On a Linear Program for Minimum Weight Triangulation

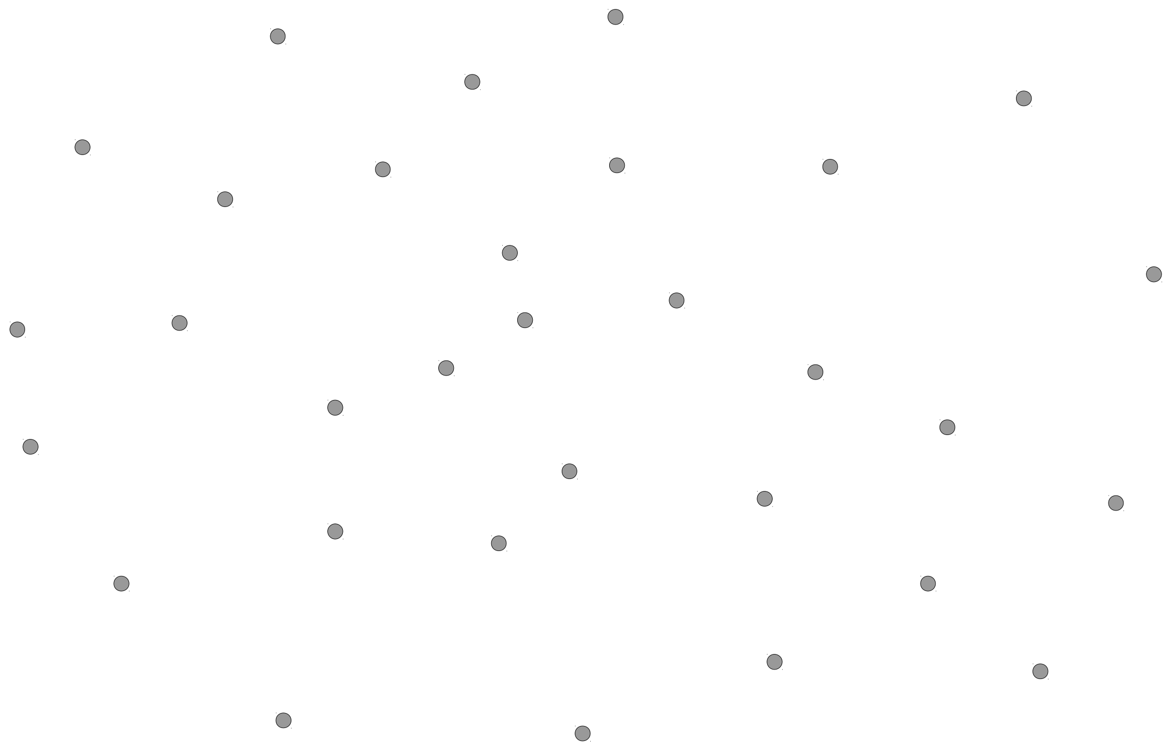
Arman Yousefi and Neal Young

University of California Riverside

January 2012

Minimum Weight Triangulation (MWT)

- Input: a set of point in 2D



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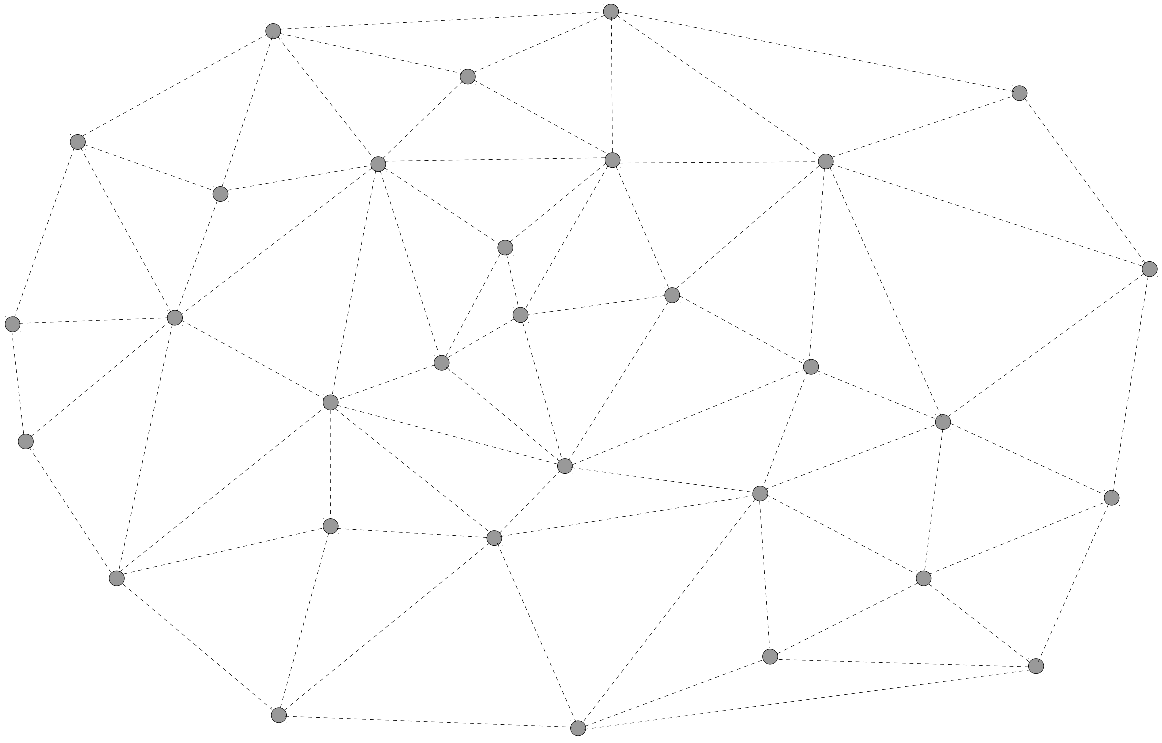
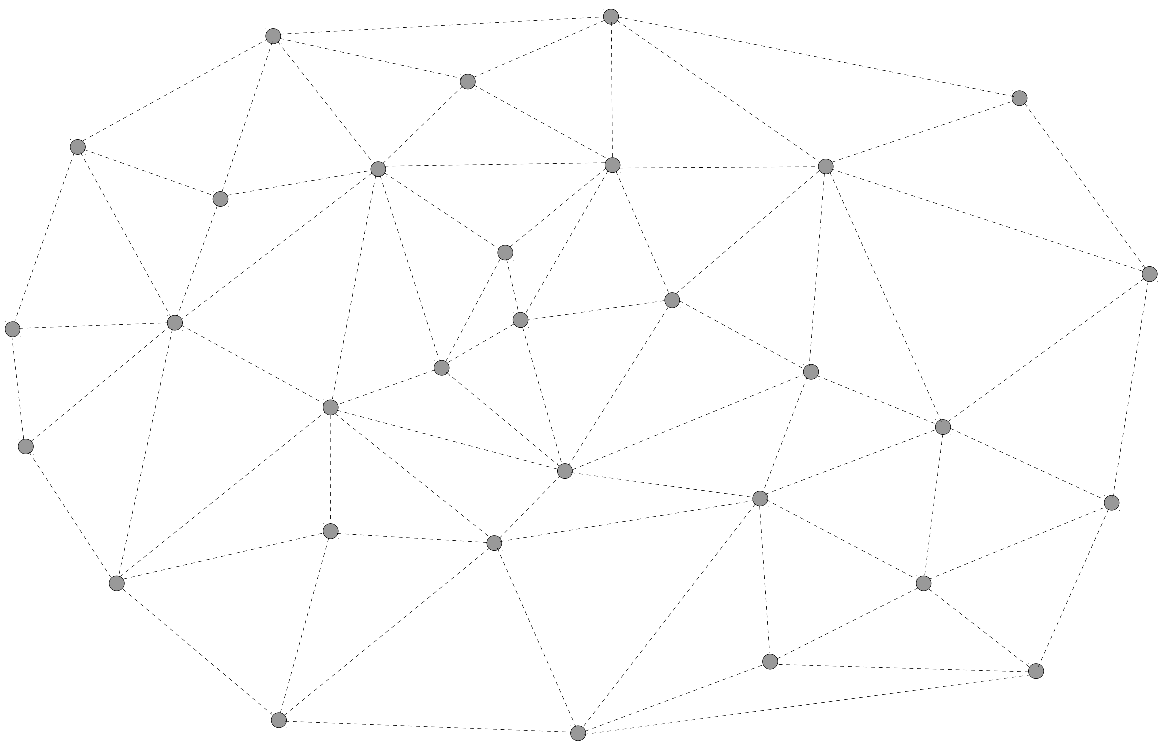


Minimum Weight Triangulation (MWT)

- Input: a set of point in 2D
- Goal: a triangulation T minimizing $\sum_{e \in T} |e|$



Outline

- Previous Results
- Linear Program
- Heuristics
- Integrality Gap

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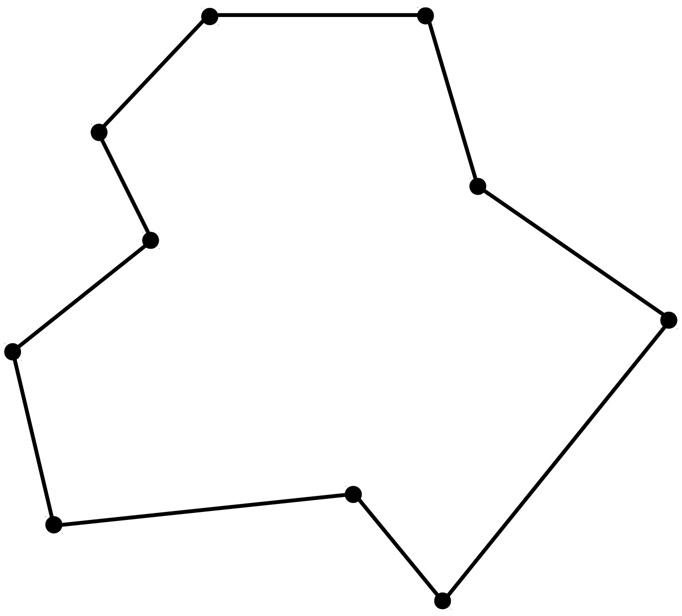
- [2006] Remy and Steger. A quasi-polynomial time approximation scheme for minimum weight triangulation. ---QPTAS

Previous Results (cont'd)

Simple Polygons

- [1979] Gilbert. New results on planar triangulations.
- [1980] Klincsek. Minimal triangulations of polygonal domains.



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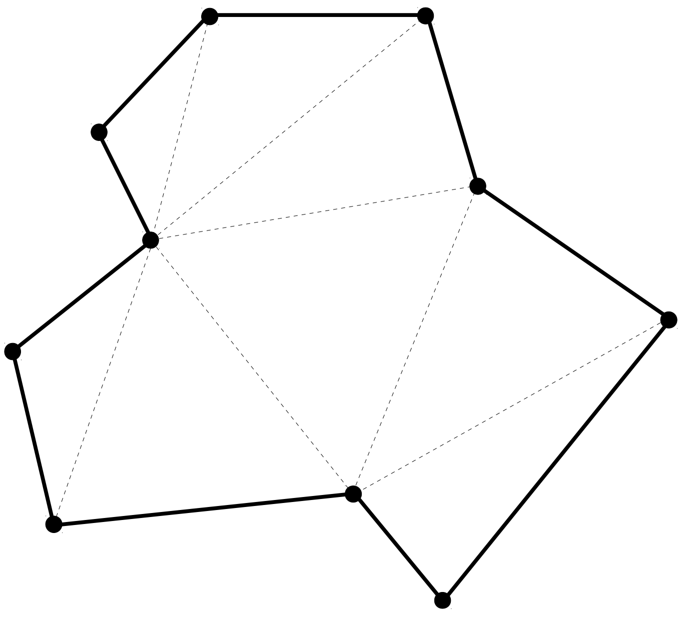

Heuristics

- Edges In:
  - β-skeleton

    [Keil '93][Yang '95][Cheng and Xu '96]
  - LMT-skeleton

    [Dickerson et al. '97][Beirouti and Snoeyink '98][Cheng et al. '96]
    [Aichholzer et al '99][Belleville et al. '96][Bose et al. '02]
  - Mutual Nearest Neighbors

    [Gilbert '79][Yang et al. '94]
- Edges Out:
  - Diamond Test

    [Das and Joseph '89][Drysdale et al. '01]

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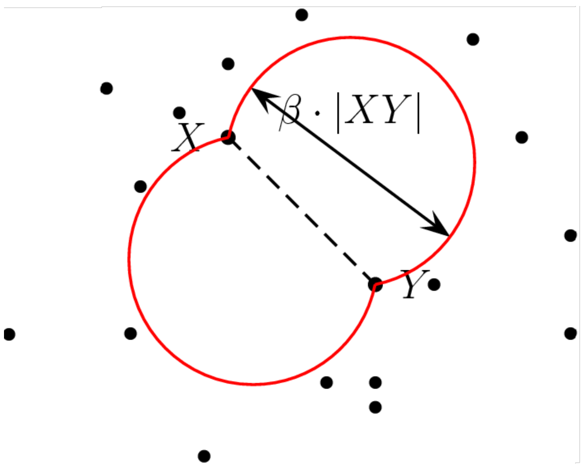

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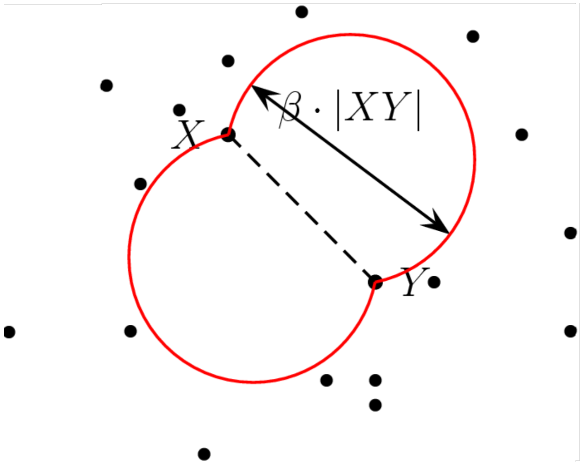

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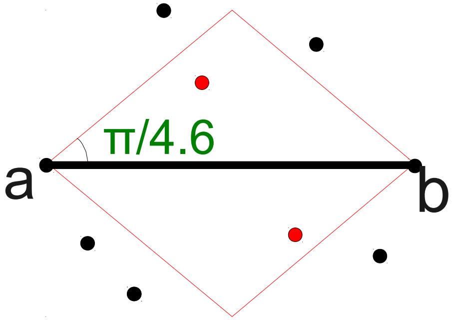

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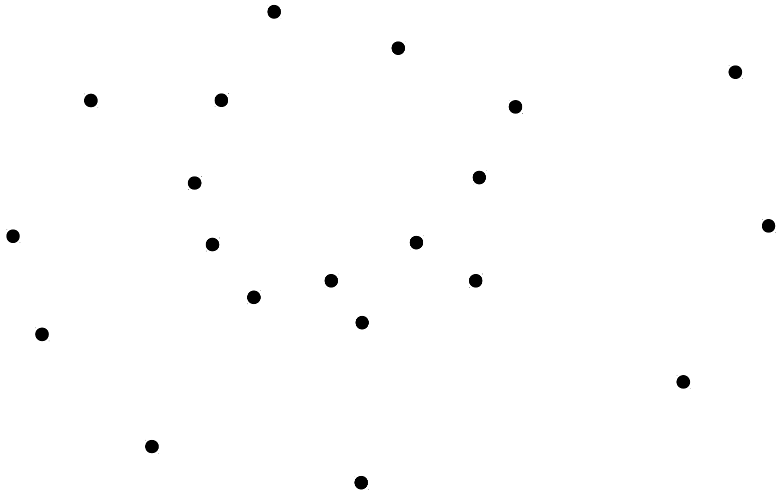

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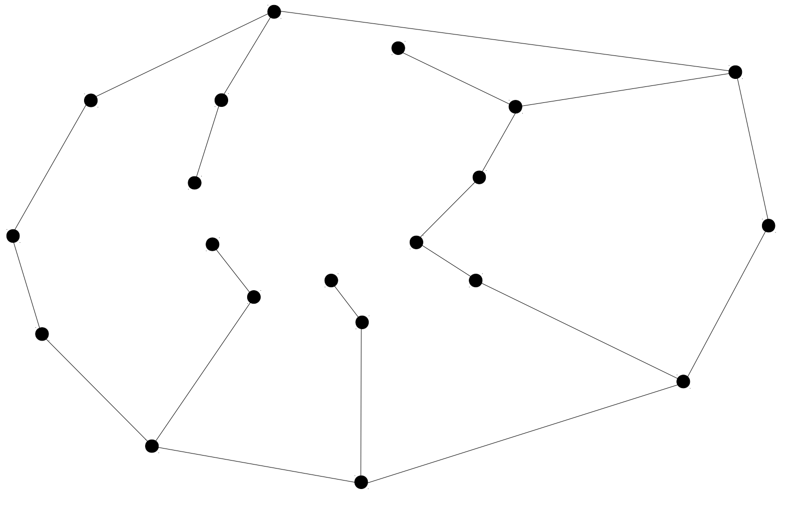

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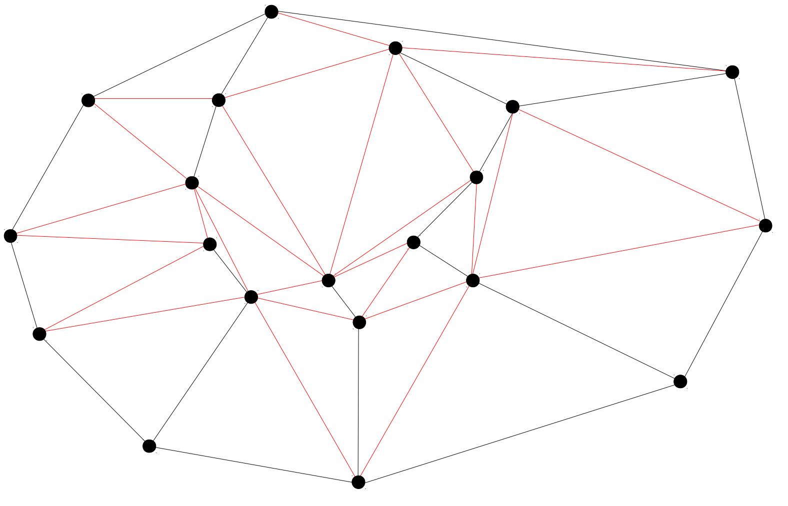

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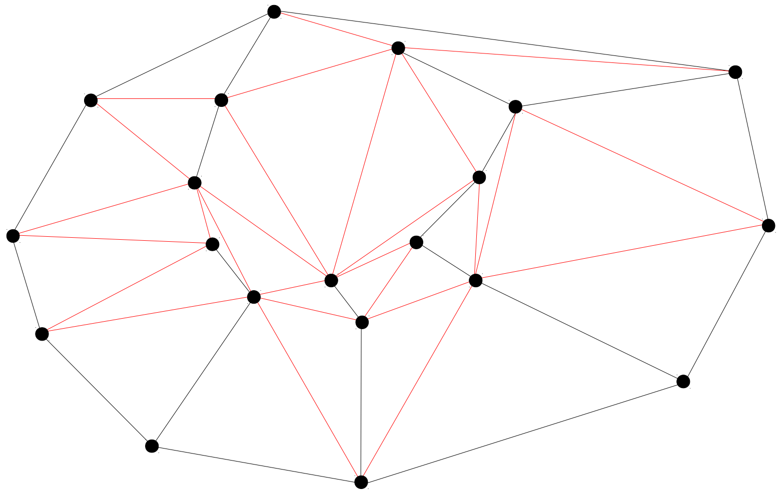

- Most random instances with 40,000 points are solvable in this way.

[Dickerson et al. '97]

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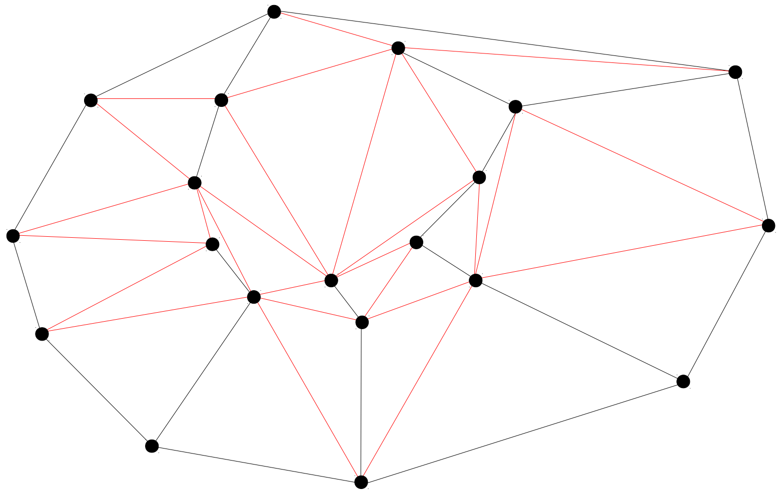

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  [Dickerson et al. '97]
- For random instances the expected number of components of LMT-skeleton is $\Omega(n)$. (with astronomically small constant 10^{-51}). [Bose et al. '02]

Linear Programs for MWT

Triangle-based LP:

- [1985] Dantzig et al. Triangulations (tilings) and certain block triangular matrices.
- [1996] Loera et al. The polytope of all triangulations of a point configuration.
- [2004] Kirsanov. Minimal discrete curves and surfaces.

Edge-based LP:

- [1997] Kyoda et al. A branch-and-cut approach for minimum weight triangulation.
- [1996] Kyoda. A study of generating minimum weight triangulation within practical time.
- [1996] Ono et al. A package for triangulations.
- [1998] Tajima. Optimality and integer programming formulations of triangulations in general dimension.
- [2000] Aurenhammer and Xu. Optimal triangulations.

