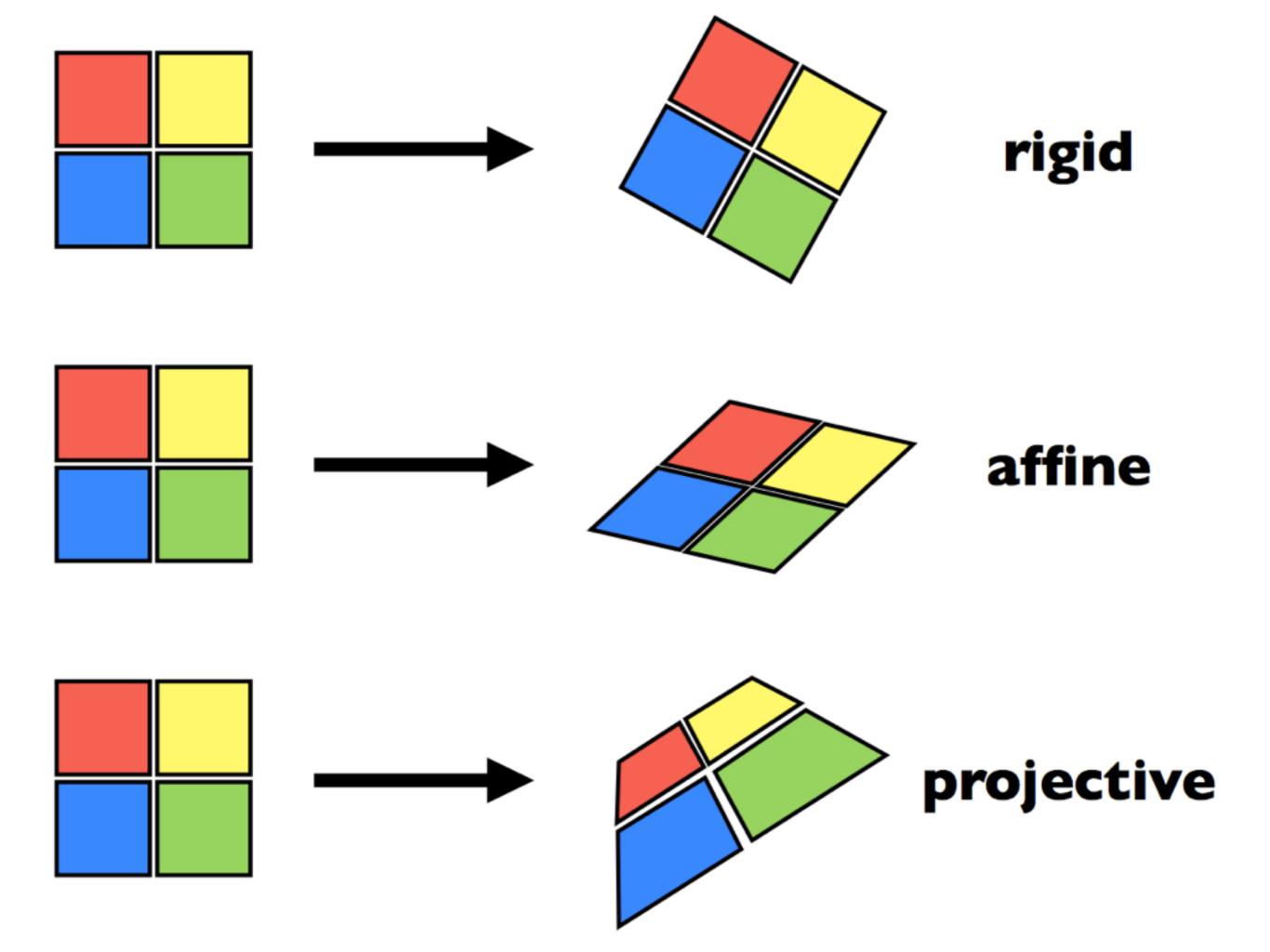
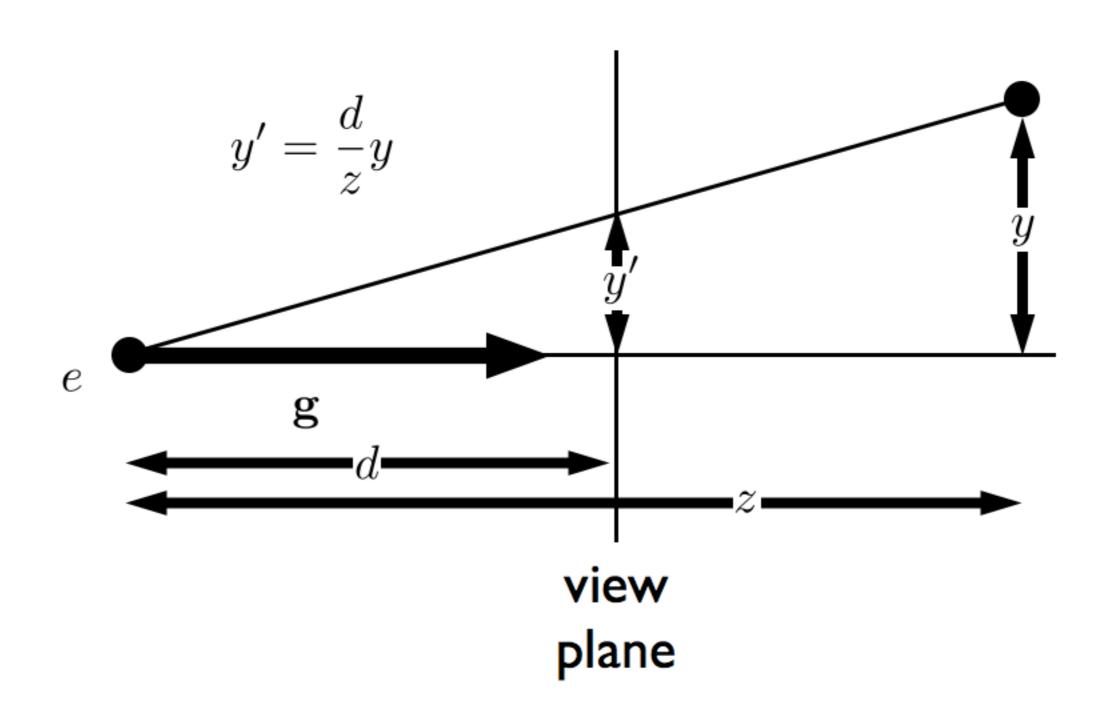
