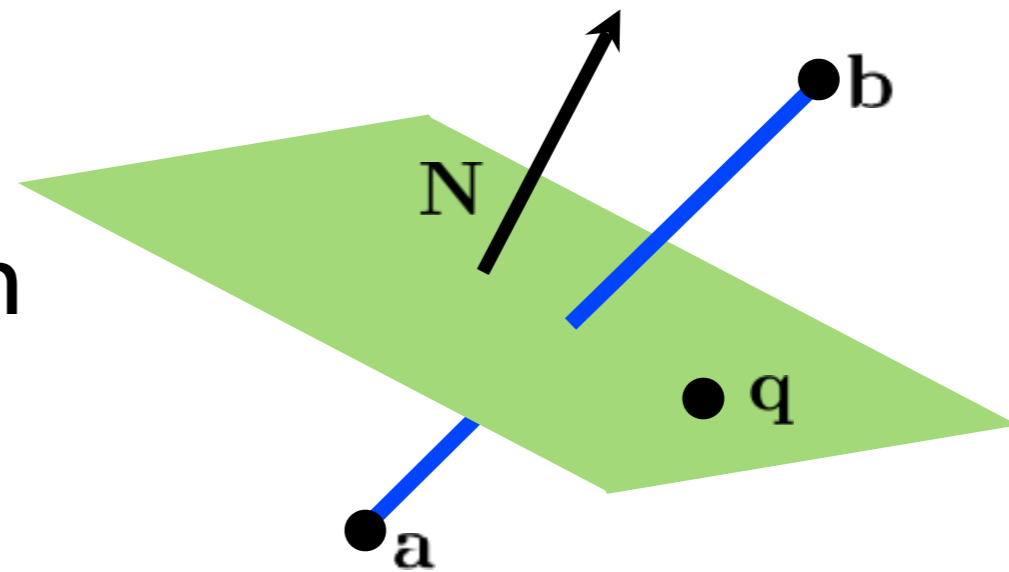
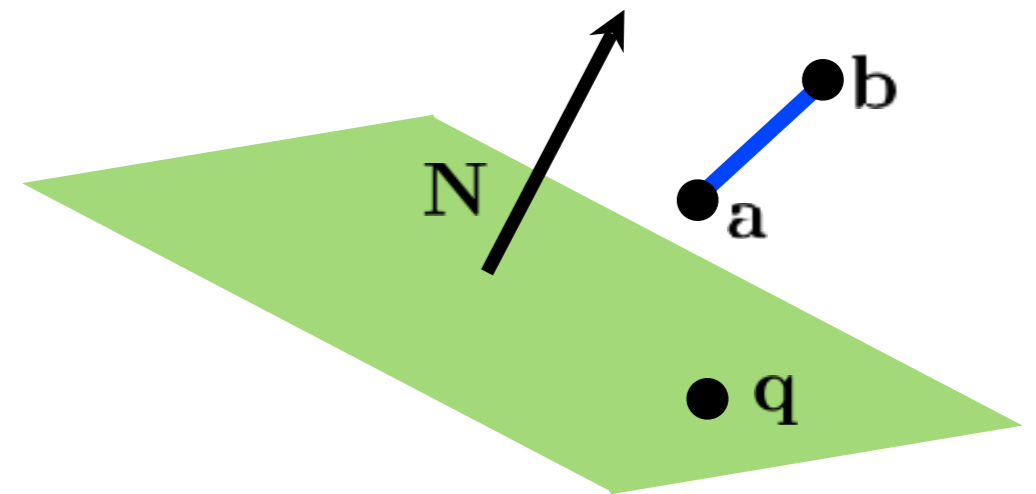
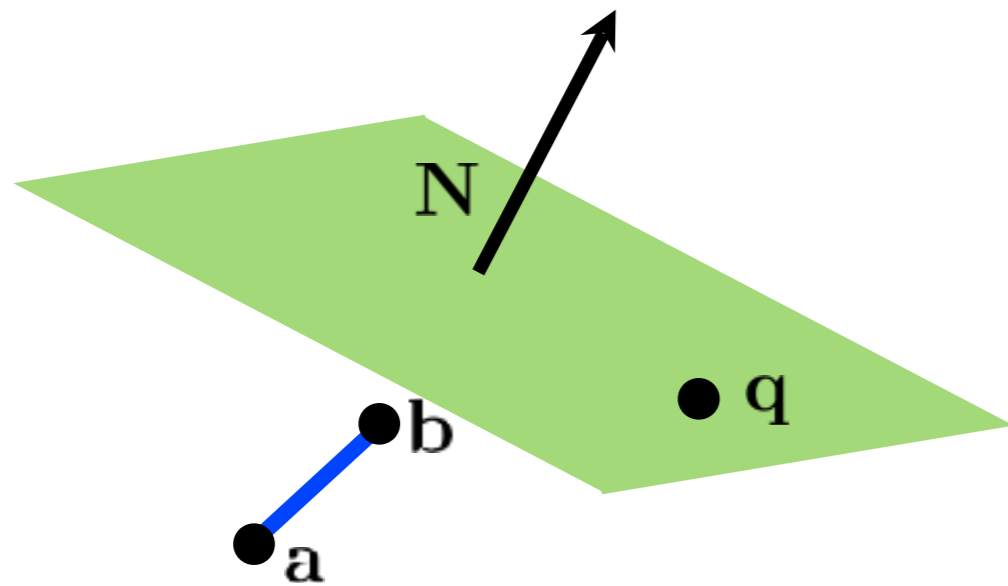
Clipping against a plane

What's the equation for
the plane through \mathbf{q}
with normal \mathbf{N} ?