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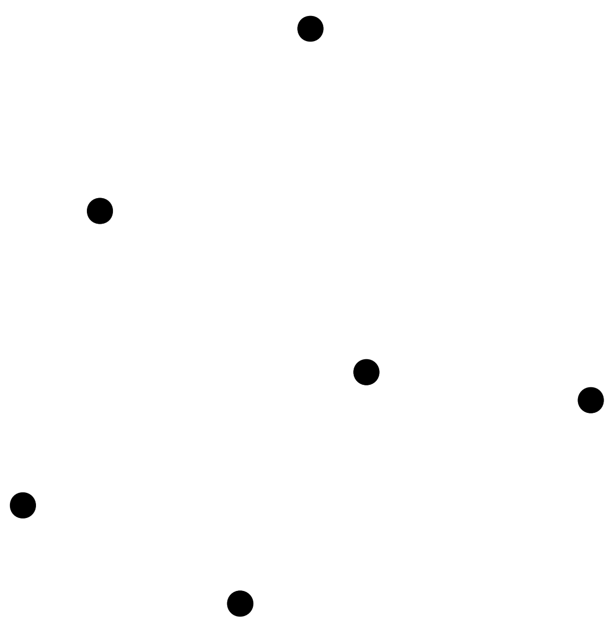

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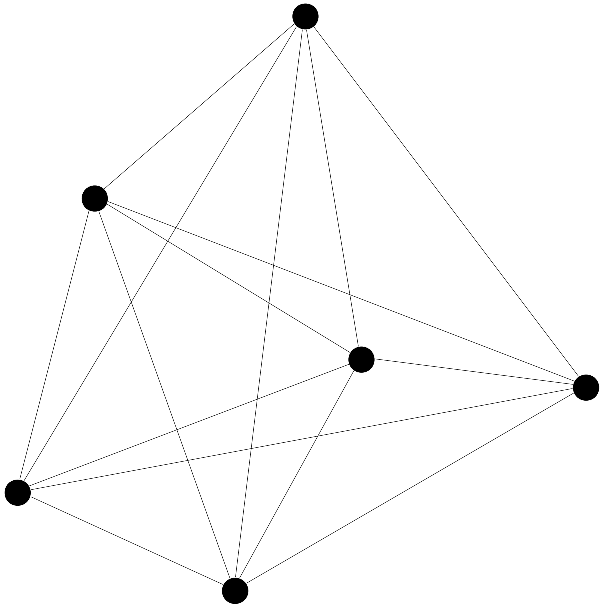

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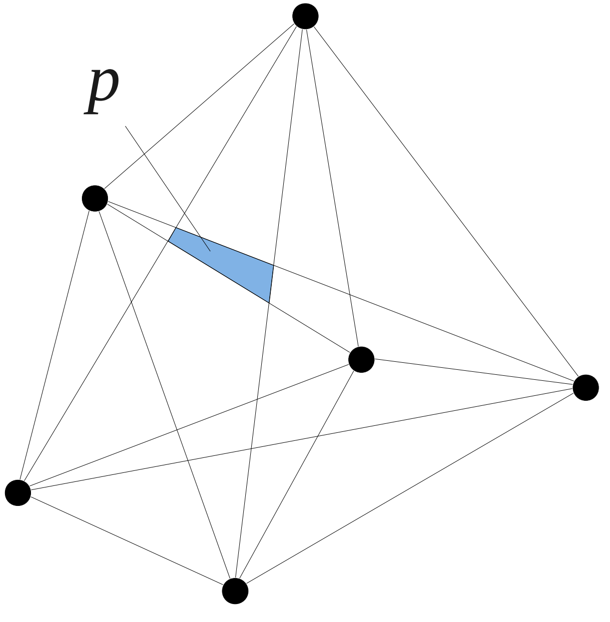

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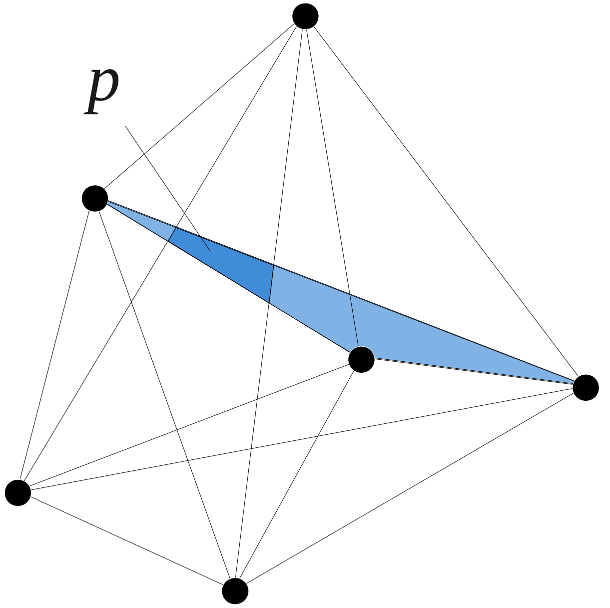

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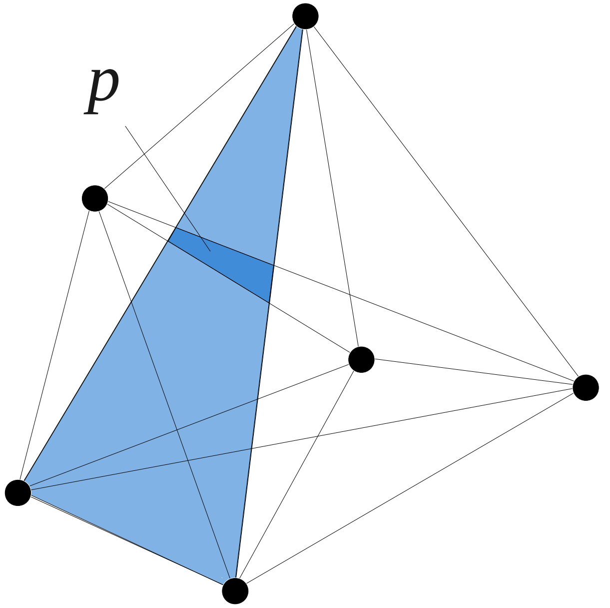

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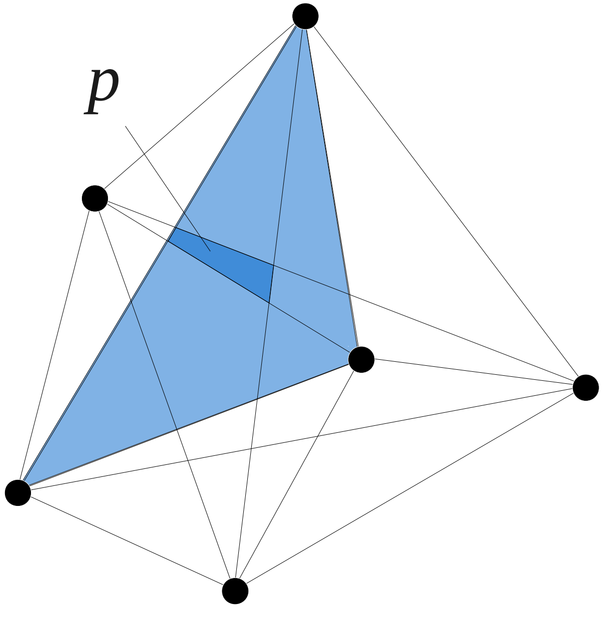

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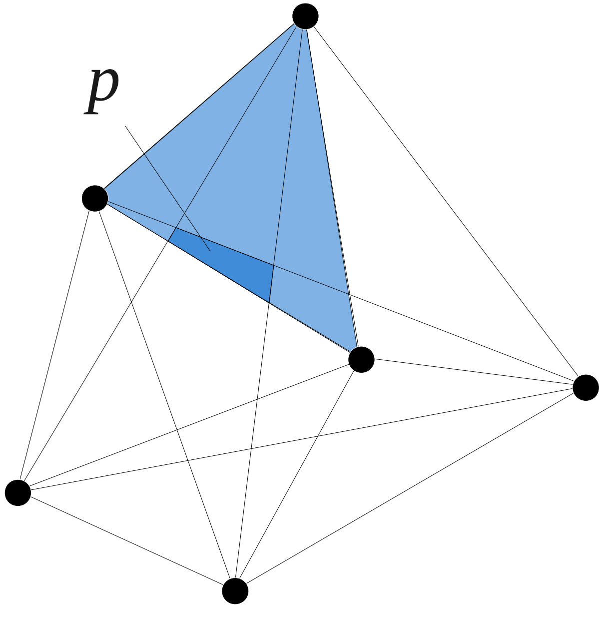

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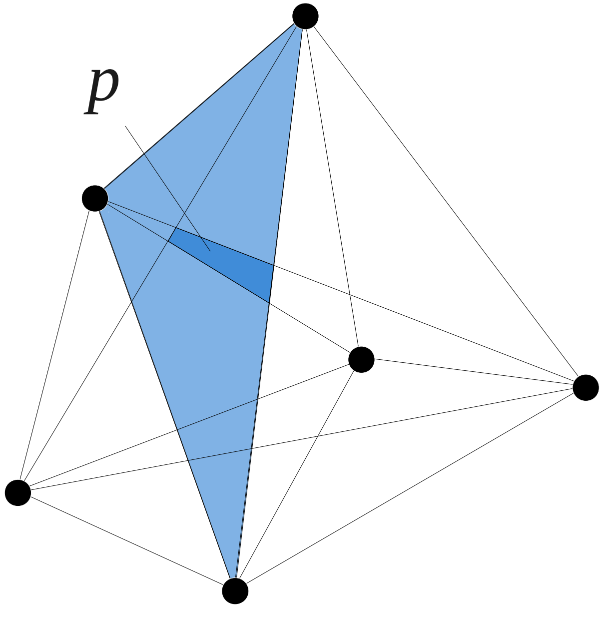

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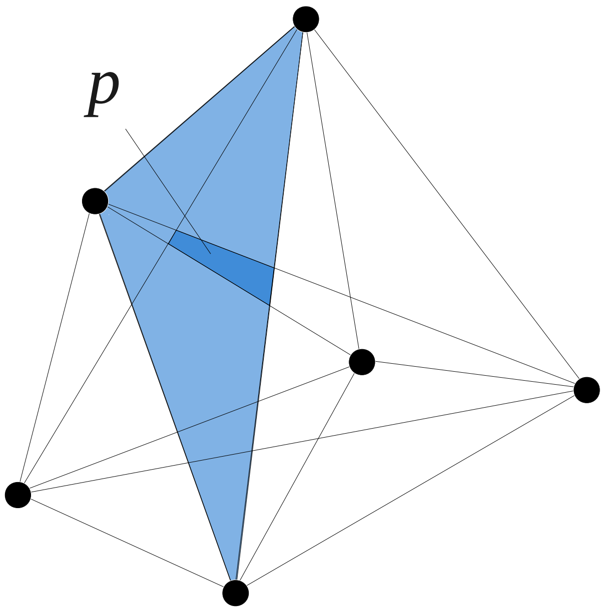

$$\sum_{t \ni p} X_t = 1, \quad \forall\, p$$

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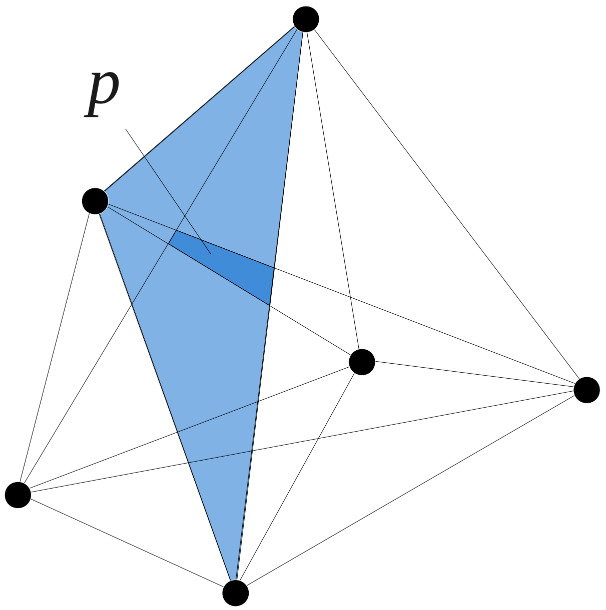

minimize $\sum_{t \in \Delta} |t| \cdot X_t$

subject to

$$\sum_{t \ni p} X_t = 1, \quad \forall\, p$$

$$X_t \in \{0, 1\}, \quad \forall\, t \in \Delta$$

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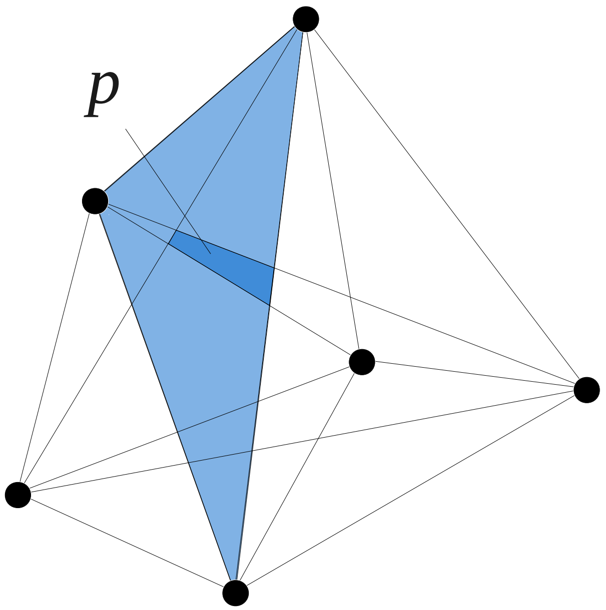

minimize $$\sum_{t \in \Delta} |t| \cdot X_t$$

subject to

$$\sum_{t \ni p} X_t = 1, \quad \forall\, p$$

$$0 \leqslant X_t \leqslant 1 \quad \forall\, t \in \Delta$$

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The integrality gap of the LP in the simple-polygon case is one.

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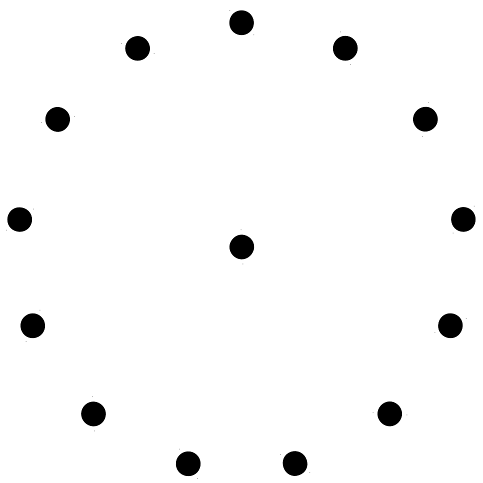

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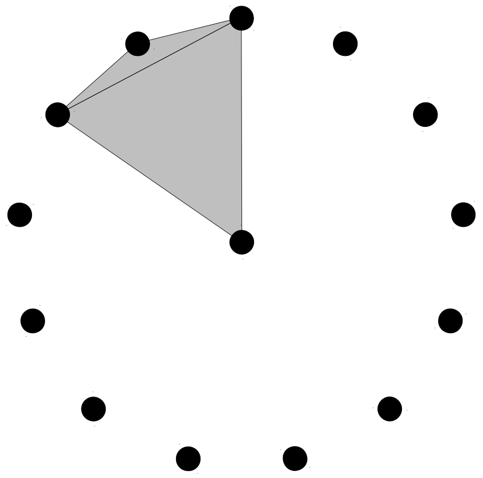

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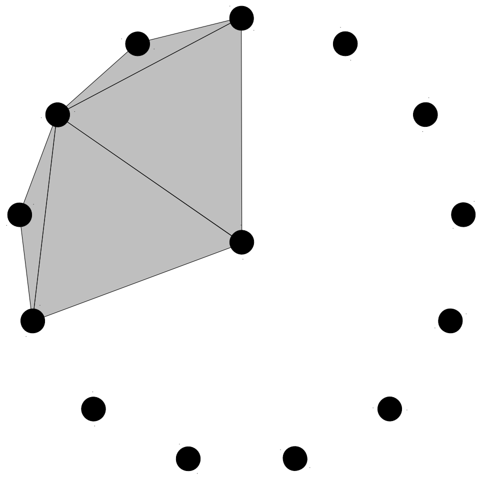

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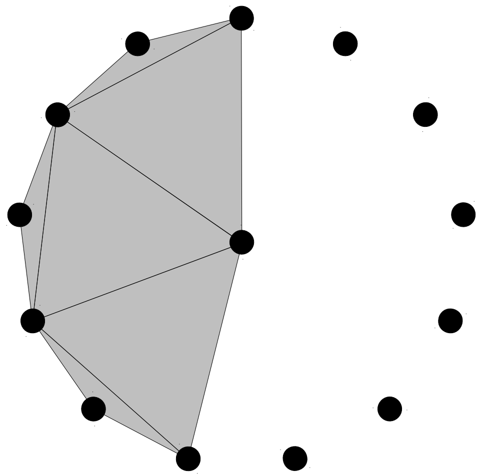

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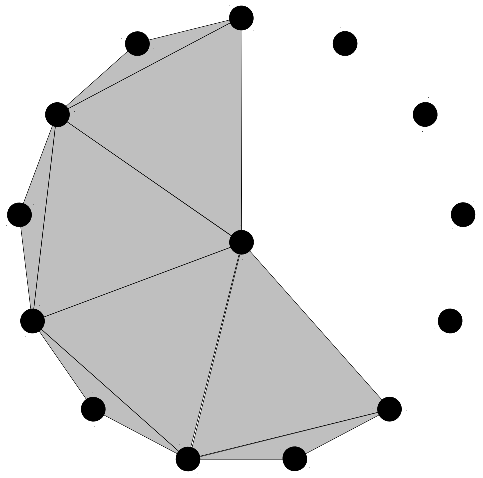

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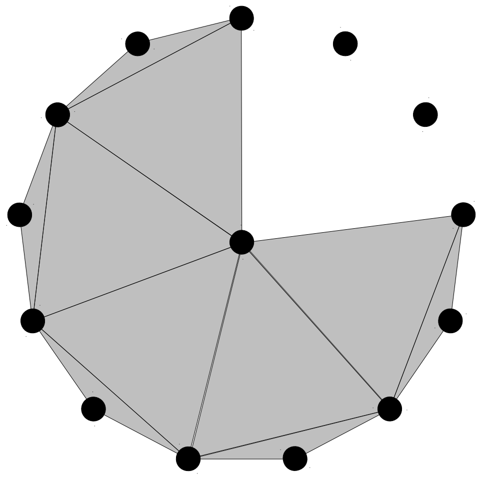

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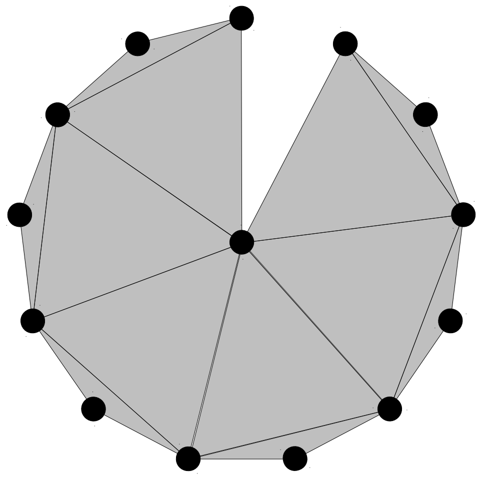

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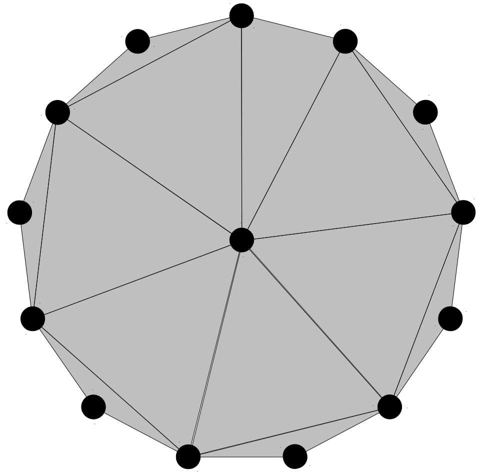

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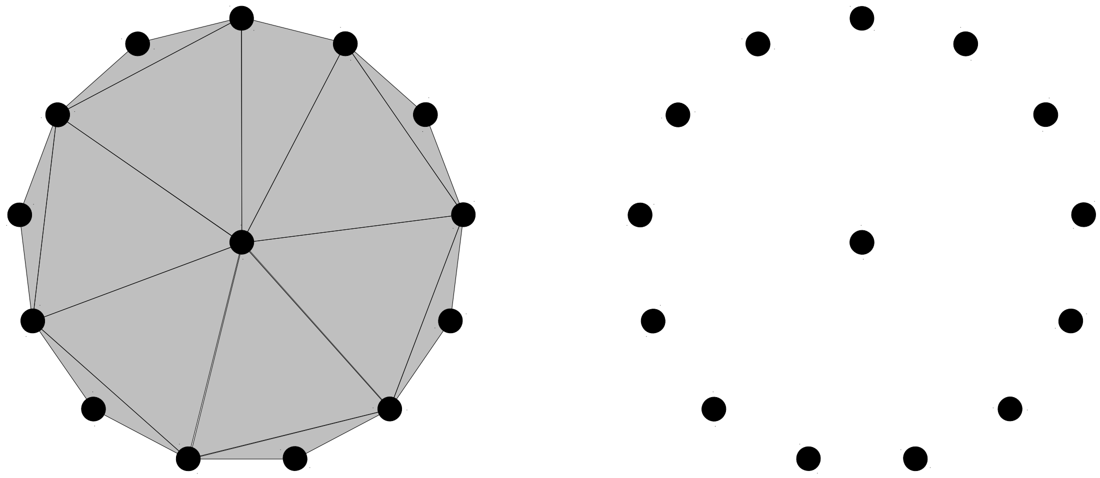

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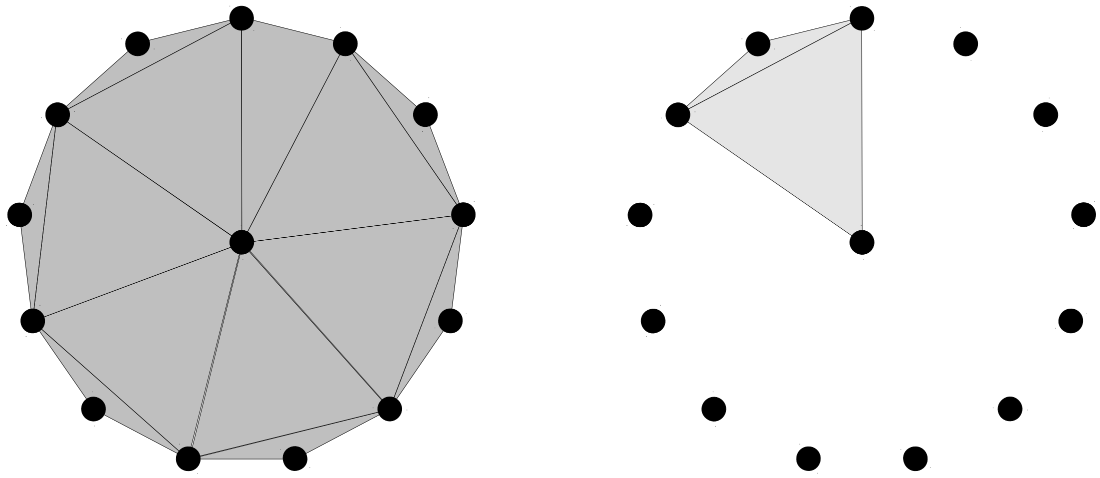

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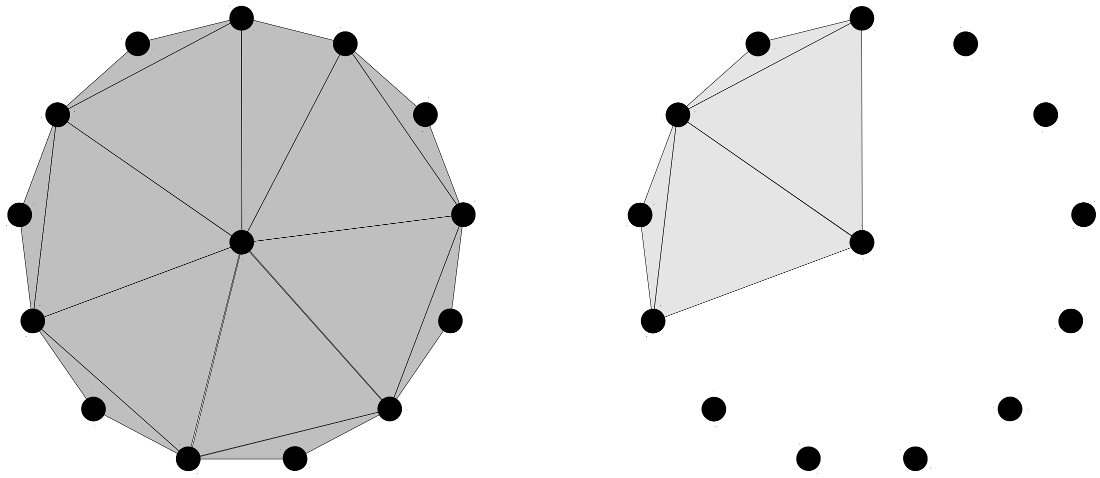