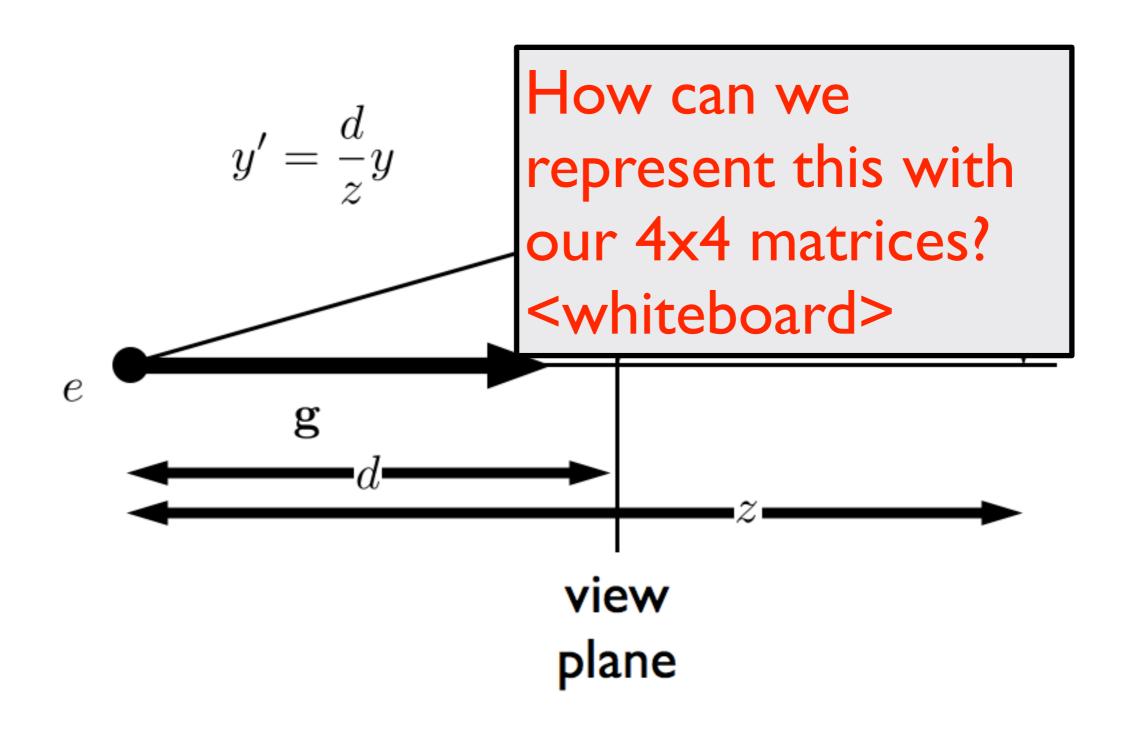
Perspective Viewing



