CS130 : Computer Graphics

Lecture 24: Curves (cont.)

Tamar Shinar
Computer Science & Engineering
UC Riverside

Blending functions are more convenient basis than monomial basis

- monomial basis

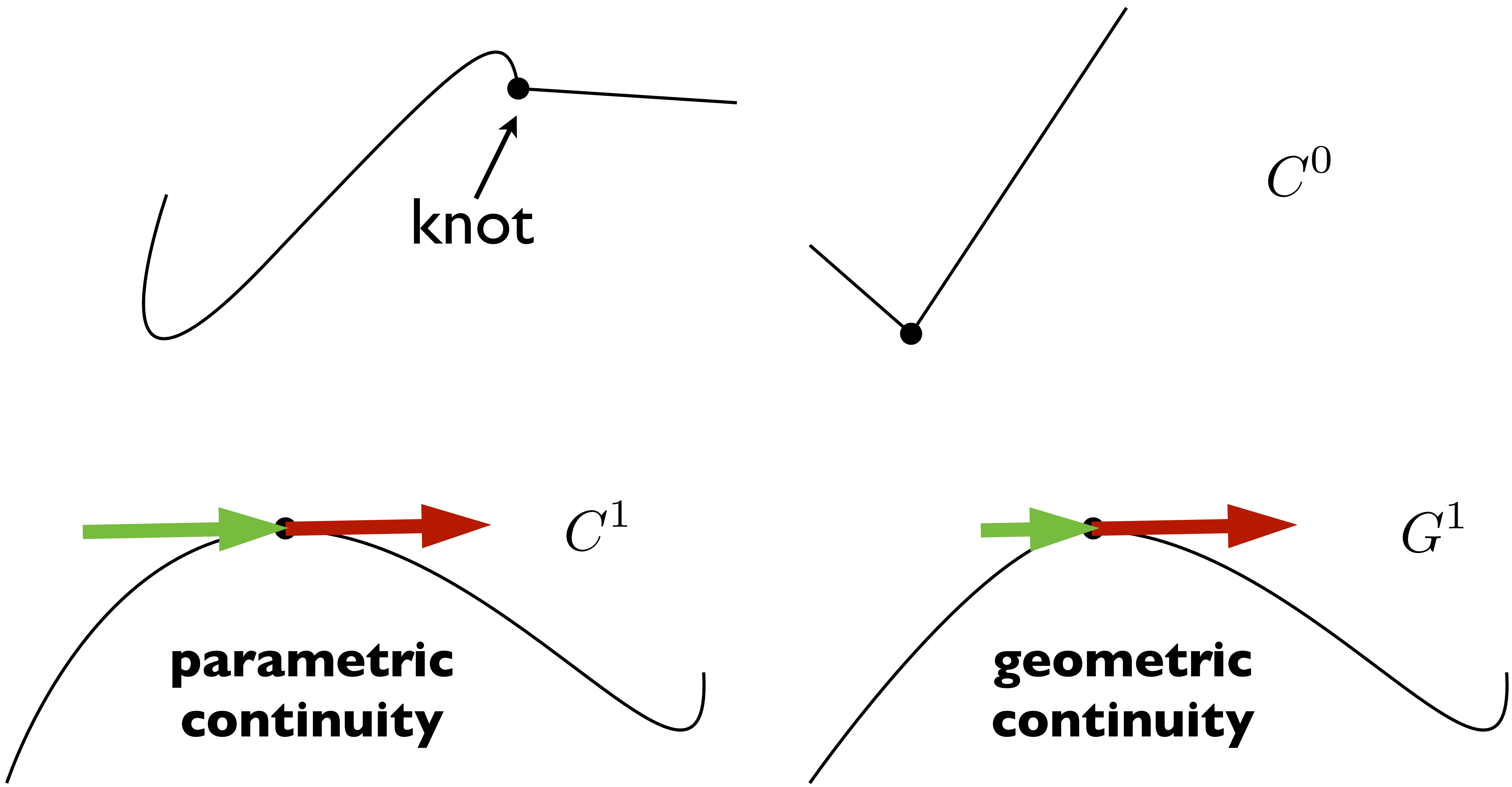
$$\mathbf{f}(u) = \mathbf{a}_0 + \mathbf{a}_1 u + \mathbf{a}_2 u^2 + \mathbf{a}_3 u^3$$

- blending functions

$$\mathbf{f}(u) = b_0(u)\mathbf{p}_0 + b_1(u)\mathbf{p}_1 + b_2(u)\mathbf{p}_2 + b_3(u)\mathbf{p}_3$$

– geometric form is more intuitive because it combines control points with blending functions

Stitching curve segments together: **continuity**



Top

C0: the curves are continuous, but have discontinuous first derivatives

Bottom

Left: At the knot, the curve has C1 continuity: the curve segments have common point and first derivative

Right: At the knot, the curve has G1 continuity: the curve segments have a common point, and parallel first derivatives of different magnitude

Cubics

$$\mathbf{f}(u) = \mathbf{a}_0 + \mathbf{a}_1 u + \mathbf{a}_2 u^2 + \mathbf{a}_3 u^3$$

- Allow up to C^2 continuity at knots
- Symmetry: specify position and derivative at the beginning and end
- good smoothness and computational properties

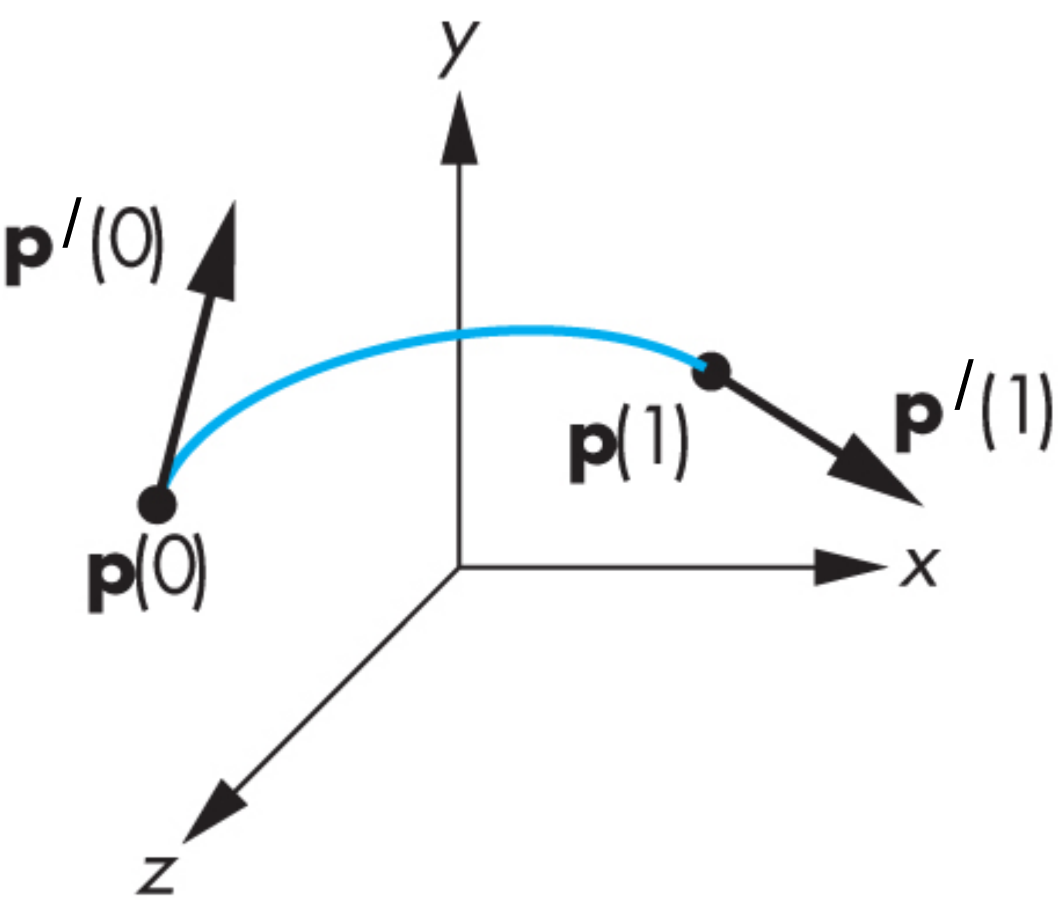
need 4 control points: might be 4 points on the curve, combination of points and derivatives, ...

