CS130 : Computer Graphics

Lecture 18: Rotations

Tamar Shinar
Computer Science & Engineering
UC Riverside

general rotations

Rotation

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

X axis

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Y axis

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Z axis

The rows and columns are orthonormal

Rotation about an arbitrary axis

Rotating about an axis by theta degrees

- Rotate about x to bring axis to xz plane
- Rotate about y to align axis with z -axis
- Rotate theta degrees about z
- Unrotate about y, unrotate about x

$$\mathbf{M} = \mathbf{Rx}^{-1}\ \mathbf{Ry}^{-1}\ \mathbf{Rz}(\theta)\ \mathbf{Ry}\ \mathbf{Rx}$$

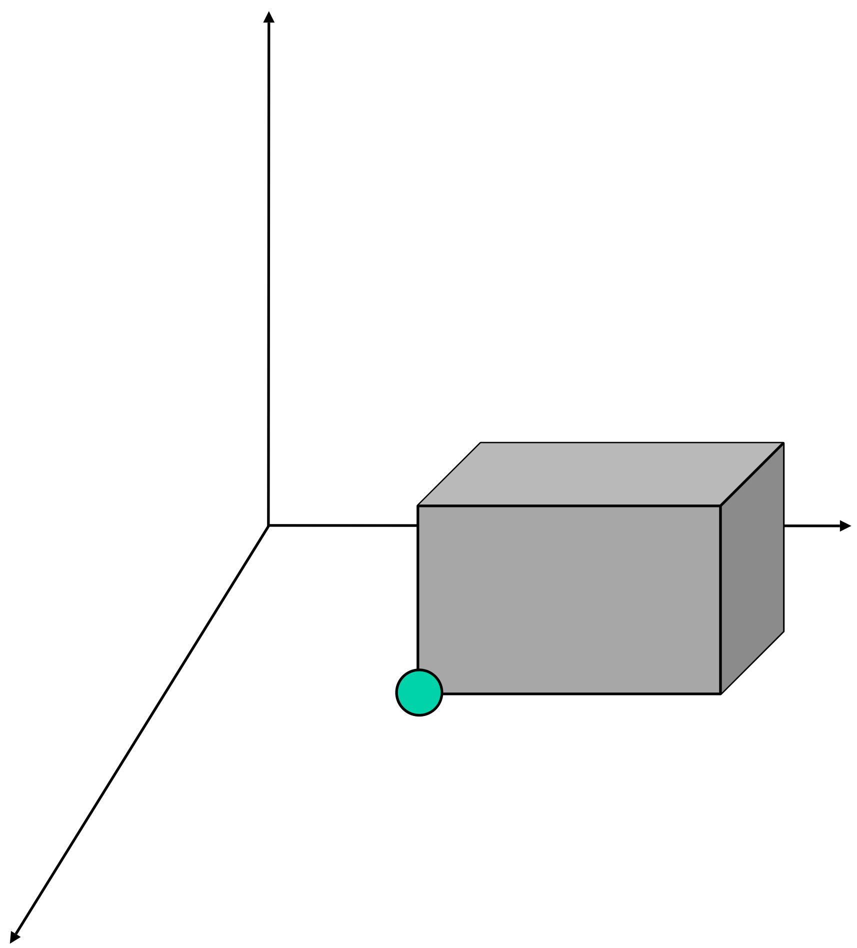
- Can you determine the values of Rx and Ry?