Projective Transformations



Projective Transformations



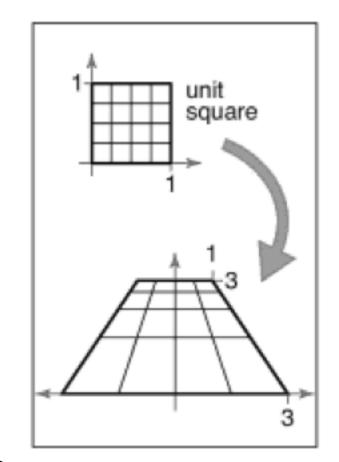
[Shirley, Marschner]

Projective Transformations

$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \\ w \end{pmatrix} \rightarrow \qquad y = \frac{\tilde{x}}{w}$$
$$z = \frac{\tilde{y}}{w}$$

Example:

$$M = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 3 & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} \end{pmatrix}$$



<whiteboard>

[Shirley, Marschner]

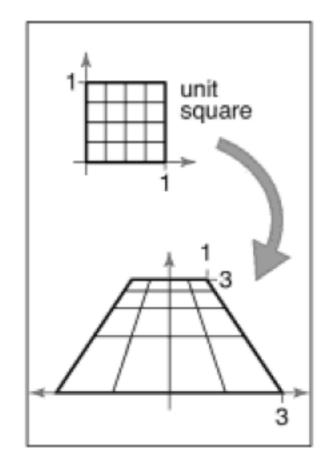
Projective Transformations

$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \\ w \end{pmatrix} \rightarrow \qquad \begin{aligned} x &= \frac{x}{w} \\ y &= \frac{\tilde{y}}{w} \\ z &= \frac{\tilde{z}}{w} \end{aligned}$$

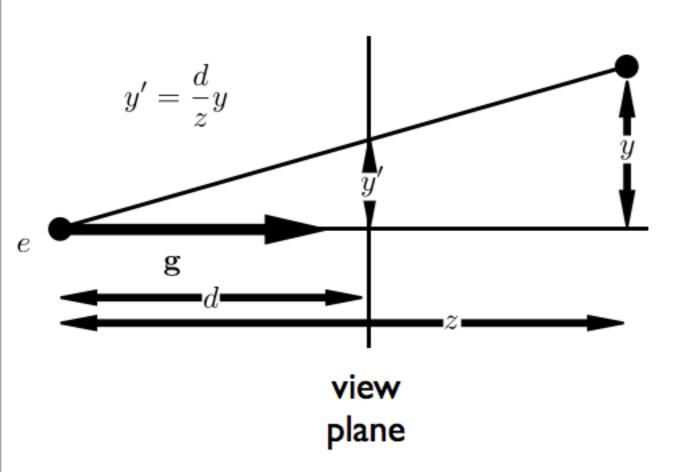
We can now implement perspective projection!

