CS130 : Computer Graphics

Lecture 4: Rasterizing 2D Lines

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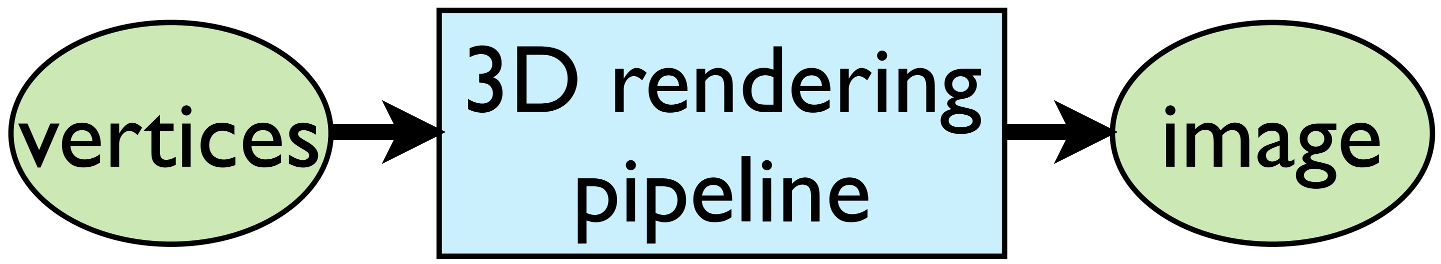
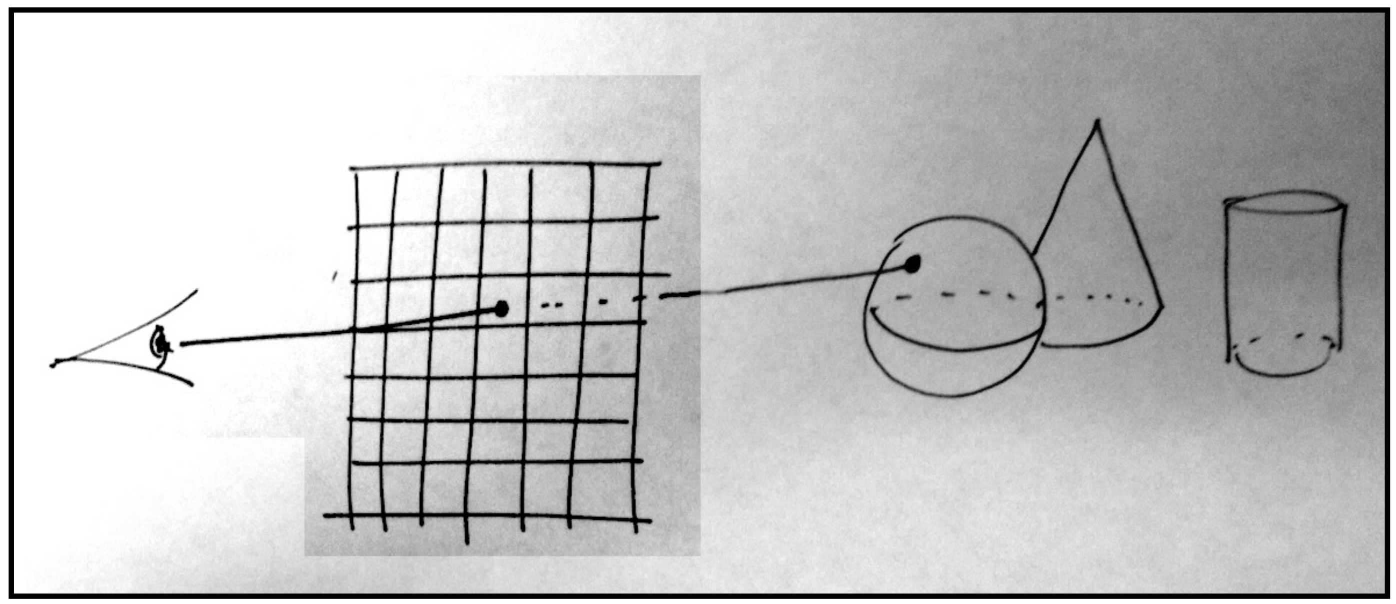
Rendering approaches

1. object-oriented



foreach object ...

2. image-oriented



foreach pixel ...

there’s more than one way to do **object-oriented rendering** – e.g., OpenGL graphics pipeline vs. Renderman

Outline

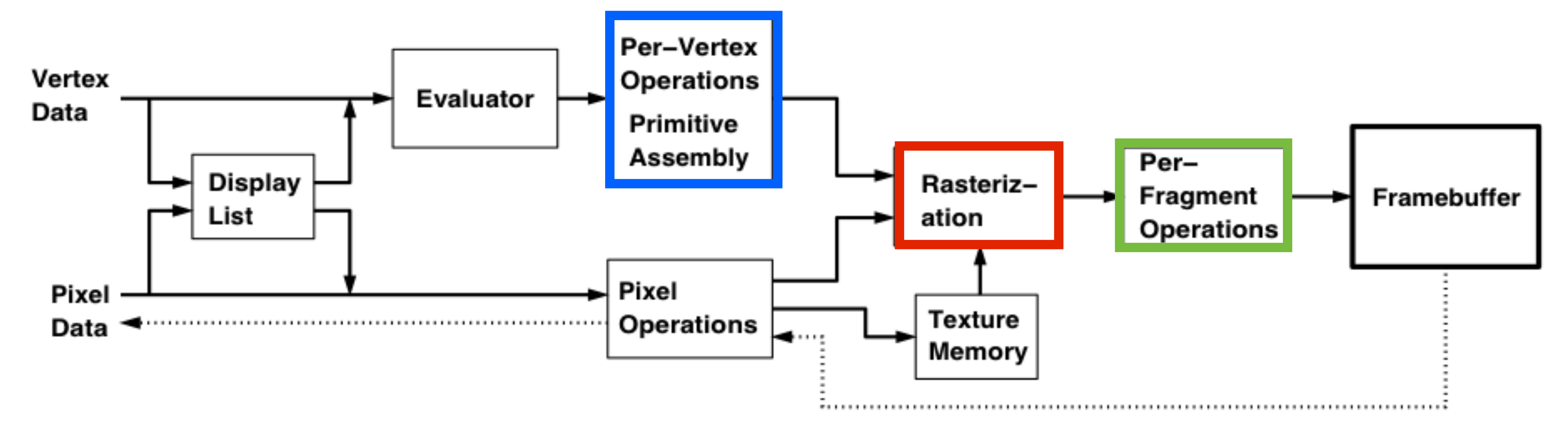


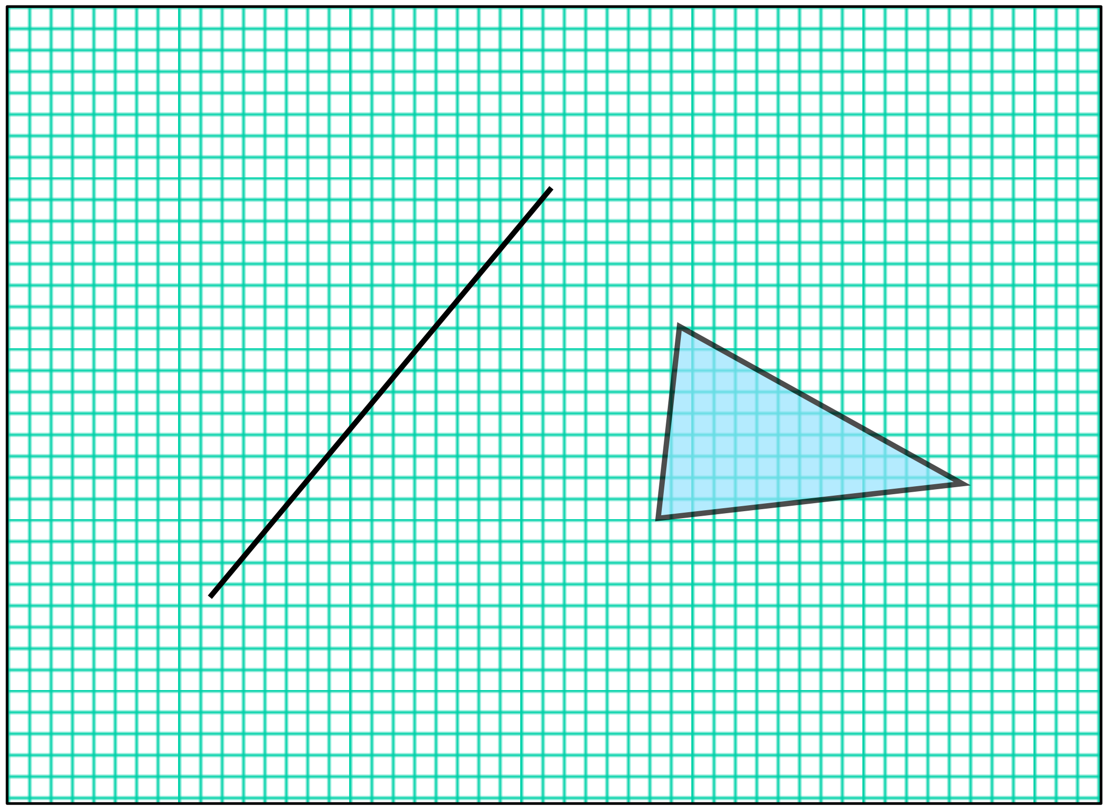
Figure 1. Block diagram of OpenGL.

rasterization - make fragments from clipped objects

clipping - clip objects to viewing volume

hidden surface removal - determine visible fragments

What is rasterization?