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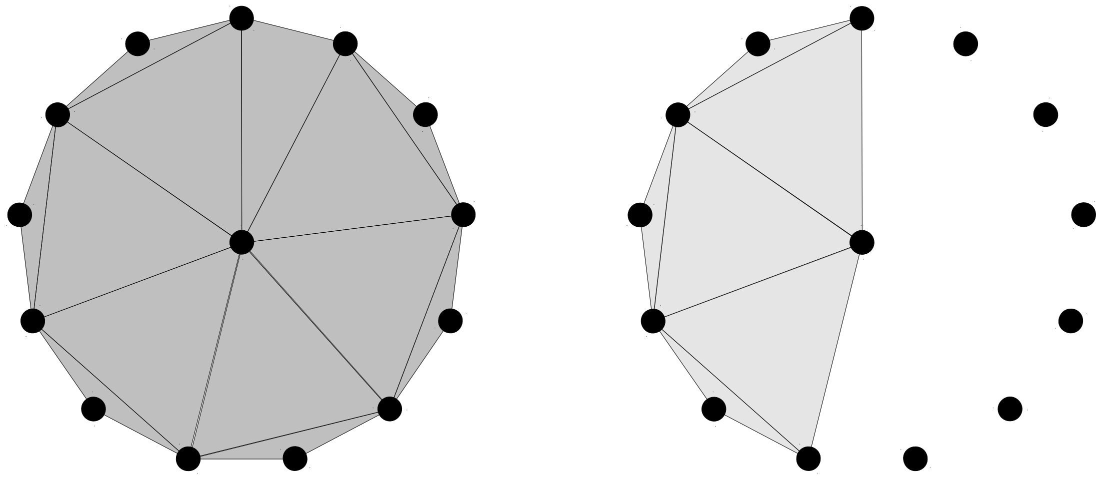

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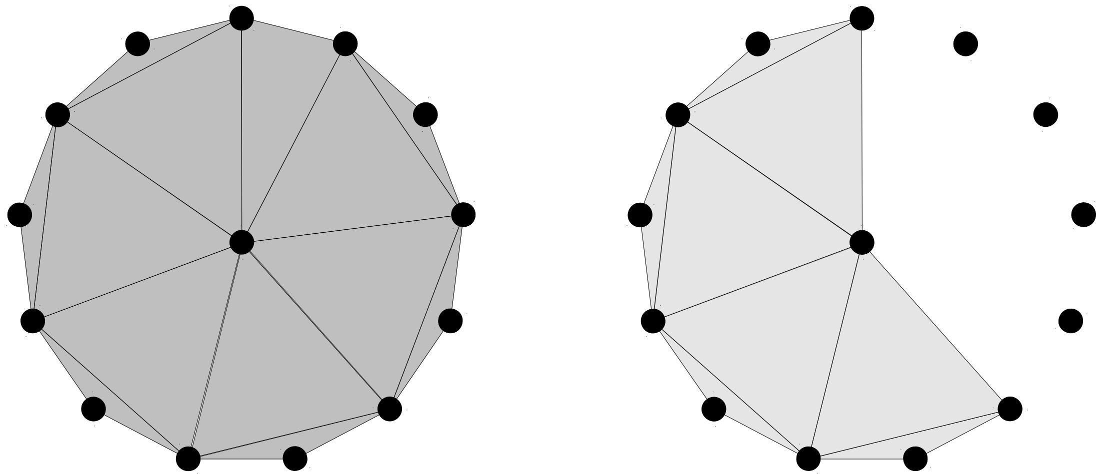

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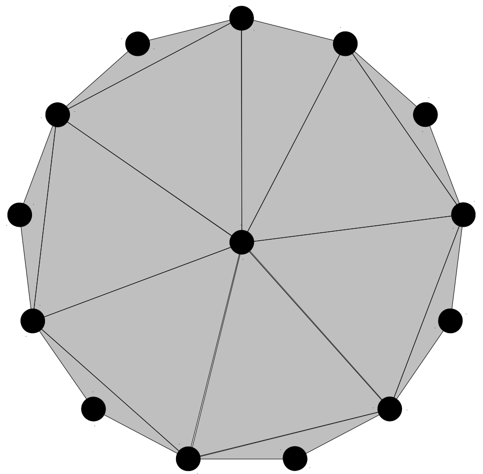

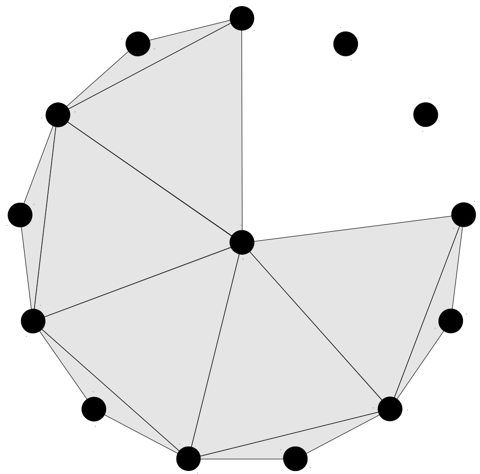

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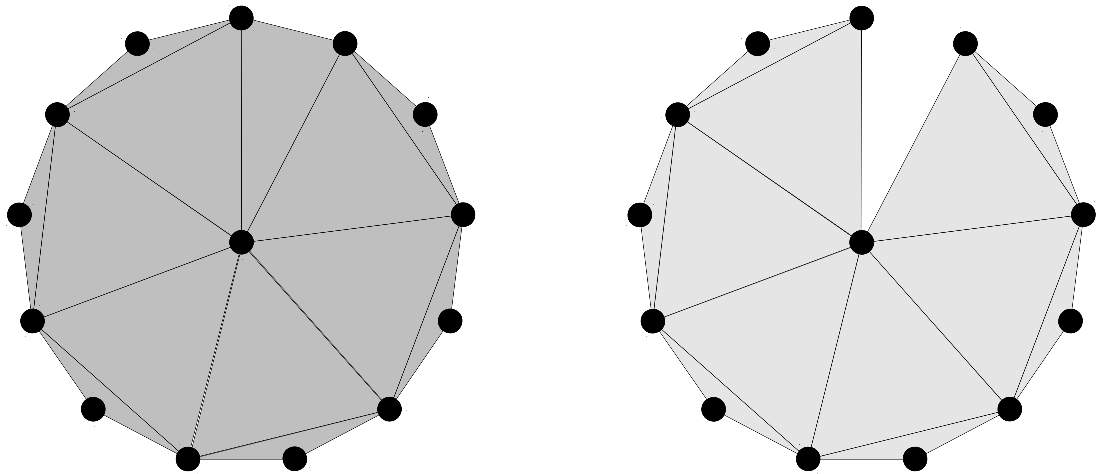

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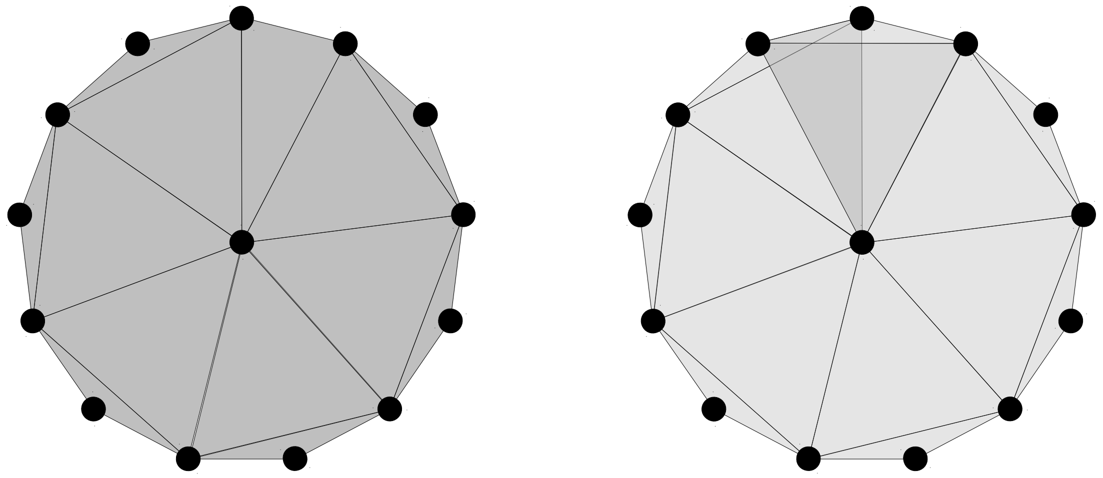

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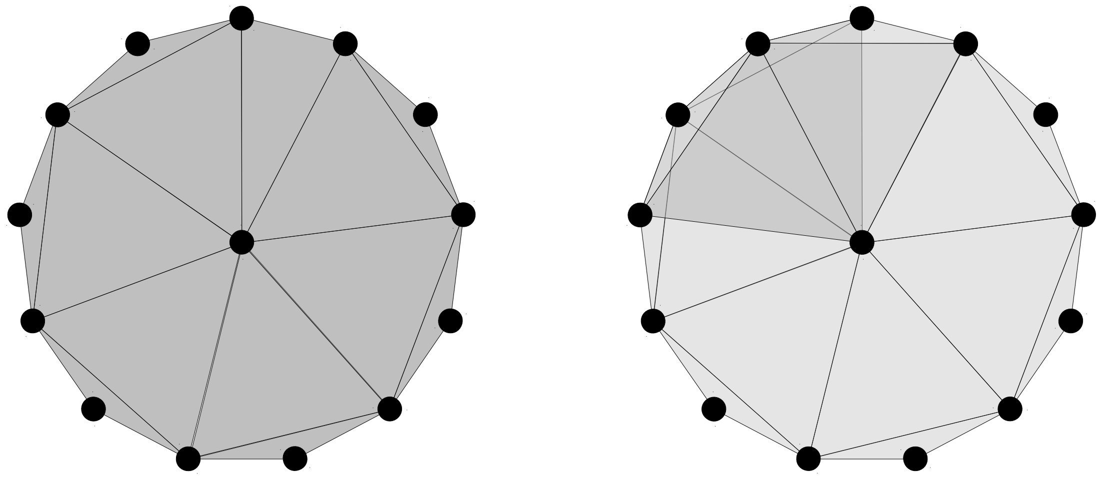

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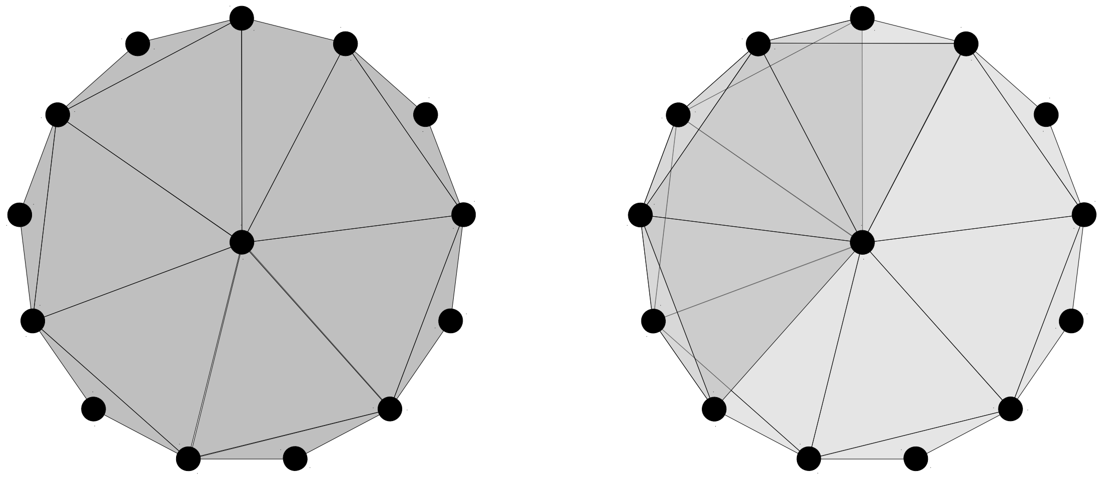

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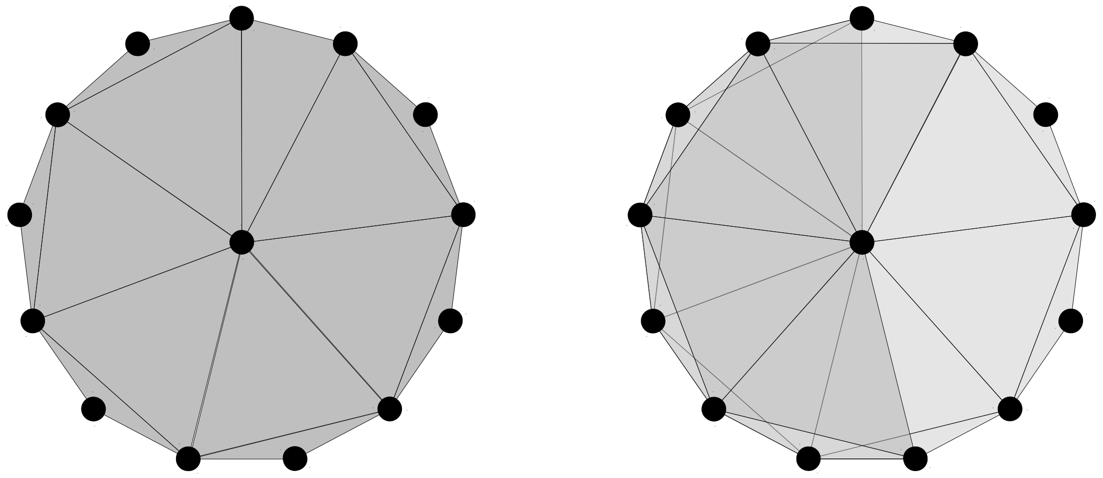

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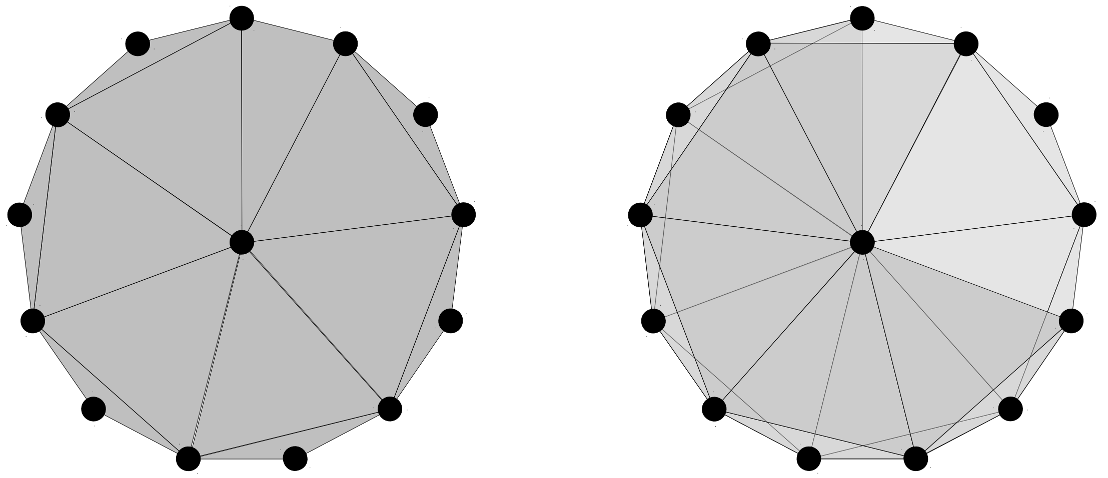

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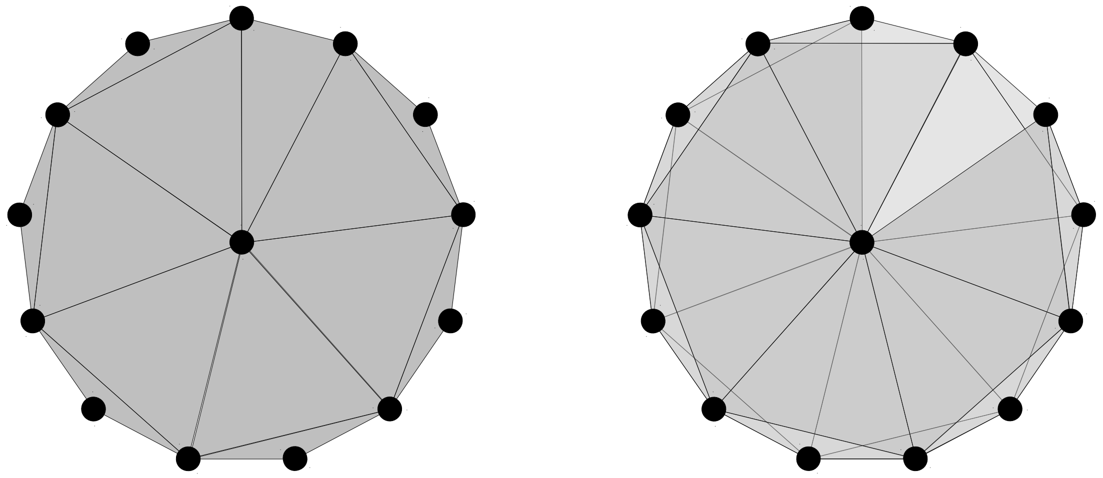

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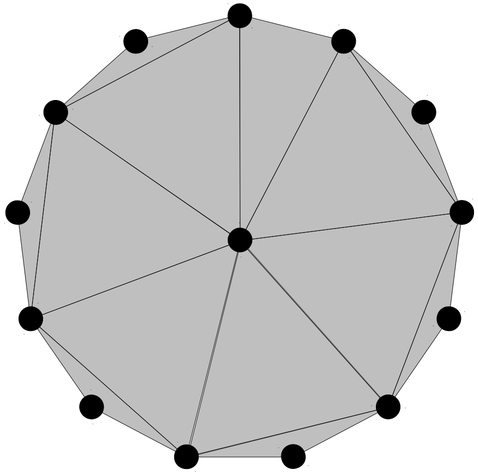

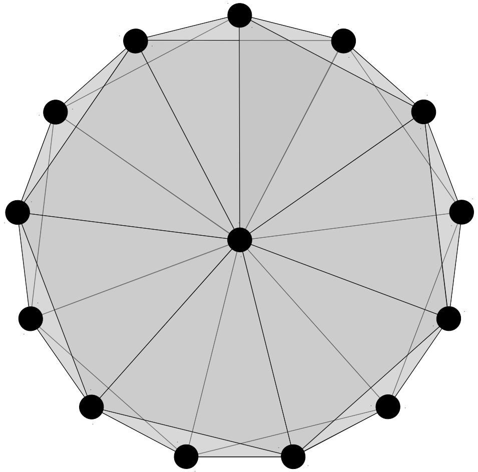

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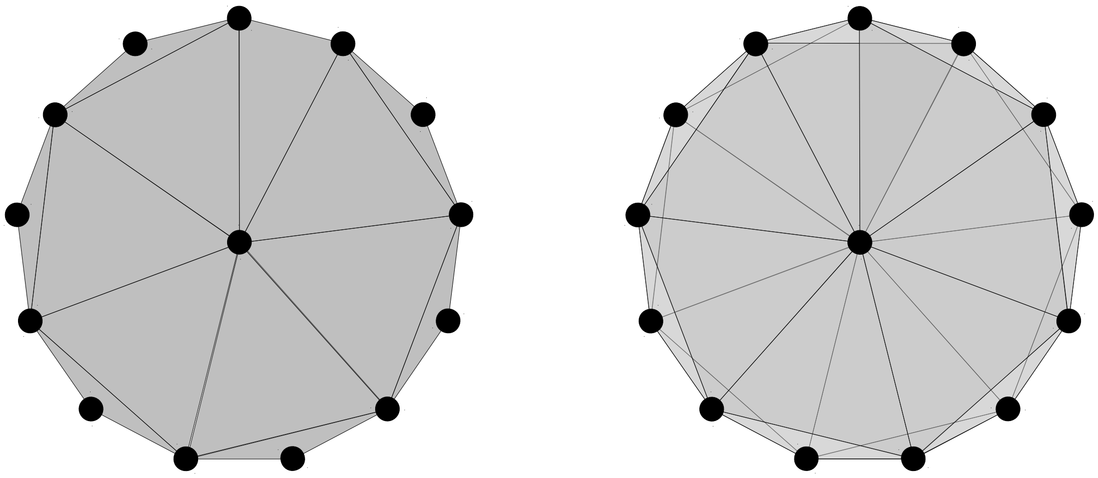

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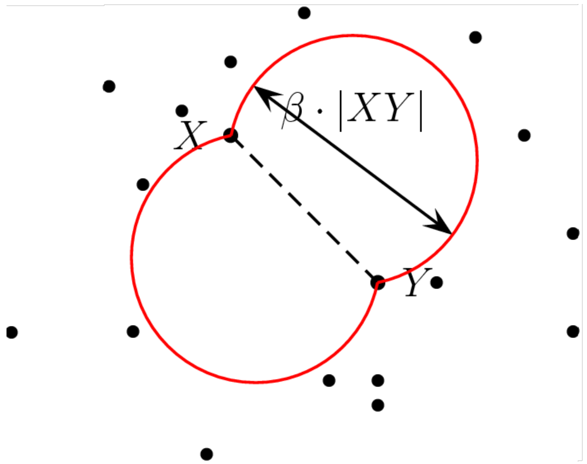
III. Given any instance, if the heuristics find the MWT, then so does the LP.