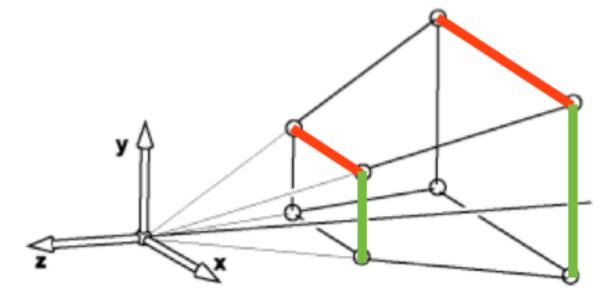
Example:

$$M = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 3 & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} \end{pmatrix}$$



Perspective Projection





both x and y get multiplied by d/z

Simple perspective projection

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \\ z/d \end{pmatrix} \implies \begin{cases} x' = \frac{d}{z}x \\ y' = \frac{d}{z}y \\ z' = \frac{d}{z}z = d \end{cases}$$

This achieves a simple perspective projection onto the view plane z = d

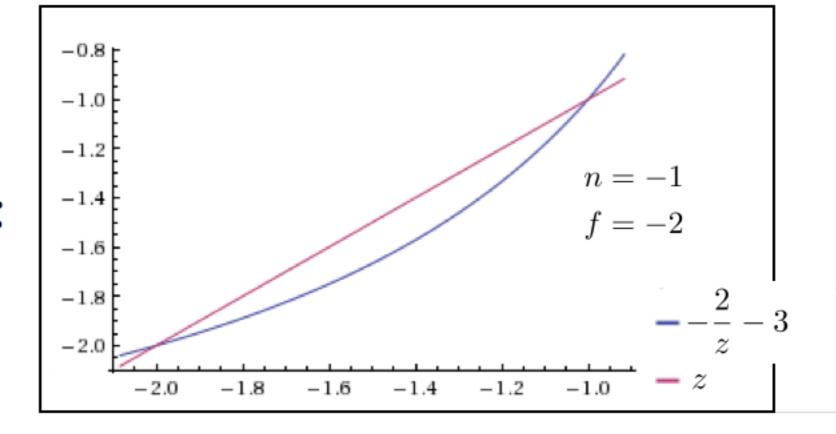
but we've lost all information about z!

<whiteboard>

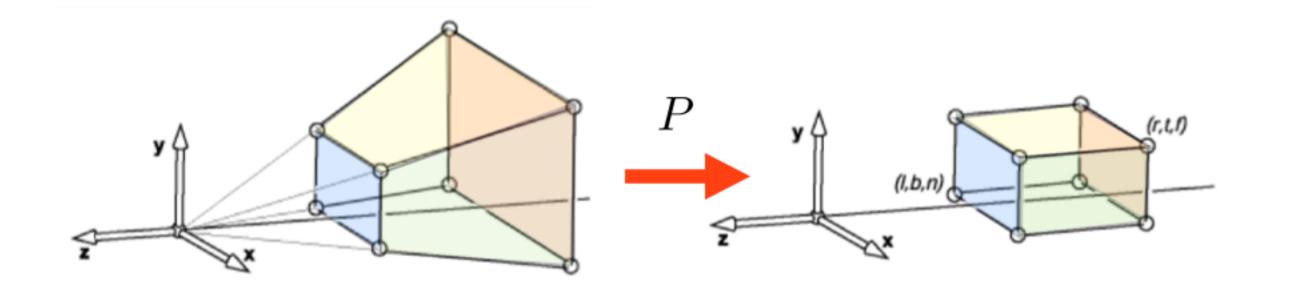
Perspective Projection

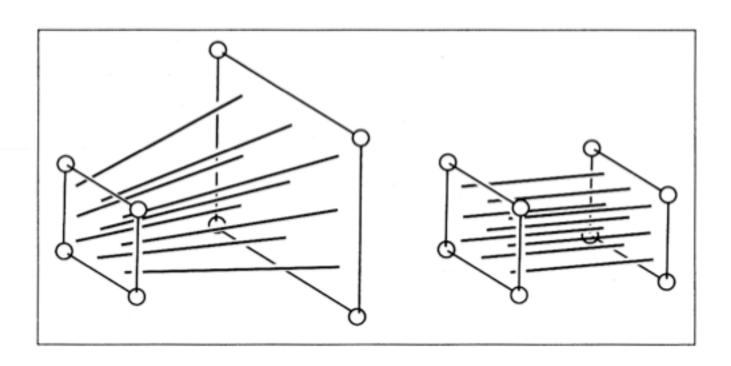
$$P = \begin{pmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad z' = (n+f) - \frac{nf}{z}$$