Rasterization is the process of determining which pixels are "covered" by the primitive

What is rasterization?

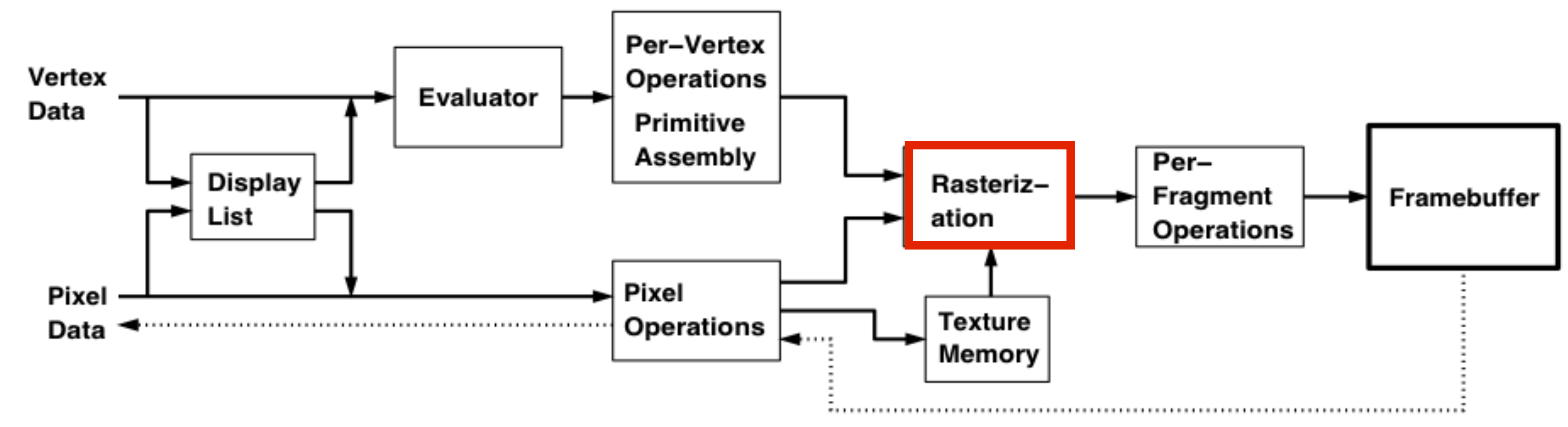


Figure 1. Block diagram of OpenGL.

- input: primitives, output: fragments
- enumerate the pixels covered by a primitive
- interpolate attributes across the primitive

– **output** 1 fragment per pixel covered by the primitive

Rasterization

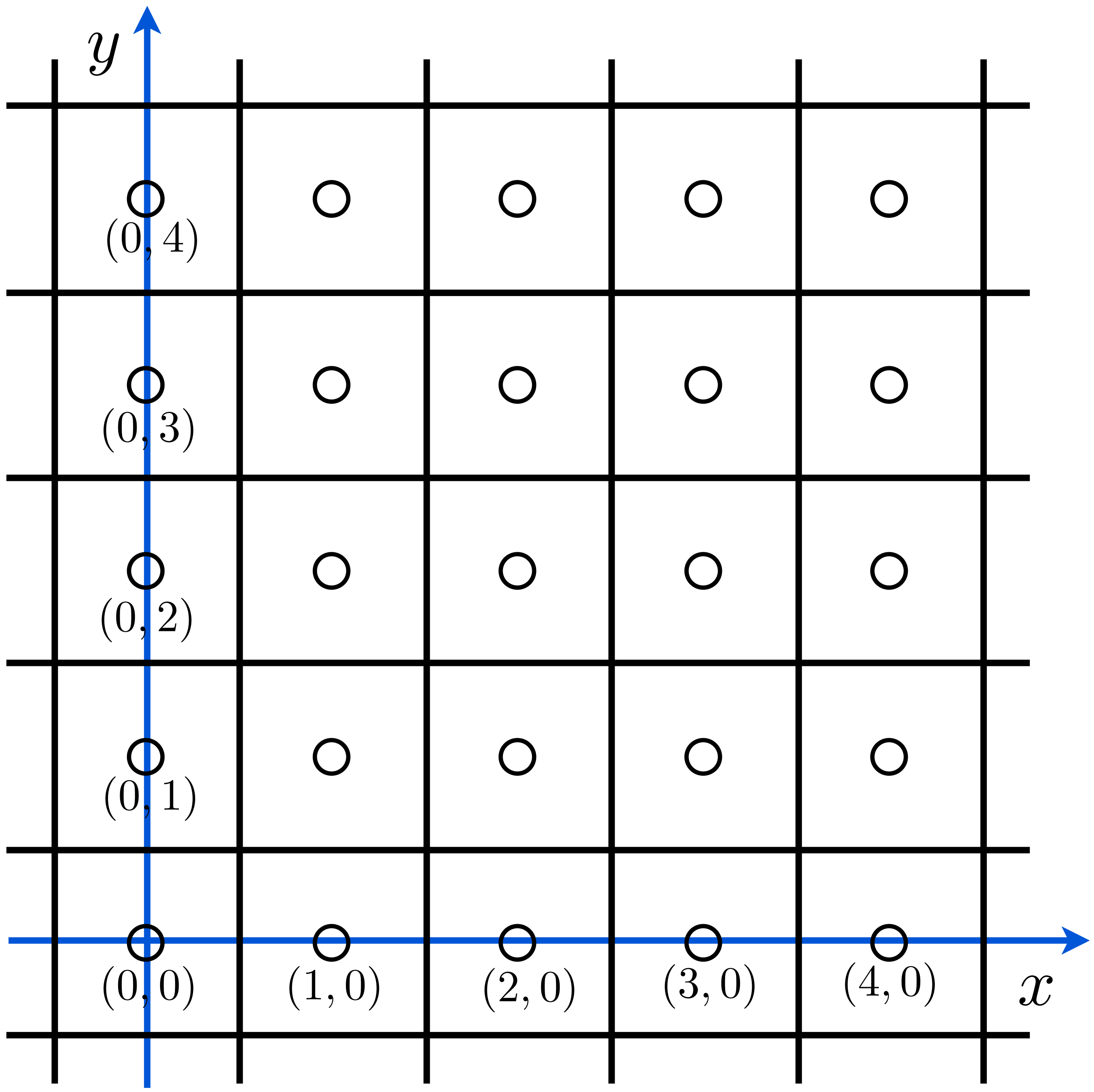
Compute integer coordinates for pixels near the 2D primitives

Algorithms are invoked many, many times and so must be efficient

Output should be visually pleasing, for example, lines should have constant density

