## Perspective Viewing



## rigid


affine

projective

## Projective Transformations



## Projective Transformations



## Projective Transformations

$$
\begin{aligned}
& \text { Example: } \\
& M=\left(\begin{array}{ccc}
2 & 0 & -1 \\
0 & 3 & 0 \\
0 & \frac{2}{3} & \frac{1}{3}
\end{array}\right) \\
& \\
& ,
\end{aligned}
$$

<whiteboard>

## Projective Transformations

Example:

$$
M=\left(\begin{array}{ccc}
2 & 0 & -1 \\
0 & 3 & 0 \\
0 & \frac{2}{3} & \frac{1}{3}
\end{array}\right)
$$



## Perspective Projection


both $x$ and $y$ get multiplied by $\mathrm{d} / \mathrm{z}$
[Shirley, Marschner]

## Simple perspective projection

$$
\left(\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 1 / d & 0
\end{array}\right)\left(\begin{array}{l}
x \\
y \\
z \\
1
\end{array}\right)=\left(\begin{array}{c}
x \\
y \\
z \\
z / d
\end{array}\right) \Rightarrow\left\{\begin{array}{l}
x^{\prime}=\frac{d}{z} x \\
y^{\prime}=\frac{d}{z} y \\
z^{\prime}=\frac{d}{z} z=d
\end{array}\right.
$$

This achieves a simple perspective projection onto the view plane $\mathbf{z}=\mathbf{d}$
but we've lost all information about $z$ !
<whiteboard>

## Perspective Projection

$$
P=\left(\begin{array}{cccc}
n & 0 & 0 & 0 \\
0 & n & 0 & 0 \\
0 & 0 & n+f & -f n \\
0 & 0 & 1 & 0
\end{array}\right) \quad z^{\prime}=(n+f)-\frac{n f}{z}
$$







$\searrow$

$$
M_{\mathrm{per}}=M_{\mathrm{orth}} P
$$



## OpenGL Perspective Viewing

glFrustum (xmin, xmax, ymin, ymax, near , far)


## Using Field of View

With glFrustum it is often difficult to get the desired view gluPerpective (fovy, aspect, near, far) often provides a better interface



## Clipping after the perspective transformation can cause problems