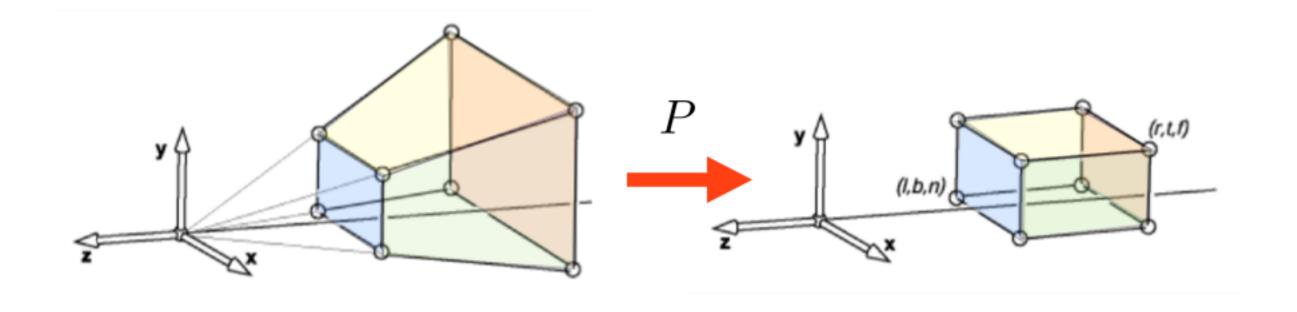
Example:



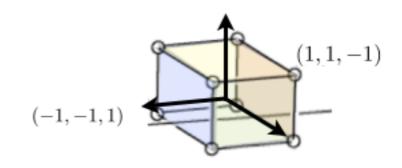
[Shirley, Marschner]

