CS130 : Computer Graphics

Lecture 23: Curves

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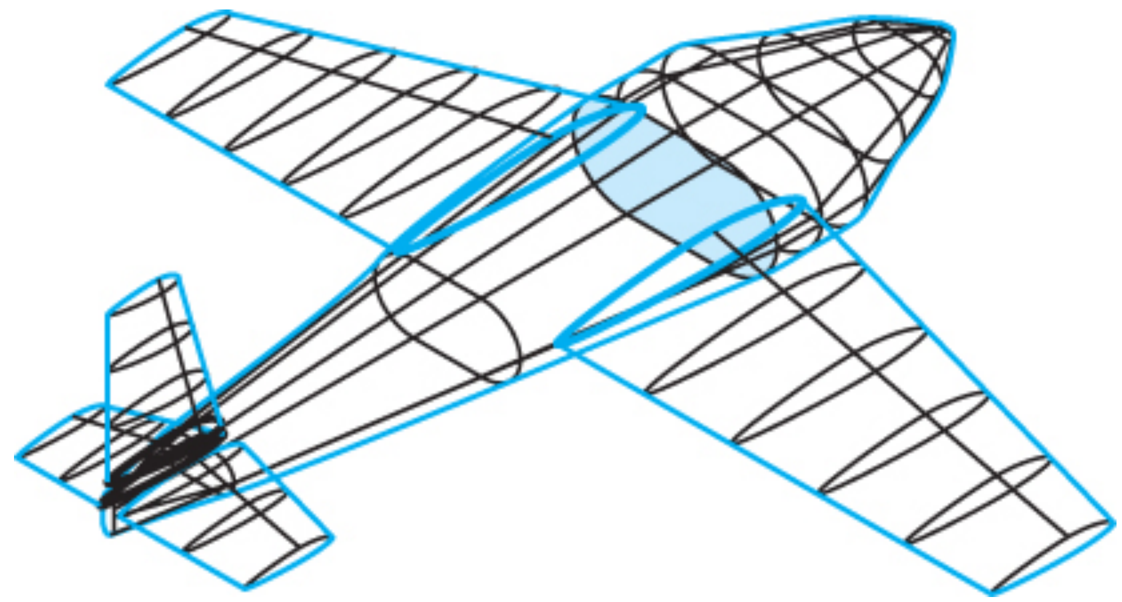
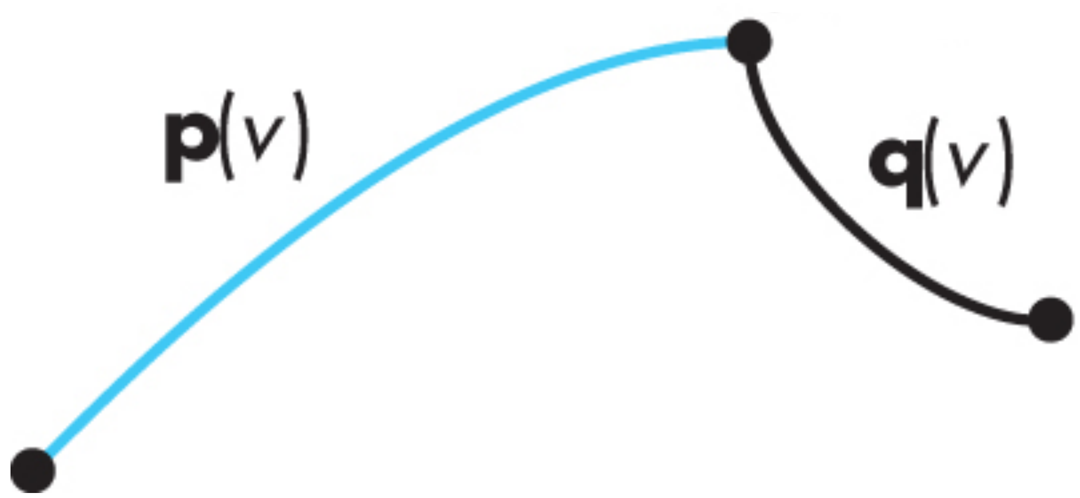
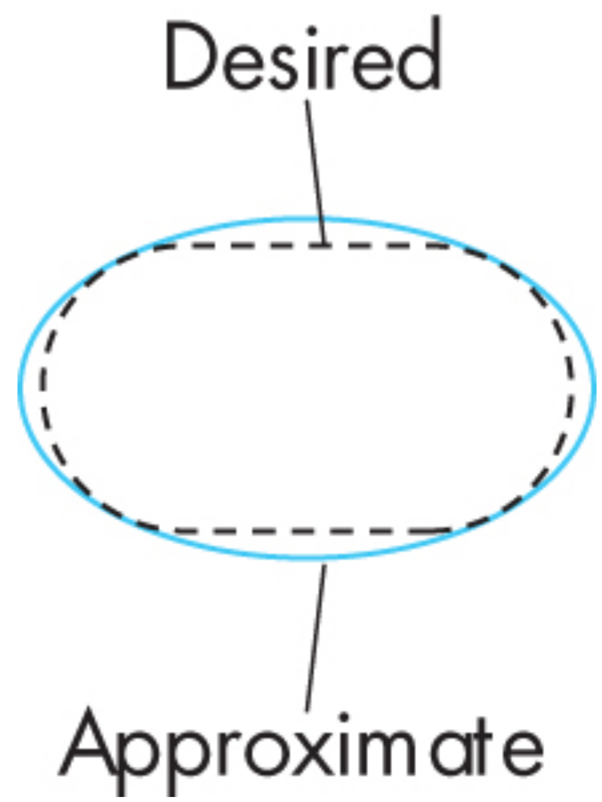
Design considerations