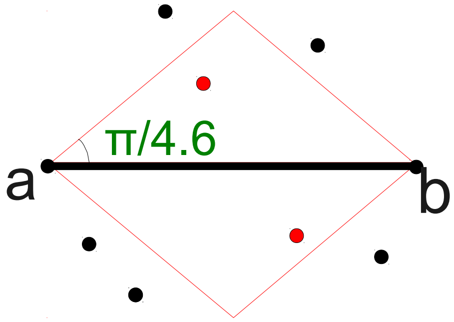
Outline

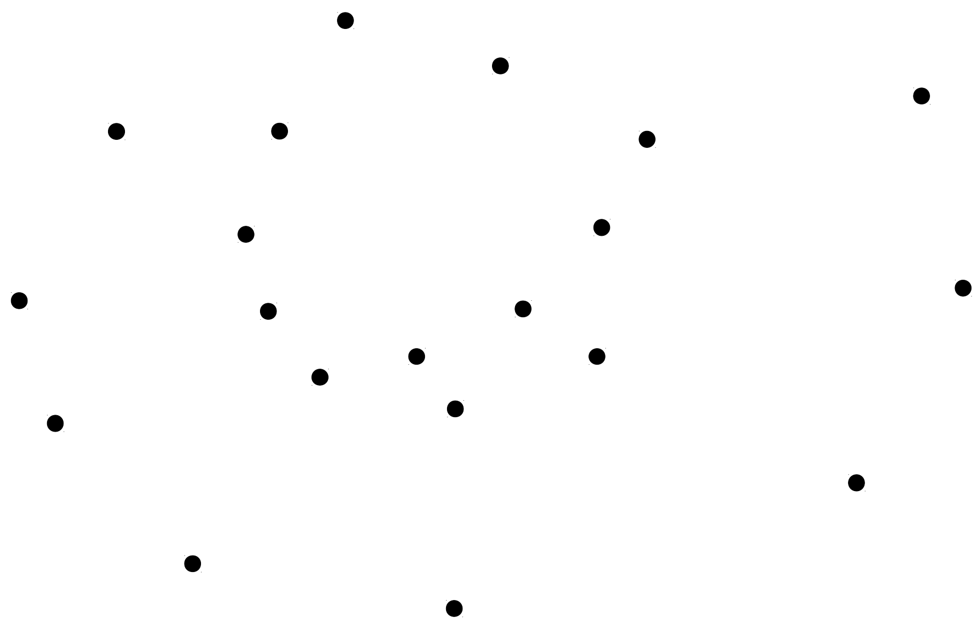
- Previous Results
- Linear Program
- Heuristics
- Integrality Gap

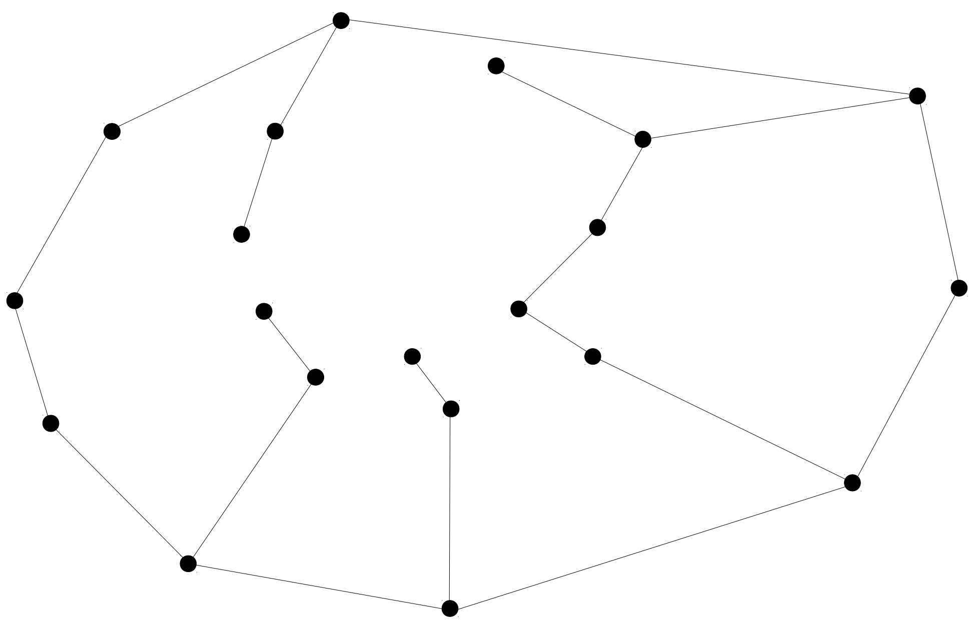
Heuristics

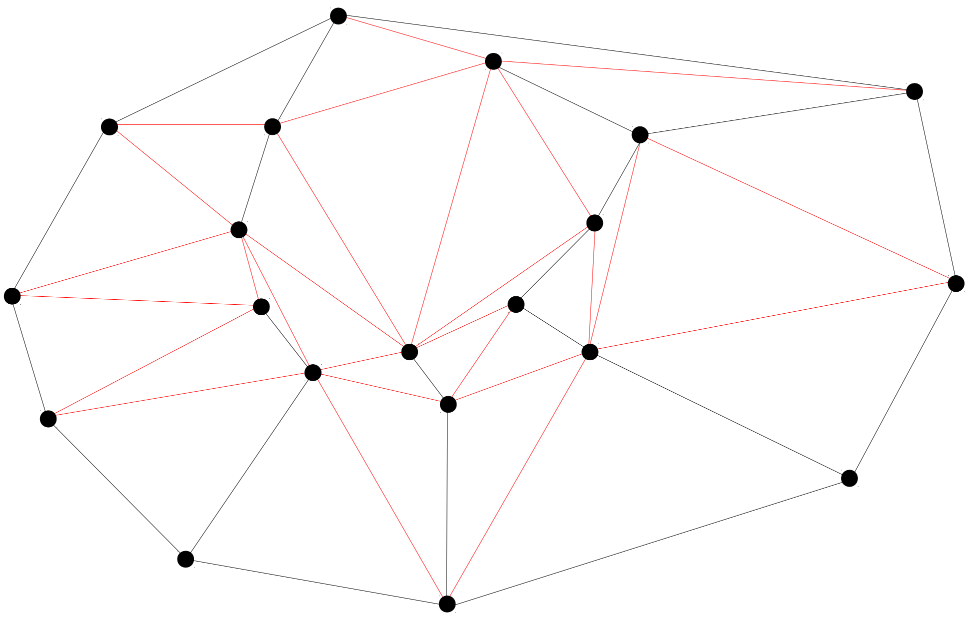
- Edges In:
  - β-skeleton
  - LMT-skeleton
  - Mutual Nearest Neighbors
- Edges Out:
  - Diamond Test



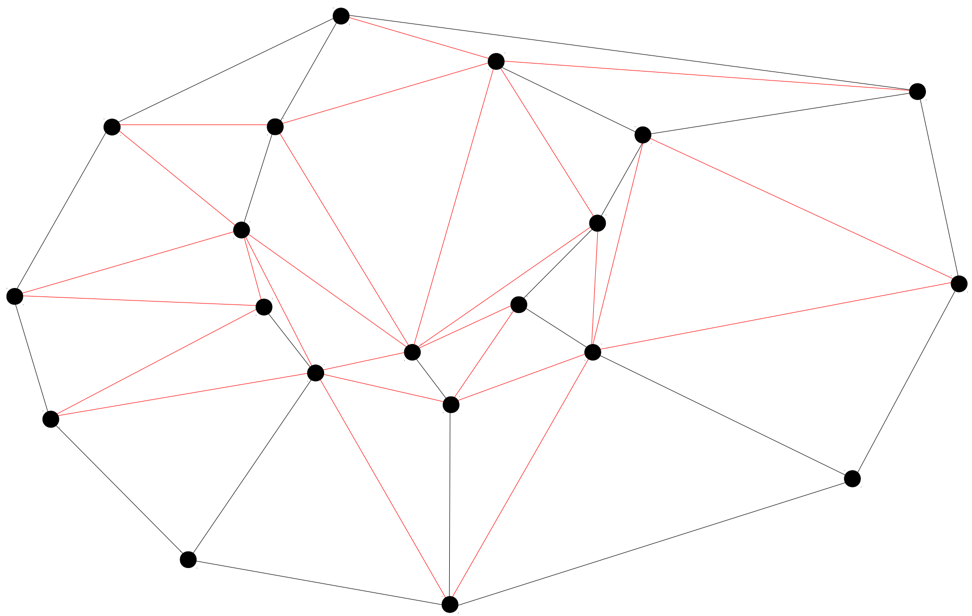








Theorem: Given any instance, if the heuristics find the MWT, then so does the LP (i.e. every optimal extreme point of the LP is the incidence vector of an MWT).



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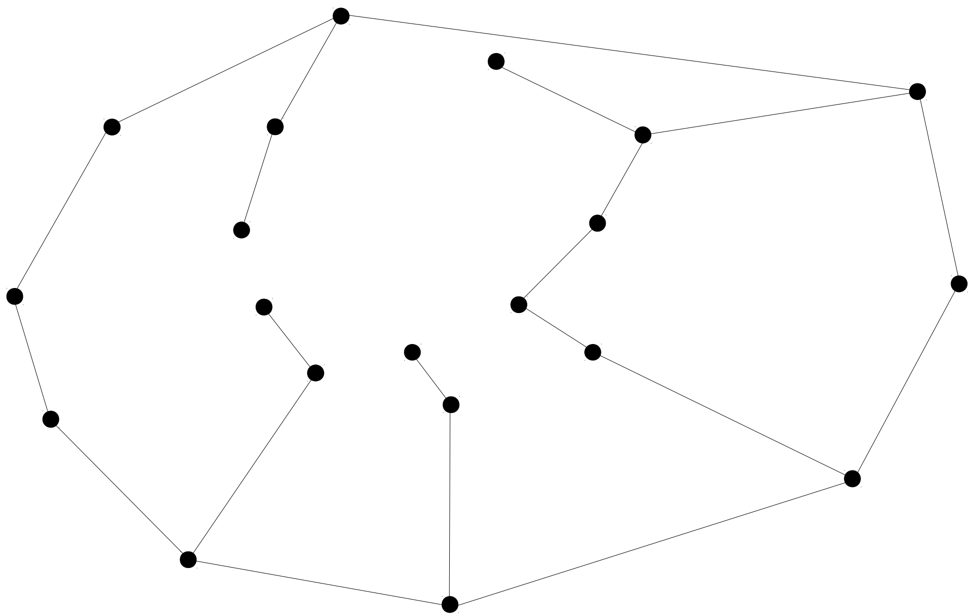

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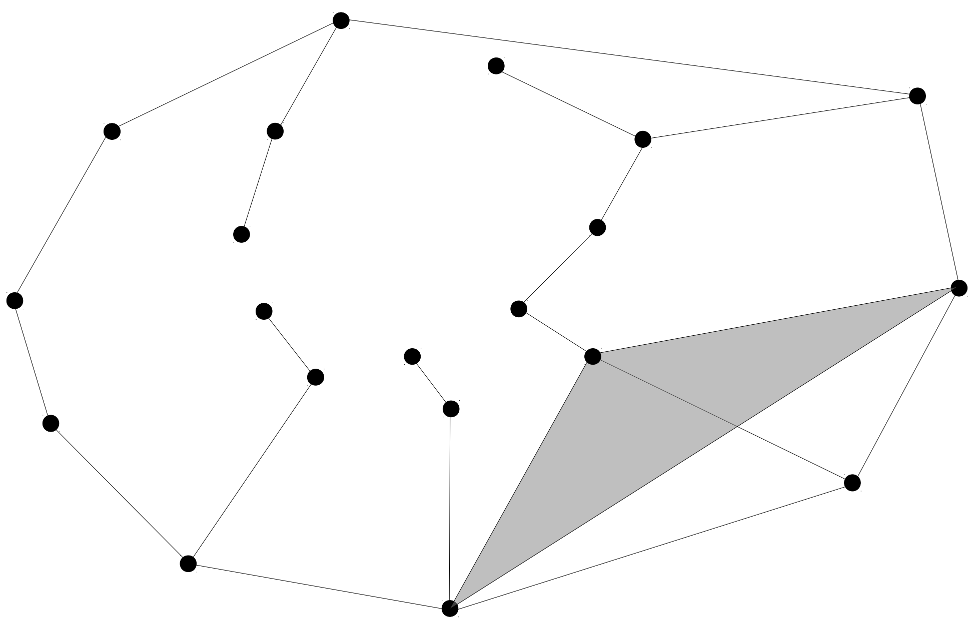

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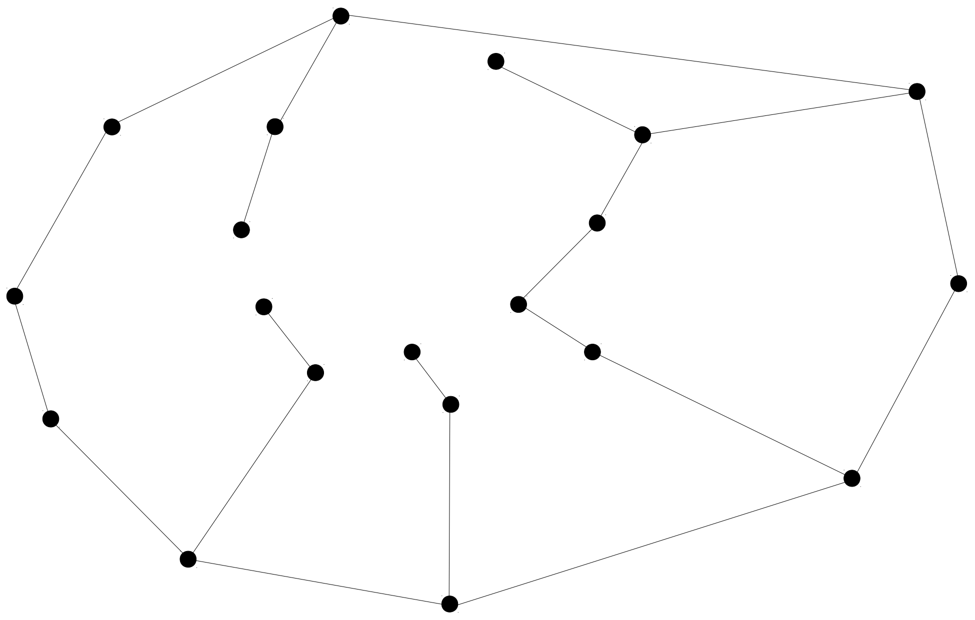

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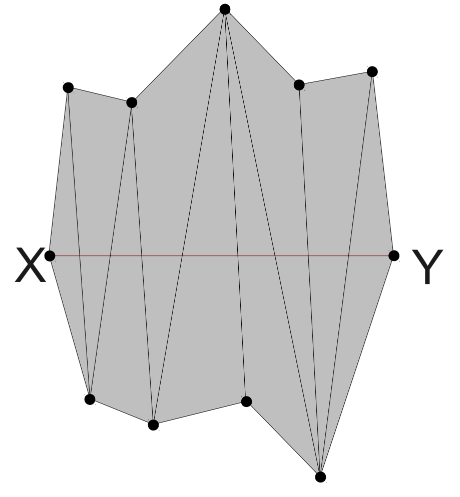

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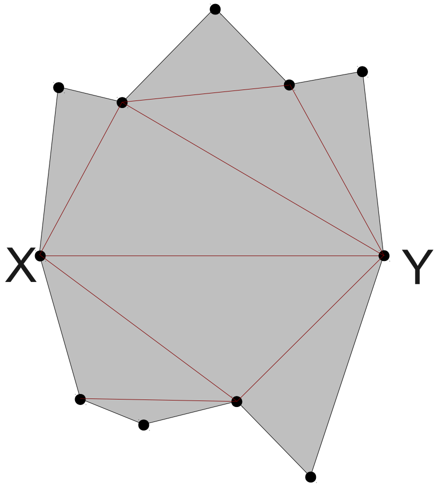

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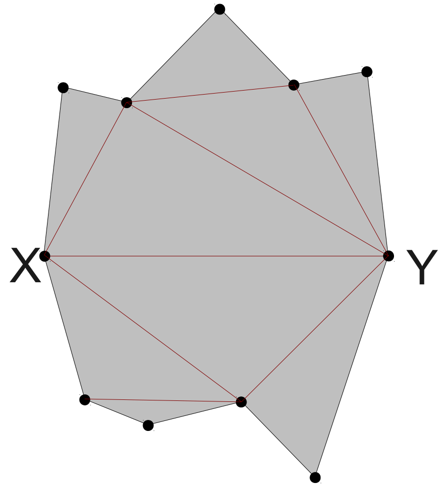

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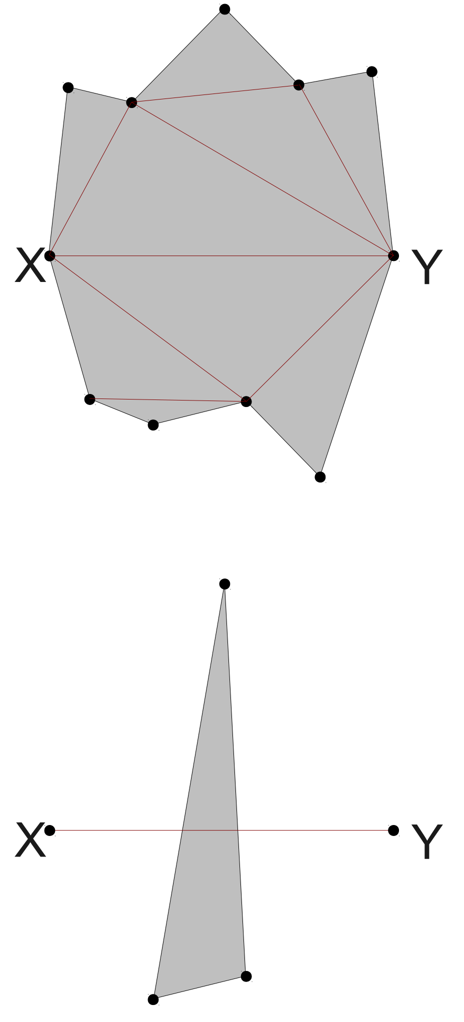

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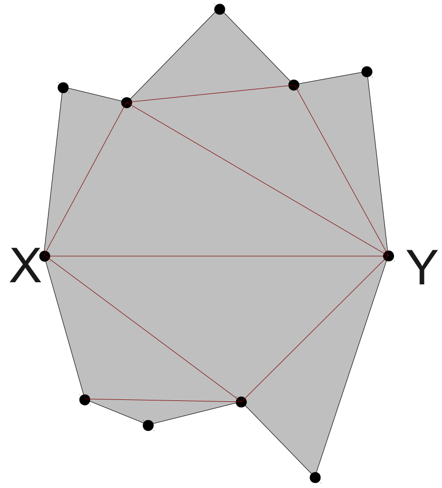

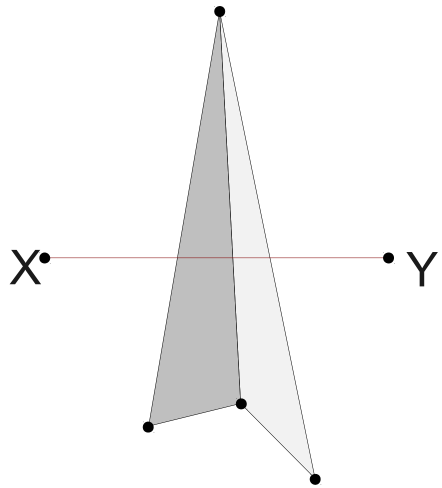

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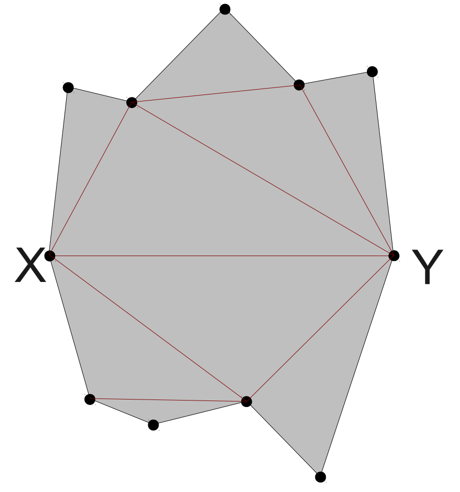

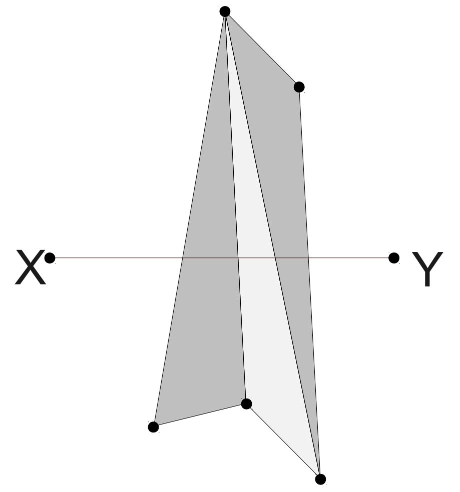

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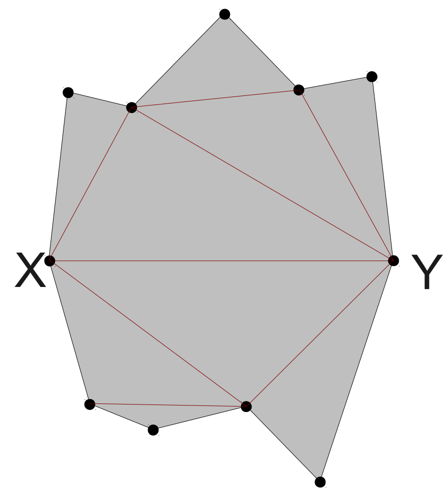

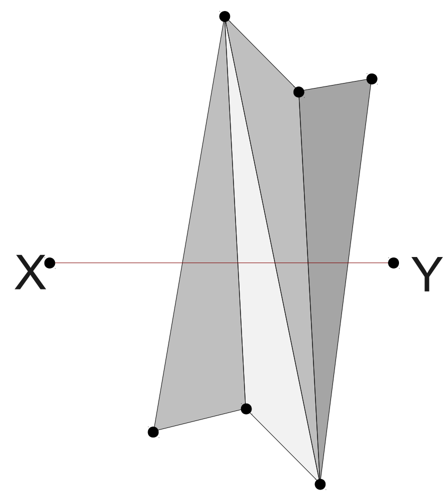

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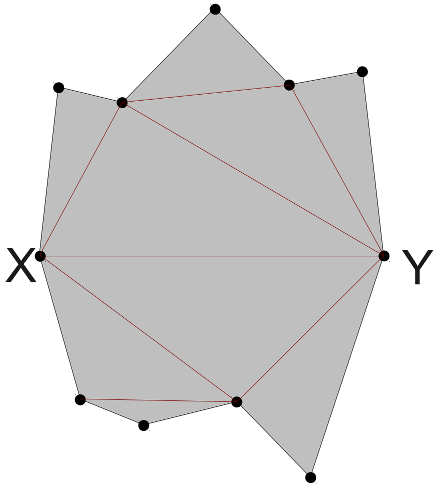

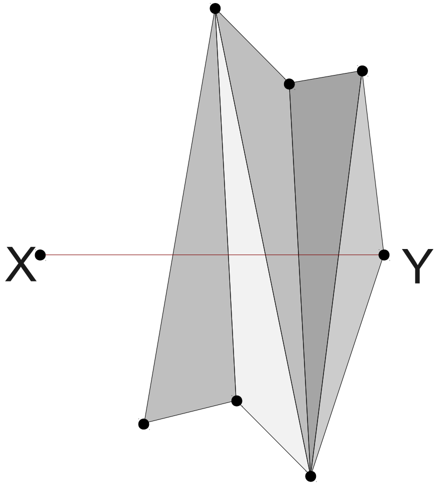

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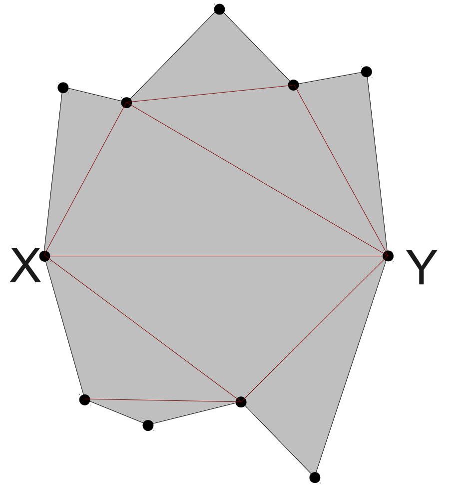

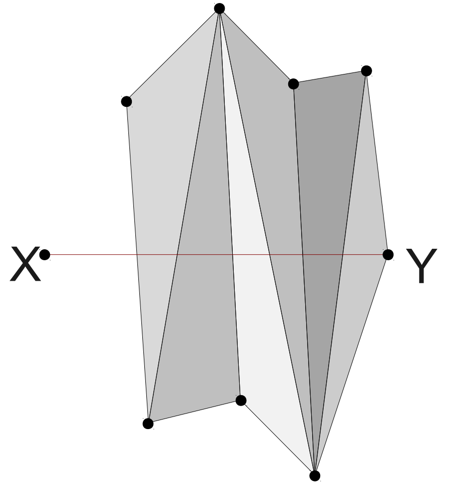

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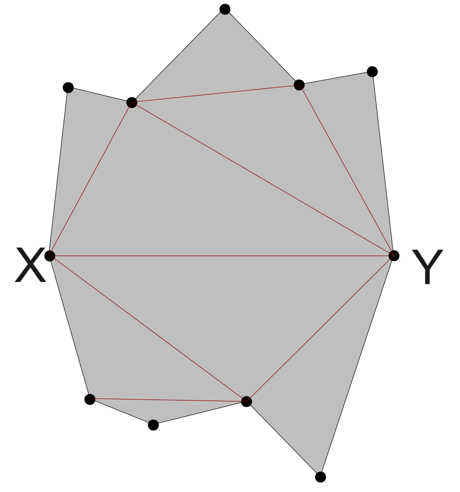