Obviously, they should be able to draw all possible 2D primitives

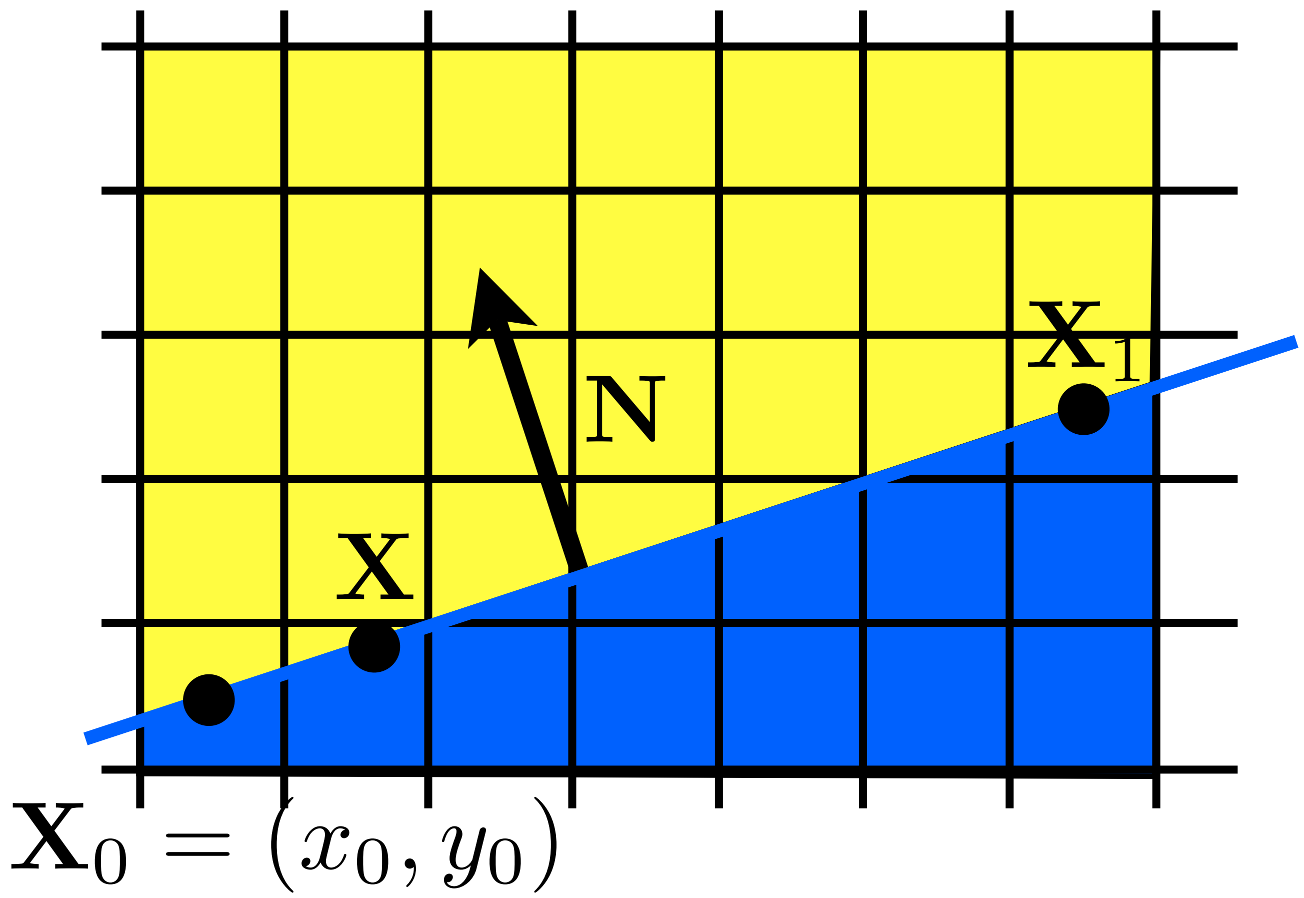
Screen coordinates



we’ll assume stuff has been converted to **normalized device coordinates**

Line Representation

Implicit Line Equation



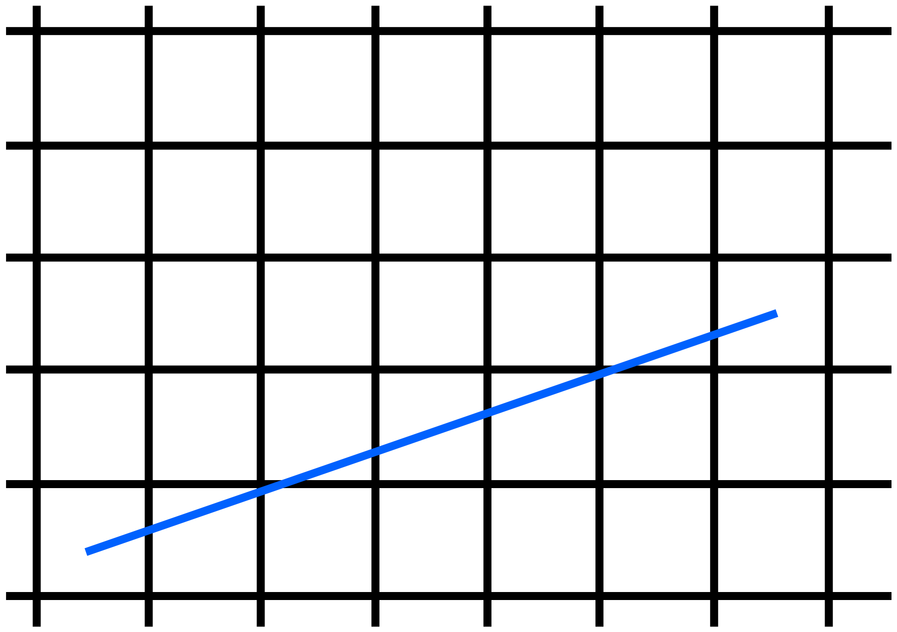
$$f(\mathbf{X}) = \mathbf{N} \cdot (\mathbf{X} - \mathbf{X}_0) = 0$$

<whiteboard>

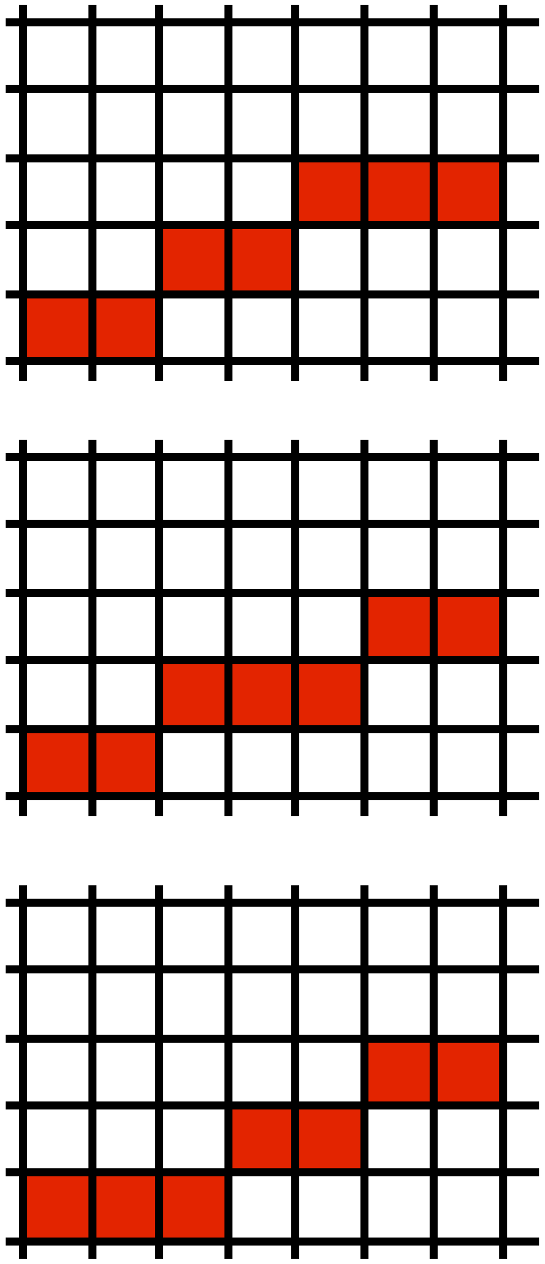
<whiteboard>: work out the implicit line equation in terms of X0 and X1

Line Drawing

Which pixels should be used to approximate a line?



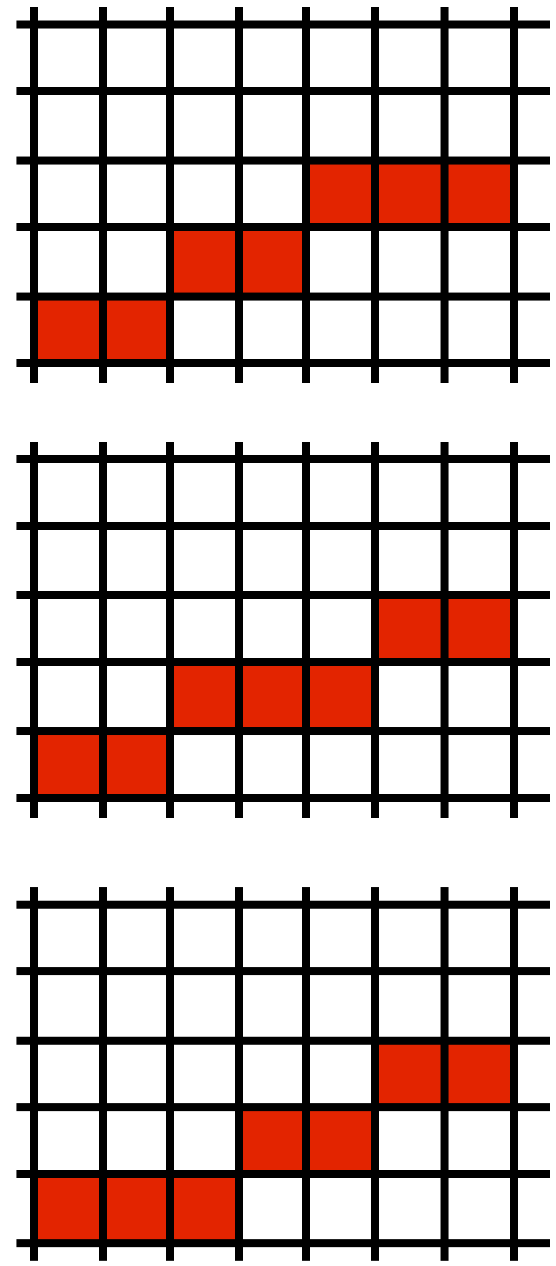
Draw the thinnest possible line that has no gaps



Line drawing algorithm

(case: 0 < m <= 1)

```
y = y0
for x = x0 to x1 do
    draw(x,y)
    if (<condition>) then
        y = y+1
```



- move from left to right
- choose between (x+1,y) and (x+1,y+1)