- local control of shape
- smoothness and continuity
- ability to evaluate derivatives
- stability
- ease of rendering



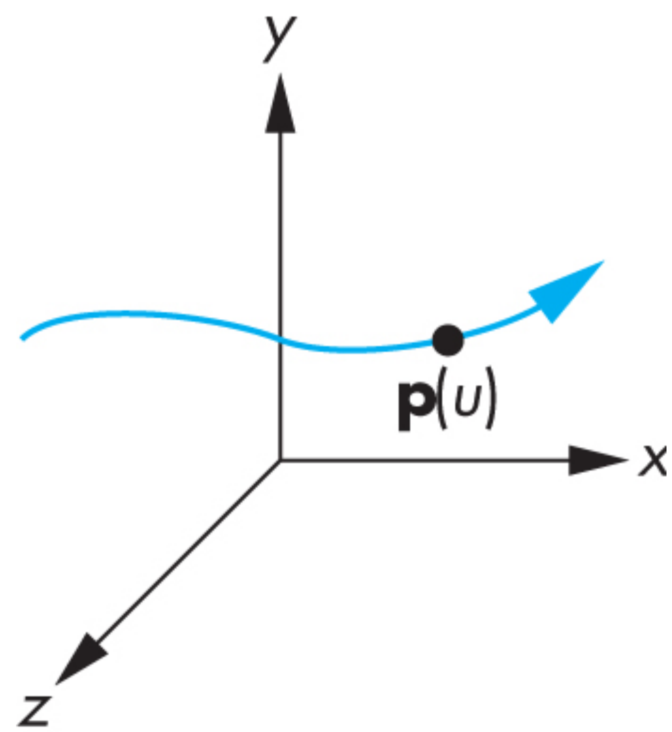


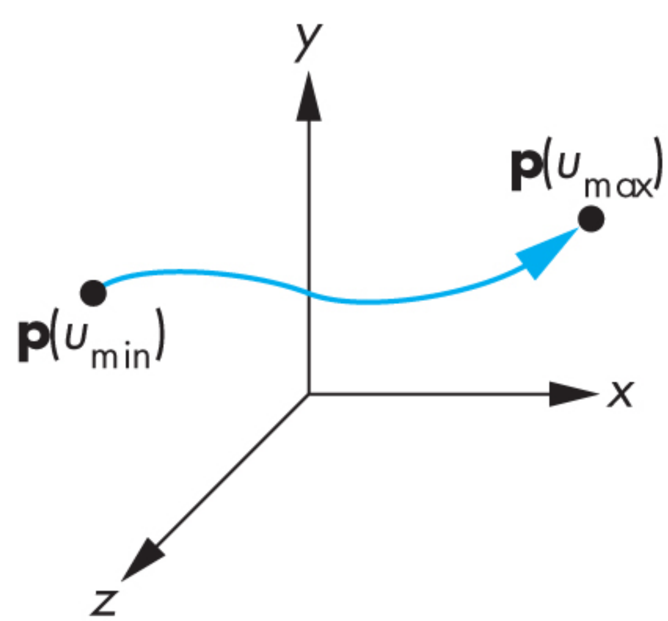


- local control - design each segment independently
- stability - small change in input values leads to small change in output

