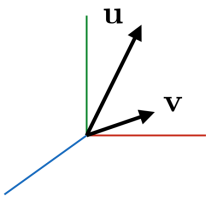
Homework 1
Math refresher

Name:

Student ID:



Answer the questions below, where

$$\mathbf{u} = \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}, \quad A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \\ 2 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

Show your work. If an answer does not exist, write "DNE" and explain.

1. Find a vector $\mathbf{v}_2$ that has the same direction as $\mathbf{v}$ and unit length.
2. What is the angle between $\mathbf{u}$ and $\mathbf{v}$?
3. Fine a vector $\mathbf{w}$ that is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.
4. What is $A^T\mathbf{v}$?
5. What is $A^TA - B$?
6. What is $AA^T - B$?

Bonus:

B1. Find a vector of the form $\alpha\mathbf{u} + \beta\mathbf{v}$ which is orthogonal to $\mathbf{v}$. (α, β are scalars.)

B2. Find two vectors $\mathbf{w}$ and $\mathbf{x}$ such that $\mathbf{w} + \mathbf{x} = \mathbf{u}$, $\mathbf{w}$ is parallel to $\mathbf{v}$, and $\mathbf{x}$ is orthogonal to $\mathbf{v}$.