draw pixels from left to right, occasionally move up

Line drawing algorithm

(case: 0 < m <= 1)

```
y = y0
for x = x0 to x1 do
    draw(x,y)
    if (<condition>) then
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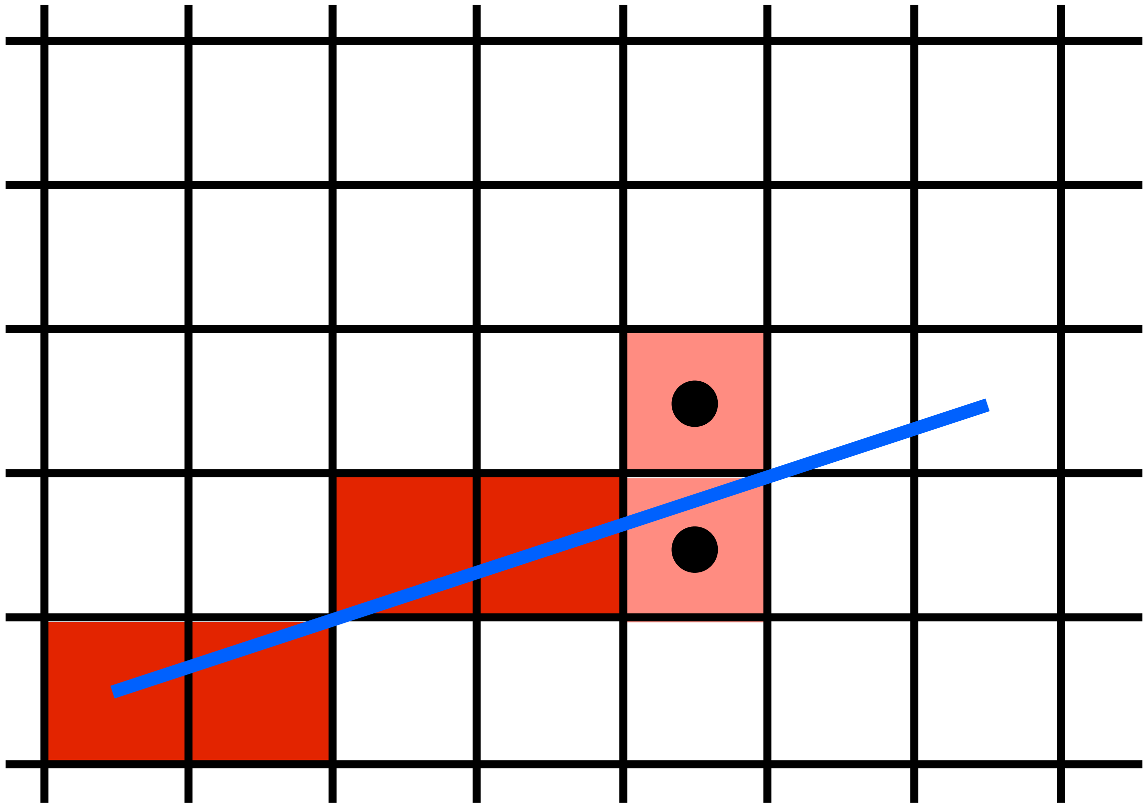
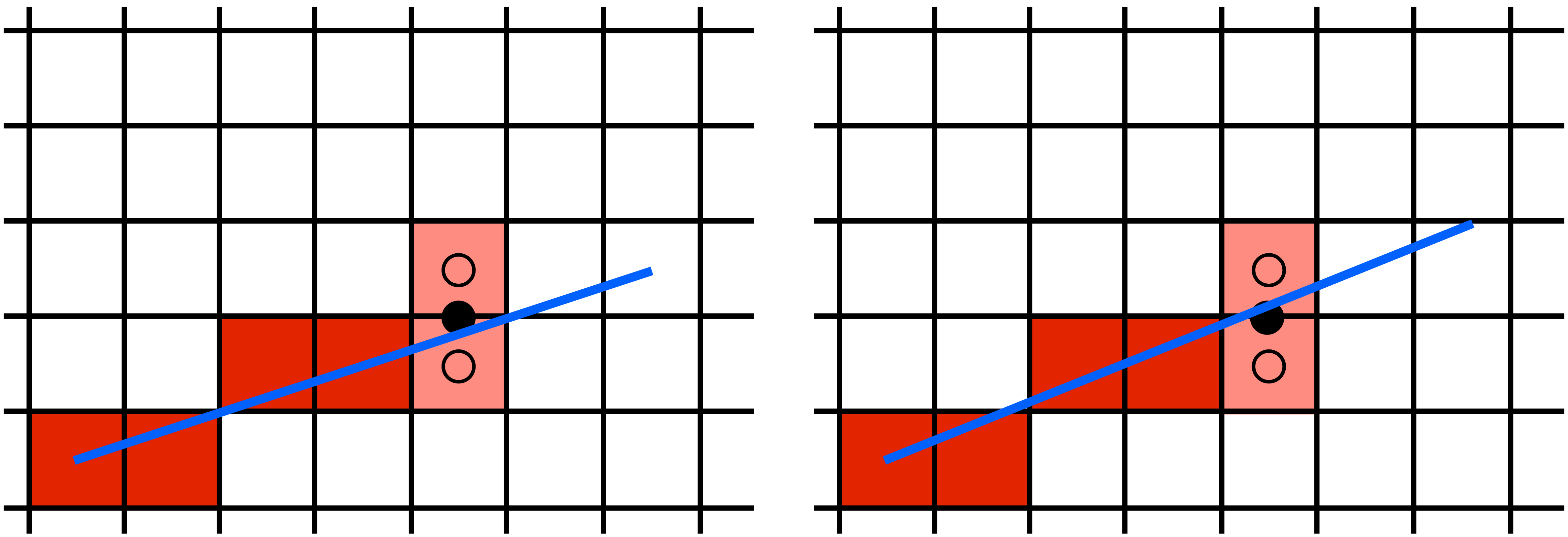


- move from left to right
- choose between (x+1,y) and (x+1,y+1)

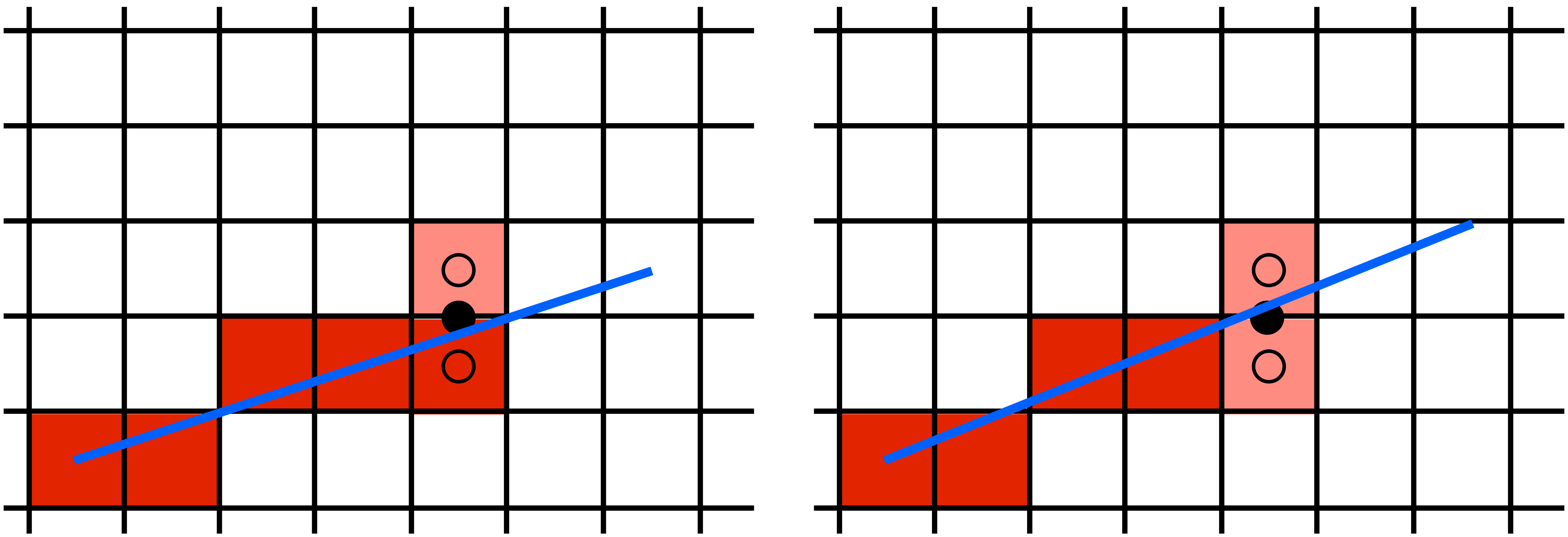
draw pixels from left to right, occasionally move up

Use the midpoint between the two pixels to choose



If the line falls **below** the midpoint, use the bottom pixel
if the line falls **above** the midpoint, use the top pixel

Use the midpoint between the two pixels to choose



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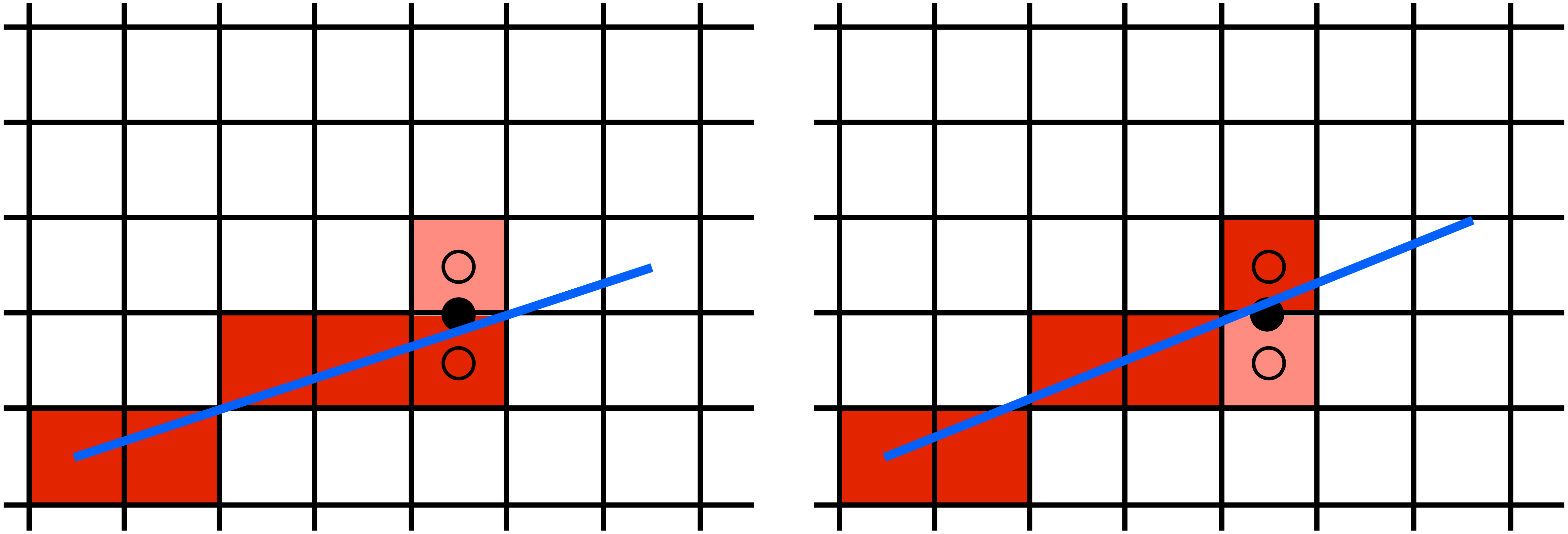

If the line falls **below** the midpoint, use the bottom pixel
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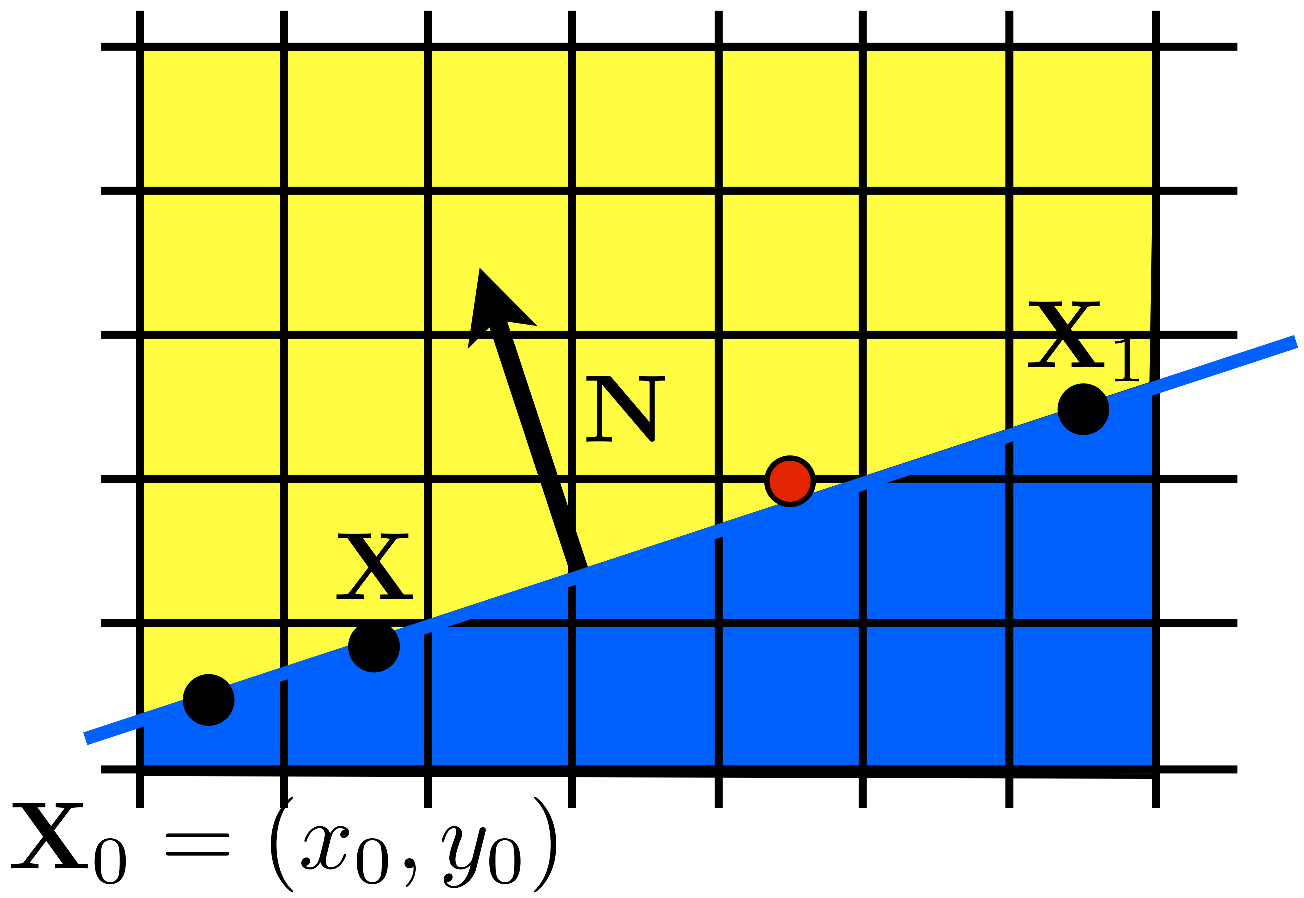