Parametric curve

$$x = x(u)$$
$$y = y(u)$$
$$z = z(u)$$





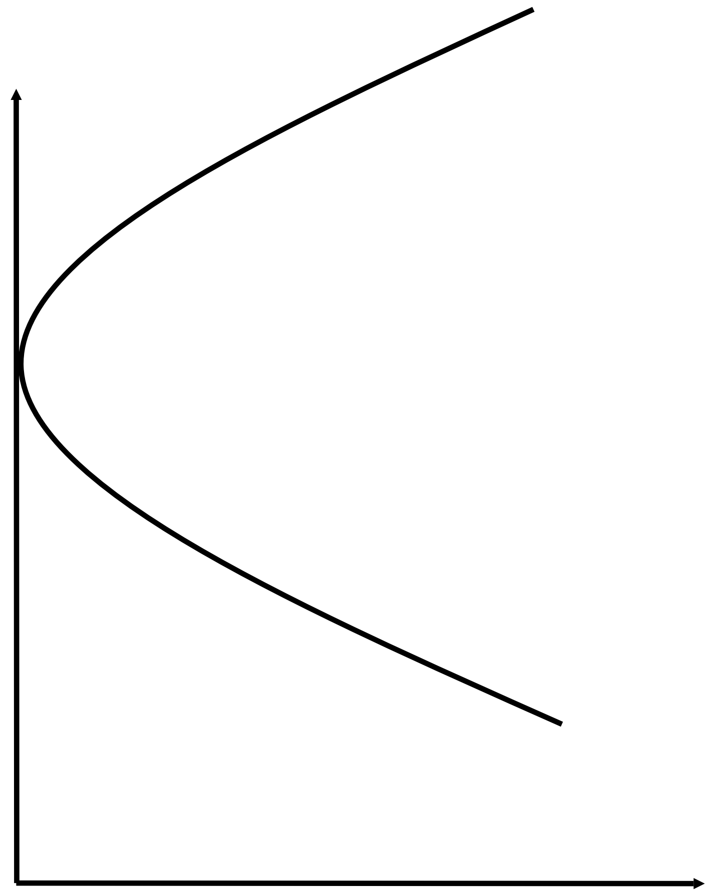
Curve segment

Parametric curve example

$$p(u) = c_0 + c_1 u + c_2 u^2$$

$$x(u) = 3u^2$$

$$y(u) = 2u + 3$$



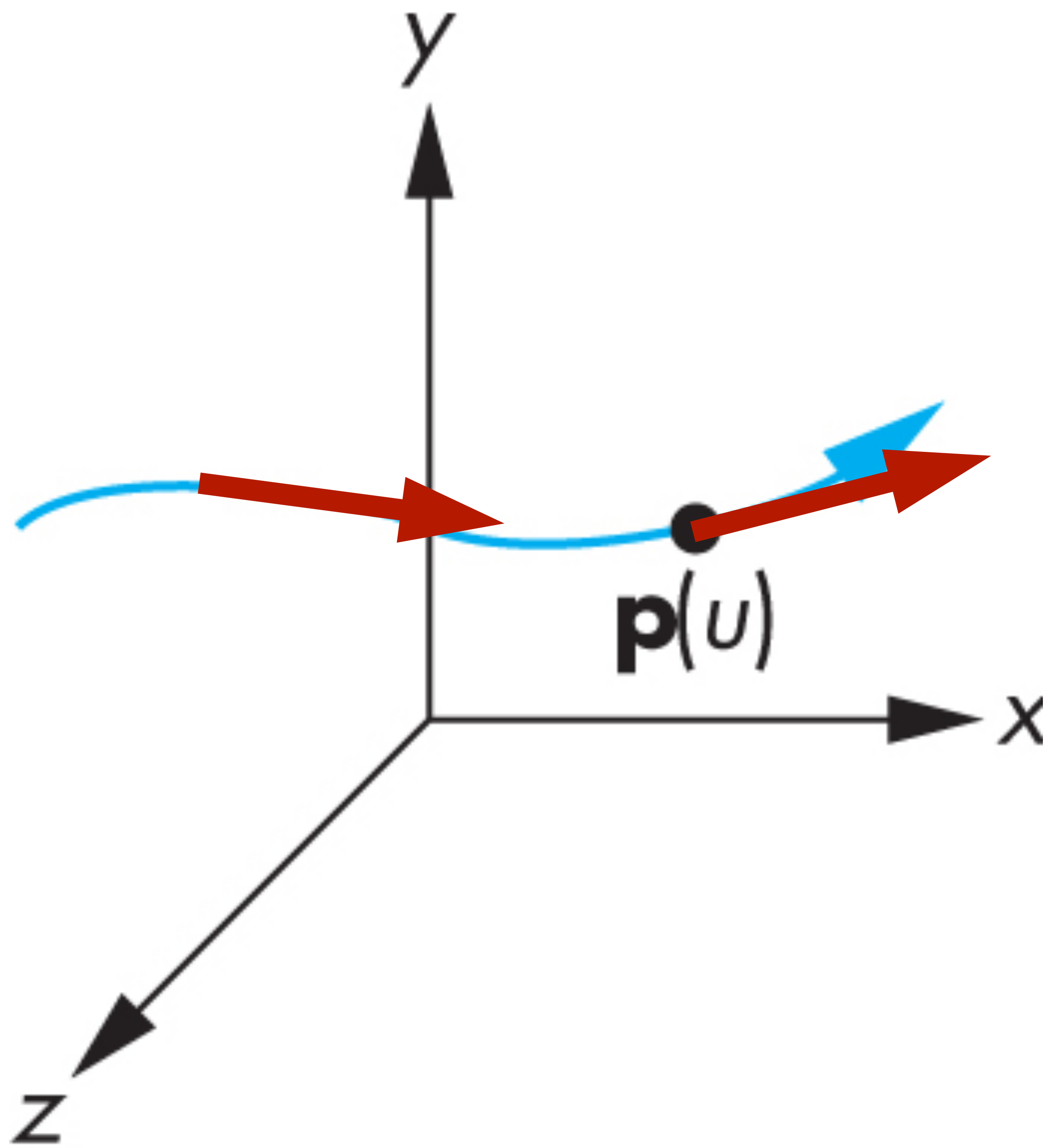
$$c_0 = ?, \quad c_1 = ?, \quad c_2 = ?$$