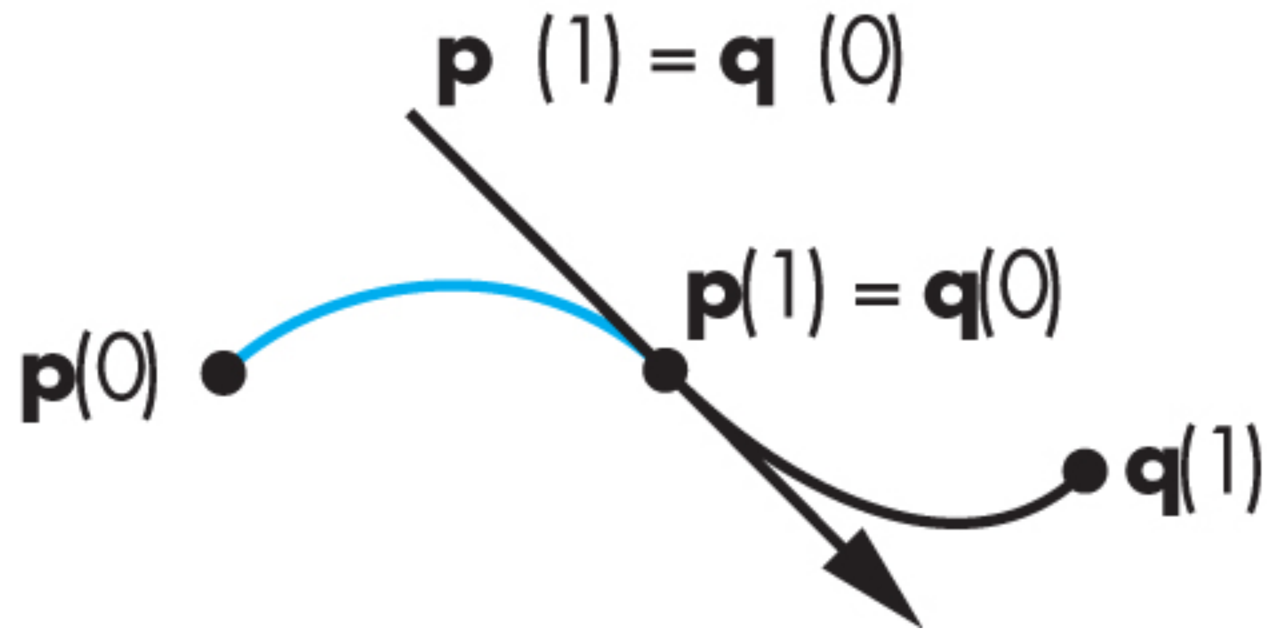
Cubic Hermite Curves

Cubic Hermite Curves

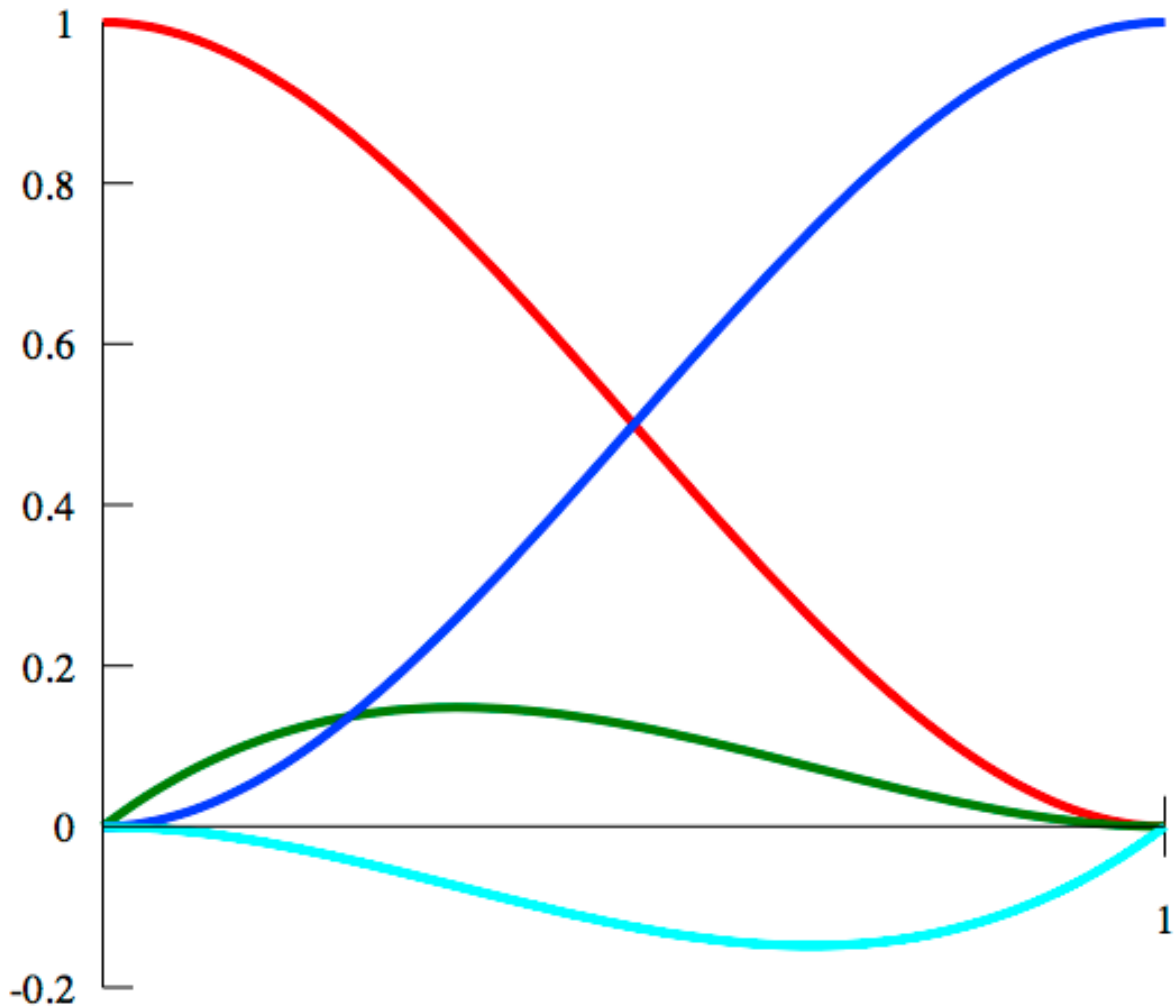
Specify endpoints and derivatives