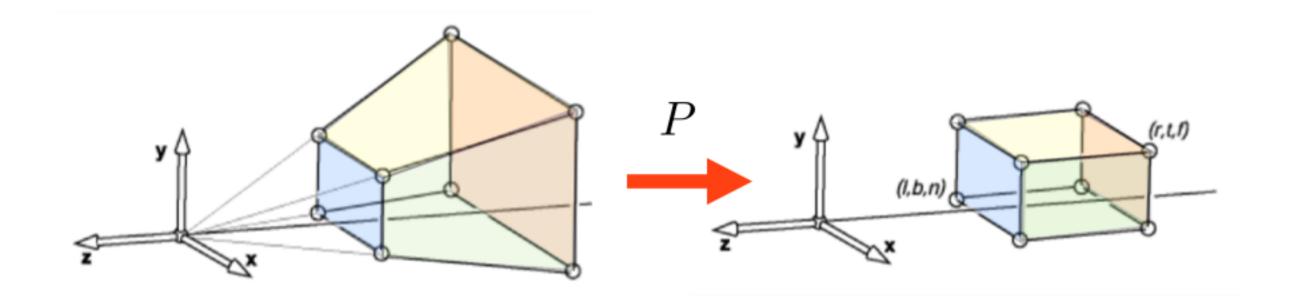








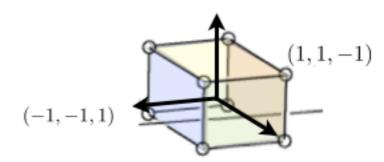






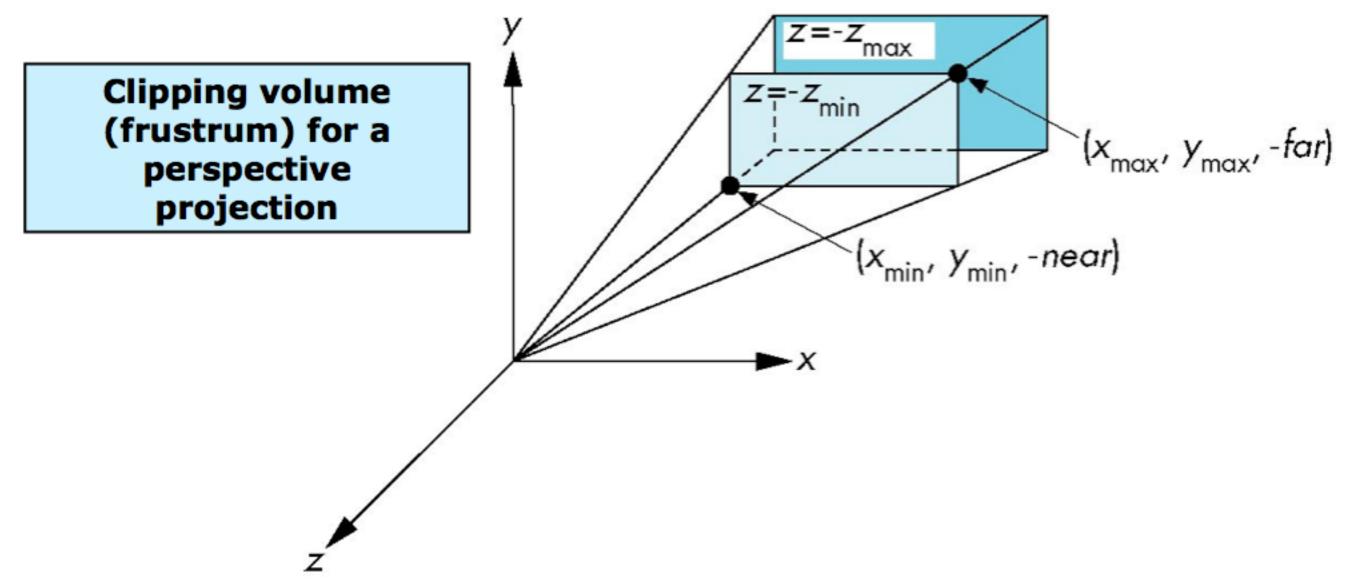


$$M_{\rm per} = M_{\rm orth} P$$



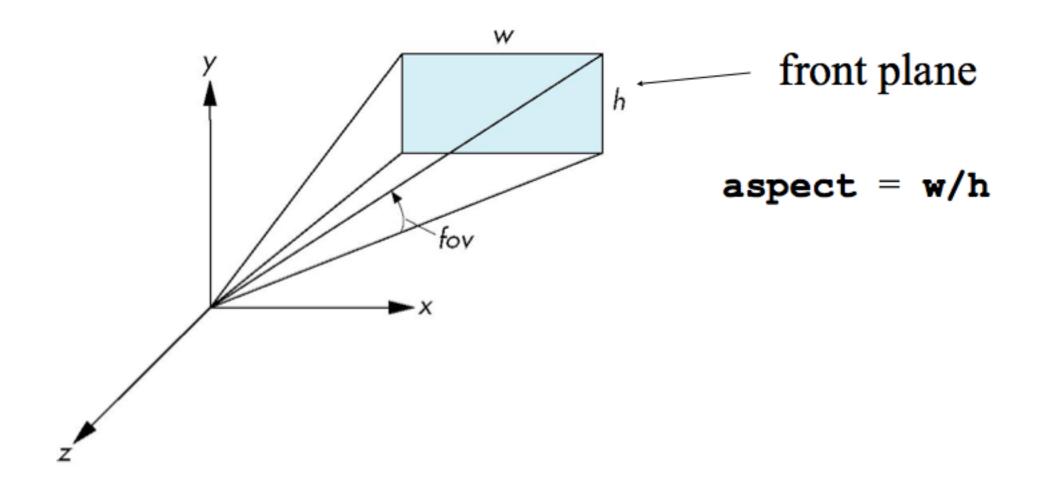
OpenGL Perspective Viewing

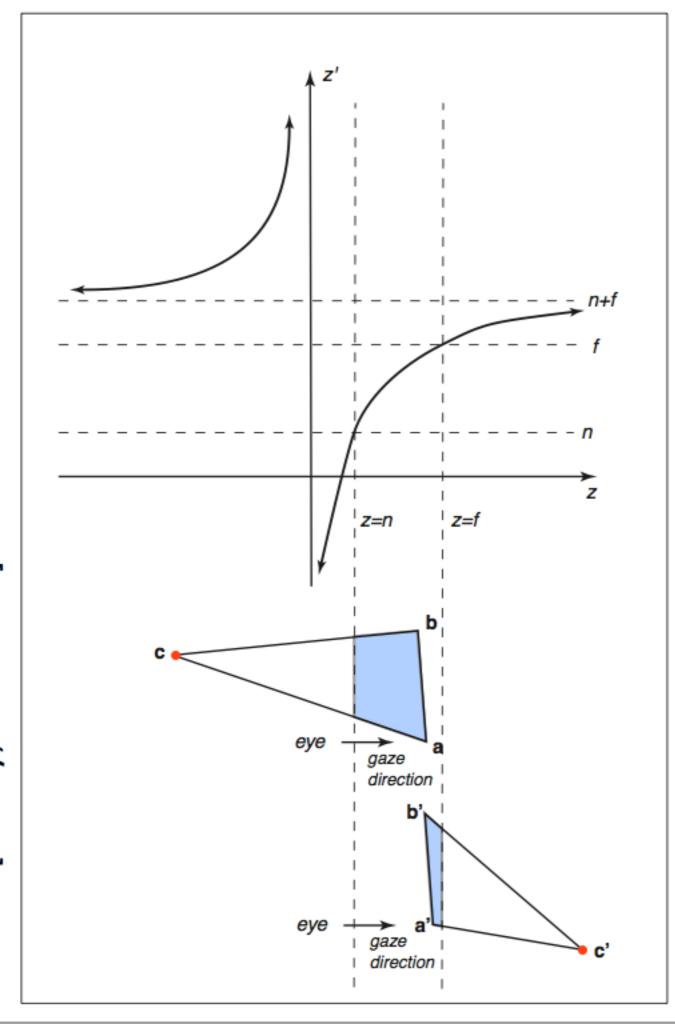
glFrustum(xmin,xmax,ymin,ymax,near,far)



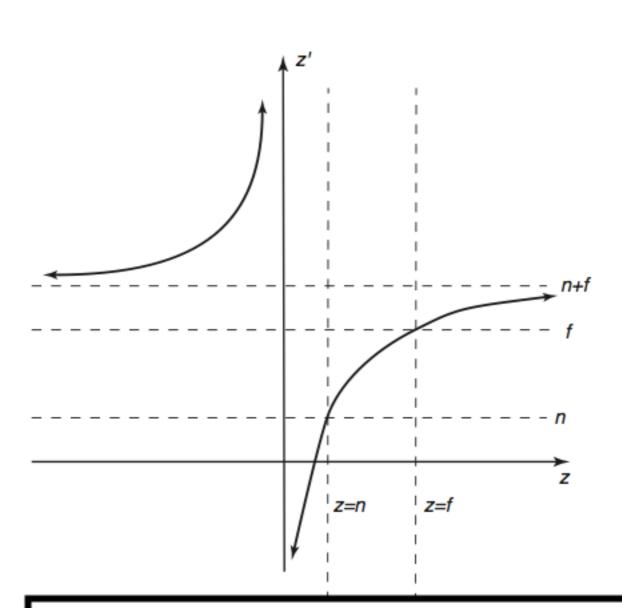
Using Field of View

With glfrustum it is often difficult to get the desired view gluPerpective (fovy, aspect, near, far) often provides a better interface





Clipping after the perspective transformation can cause problems



OpenGL clips **after** projection and **before** perspective division

$$-w \le x \le w$$

$$-w \le y \le w$$

$$-w \le z \le w$$