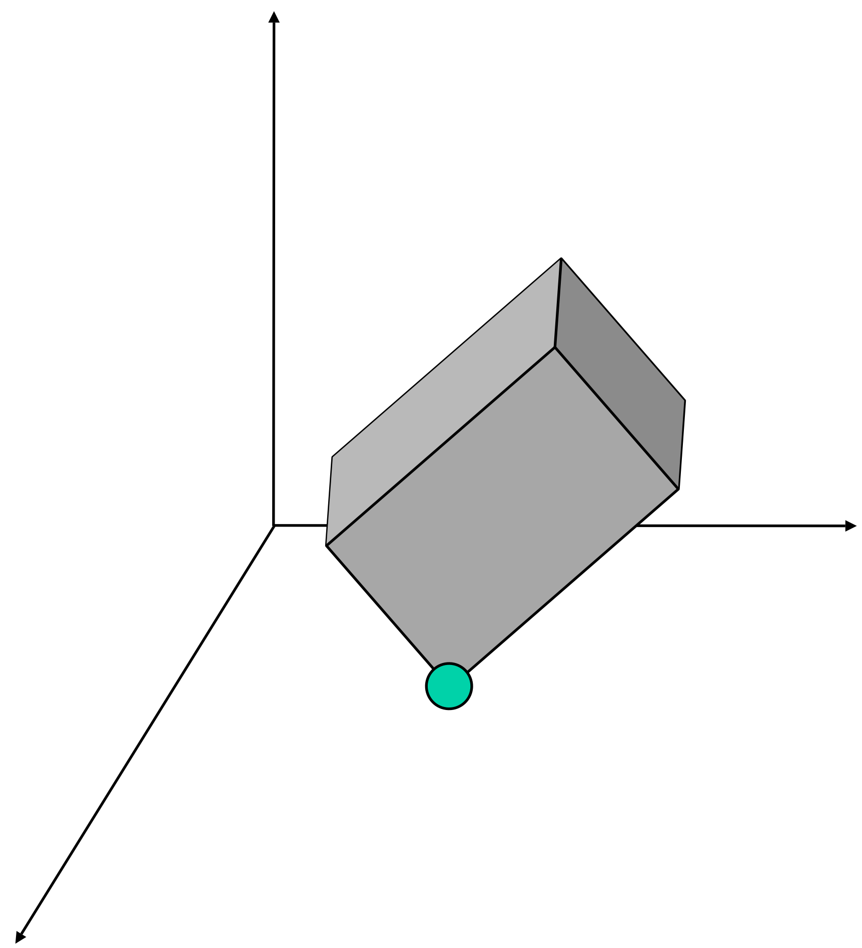
Composite Transformations

- Rotating about a fixed point

  - **basic** rotation alone will rotate about origin

  but we want:

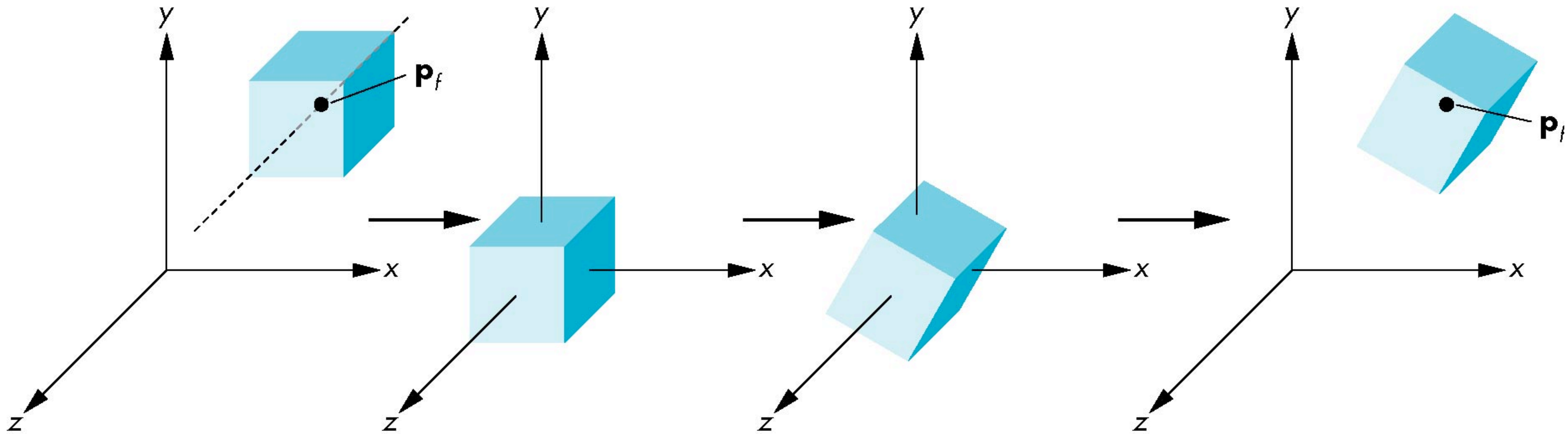




Composite Transformations

- Rotating about a fixed point
  - Move fixed point (px,py,pz) to origin
  - Rotate by desired amount
  - Move fixed point back to original position