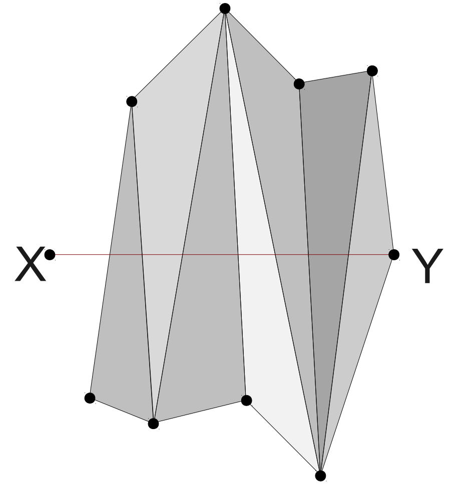

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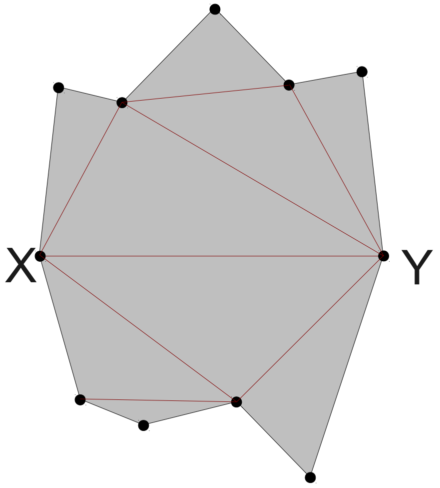

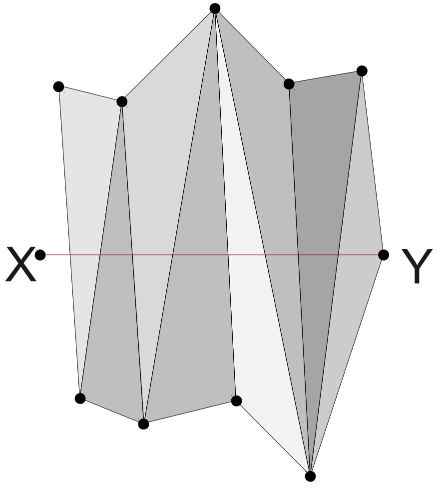

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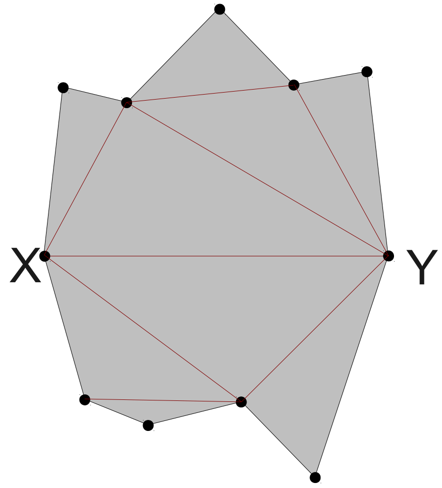

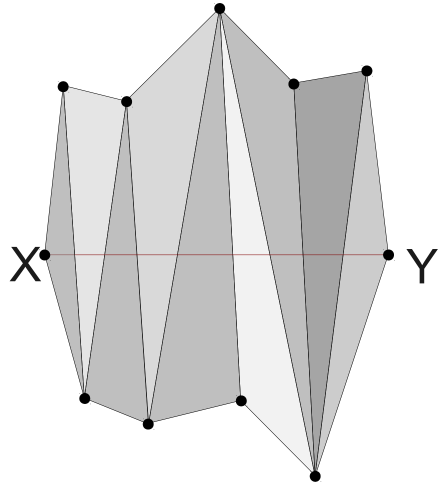

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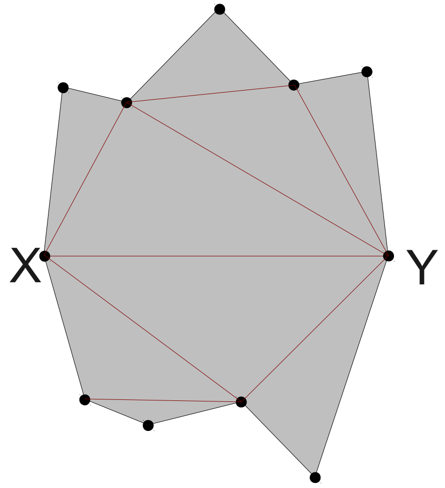

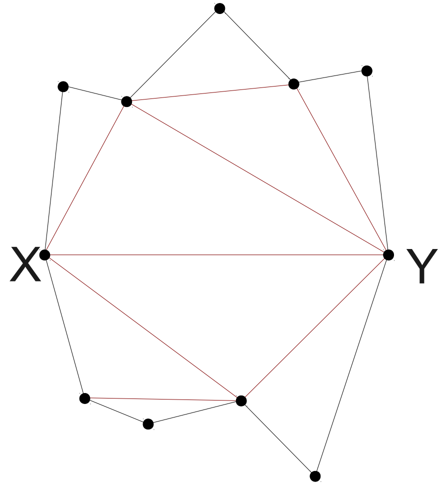

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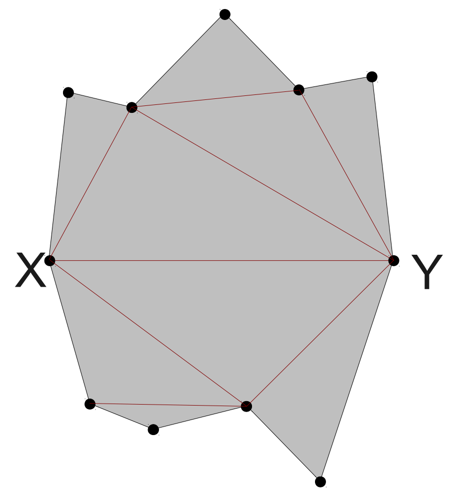

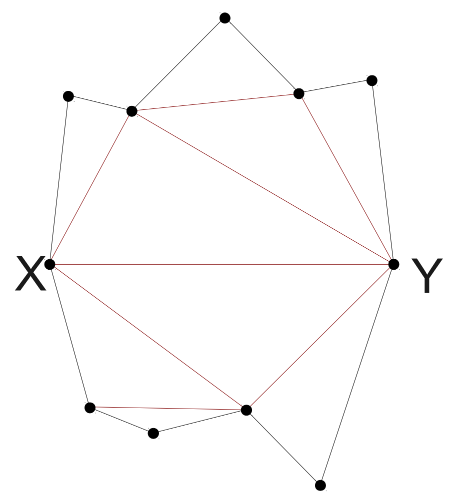

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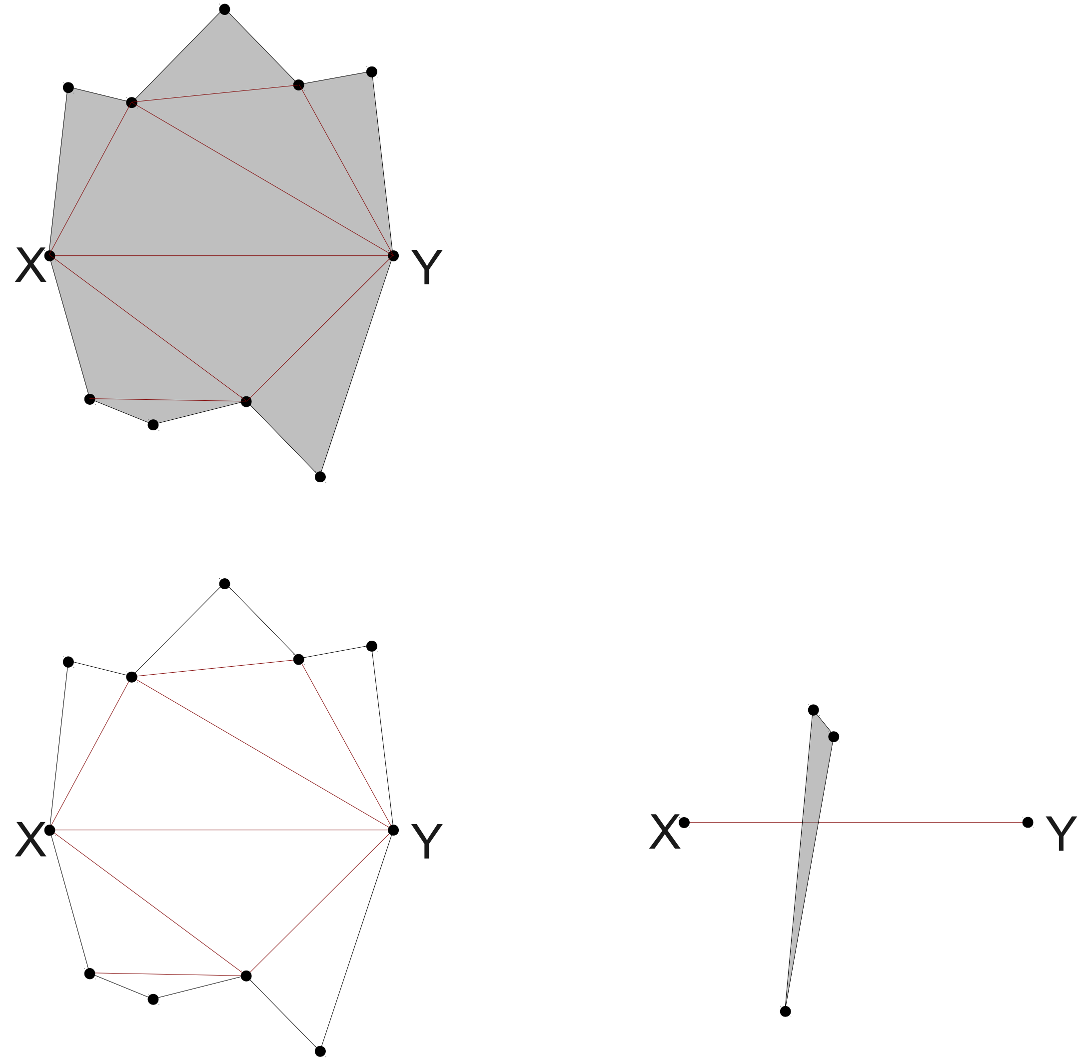

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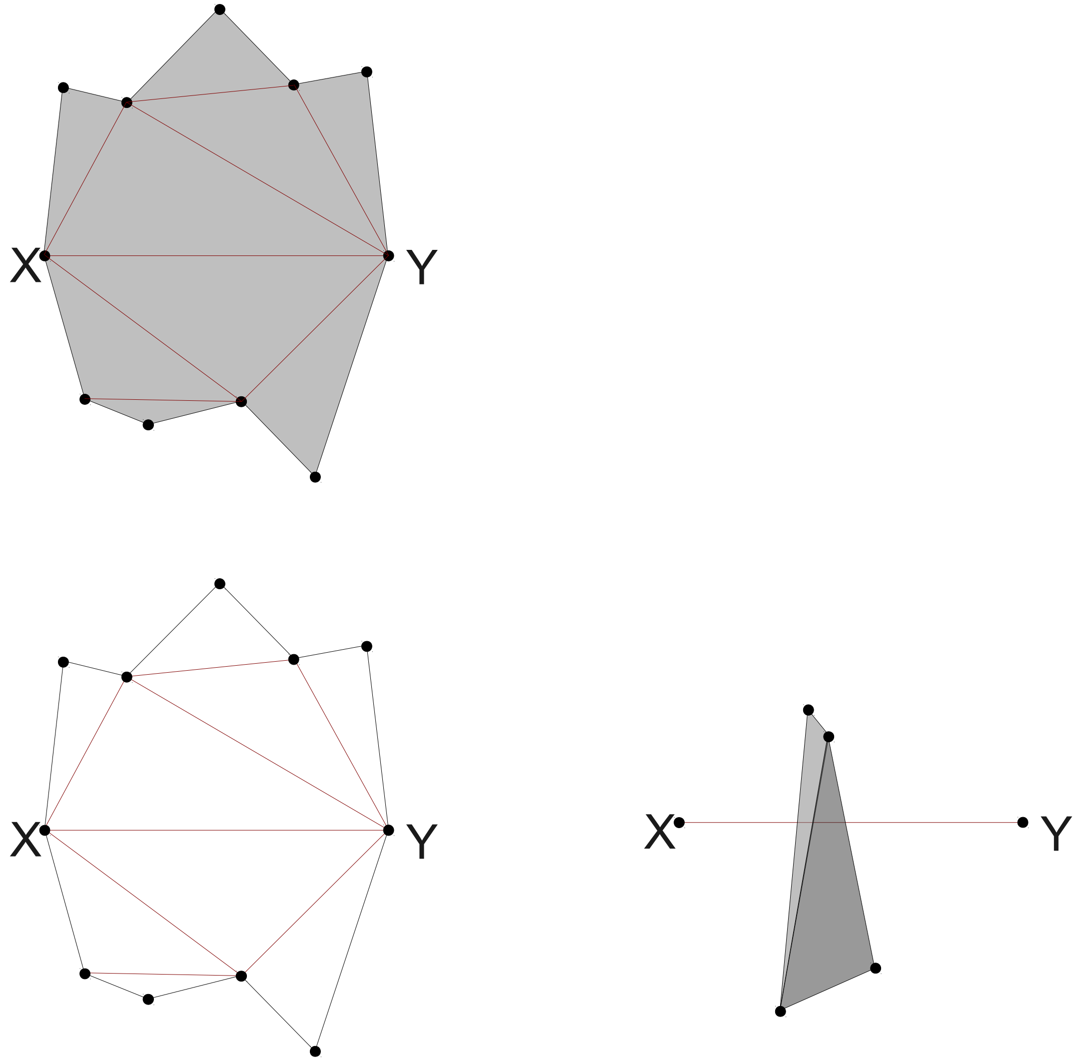

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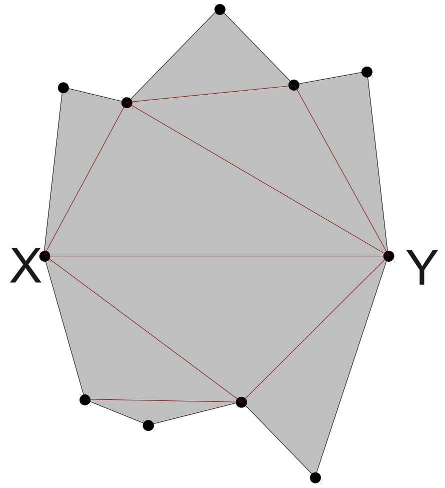

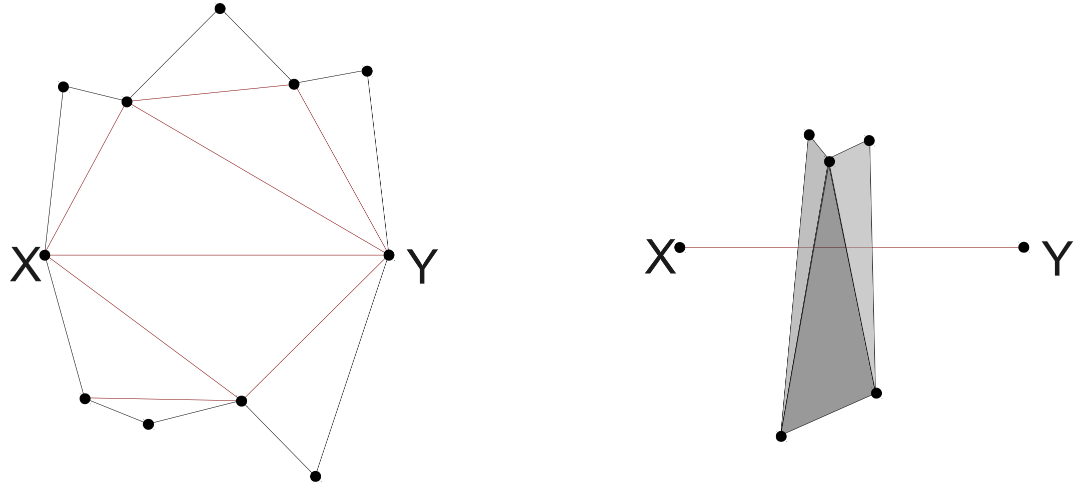

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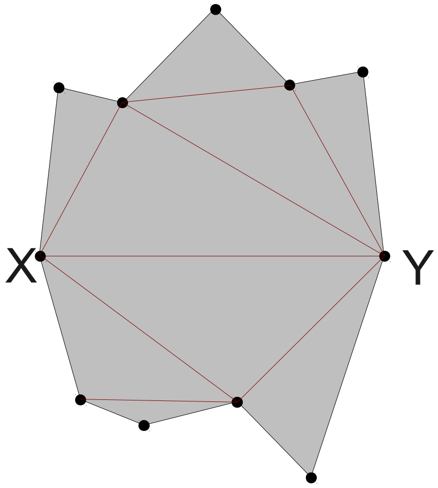

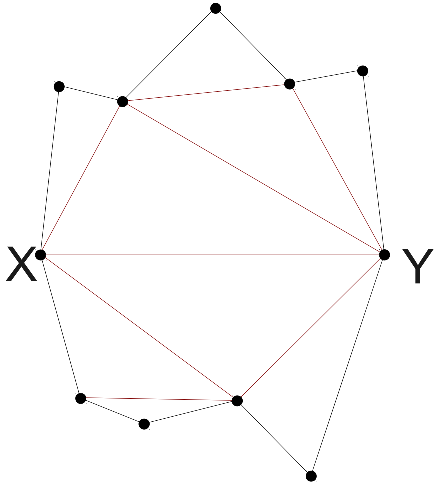

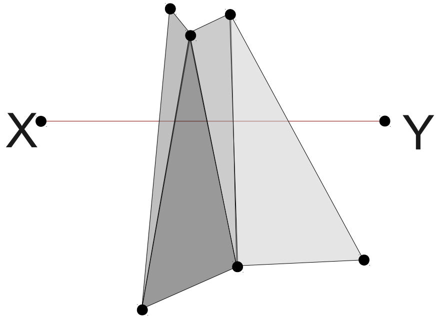

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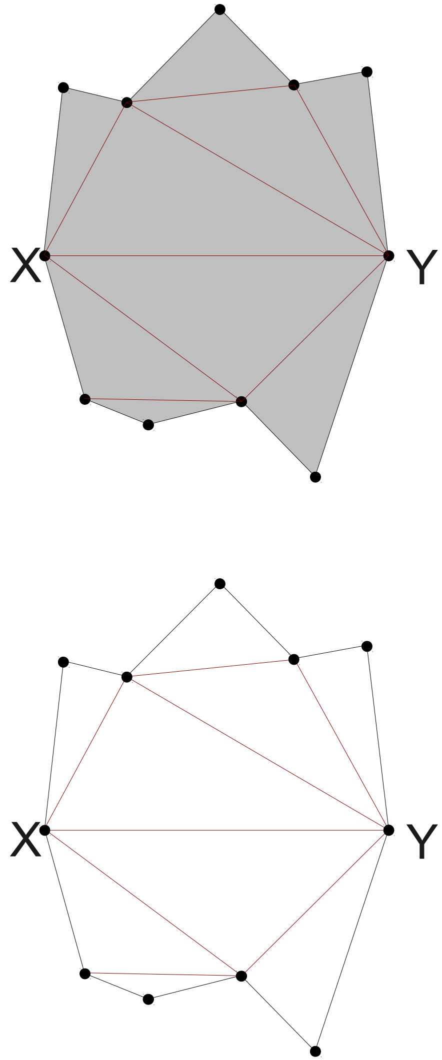

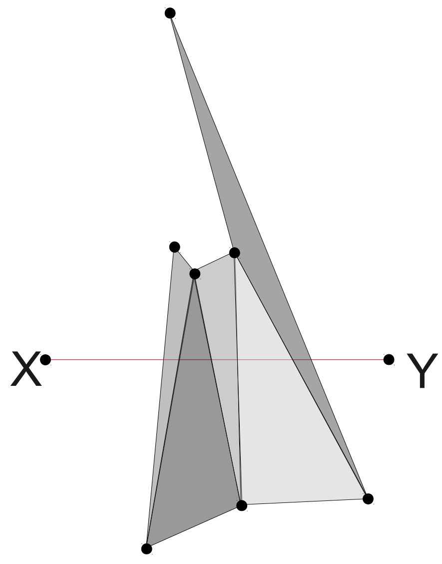

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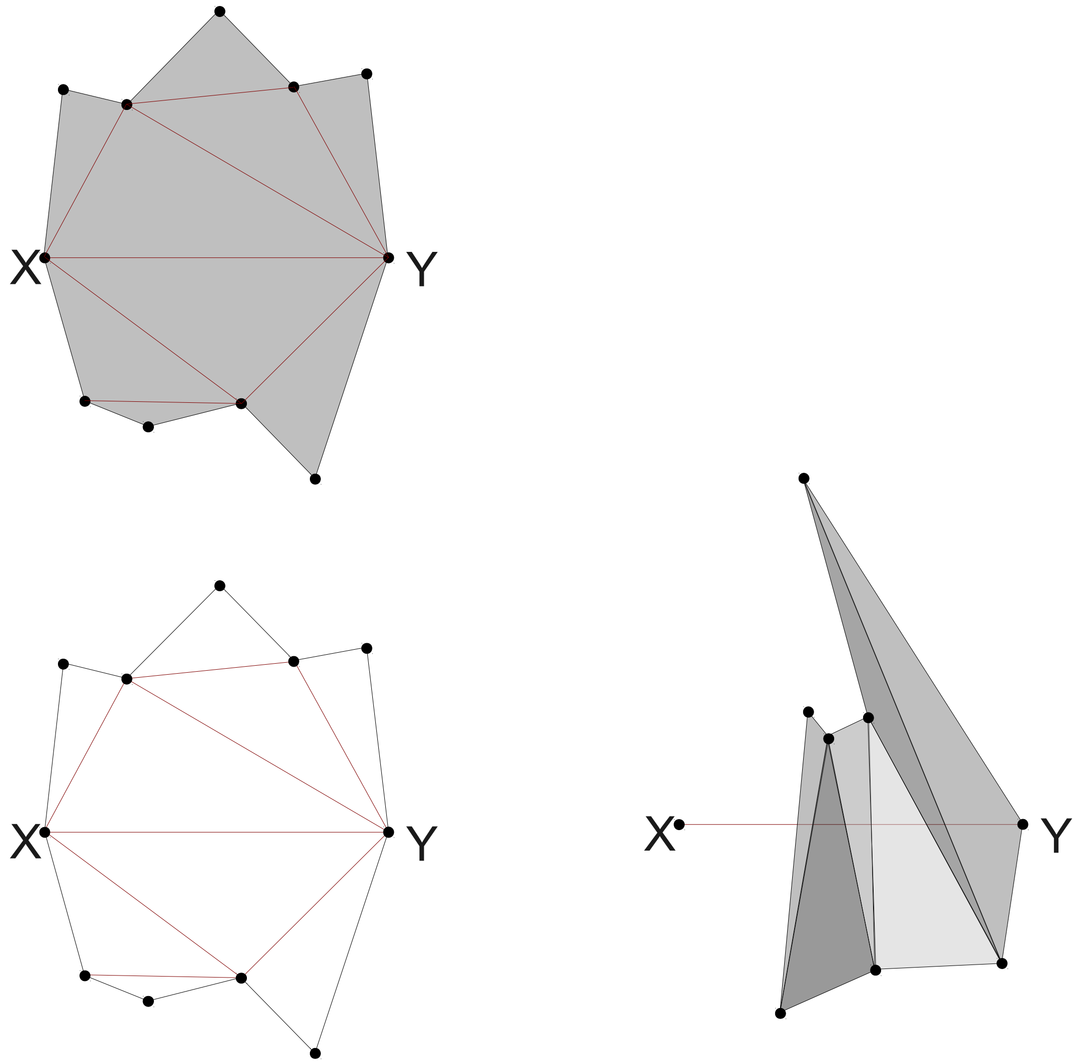