$$f(\mathbf{p}) = \mathbf{N} \cdot (\mathbf{p} - \mathbf{q}) = 0$$



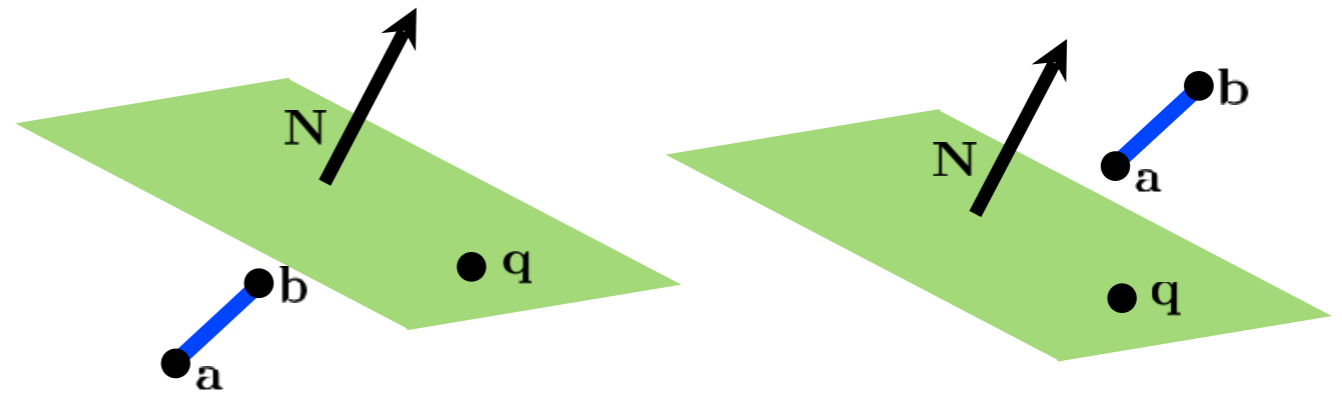
Intersection of line and plane



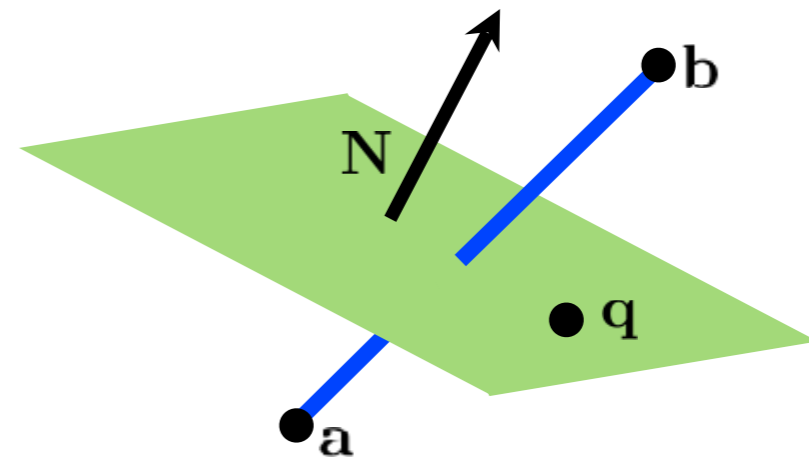
How can we distinguish between these cases?

Intersection of line and plane

$$f(\mathbf{a})f(\mathbf{b}) \geq 0$$

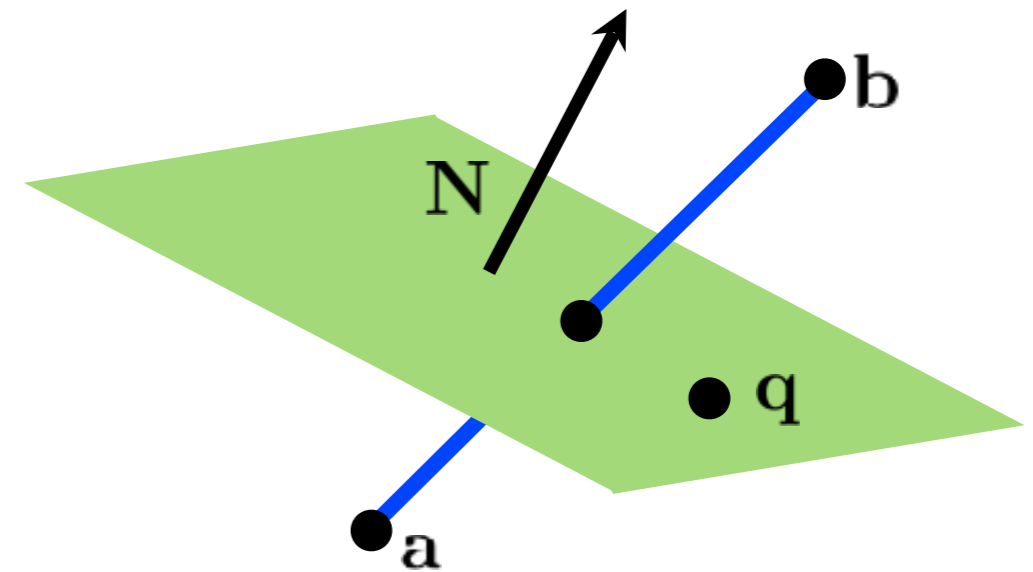


$$f(\mathbf{a})f(\mathbf{b}) < 0$$



Intersection of line and plane

How can we find the intersection point?



<whiteboard>

Clip against view volume

$$s = \frac{\mathbf{N} \cdot (\mathbf{q} - \mathbf{c})}{\mathbf{N} \cdot (\mathbf{b} - \mathbf{c})}$$

$$t = \frac{\mathbf{N} \cdot (\mathbf{q} - \mathbf{a})}{\mathbf{N} \cdot (\mathbf{b} - \mathbf{a})}$$

need to generate new
triangles