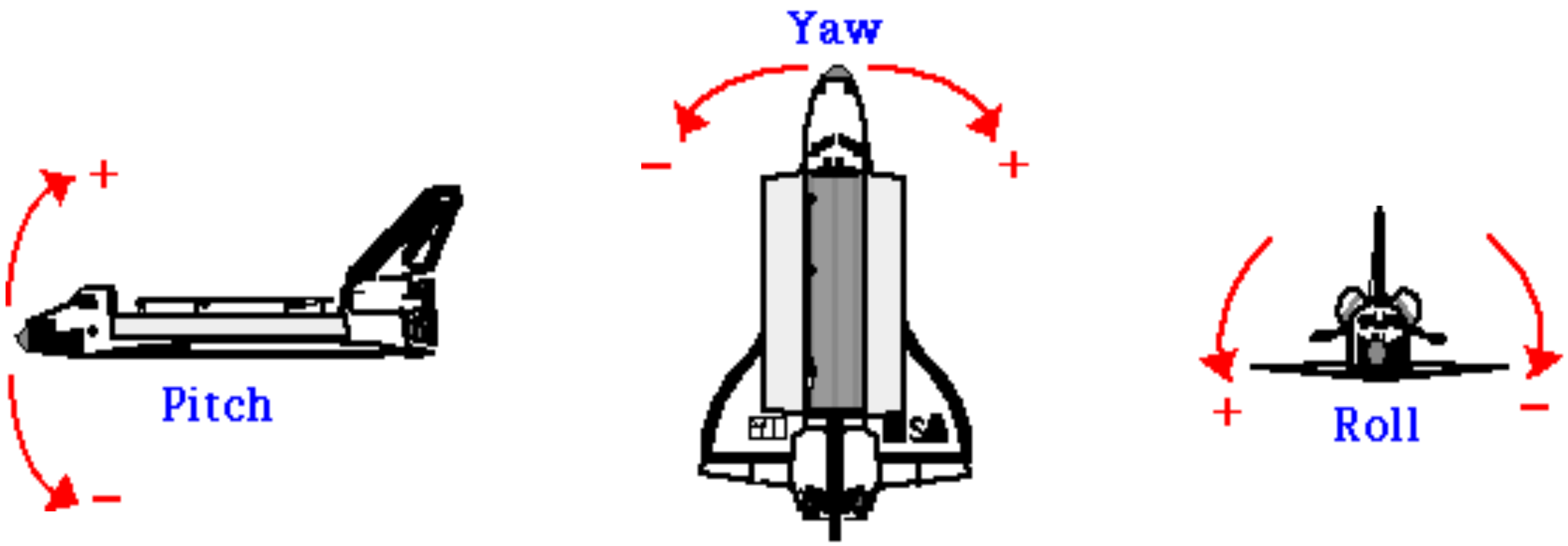
$$\mathbf{M} = \mathbf{T}(px, py, pz)\, \mathbf{R_z}(\theta)\, \mathbf{T}(-px, -py, -pz)$$