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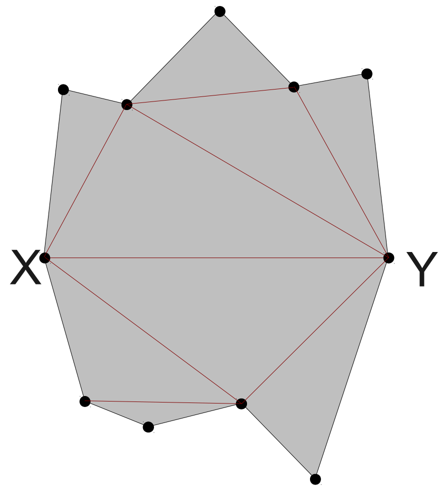

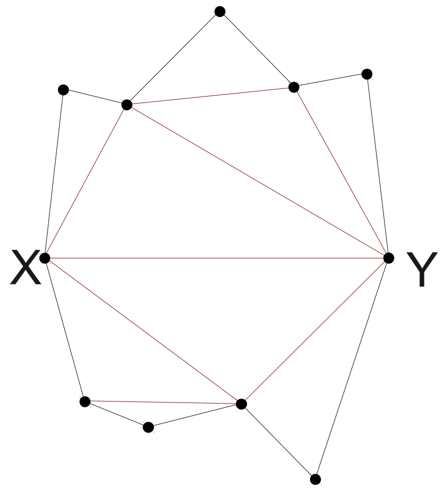

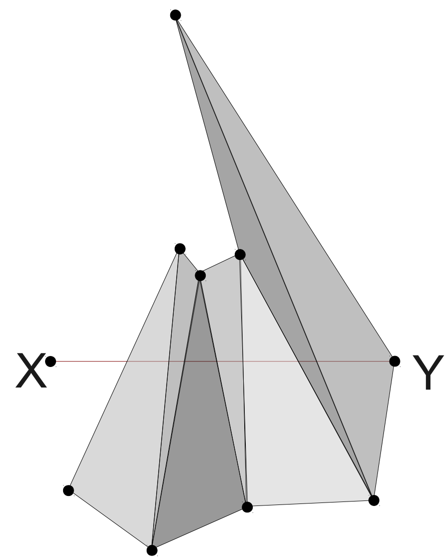

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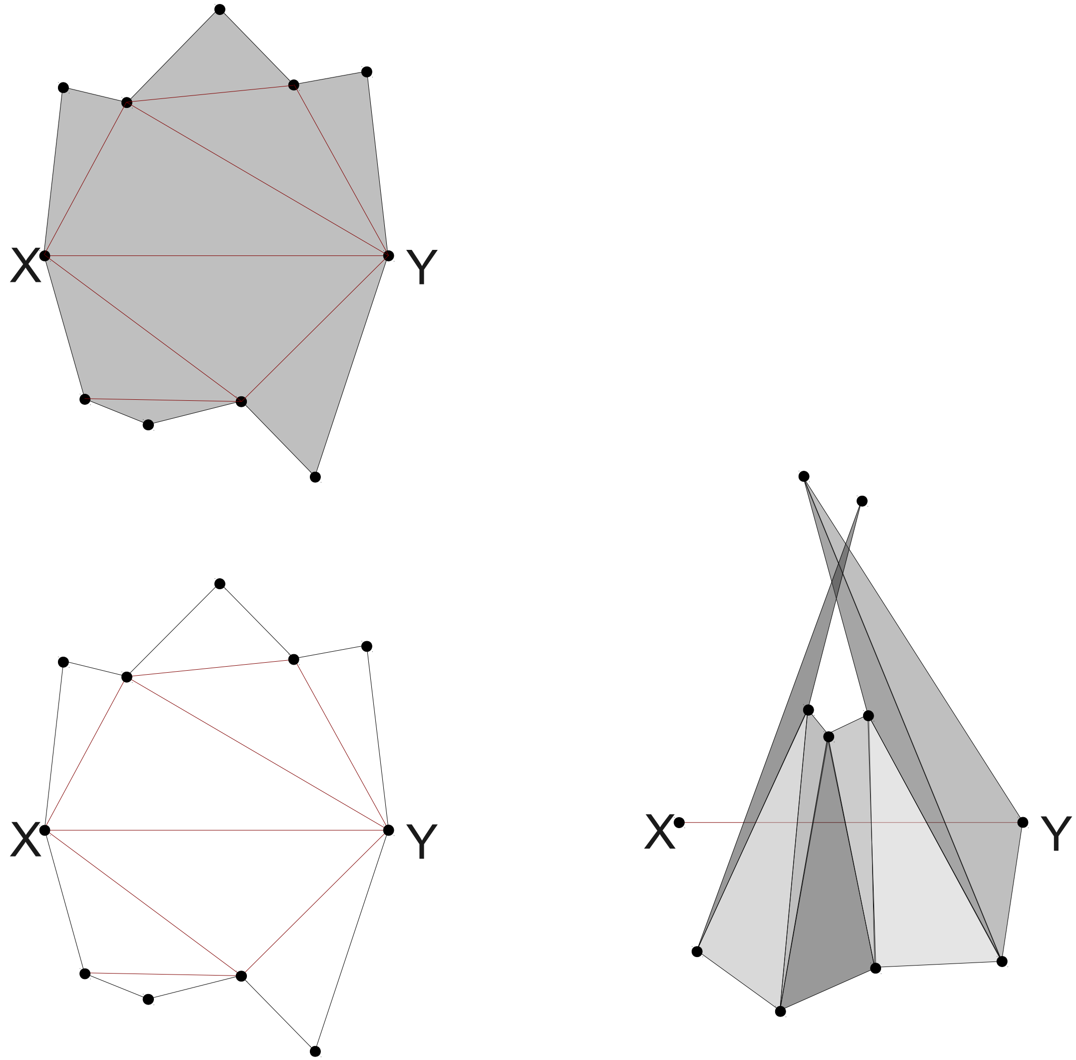

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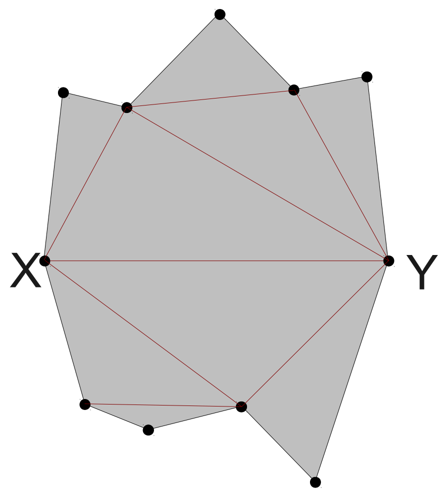

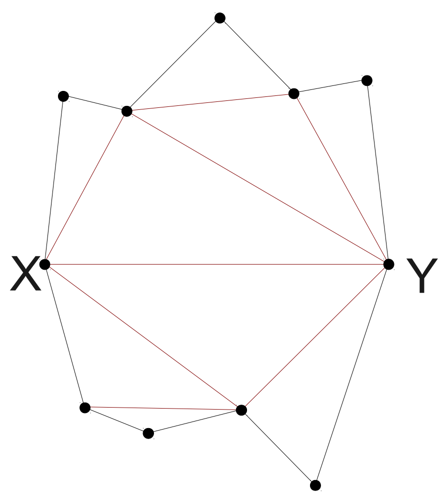

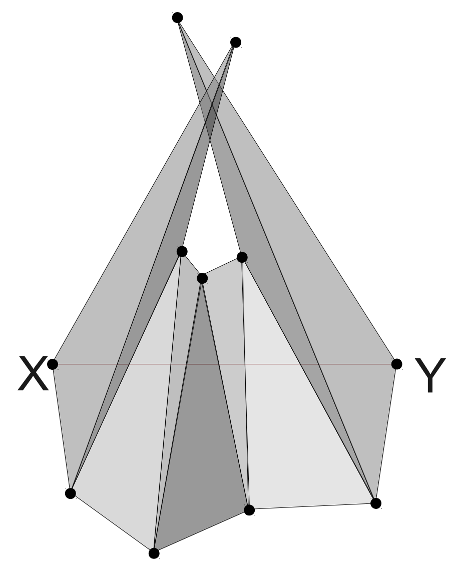

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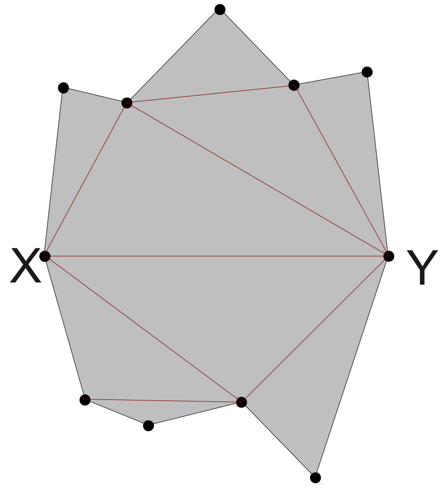

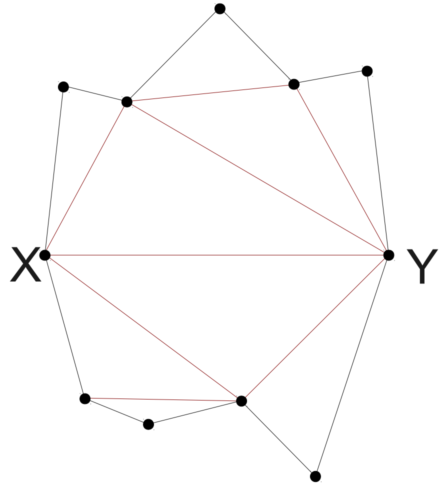

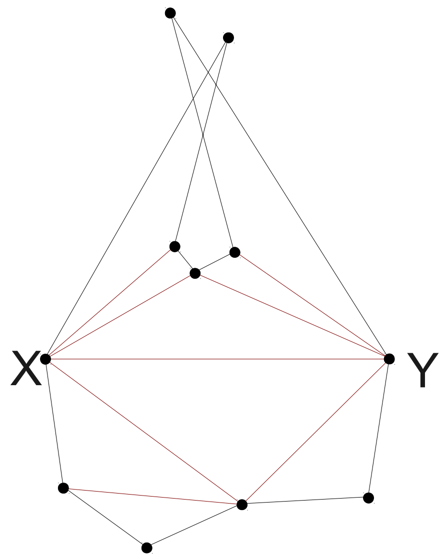

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[Levcopoulos and Krznaric '96]:

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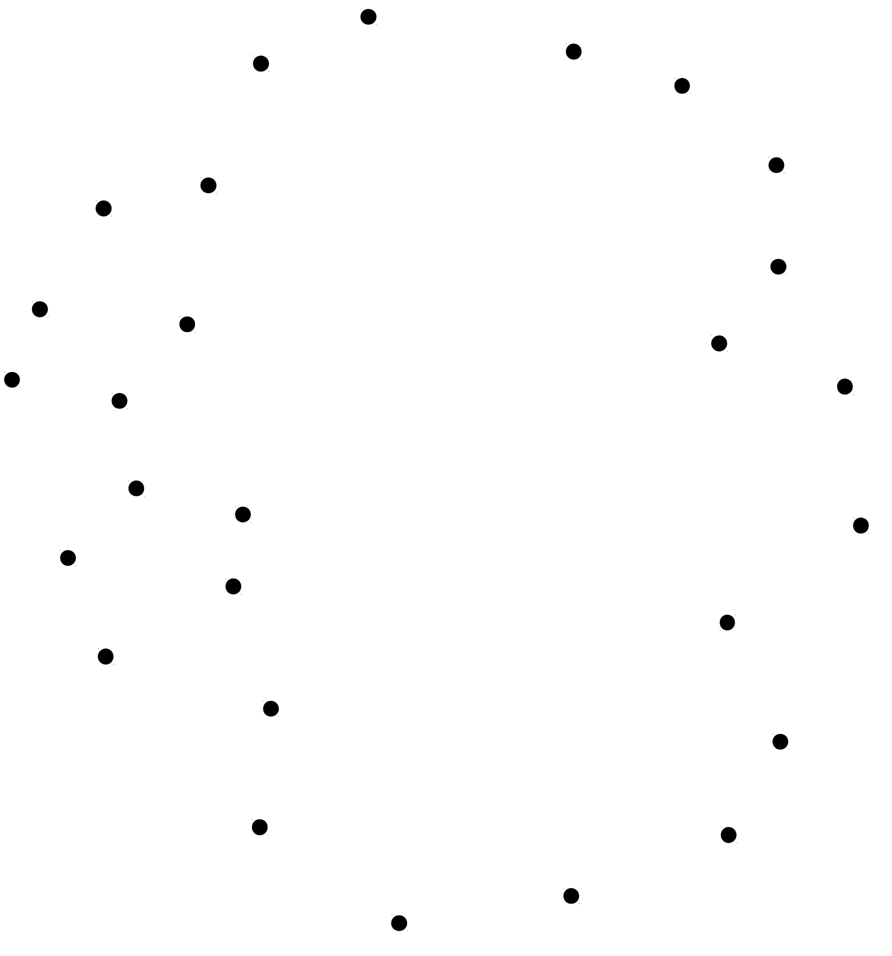

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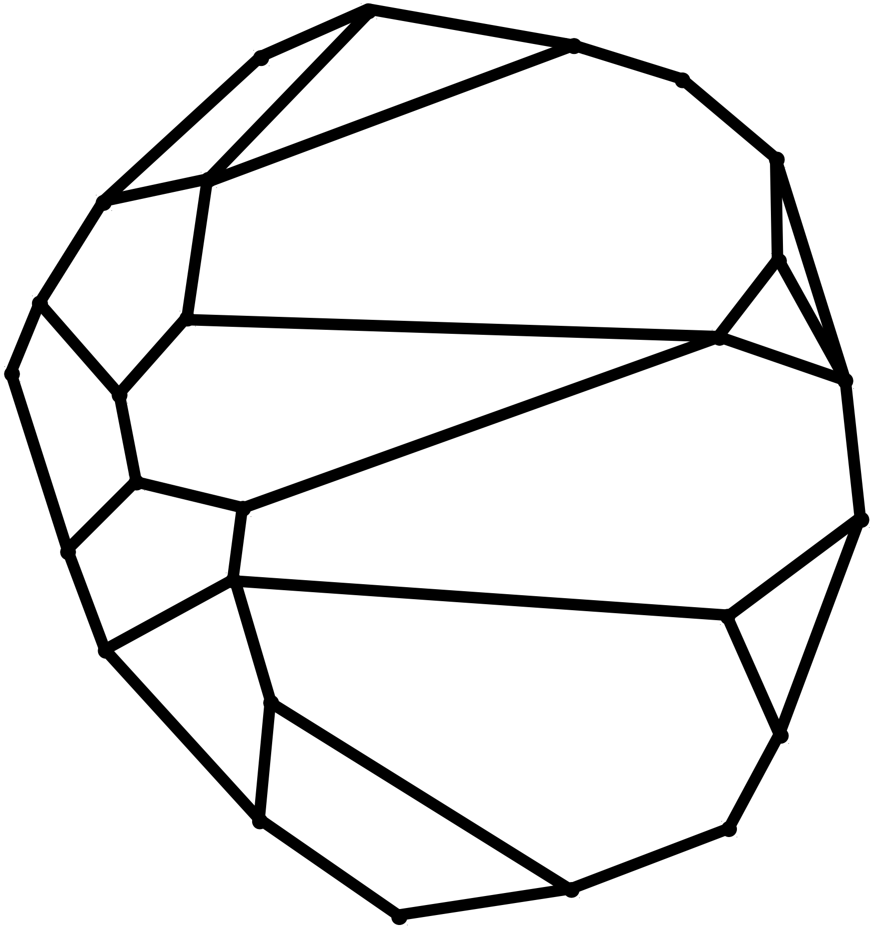

LK Convex Partition

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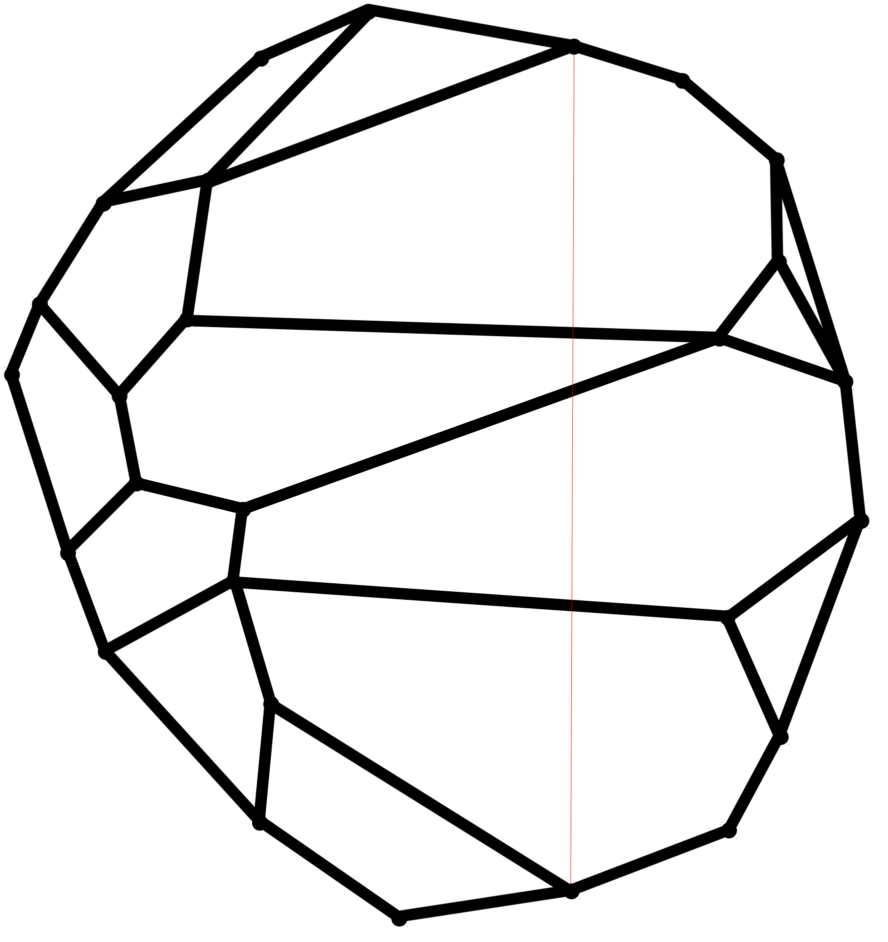

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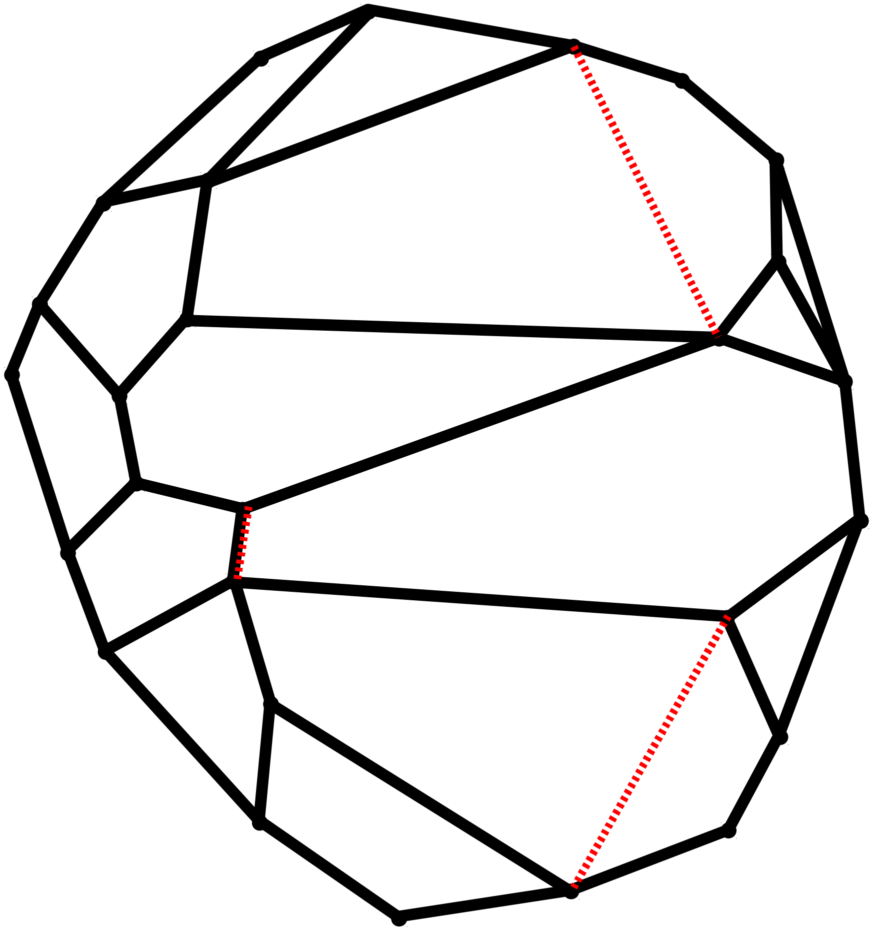
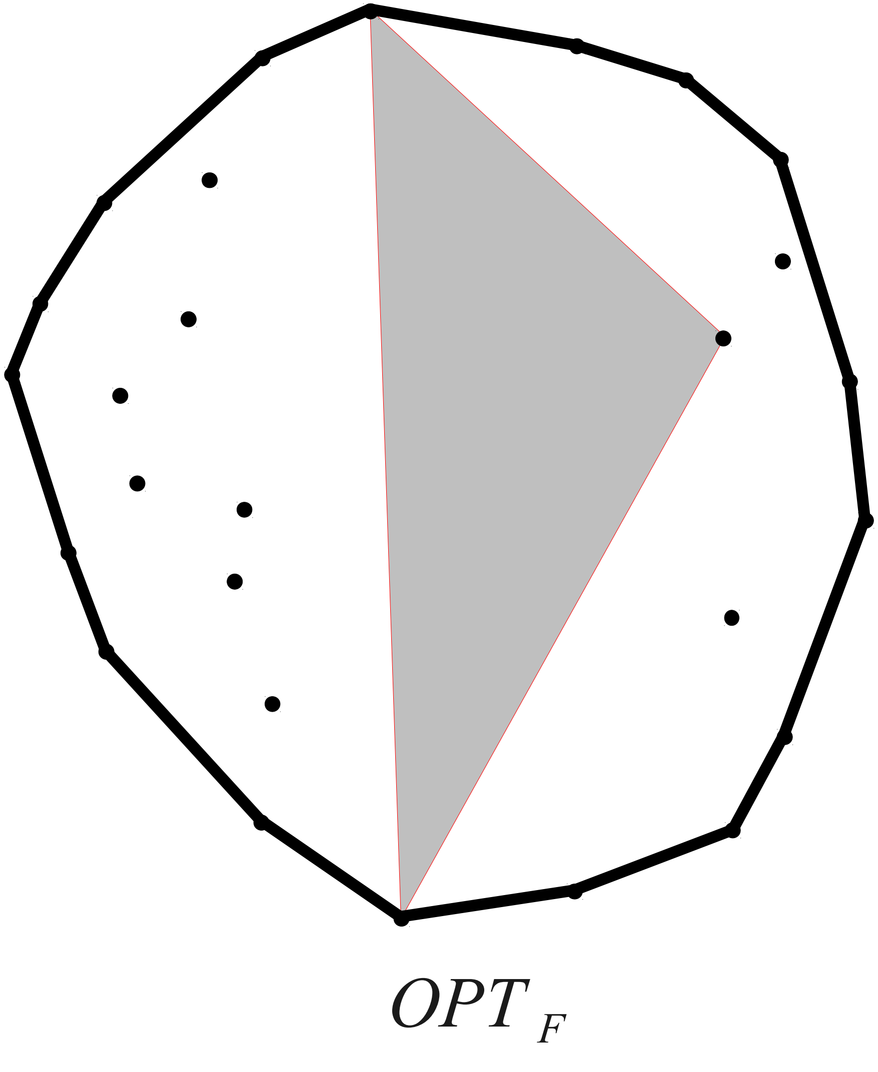