construct curve with C^1 continuity





Hermite blending functions



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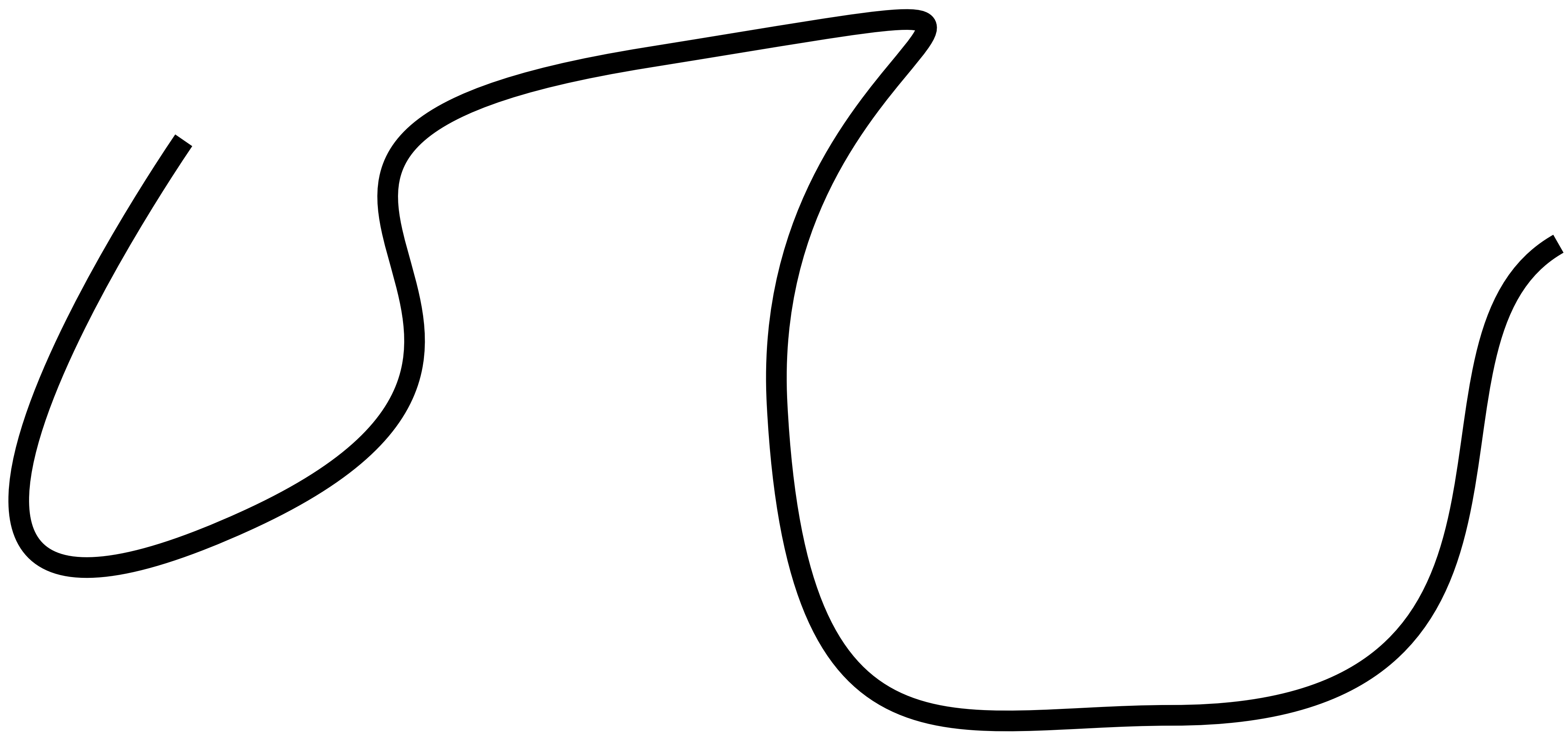
$$b_0(u) = 2u^3 - 3u^2 + 1$$