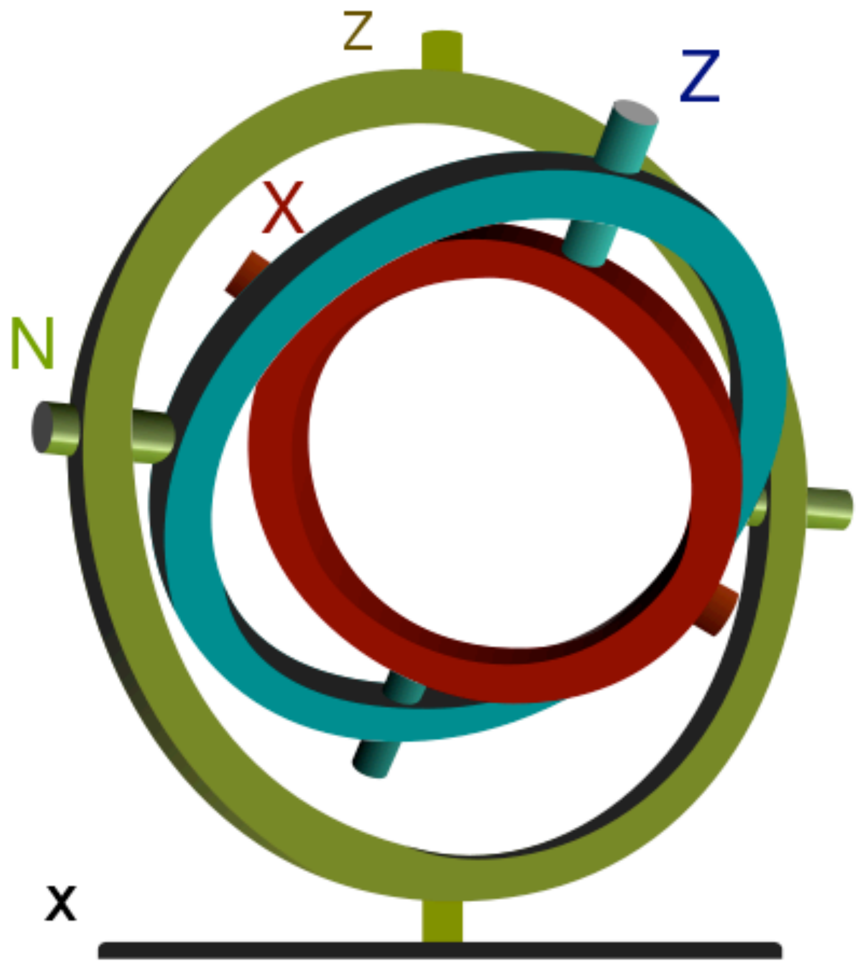
euler angles

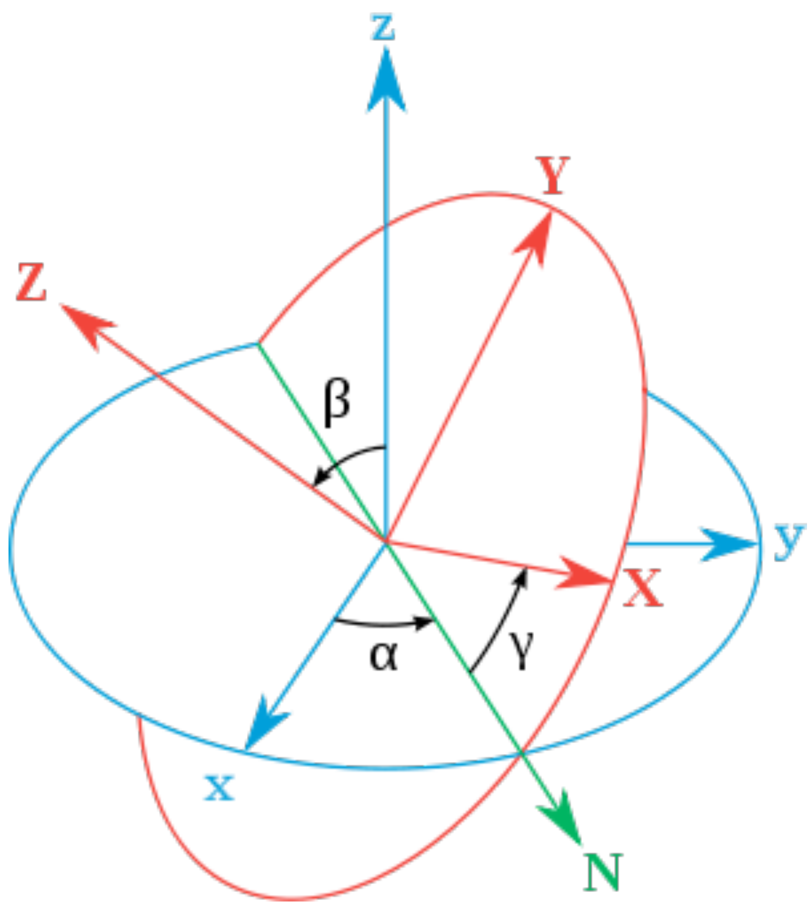
Euler Angles

- A general rotation is a combination of three elementary rotations: around the x-axis (x-roll) , around the y-axis (y-pitch) and around the z-axis (z-yaw).



Gimbal and Euler Angles





Z-X’-Z”