Use the midpoint between the two pixels to choose



implicit line equation:

$$f(\mathbf{X}) = \mathbf{N} \cdot (\mathbf{X} - \mathbf{X}_0) = 0$$

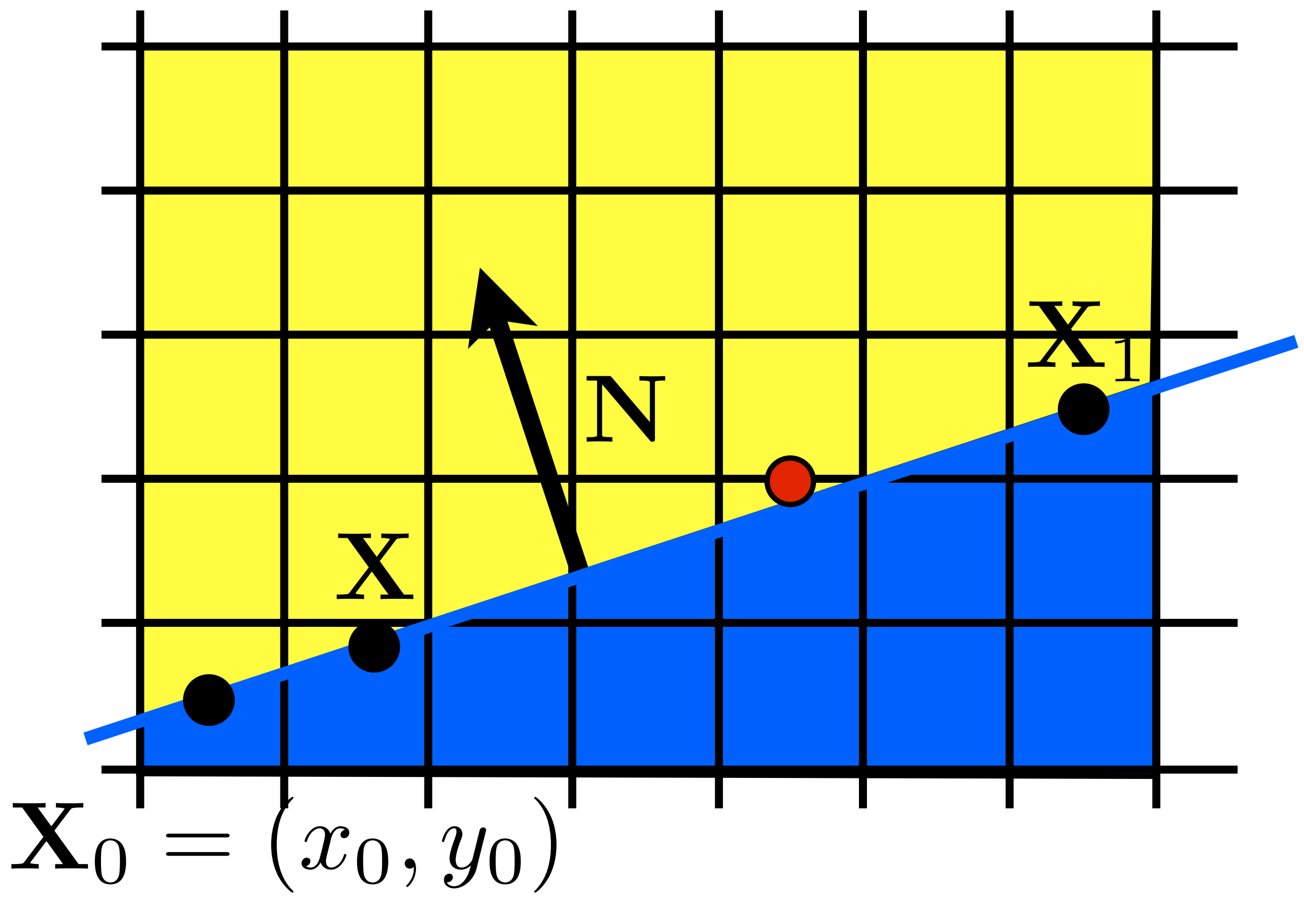
<whiteboard>

evaluate f at midpoint:

$$f(x, y + \frac{1}{2}) \; ? \; 0$$

<whiteboard>: work out the implicit line equation in terms of X0 and X1
Question: will f(x,y+1/2) be > 0 or < 0?

Use the midpoint between the two pixels to choose



implicit line equation:

$$f(\mathbf{X}) = \mathbf{N} \cdot (\mathbf{X} - \mathbf{X}_0) = 0$$

evaluate f at midpoint:

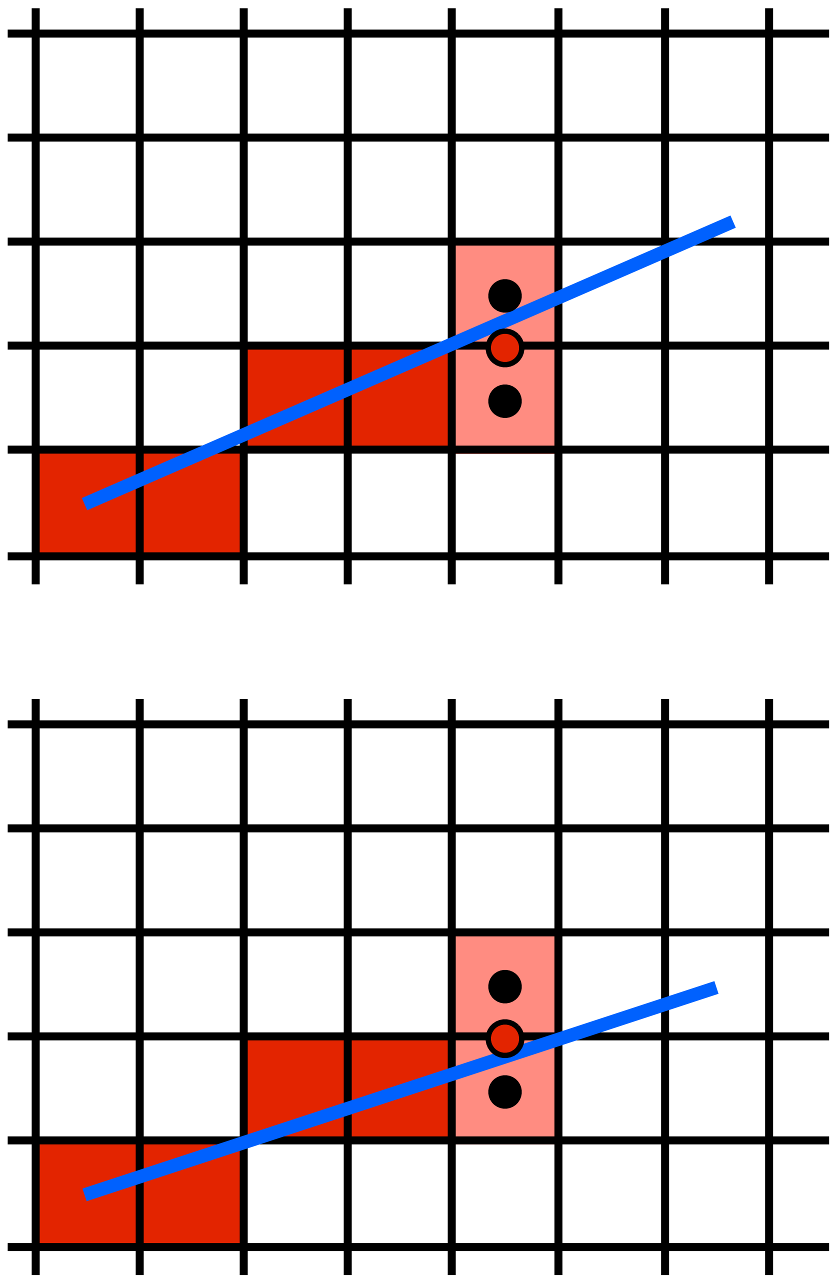
$$f(x, y + \frac{1}{2}) \ > \ 0$$

this means midpoint is above the line -> line is closer to bottom pixel

Line drawing algorithm

(case: 0 < m <= 1)

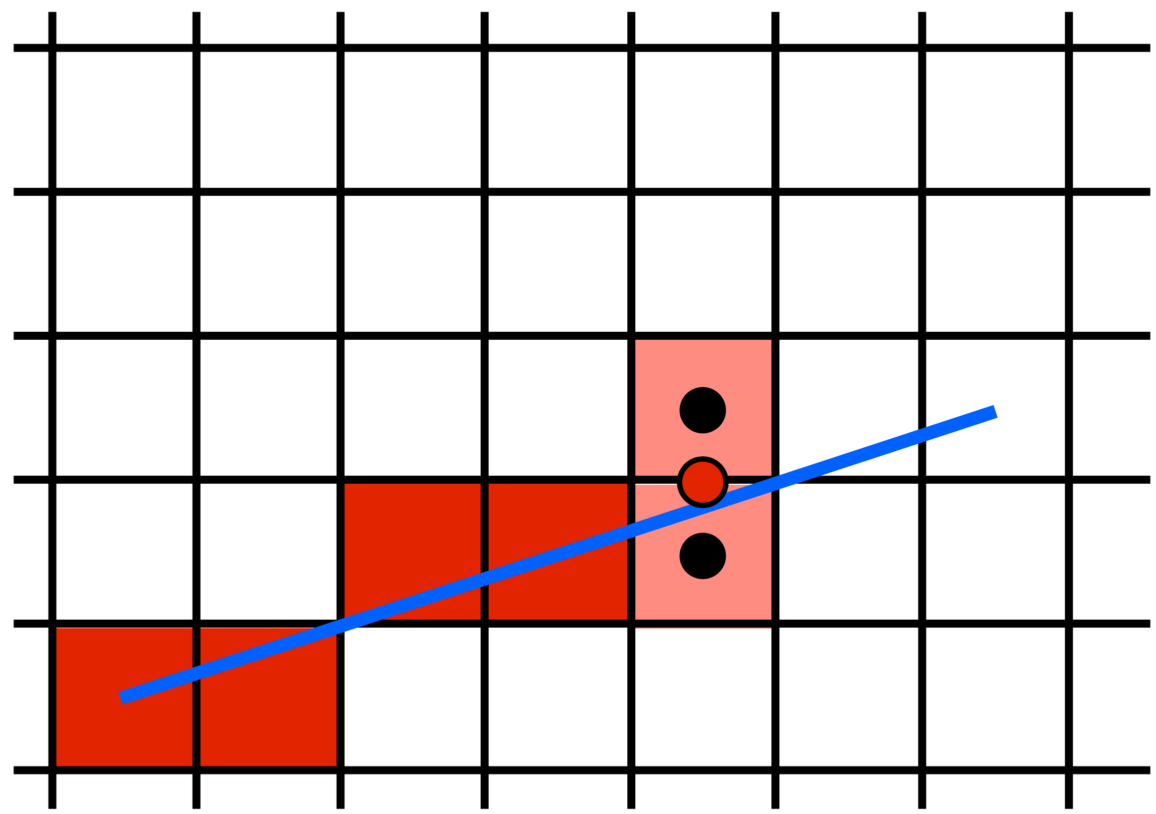
```
y = y0
for x = x0 to x1 do
    draw(x,y)
    if (f(x+1, y+1/2) < 0) then
        y = y+1
```



can now fill in the **condition**

We can make the Midpoint Algorithm more efficient