$$b_1(u) = -2u^3 + 3u^2$$

$$b_2(u) = u^3 - 2u^2 + u$$

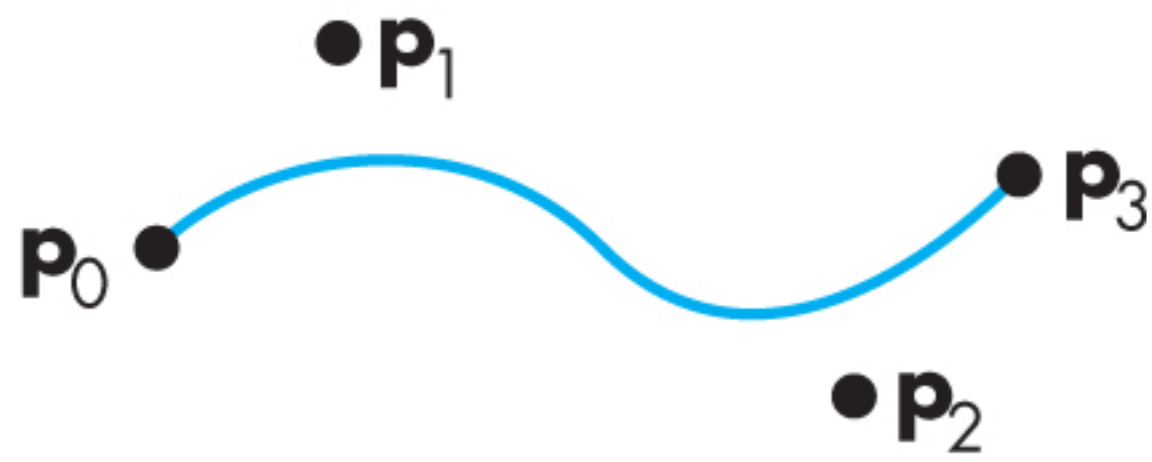
$$b_3(u) = u^3 - u^2$$

Example: keynote curve tool



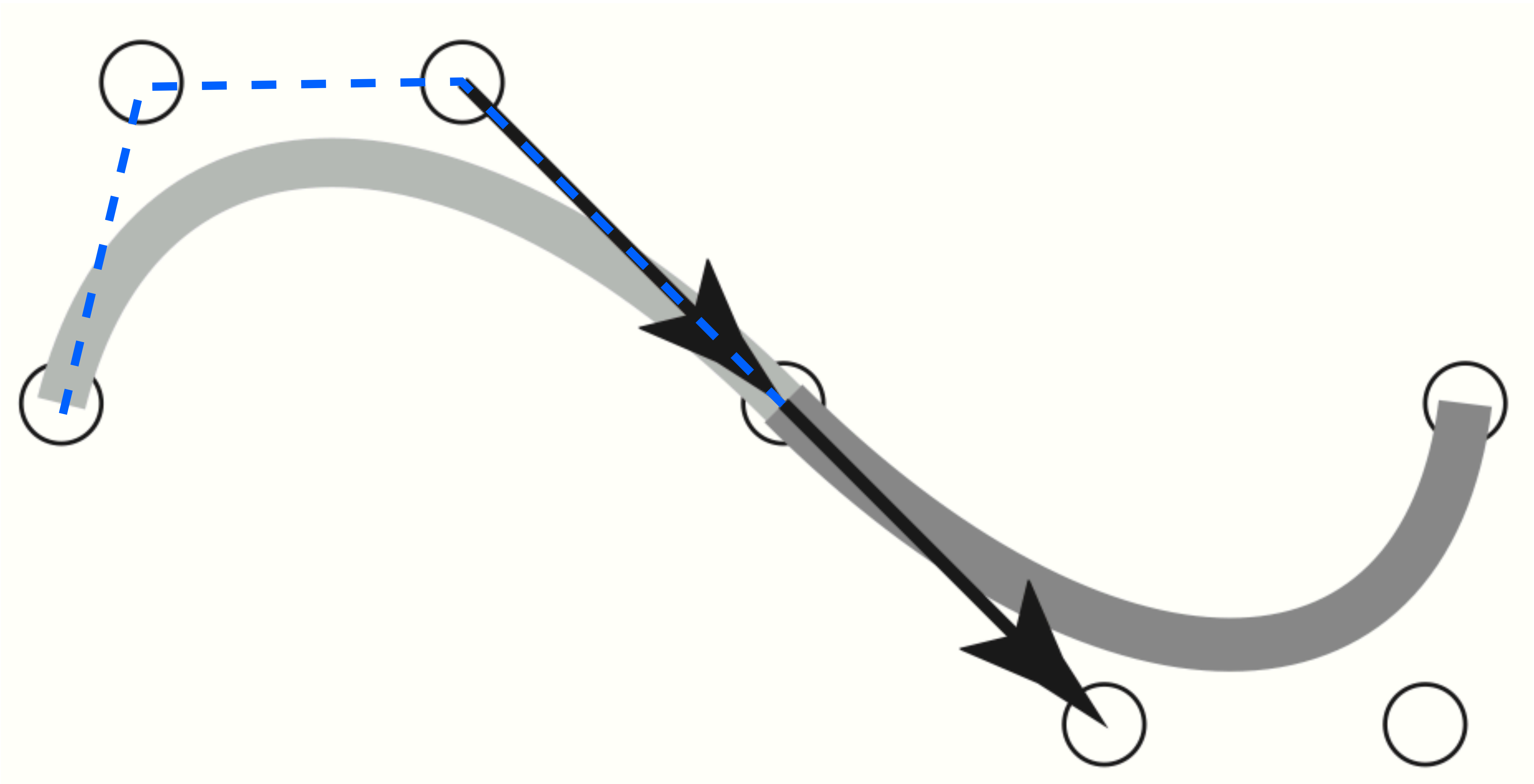
Control points

Interpolating

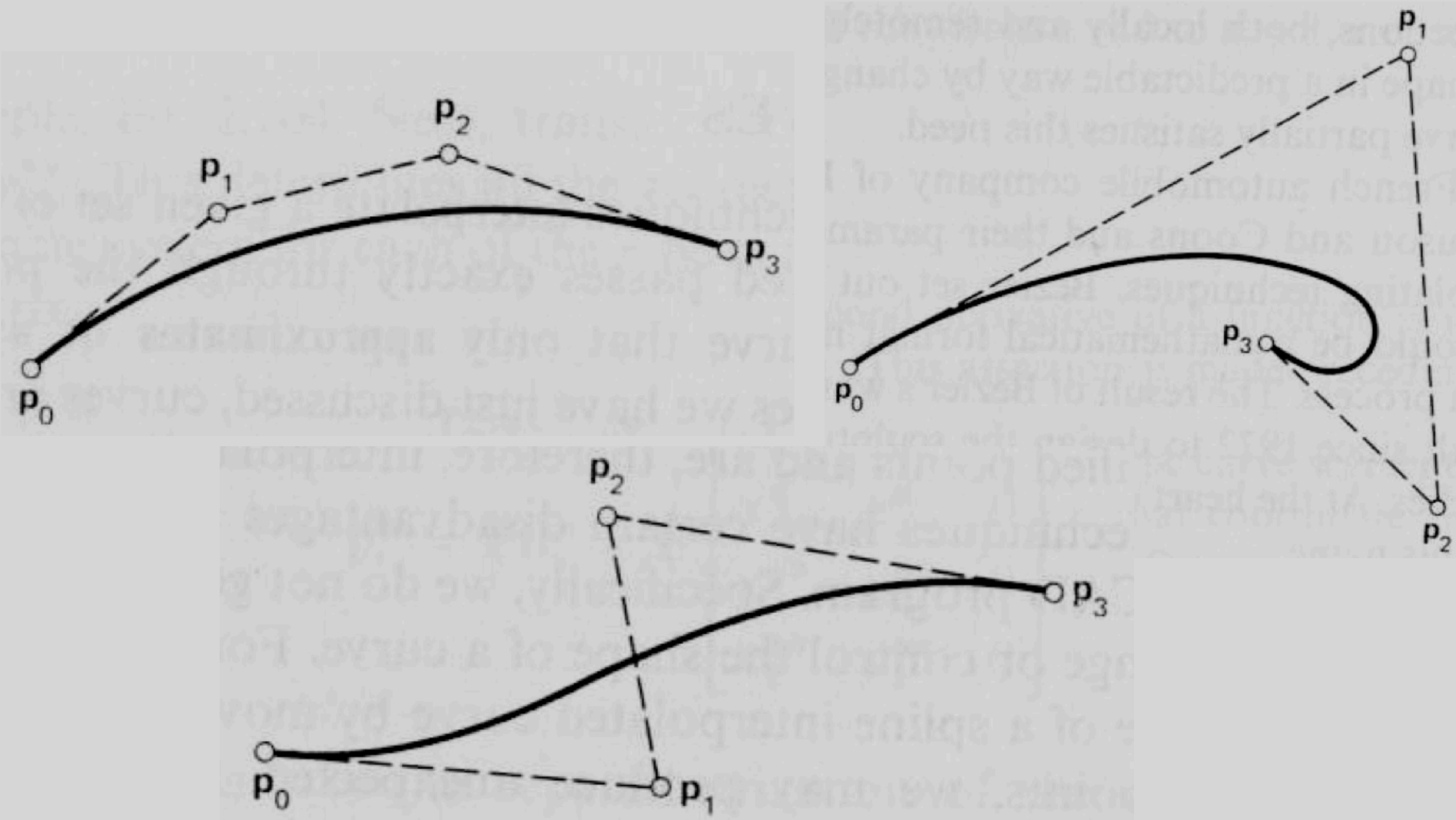


Approximating
(non-interpolating)