- this is a curve in 2D
- for a curve in 3D, we would also have z(u) = ...

Parametric curve - tangent vector

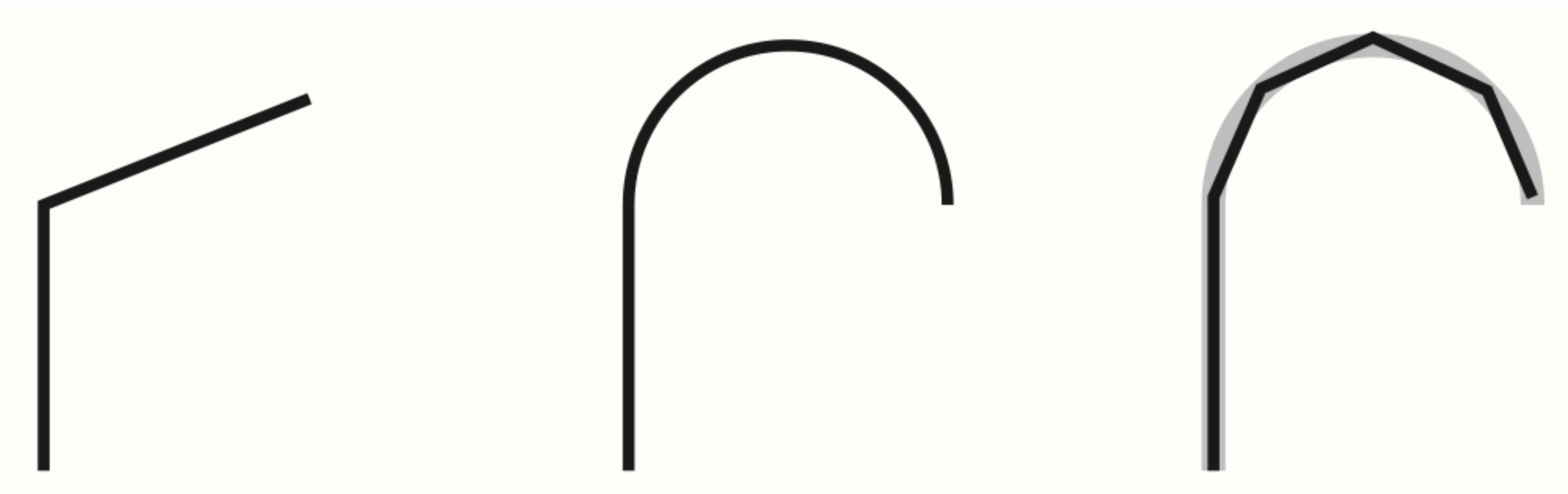
$$x = x(u)$$
$$y = y(u)$$
$$z = z(u)$$

$$\boxed{\mathbf{t} = \begin{pmatrix} x'(u) \\ y'(u) \\ z'(u) \end{pmatrix}}$$