LK Convex Partition

High-Level Idea



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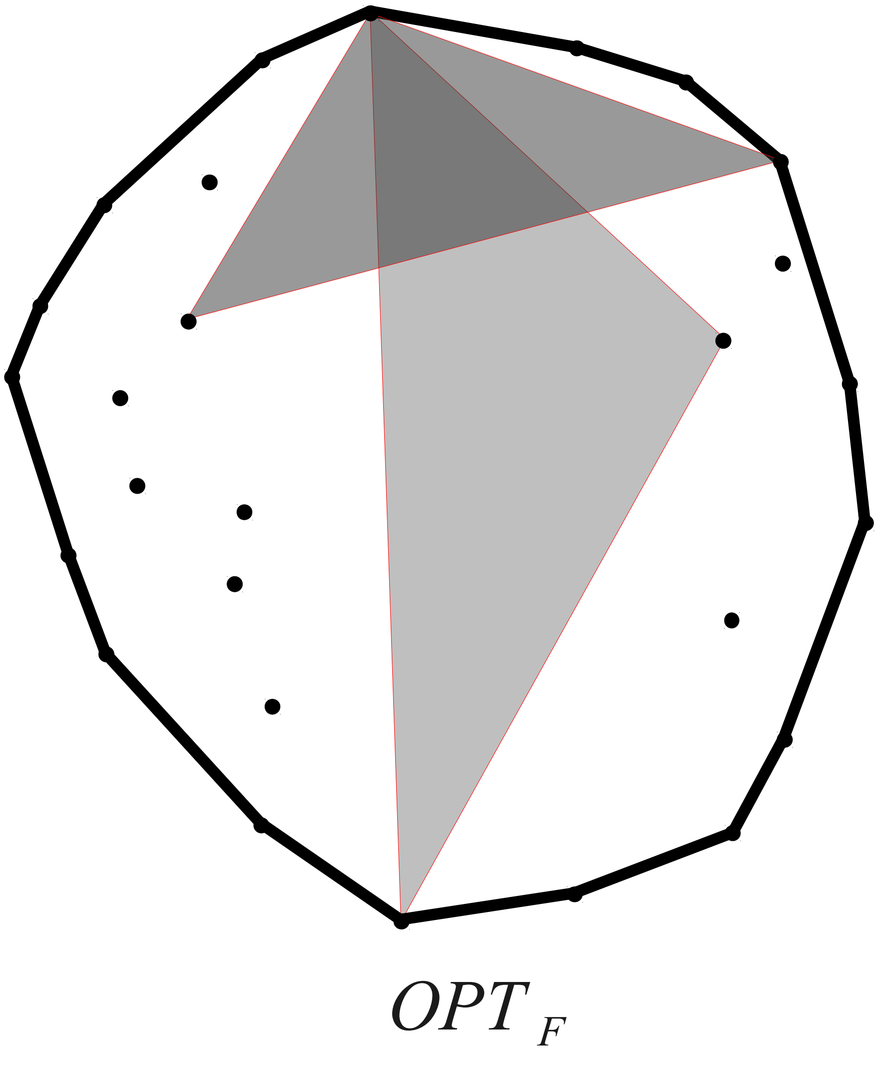
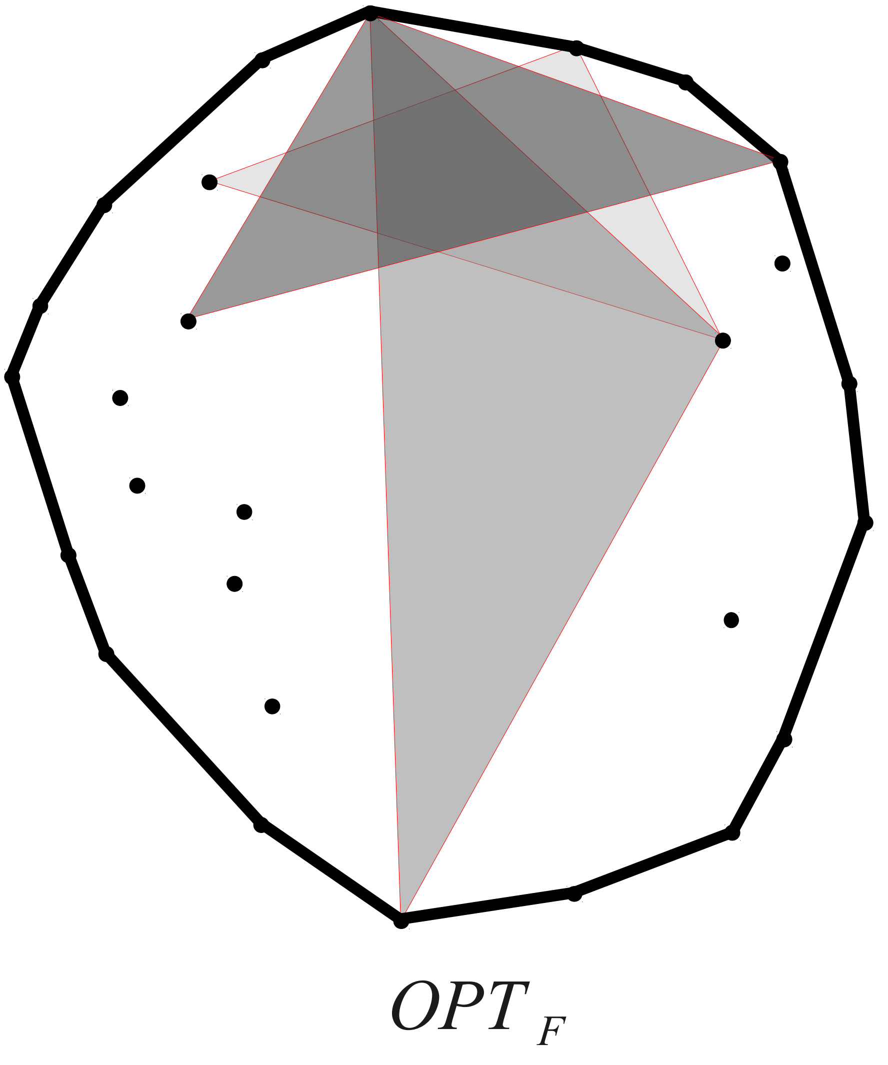


High-Level Idea



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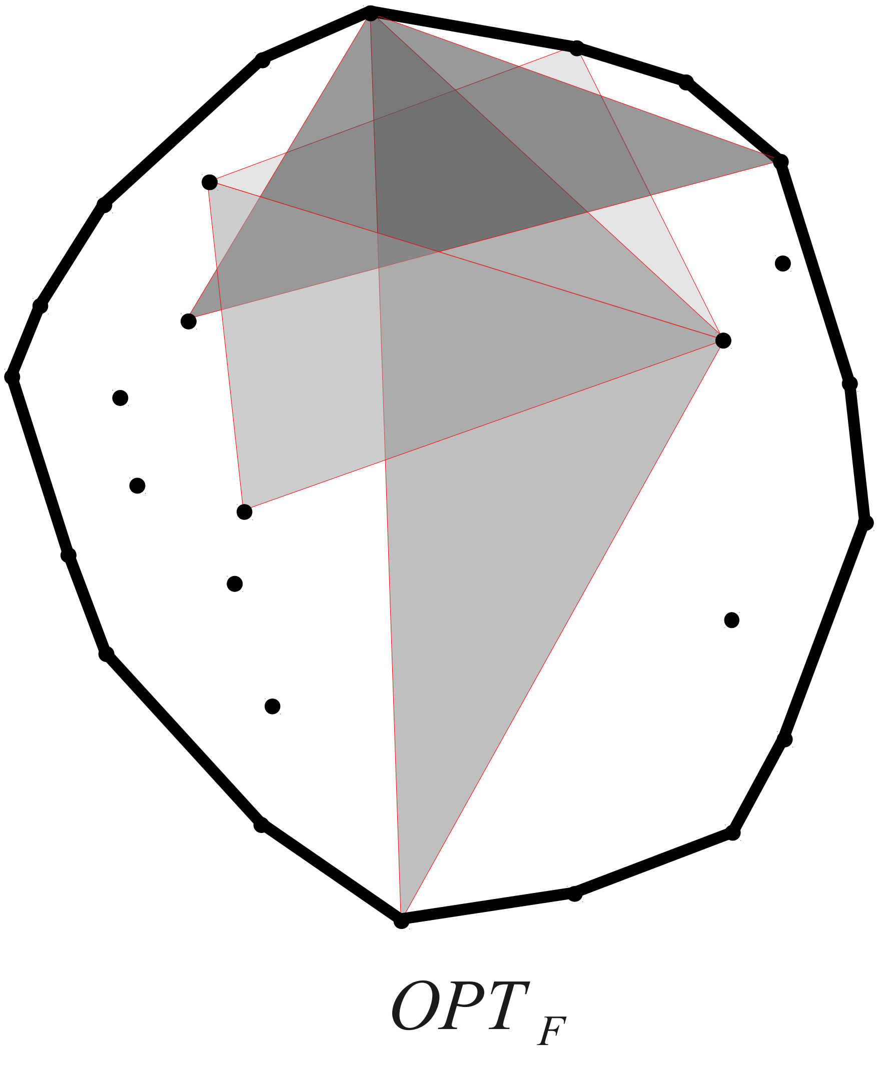

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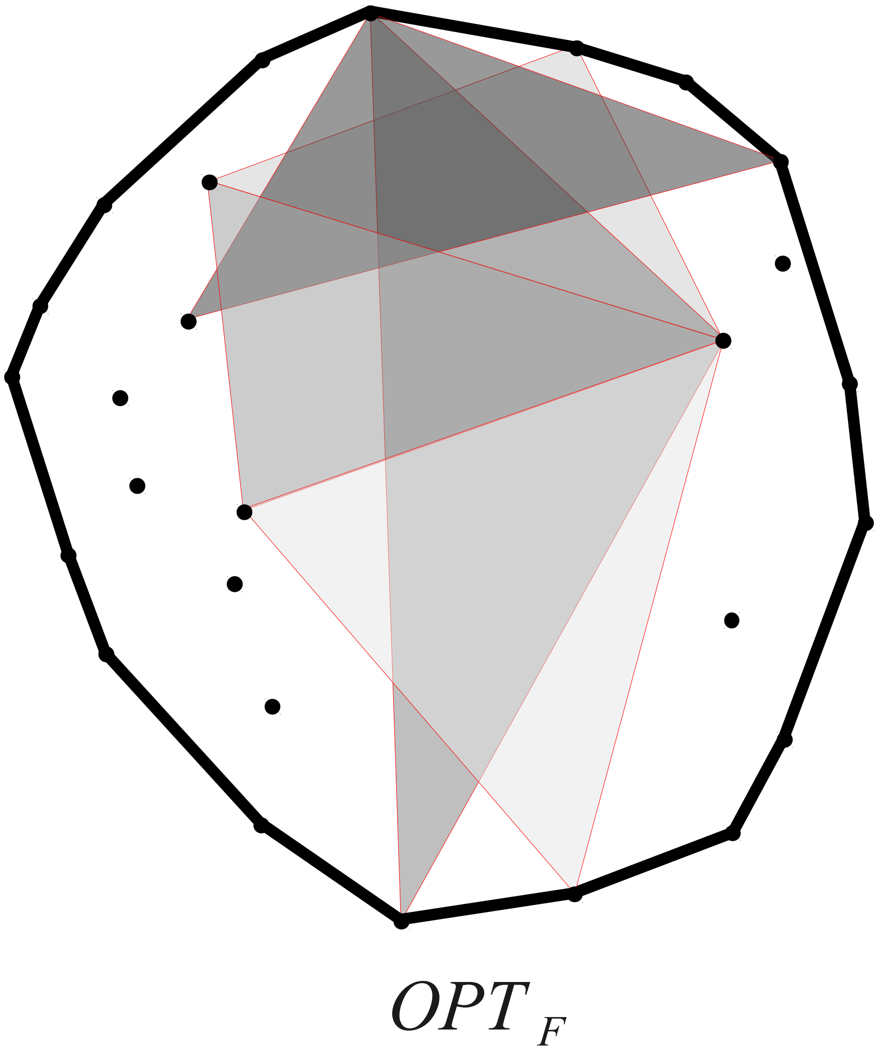

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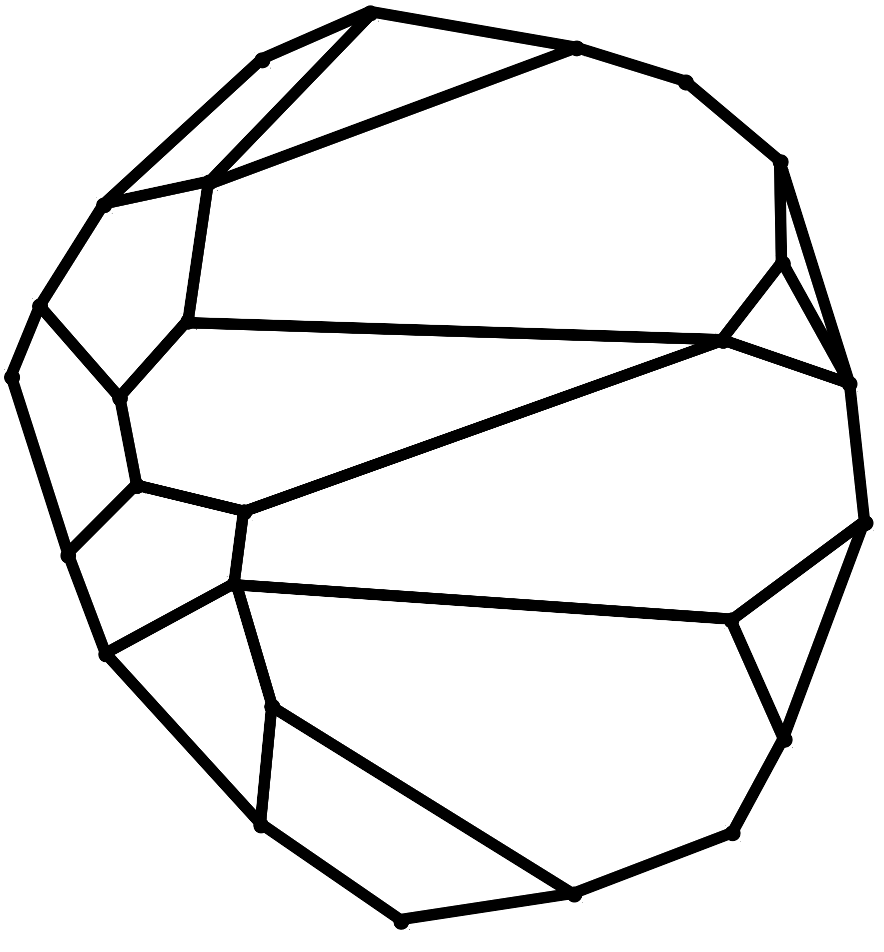
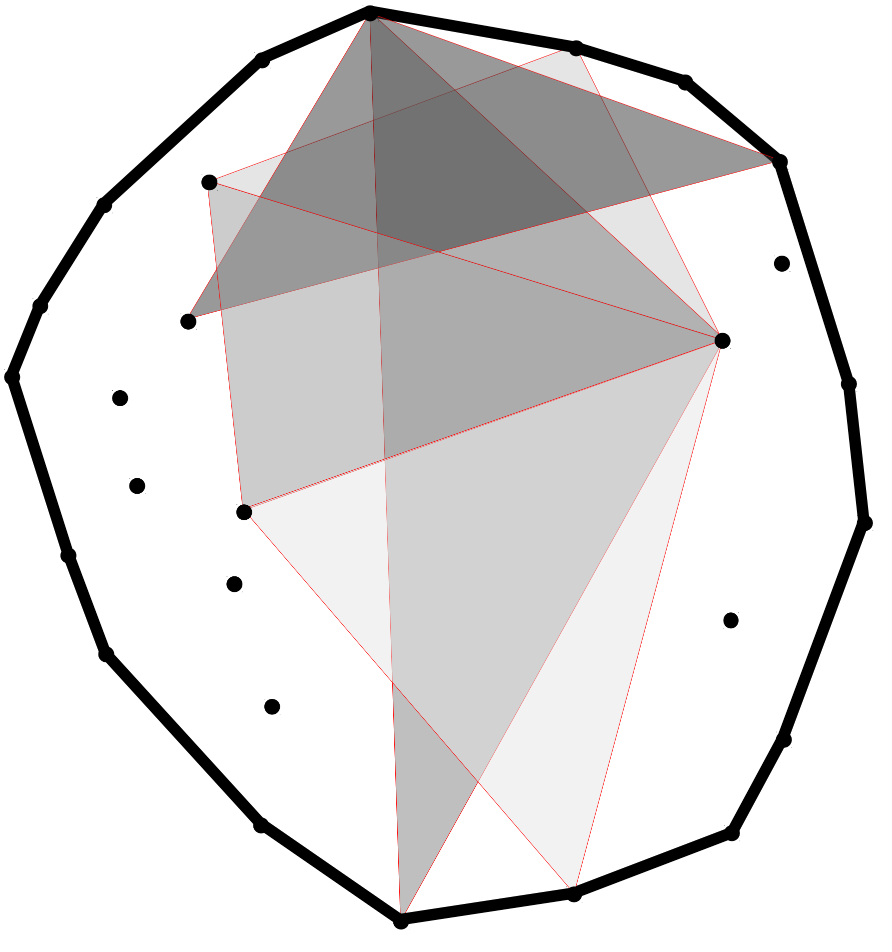
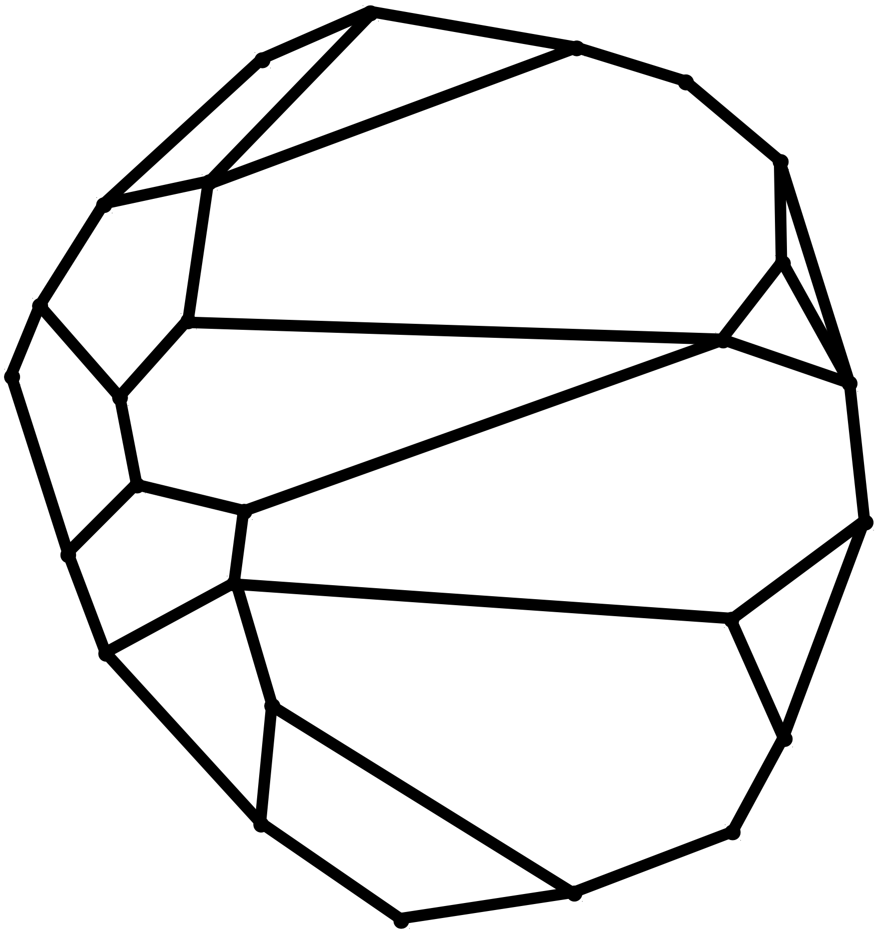