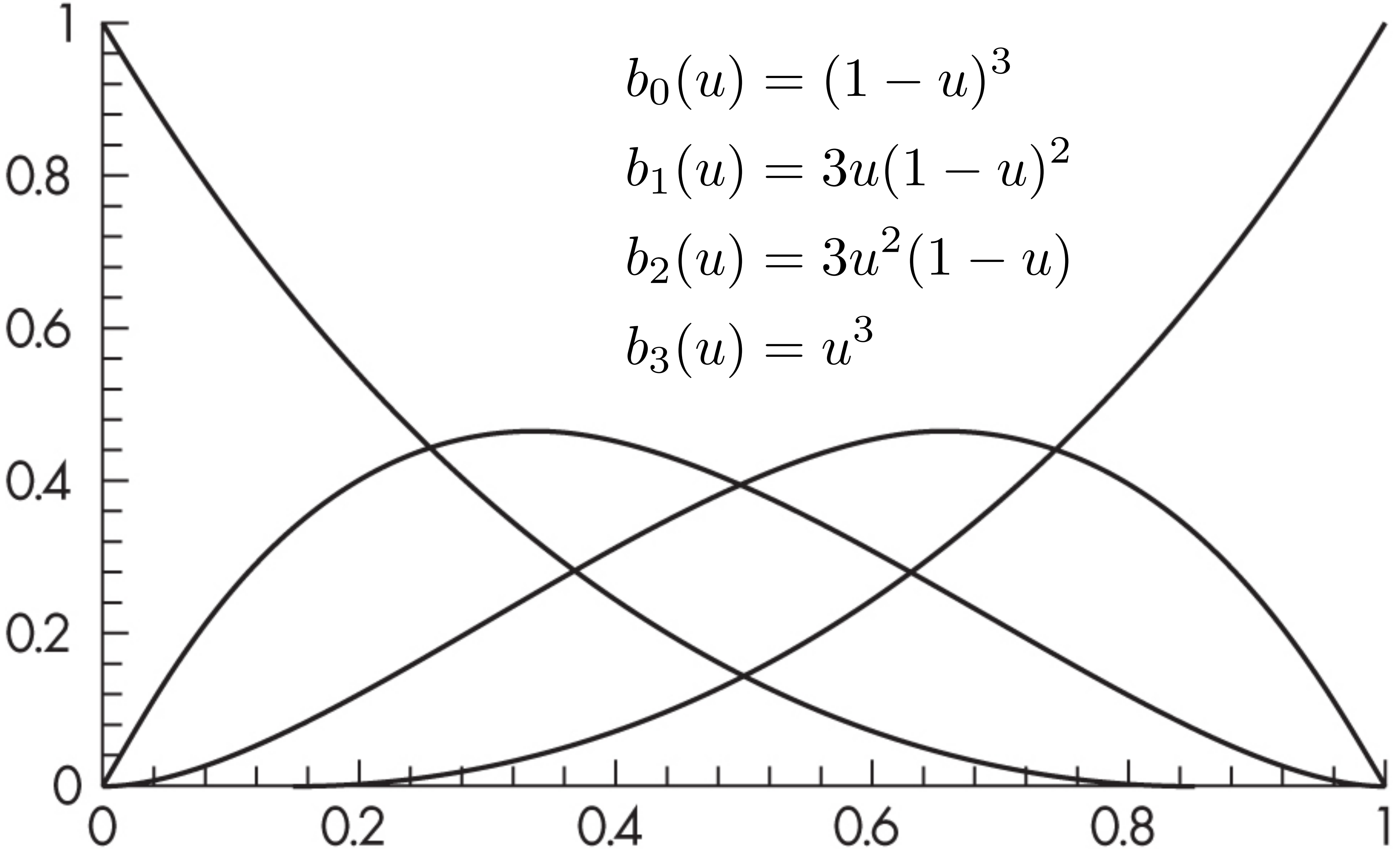
Cubic Bezier Curves



Bezier Curve Examples



Bezier blending functions



$$b_0(u) = (1-u)^3$$
$$b_1(u) = 3u(1-u)^2$$
$$b_2(u) = 3u^2(1-u)$$
$$b_3(u) = u^3$$

Bernstein Polynomials

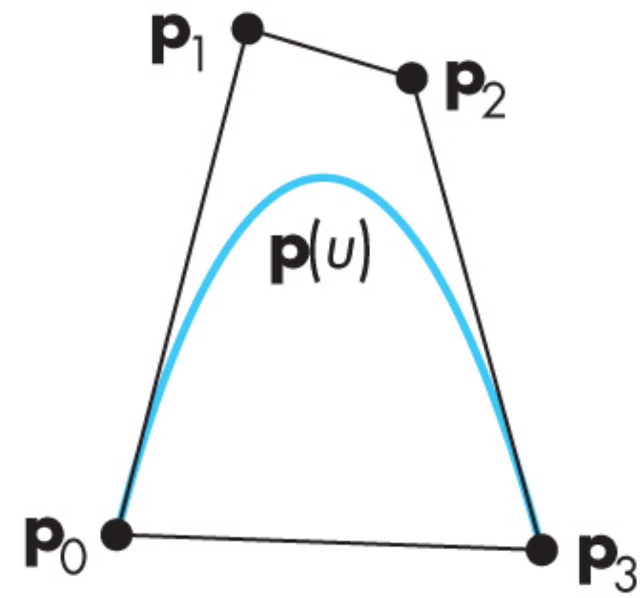
- The blending functions are a special case of the Bernstein polynomials

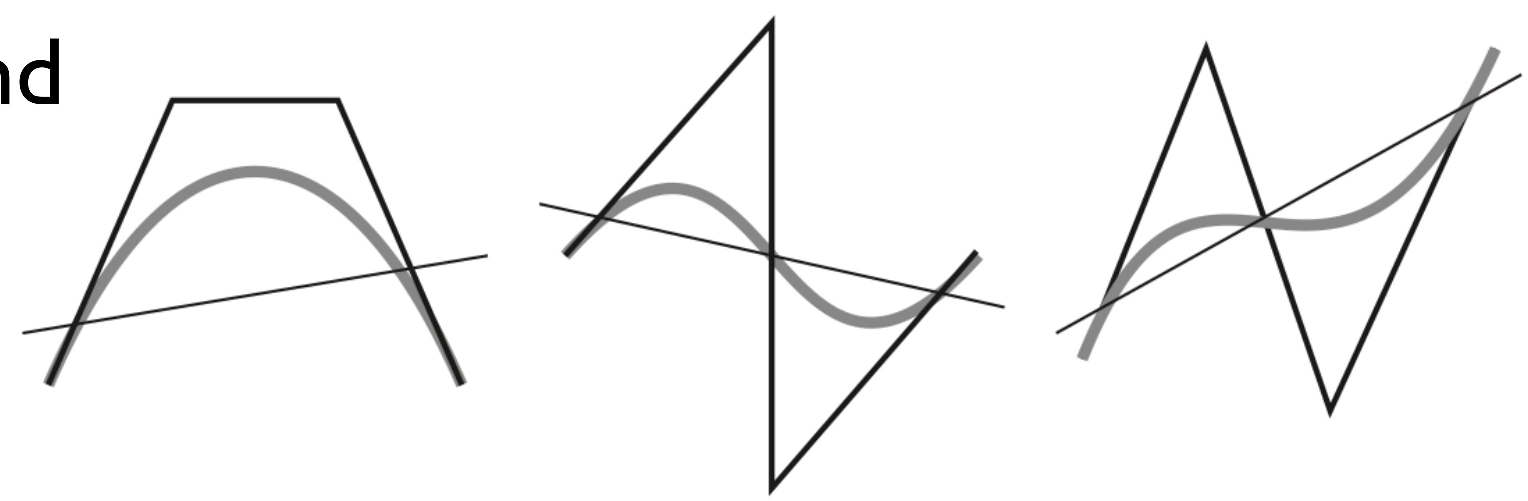
$$b_{\mathrm{kd}}(u) = \frac{d!}{k!(d-k)!} u^k (1-u)^{d-k}$$