– tangent vector

Piecewise Parametric Representations



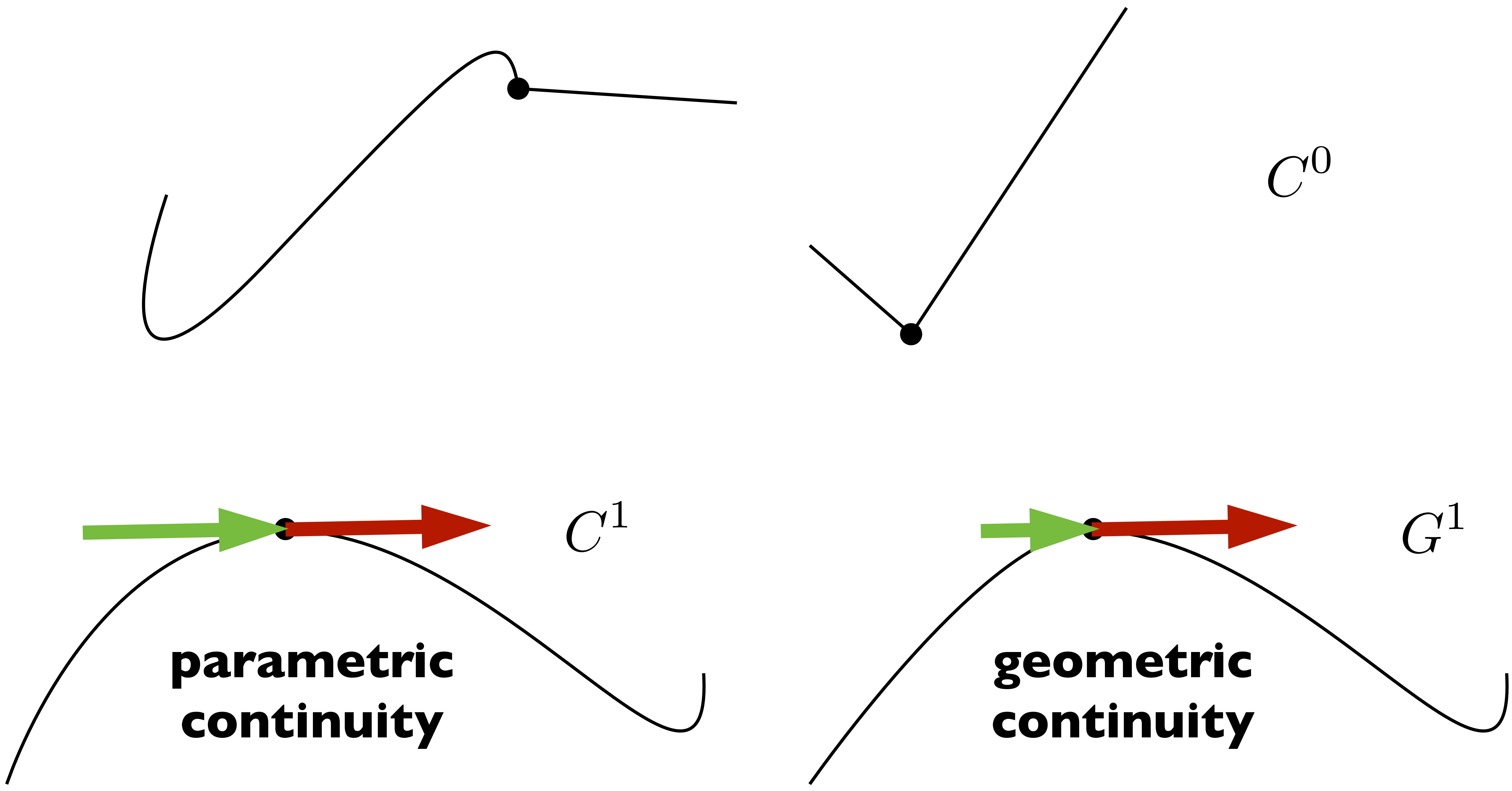
$$\mathbf{f}(u) = \begin{cases} \mathbf{f}_1(2u) & u \le 0.5 \\ \mathbf{f}_2(2u-1) & u > 0.5 \end{cases}$$