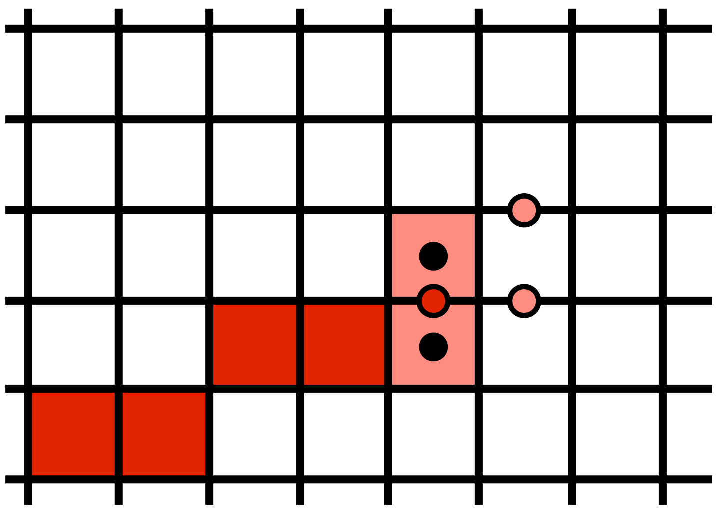
```
y = y0
for x = x0 to x1 do
    draw(x,y)
    if (f(x+1, y+1/2) < 0) then
        y = y+1
```



in each iteration we draw the **current** pixel and we evaluate the line equation at the **next** midpoint halfway above the **current** pixel

We can make the Midpoint Algorithm more efficient

by making it incremental!



$$f(x,y) = (y_0 - y_1)x + (x_1 - x_0)y + x_0y_1 - x_1y_0 = 0$$

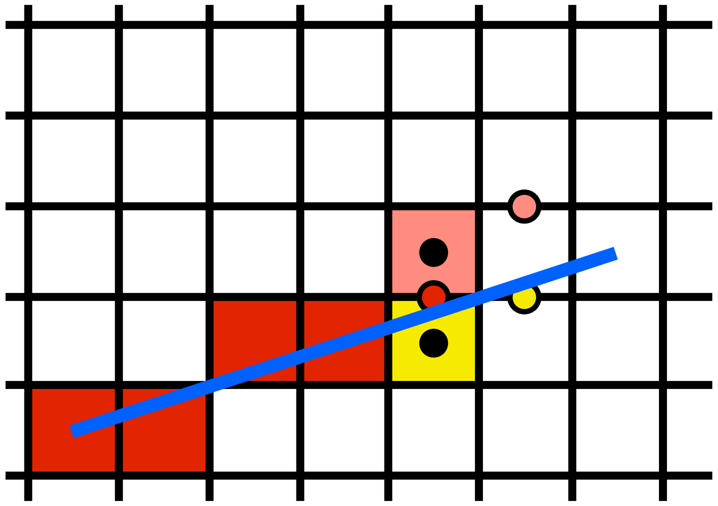
$$f(x+1,y) = f(x,y) + (y_0 - y_1)$$

$$f(x+1,y+1) = f(x,y) + (y_0 - y_1) + (x_1 - x_0)$$

Assume we have drawn the last red pixel and evaluated the line equation at the next (Red) midpoint
There are two possible outcomes:
1. we will choose the bottom pixel. In this case the next midpoint will be at the same level (x+1,y)
2. we will choose the top pixel. In this case the next midpoint will be one level up (x+1, y+1)
The line equation at these next midpoints can be evaluated incrementally using the update formulas shown.