- These polynomials give the blending polynomials for any degree Bezier form
  - All roots at 0 and 1
  - For any degree they all sum to 1
  - They are all between 0 and 1 inside (0,1)

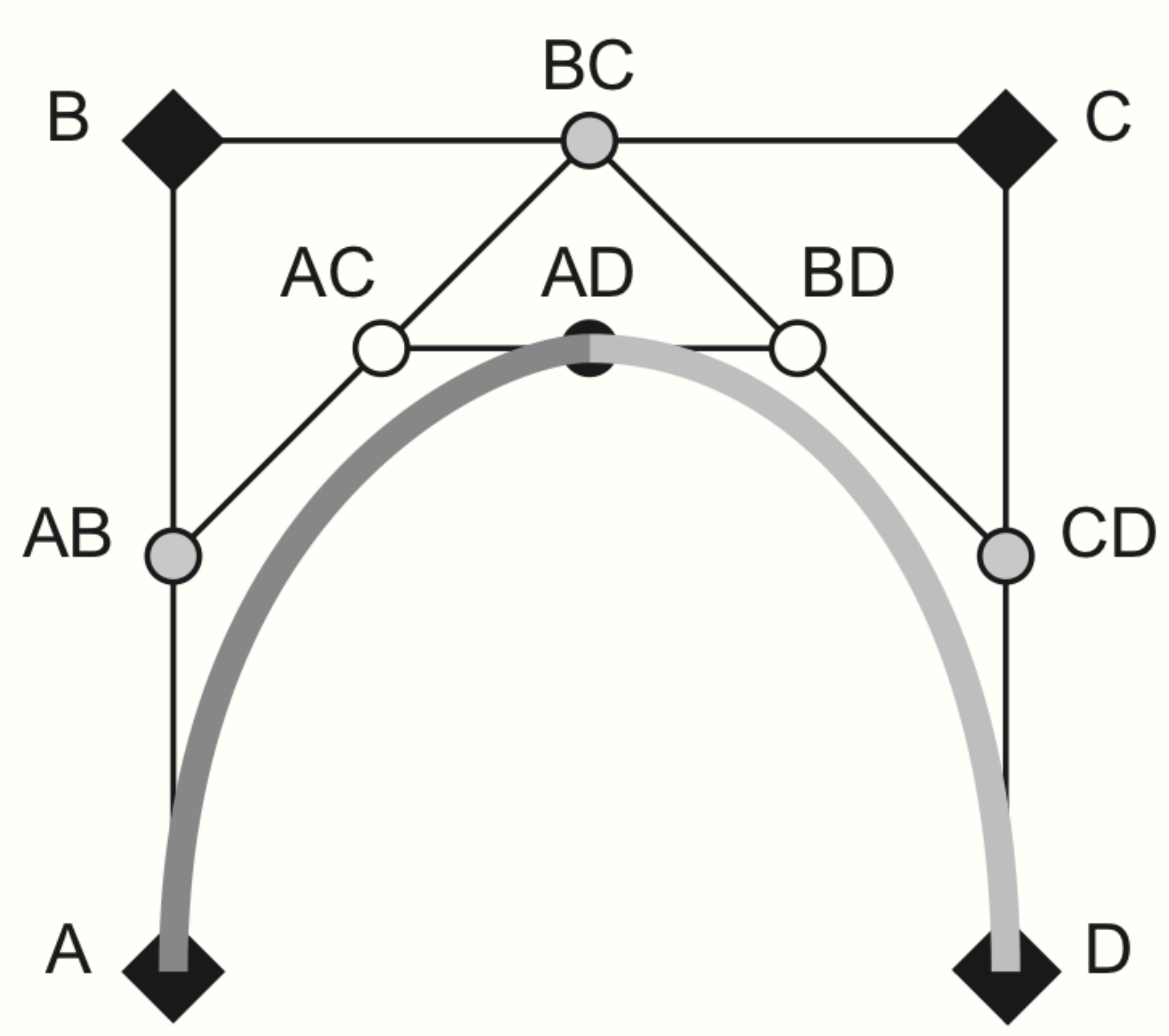
Bezier Curve Properties

- curve lies in the convex hull of the data
- variation diminishing
- symmetry
- affine invariant
- efficient evaluation and subdivision

