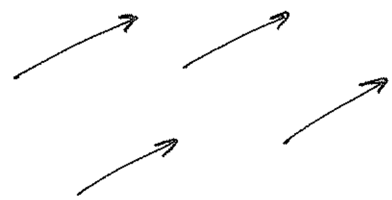
Lecture 3 Notes — Math Review

1. points — locations $\cdot P$ $\cdot Q$

   P, Q, R

2. vectors – direction & magnitude $\underset{\sim}{u}, \underset{\sim}{v}, \underset{\sim}{w}$
   - no notion of location
   - all relative



- vector addition
- scalar multiplication (+, −) scalars : α, β, γ
- vector space
  - coordinate system & basis vectors.

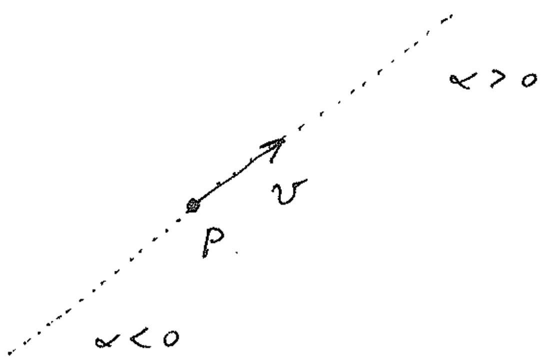
$$\boxed{\underset{\sim}{a} = (a_1, a_2, a_3) = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}}$$

3. vector space vs. affine space

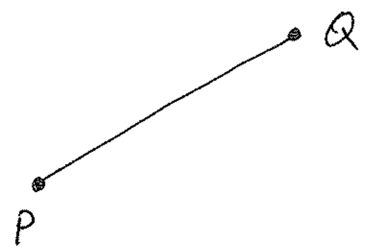
   ↳ point + vector

4. lines

$$P(\alpha) = P + \alpha \underset{\sim}{v}$$



line segments



$$(1-\alpha)P + \alpha Q$$

$$0 \le \alpha \le 1$$

equivalent :

$$P + \alpha\underbrace{(Q - P)}_{= \underset{\sim}{v}}$$

5. Dot Product

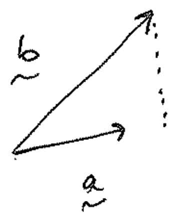
$$\underset{\sim}{a} \cdot \underset{\sim}{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 \qquad \underset{1\times 3}{\begin{pmatrix} a_1 & a_2 & a_3 \end{pmatrix}} \underset{3\times 1}{\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}}$$

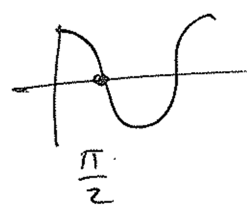
$$\underset{\sim}{a} \cdot \underset{\sim}{b} = \underset{\sim}{a}^T \underset{\sim}{b}$$

$$\underset{\sim}{a} \cdot \underset{\sim}{a} = \overset{a_1^2 + a_2^2 + a_3^2}{a_1 a_1 + a_2 a_2 + a_3 a_3} = \| a \|^2$$

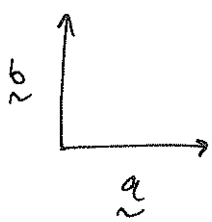
$$\underset{\sim}{a} \cdot \underset{\sim}{b} = \| a \| \, \| b \| \cos\theta$$

<u>geometric interpretation</u>: ($\underset{\sim}{a}$ unit vector)



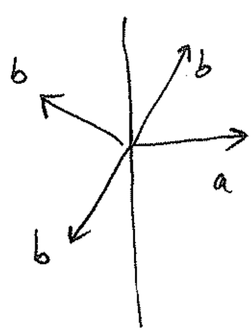


Q.

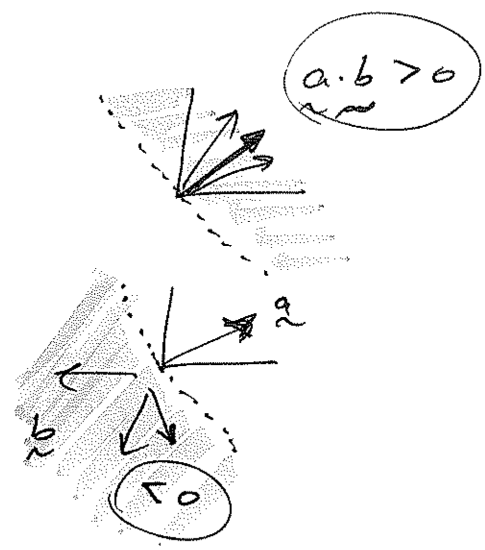


$$\underset{\sim}{a} \cdot \underset{\sim}{b} = ?$$

Q



$$\underset{\sim}{a} \cdot \underset{\sim}{b} \overset{?}{\substack{> \\ < \\ =}} 0$$



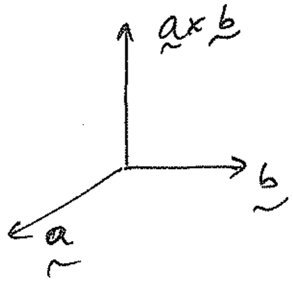
region of −ive dot product

Cross Product

$$\underset{\sim}{a} \times \underset{\sim}{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = (a_2 b_3 - a_3 b_2)\hat{i} + (a_3 b_1 - a_1 b_3)\hat{j} + (a_1 b_2 - a_2 b_1)\hat{k}$$

result of cross product is another vector!

Right-hand rule:

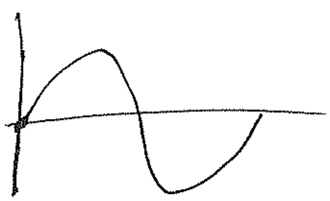


$$\|\underset{\sim}{a} \times \underset{\sim}{b}\| = \|a\| \, \|b\| \sin\theta$$

magnitude of the resulting vector

direction is given by right-hand rule.

Q $\underset{\sim}{a} \times \underset{\sim}{a} = ?$



$= 0$

$\sin\theta$

matrices

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

a_{ij}

ith row

j^{th} column

$$\begin{array}{cc} \text{2 rows} & \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} \\ & \text{3 columns} \end{array}$$

2×3 matrix

matrix multiplication

$$\begin{array}{cc} A & B \\ m \times k & k \times n \end{array}$$

- you can't just multiply any two matrices
- they have to be $\boxed{\text{compatible}}$.

$y = Ax$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix} x_1 + \begin{pmatrix} a_{12} \\ a_{22} \\ a_{32} \end{pmatrix} x_2 + \begin{pmatrix} a_{13} \\ a_{23} \\ a_{33} \end{pmatrix} x_3$$

matrix mult.
as a sequence
of transformations

$$f_1(x) = Ax$$

$$f_2(x) = Bx$$

$$f(x) = f_1(f_2(x)) = A(f_2(x))$$

$$= (AB)x$$

$$C = AB$$

$$f(x) = (AB)x$$

$$f(x) = Cx$$

$$f_1 \circ f_2$$

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transpose of a matrix

$$a_{ji} \leftarrow a_{ij}$$