We can make the Midpoint Algorithm more efficient

$$f(x+1, y+\frac{1}{2}) > 0$$



$$f(x,y) = (y_0 - y_1)x + (x_1 - x_0)y + x_0y_1 - x_1y_0 = 0$$

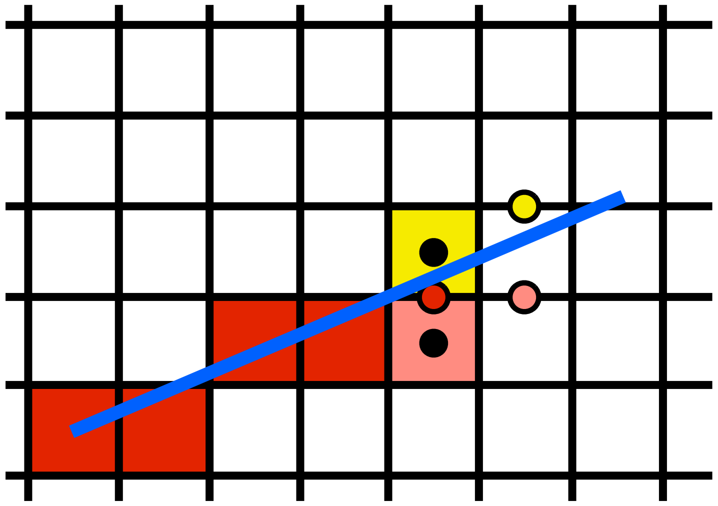
$$f(x+1, y) = f(x,y) + (y_0 - y_1)$$

$$f(x+1, y+1) = f(x,y) + (y_0 - y_1) + (x_1 - x_0)$$

As we move over one pixel to the right, we will choose either (x+1,y) (yellow) or (x+1,y+1) (green) and the next midpoint we will evaluate will be eiterh

We can make the Midpoint Algorithm more efficient

$$f(x+1, y+\frac{1}{2}) < 0$$



$$f(x,y) = (y_0 - y_1)x + (x_1 - x_0)y + x_0 y_1 - x_1 y_0 = 0$$

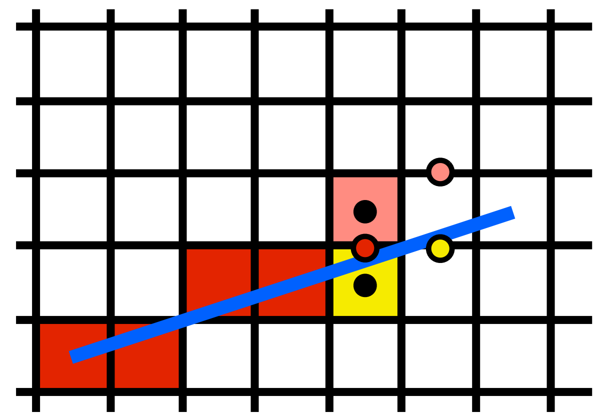
$$f(x+1, y) = f(x,y) + (y_0 - y_1)$$

$$f(x+1, y+1) = f(x,y) + (y_0 - y_1) + (x_1 - x_0)$$

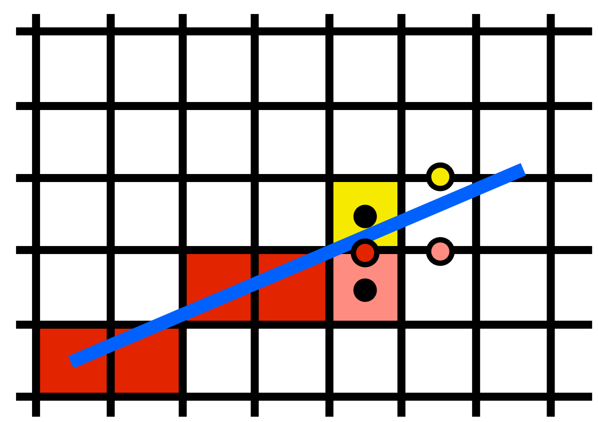
As we move over one pixel to the right, we will choose either (x+1,y) (yellow) or (x+1,y+1) (green) and the next midpoint we will evaluate will be eiterh

We can make the Midpoint Algorithm more efficient