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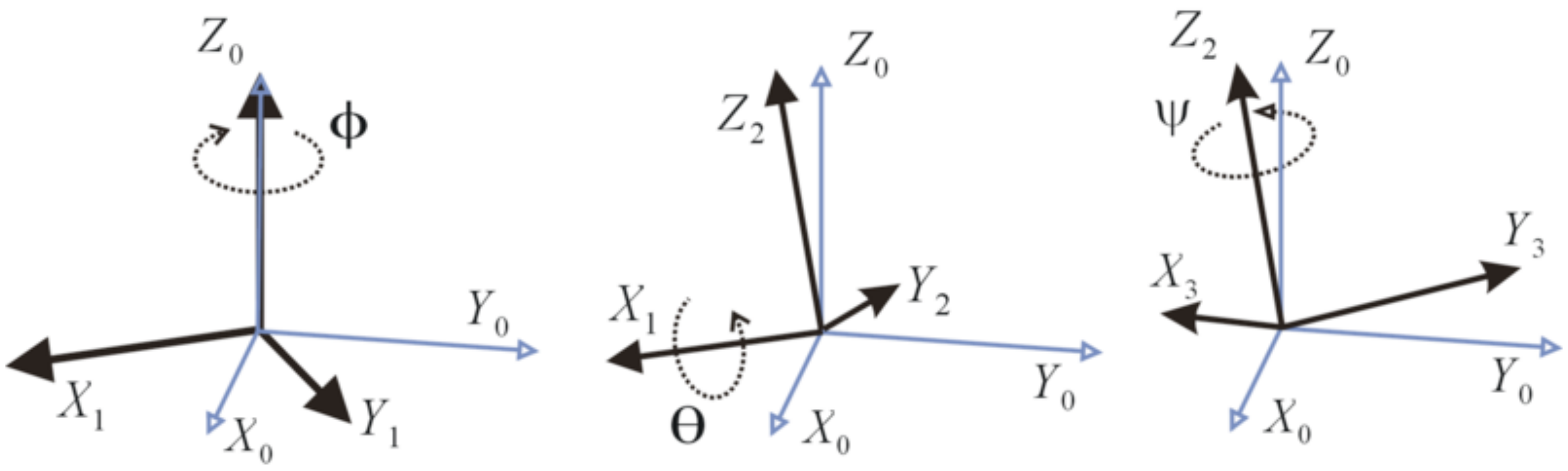