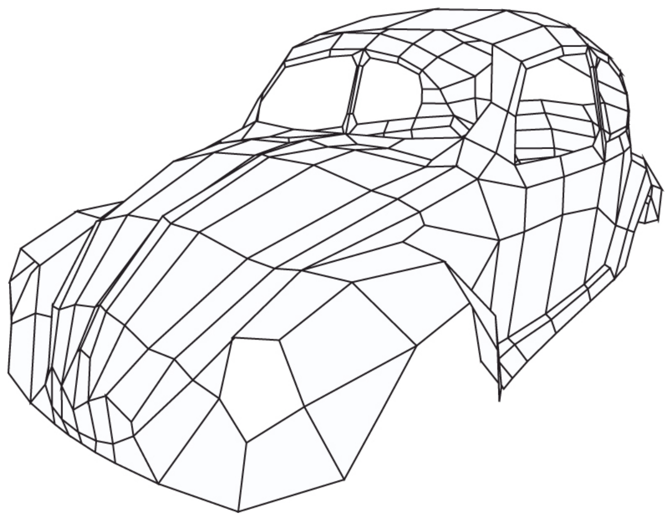


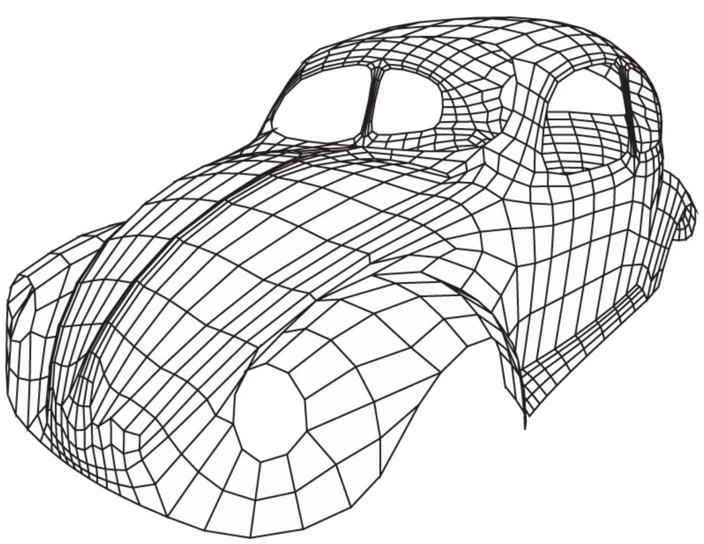


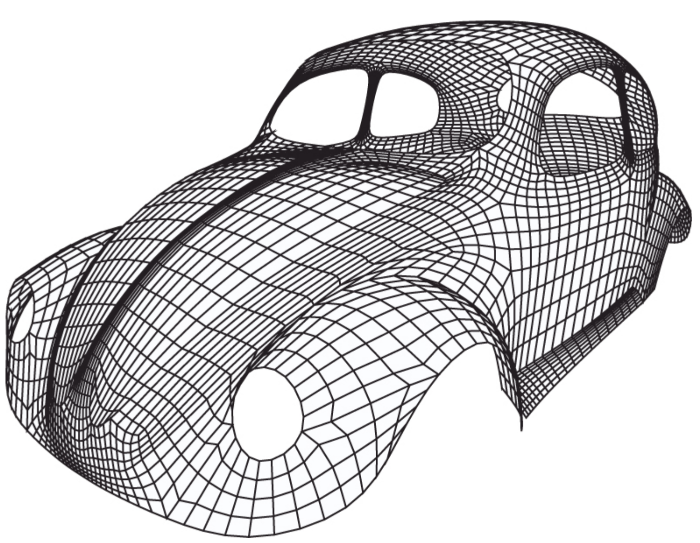
Bezier subdivision



Recursive Subdivision



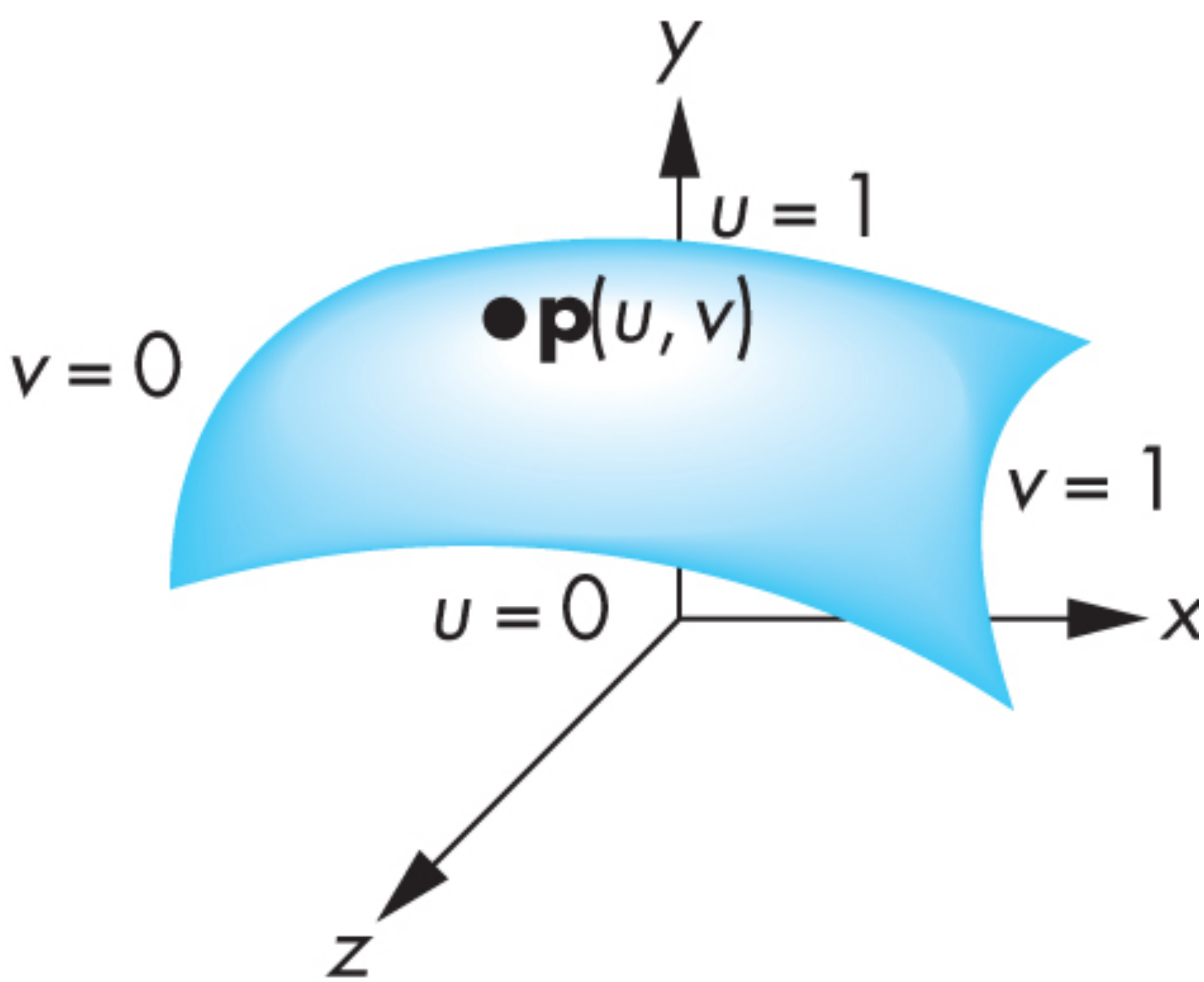




Surfaces

Parametric Surface

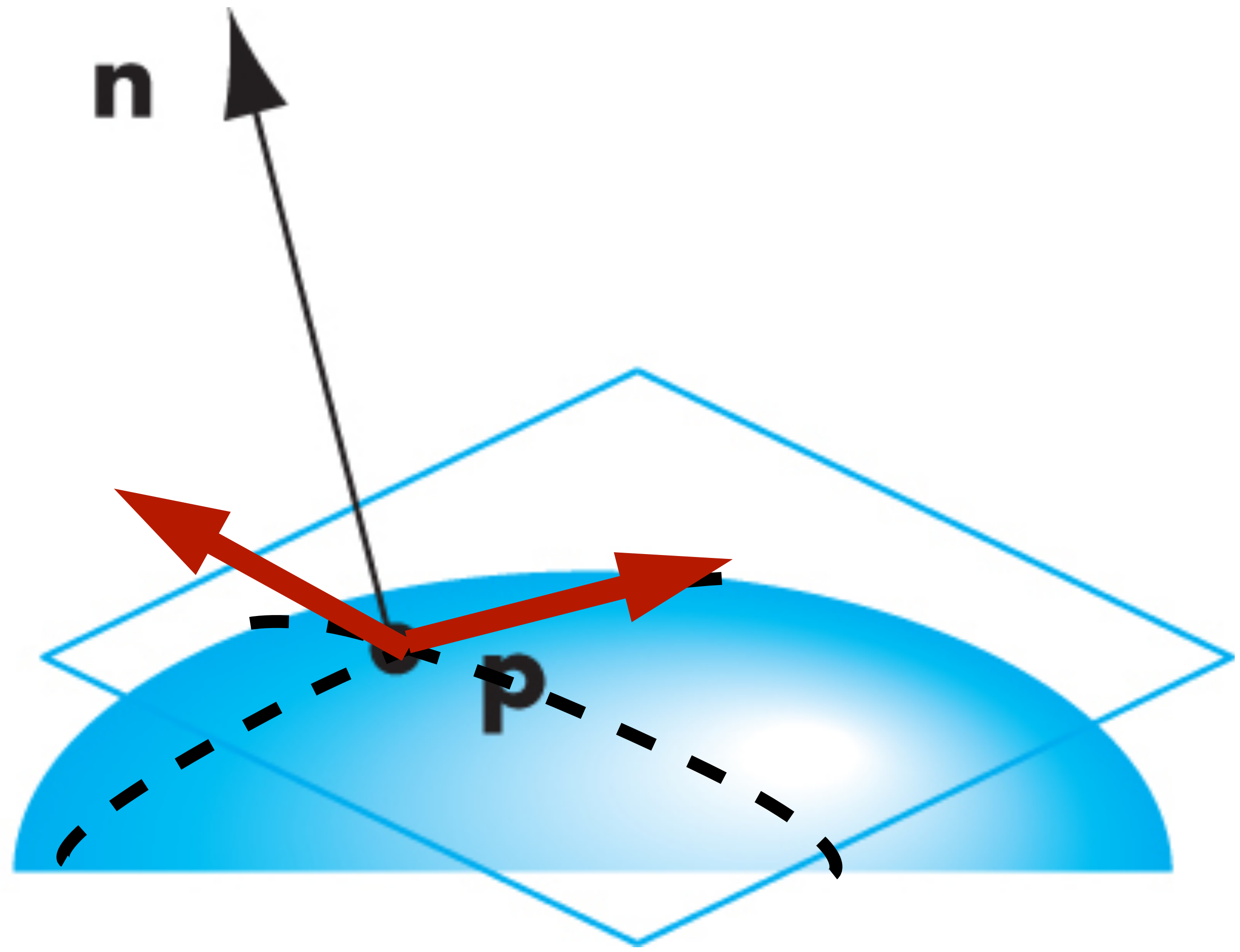
$$x = x(u, v)$$
$$y = y(u, v)$$
$$z = z(u, v)$$



Parametric Surface - tangent plane

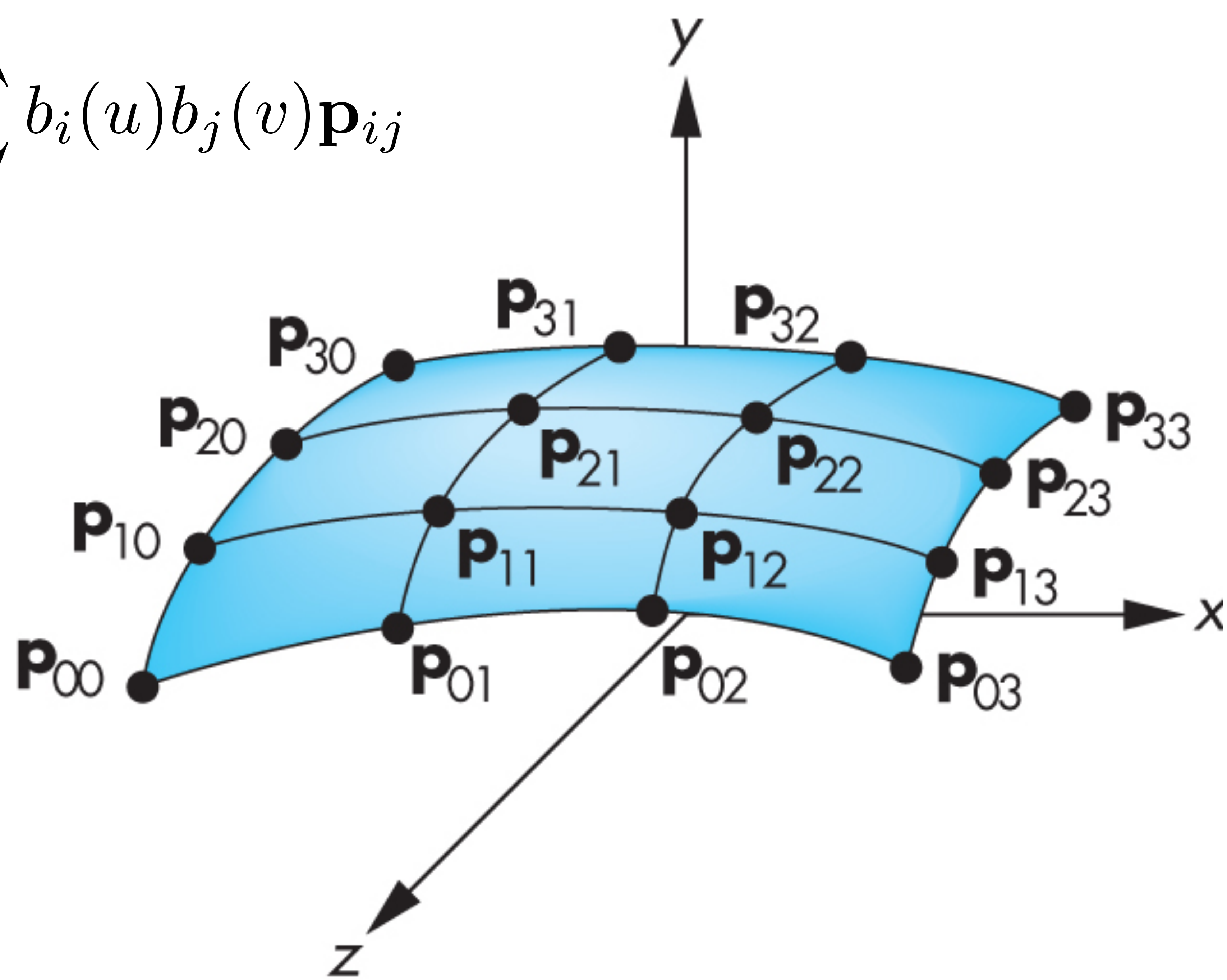
$$\mathbf{t}_u = \begin{pmatrix} \frac{\partial x}{\partial u} \\ \frac{\partial y}{\partial u} \\ \frac{\partial z}{\partial u} \end{pmatrix}$$

$$\mathbf{t}_v = \begin{pmatrix} \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial v} \\ \frac{\partial z}{\partial v} \end{pmatrix}$$



Bicubic Surface Patch

$$\mathbf{f}(u,v) = \sum_i \sum_j b_i(u) b_j(v) \mathbf{p}_{ij}$$



Bezier Surface Patch

$$\mathbf{f}(u,v) = \sum_i \sum_j b_i(u) b_j(v) \mathbf{p}_{ij}$$

Patch lies in convex hull