```
y = y0
d = f(x0+1,y0+1/2)
for x = x0 to x1 do
    draw(x,y)
    if (d<0) then
        y = y+1
        d = d+(y0-y1)+(x1-x0)
    else
        d = d+(y0-y1)
```



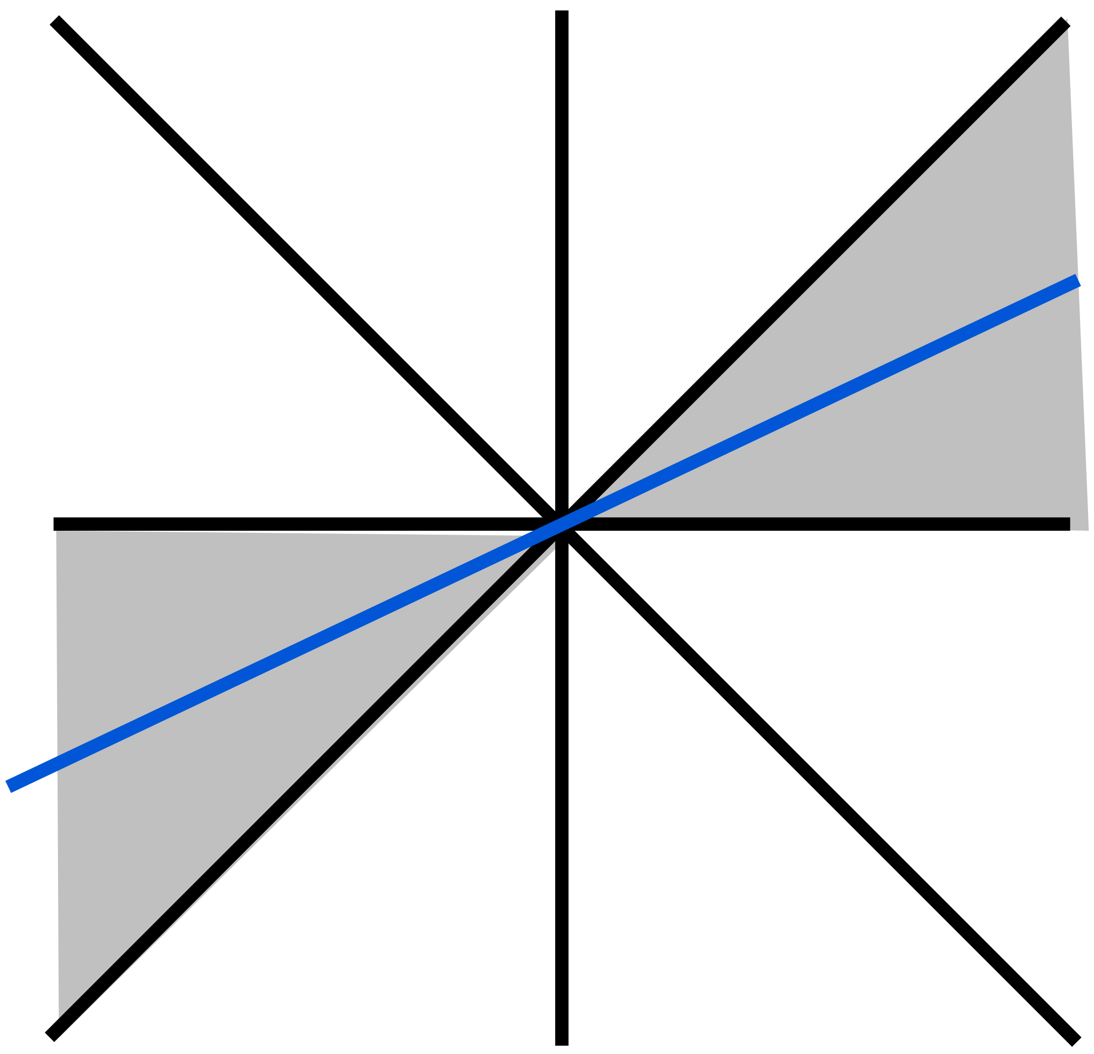
$$f(x+1,y) = f(x,y) + (y_0 - y_1)$$



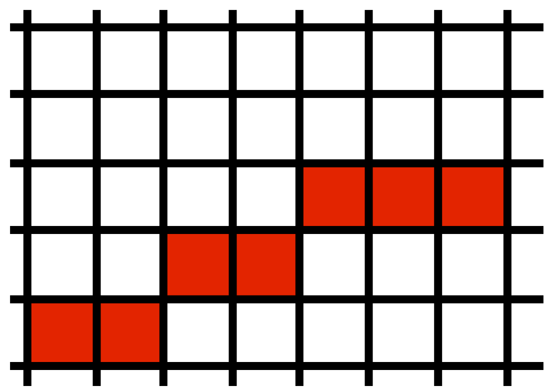
$$f(x+1,y+1) = f(x,y) + (y_0 - y_1) + (x_1 - x_0)$$

algorithm is **incremental** and uses only **integer arithmetic**

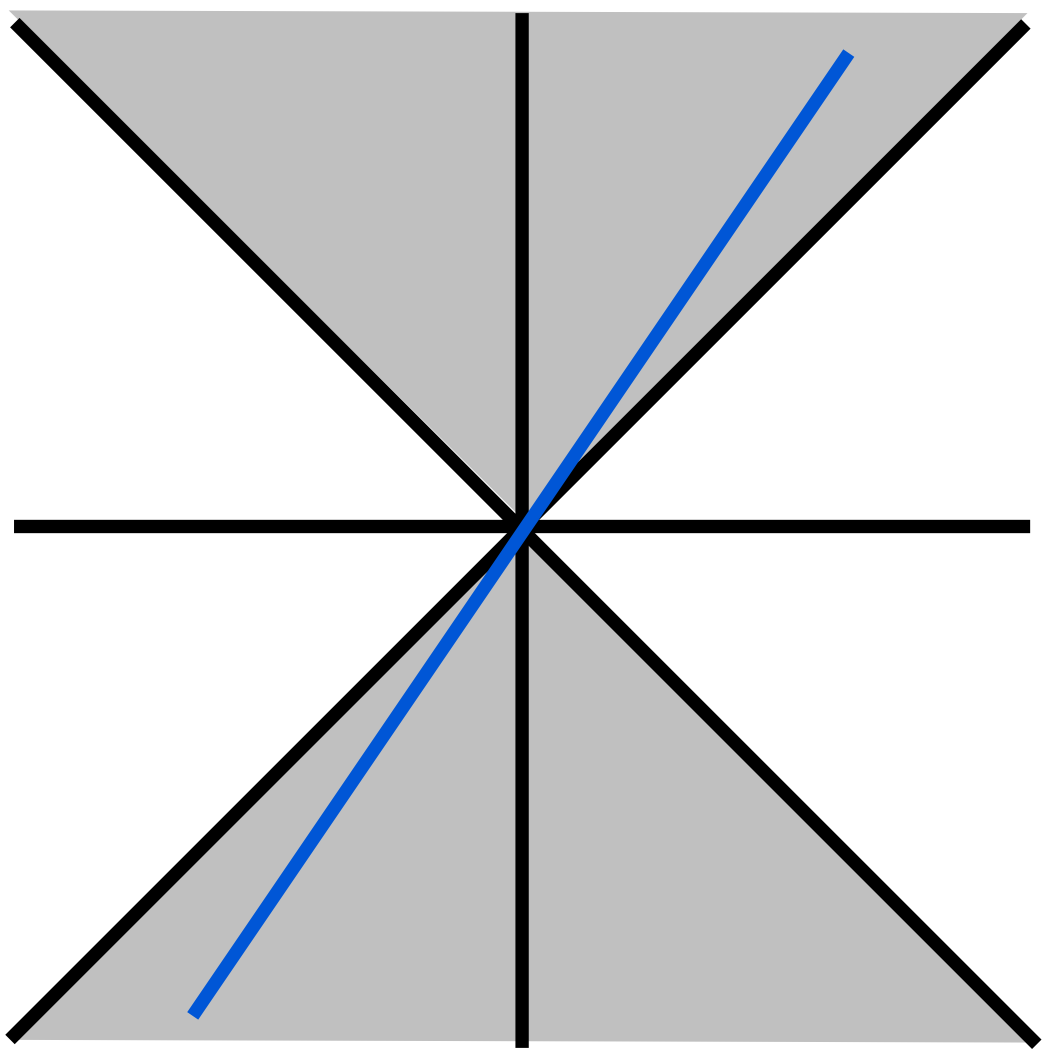
Adapt Midpoint Algorithm for other cases



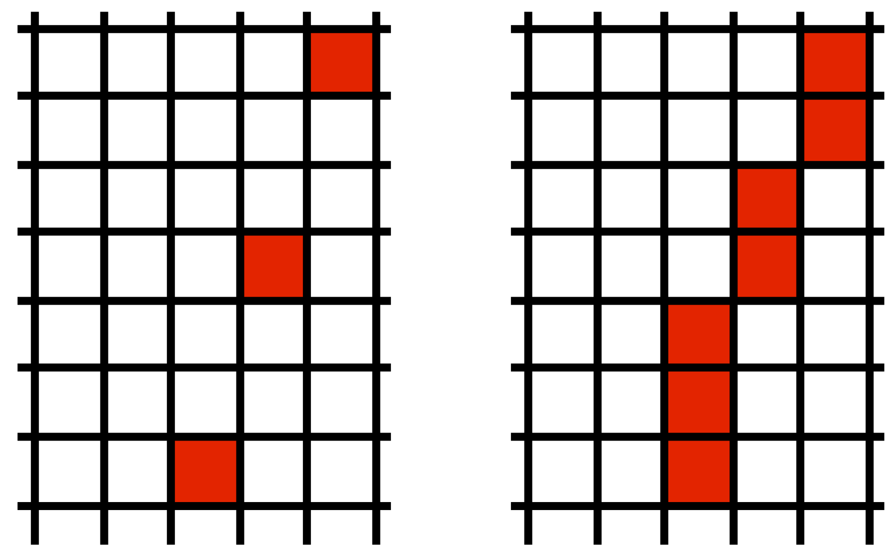
case: 0 < m <= 1



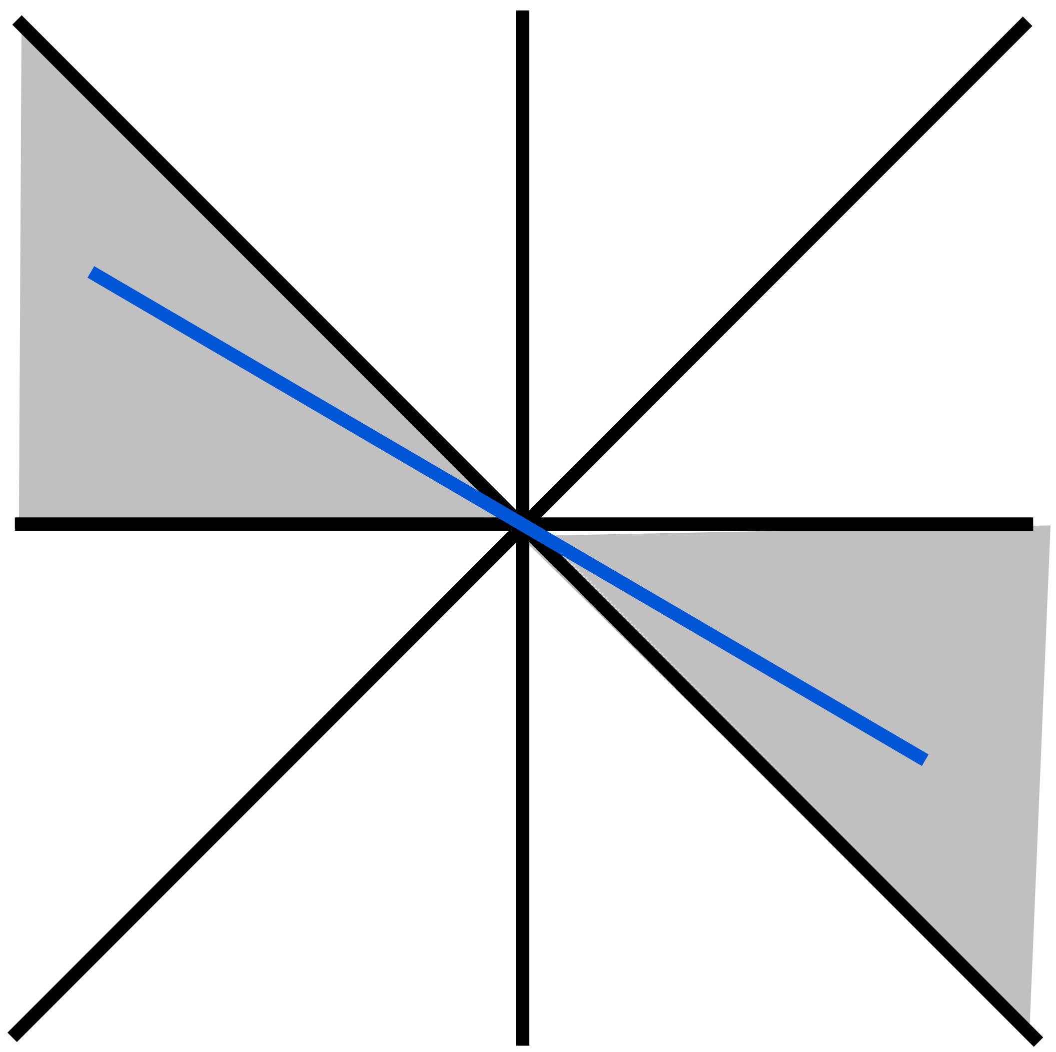
Adapt Midpoint Algorithm for other cases



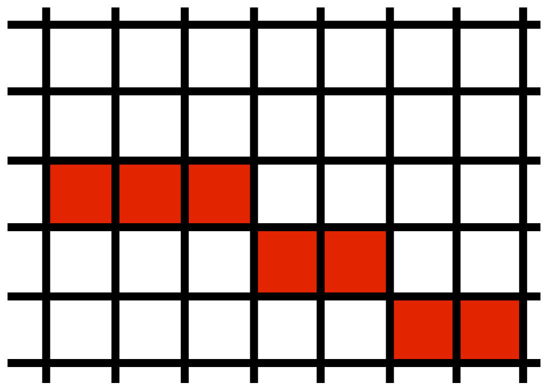
case: 1 <= m



Adapt Midpoint Algorithm for other cases



case: -1 <= m < 0



Line drawing references

- the algorithm we just described is the *Midpoint Algorithm* (Pitteway, 1967), (van Aken and Novak, 1985)
- draws the same lines as the *Bresenham Line Algorithm* (Bresenham, 1965)