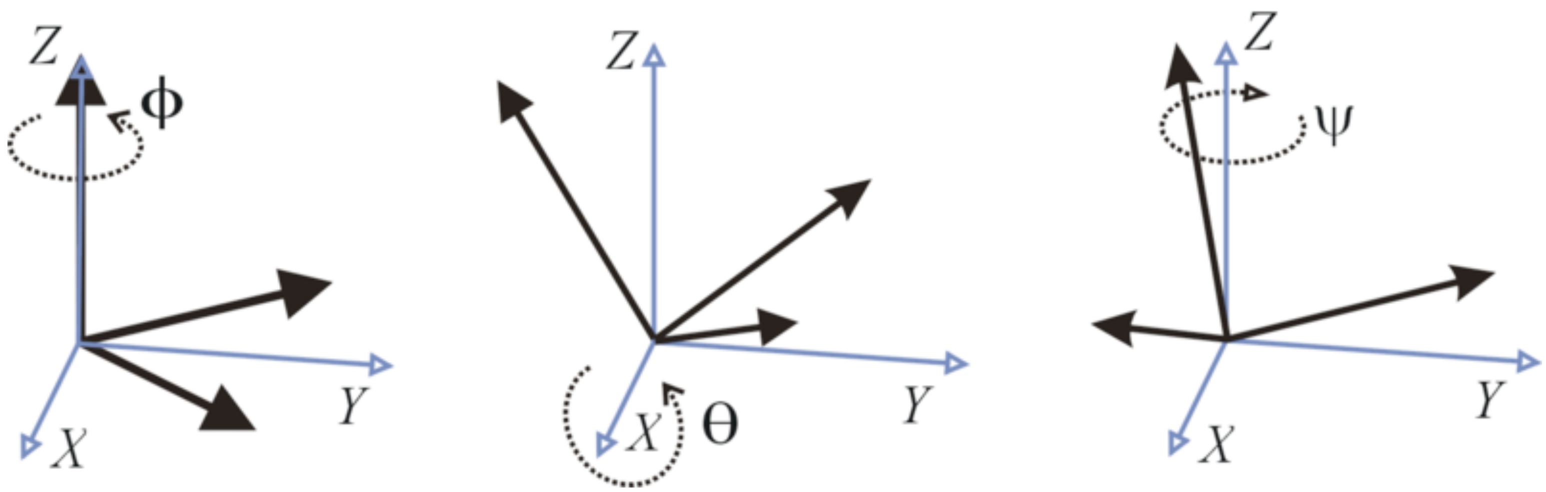
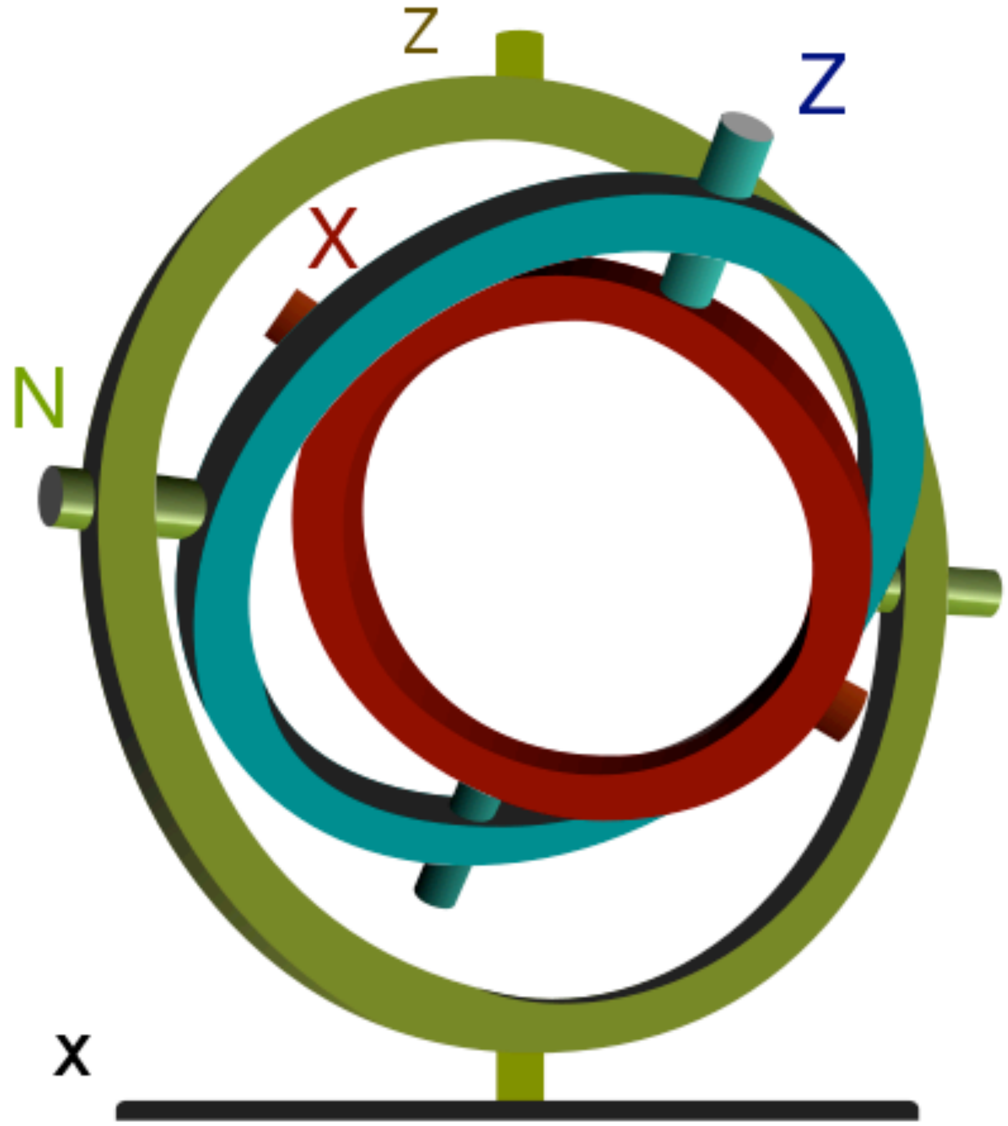
intrinsic