continuity

$$\mathbf{f}_1(1) = \mathbf{f}_2(0)$$

right: use simpler curves, but more of them to get the accuracy

Continuity



Top

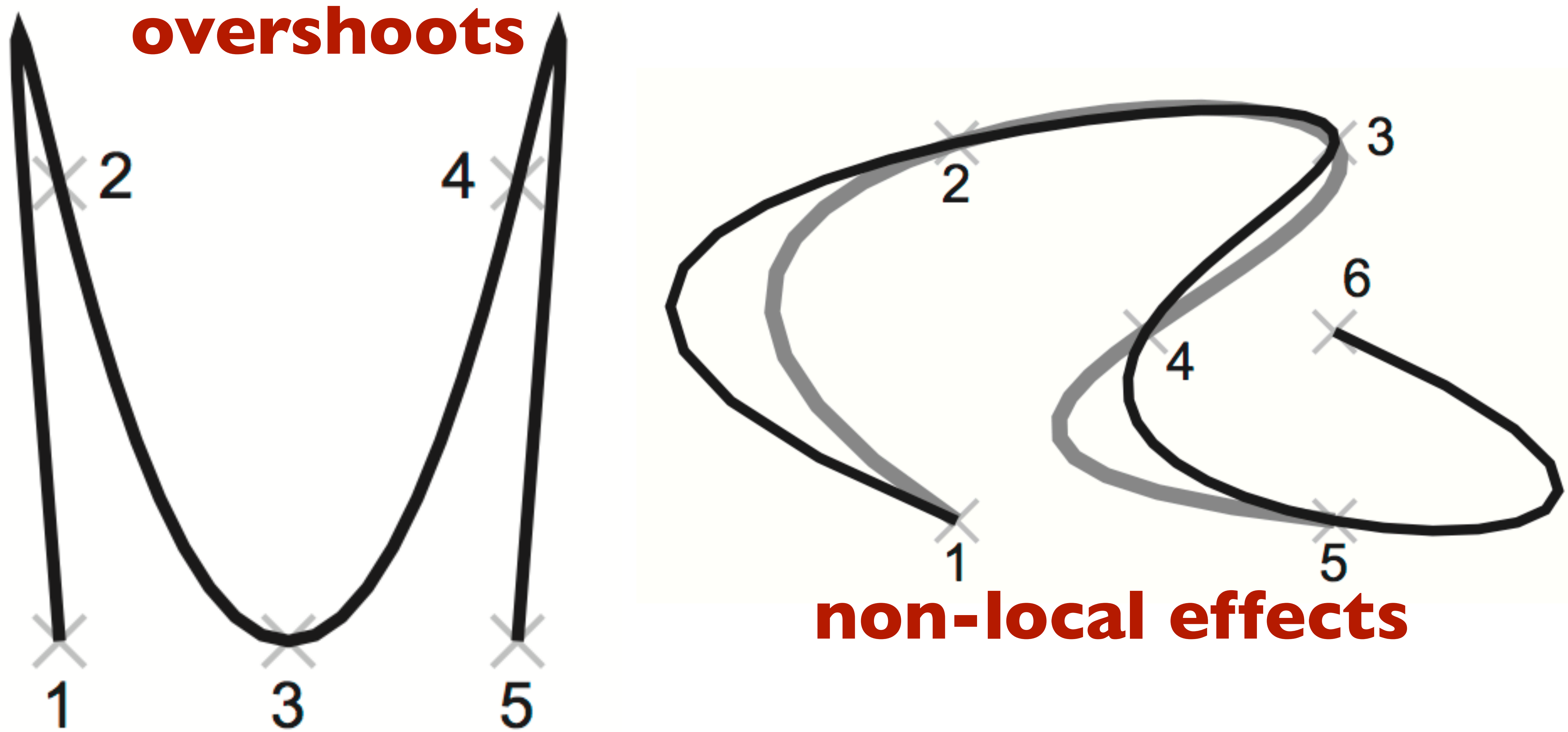
C0: the curves are continuous, but have discontinuous first derivatives

Bottom

Left: At the knot, the curve has C1 continuity: the curve segments have common point and first derivative

Right: At the knot, the curve has G1 continuity: the curve segments have a common point, and parallel first derivatives of different magnitude

higher order interpolating polynomials



These images demonstrate problems with using higher order polynomials:
- overshoots
- non-local effects (in going from the 4th order polynomial in grey to the 5th order polynomial in black)