extrinsic





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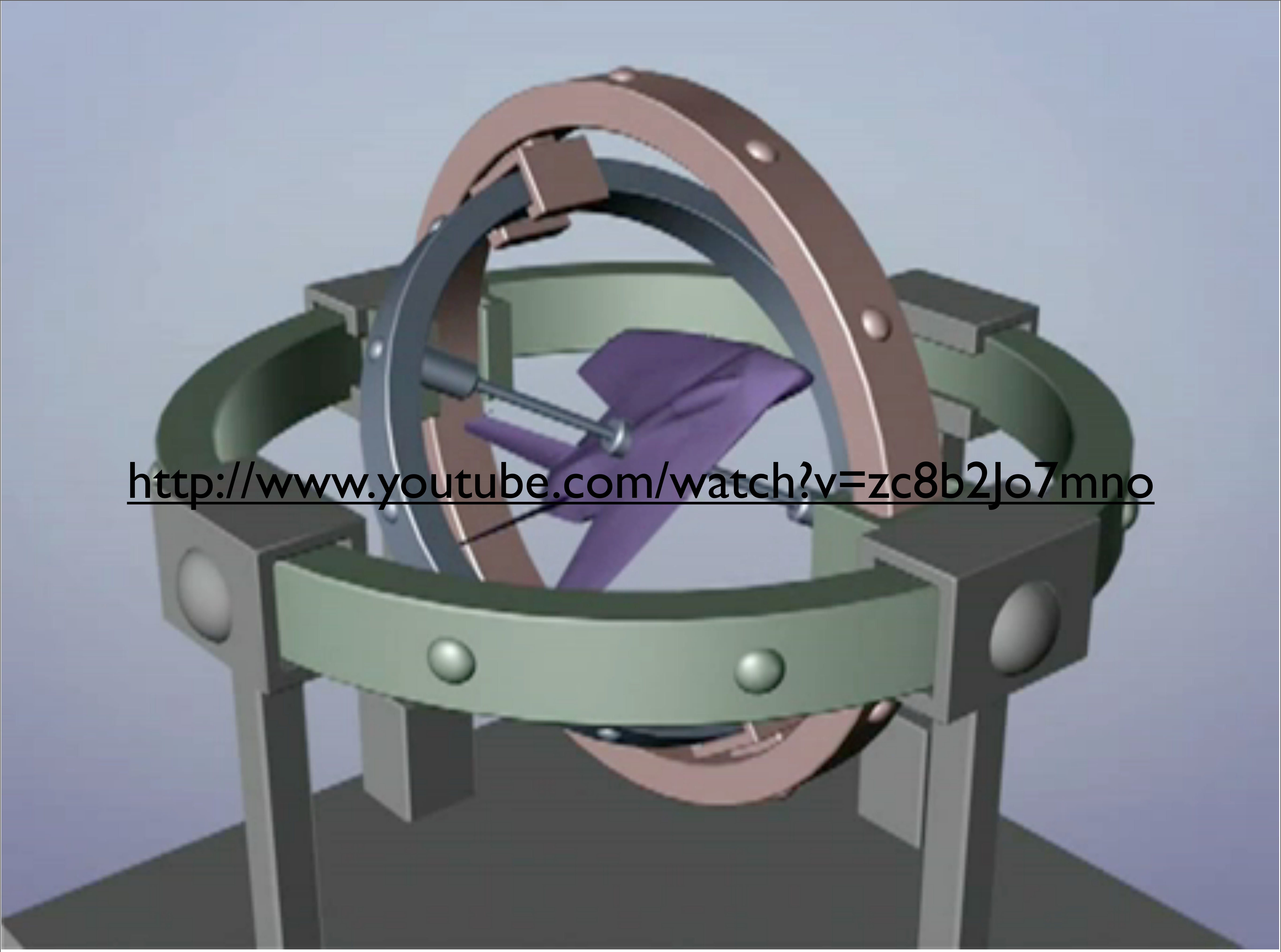
extrinsic – rotations about the reference axes
intrinsic – rotations about the object fixed axes



http://www.youtube.com/watch?v=zc8b2Jo7mno

quaternions

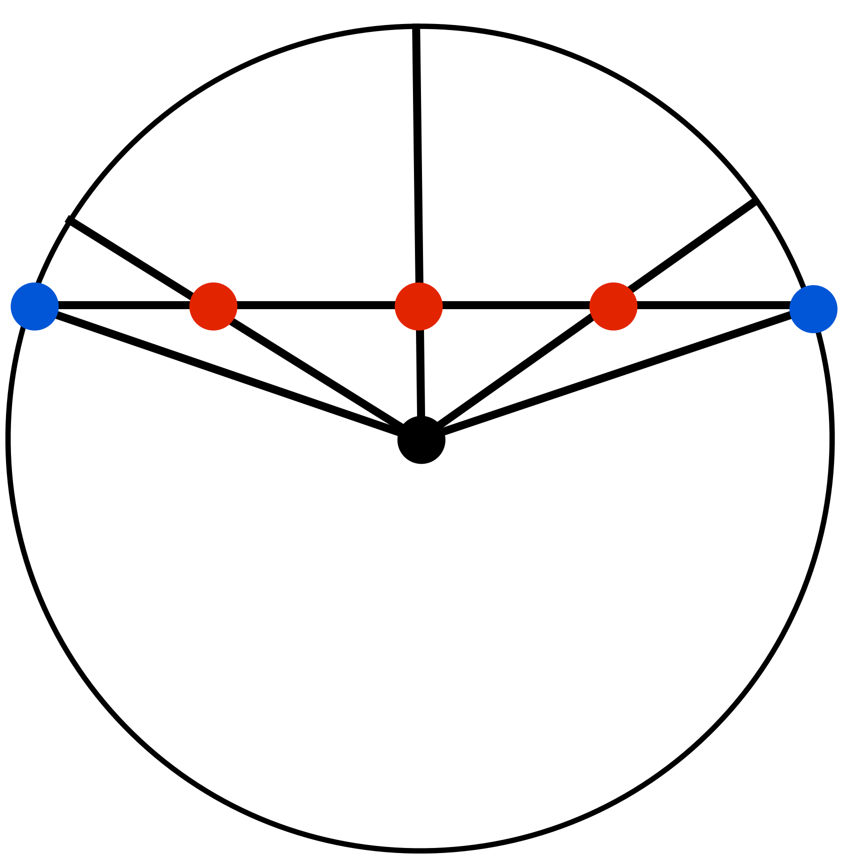

Here as he walked by
on the 16th of October 1843
Sir William Rowan Hamilton
in a flash of genius discovered
the fundamental formula for
quaternion multiplication
$i^2 = j^2 = k^2 = ijk = -1$
& cut it on a stone of this bridge

Quaternions

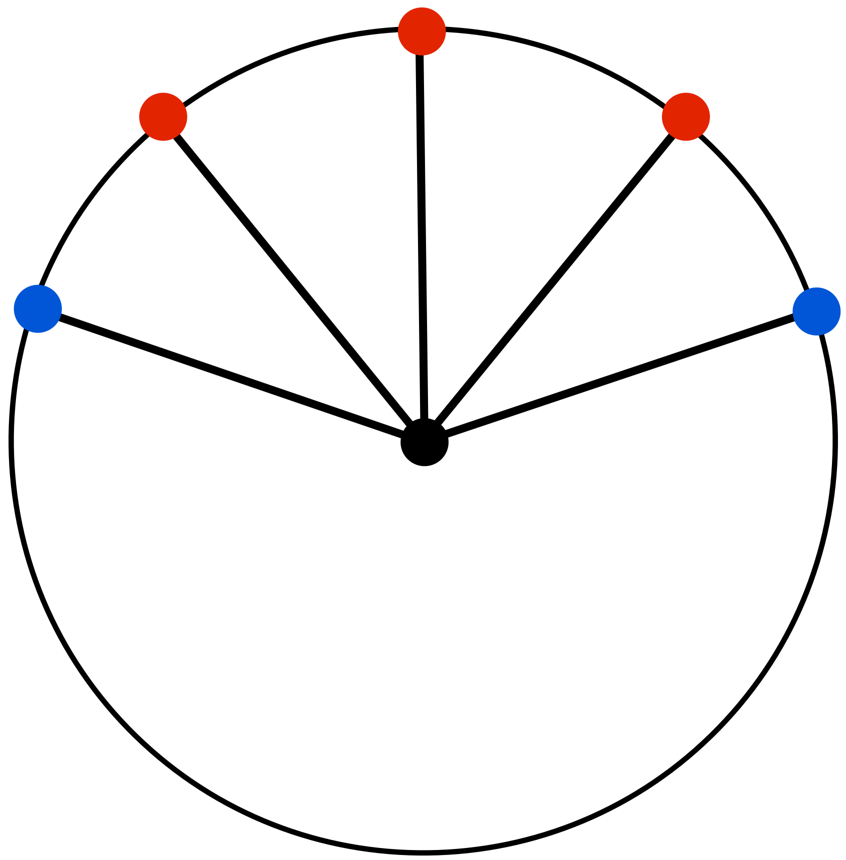
- axis/angle representation
- interpolates smoothly
- easy to compose

<whiteboard>

Quaternion Interpolation



linear



spherical linear
"slerp"

linear: treat quaternions as 4-vectors, note non-uniform speed
spherical linear: constant speed

Rotations in Reality

- It's easiest to express rotations in Euler angles or Axis/angle
- We can convert to/from any of these representations
- Choose the best representation for the task
  - input:Euler angles
  - interpolation: quaternions
  - composing rotations: quaternions, orientation matrix