LK Convex Partition



OPT_F

High-Level Idea



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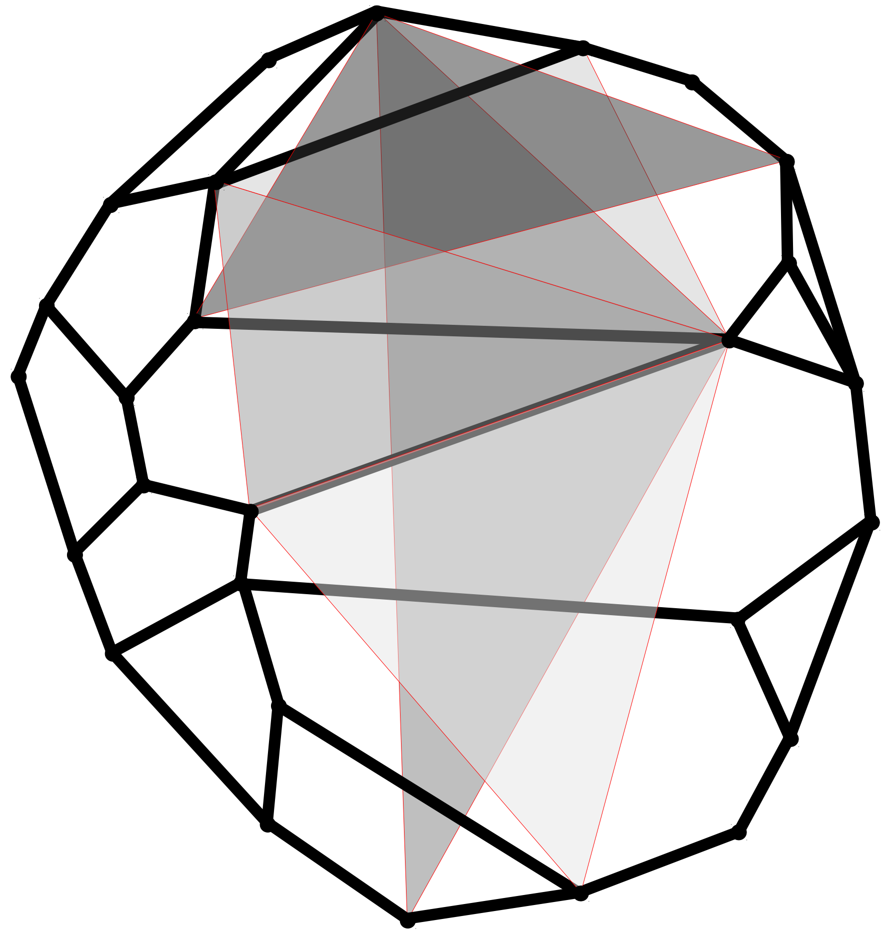
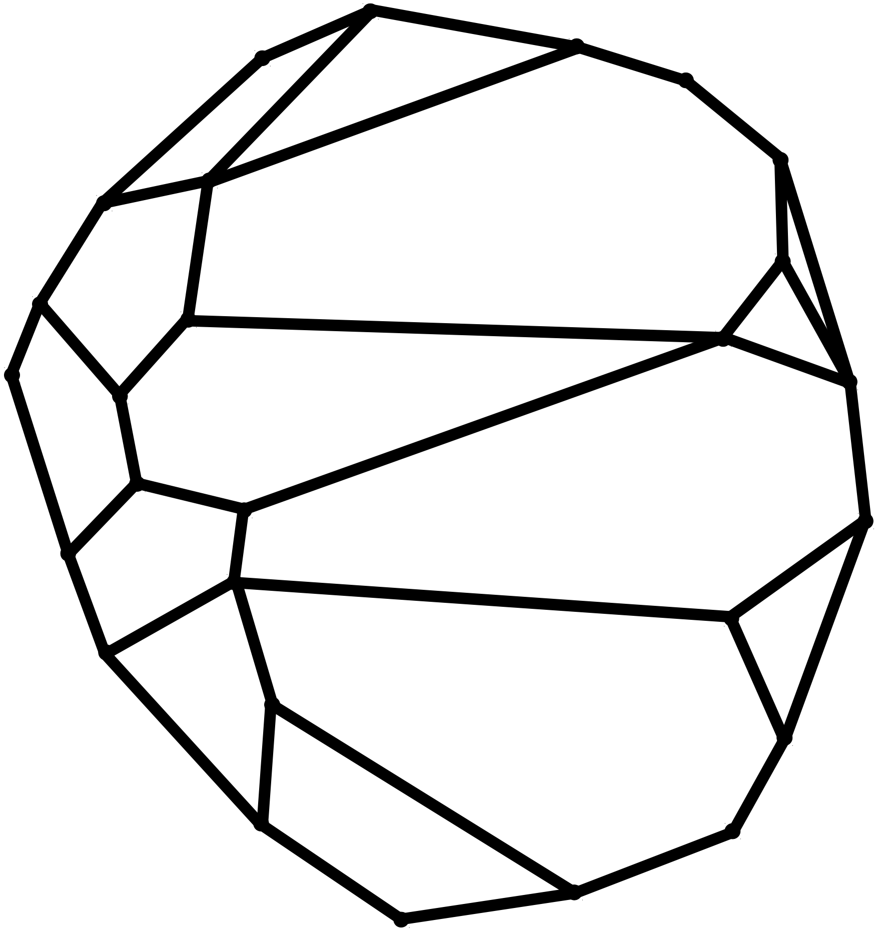
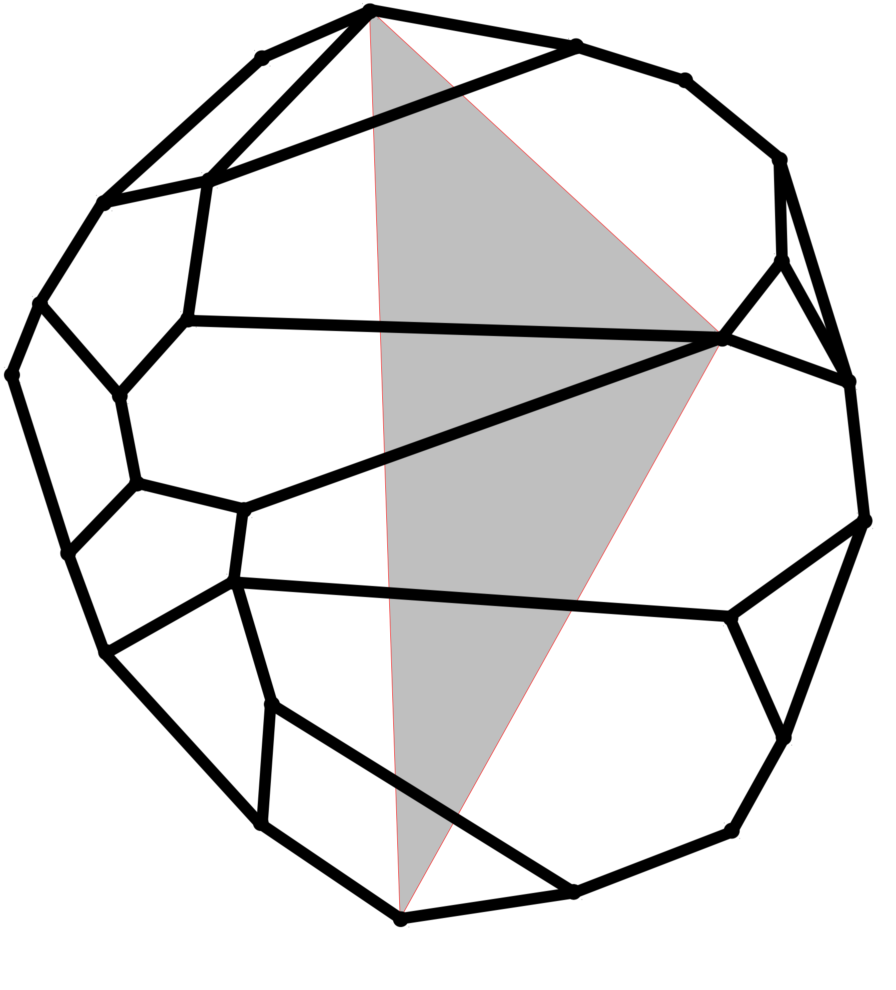


OPT_F

Proof Overview



Proof Overview



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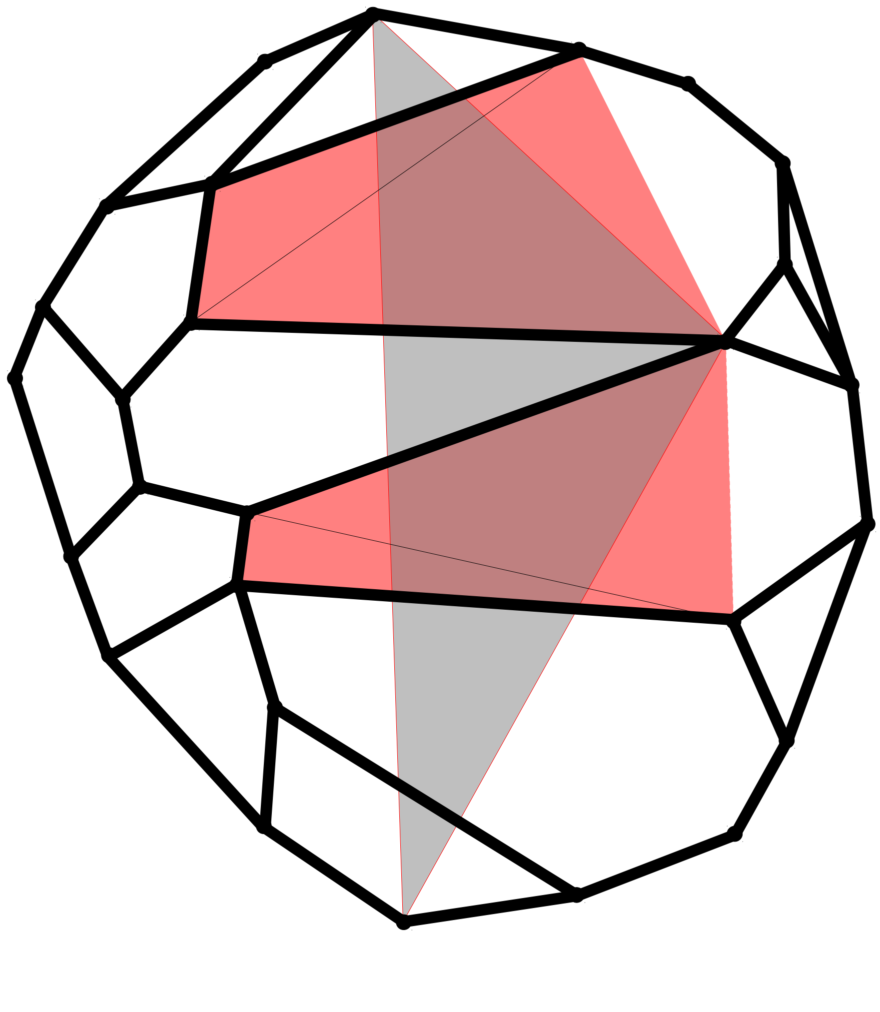
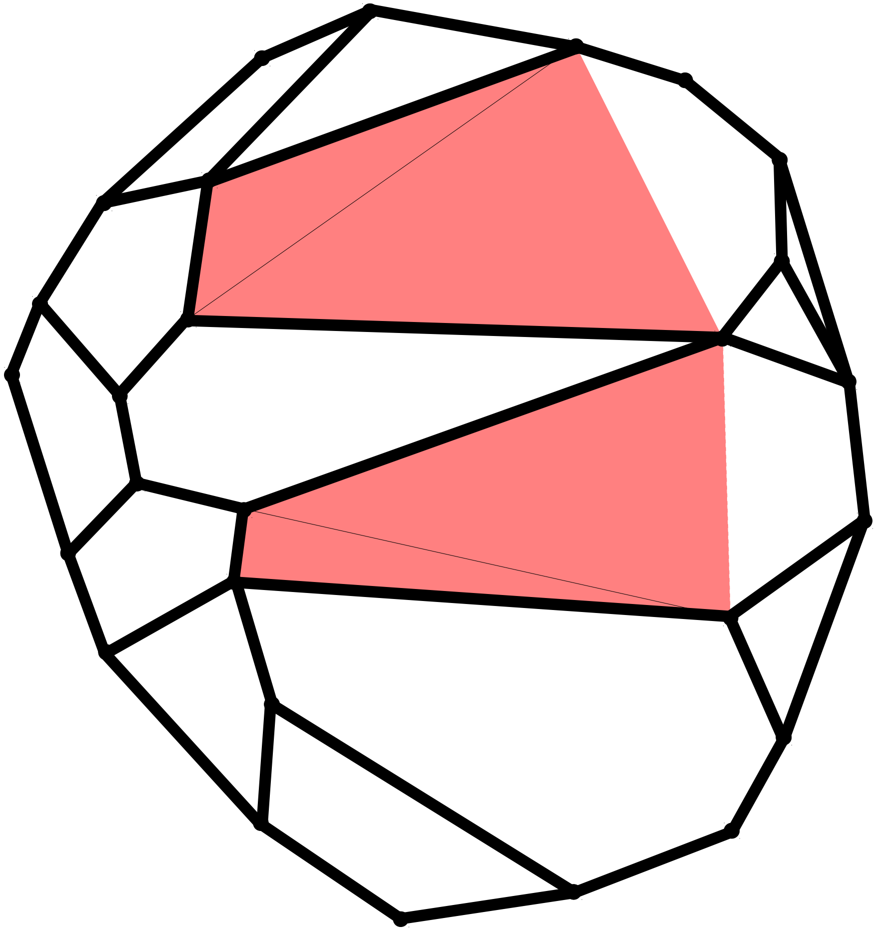
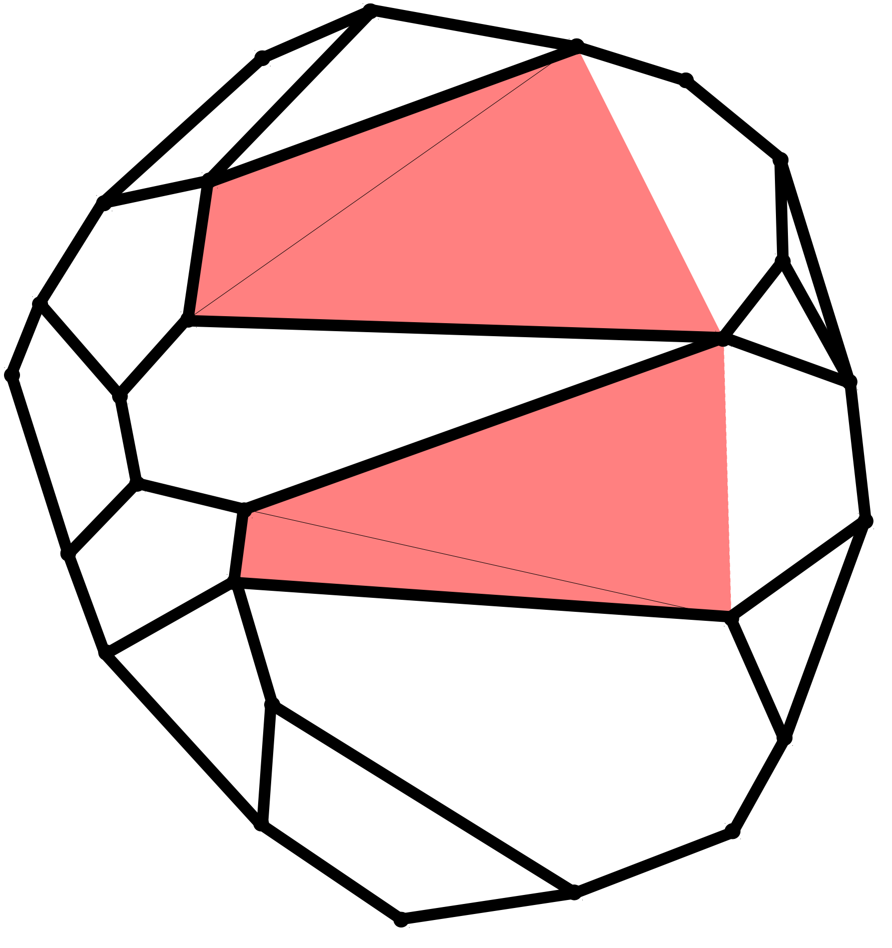


Proof Overview



Proof Overview

I. Feasibility: New triangles cover every point with weight one.

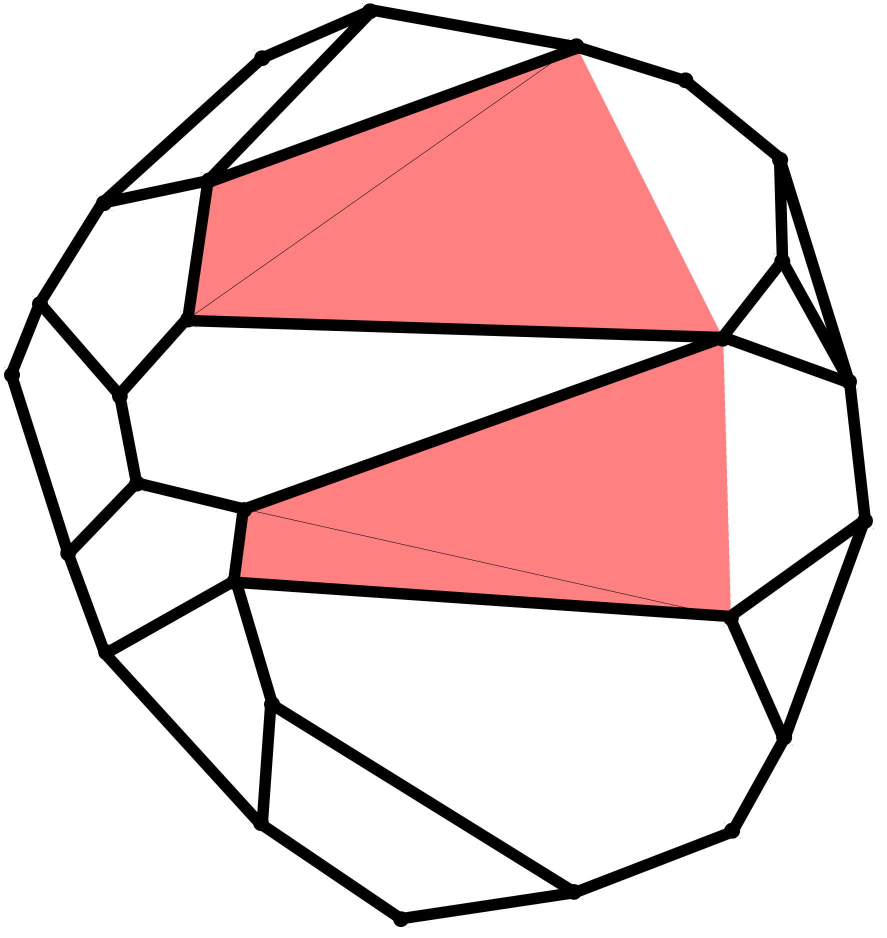


Proof Overview

I. Feasibility: New triangles cover every point with weight one.

II. Cost Bound: The total cost of new triangles is bounded by

$$O\left(\left|OPT_F\right|\right)$$



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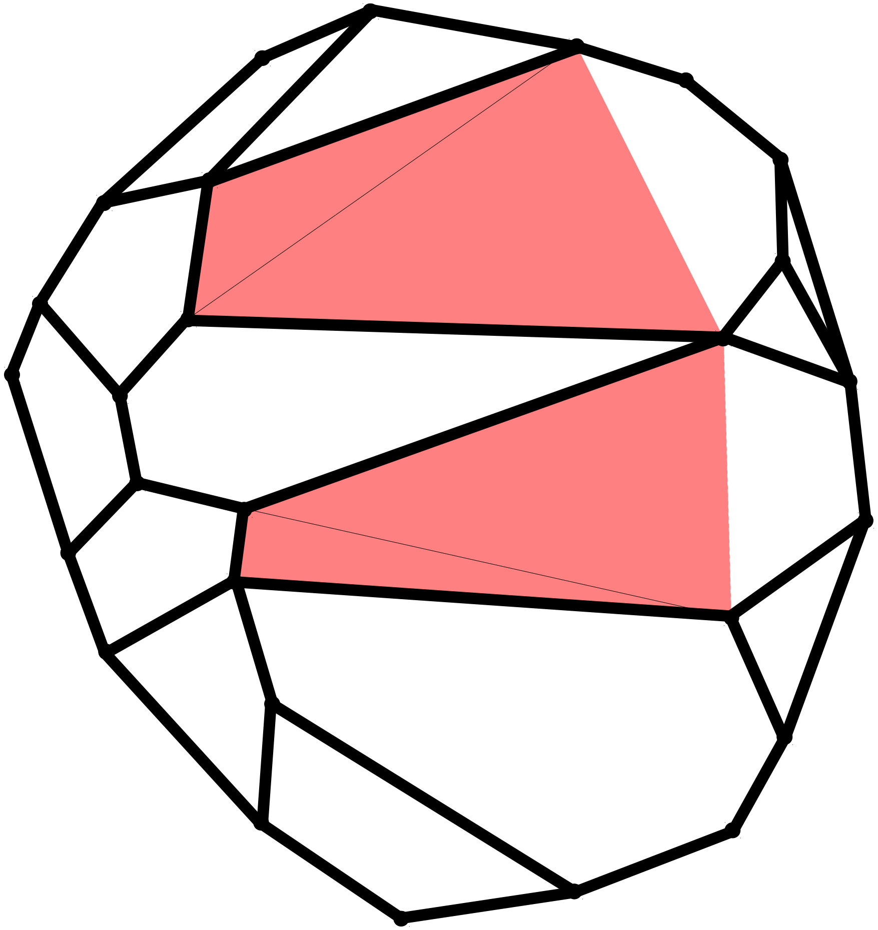
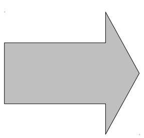
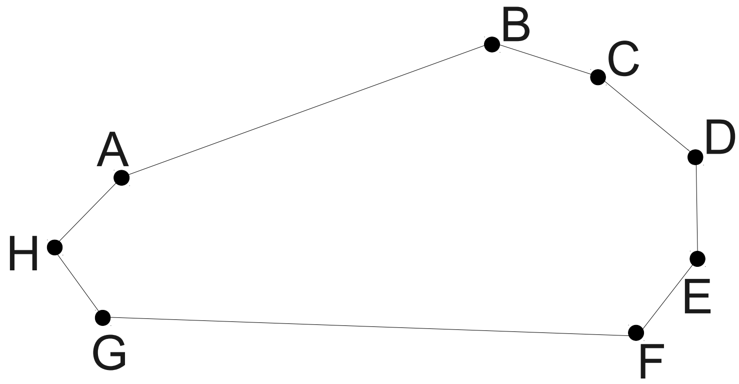


$$\Rightarrow \frac{\left|OPT_I\right|}{\left|OPT_F\right|} = O(1)$$



How to Break Triangles?



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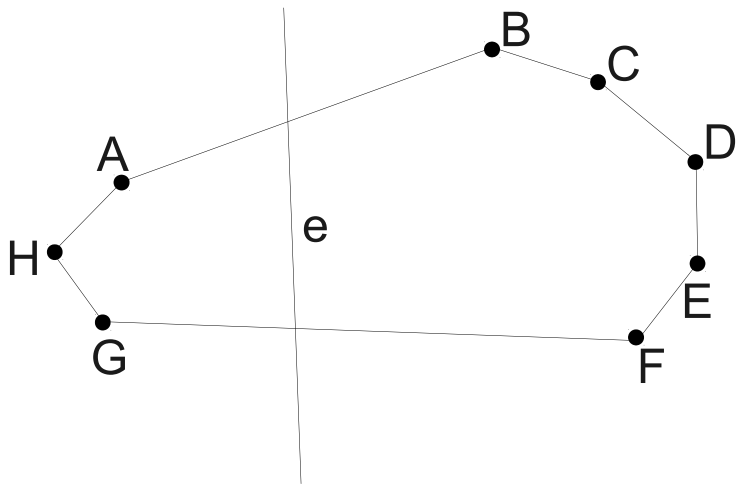

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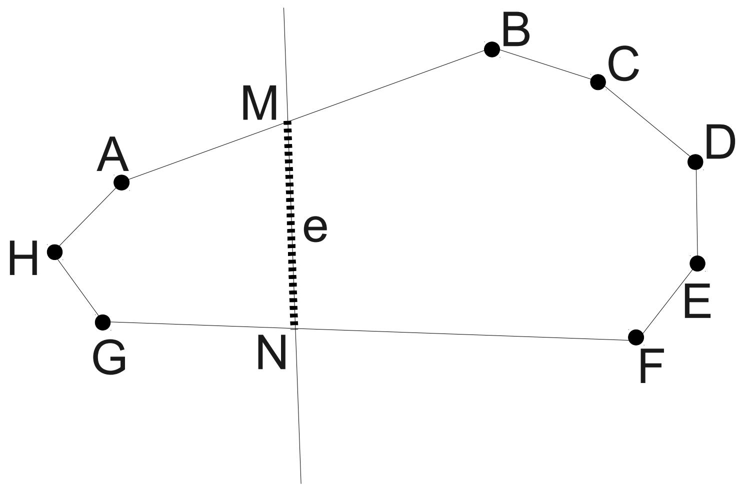

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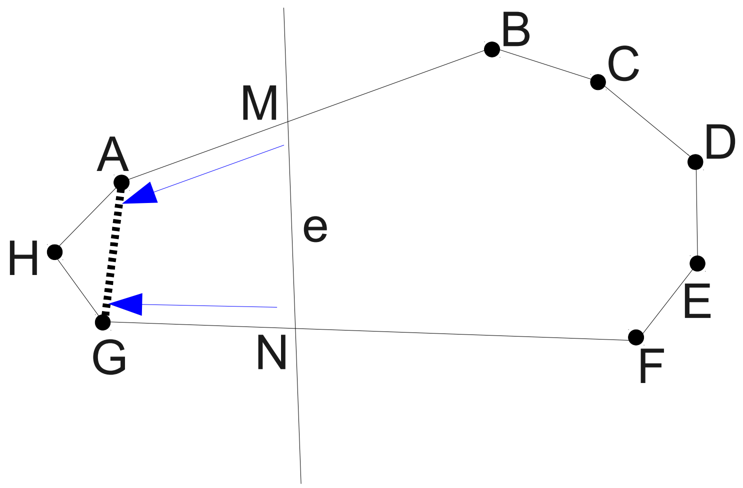

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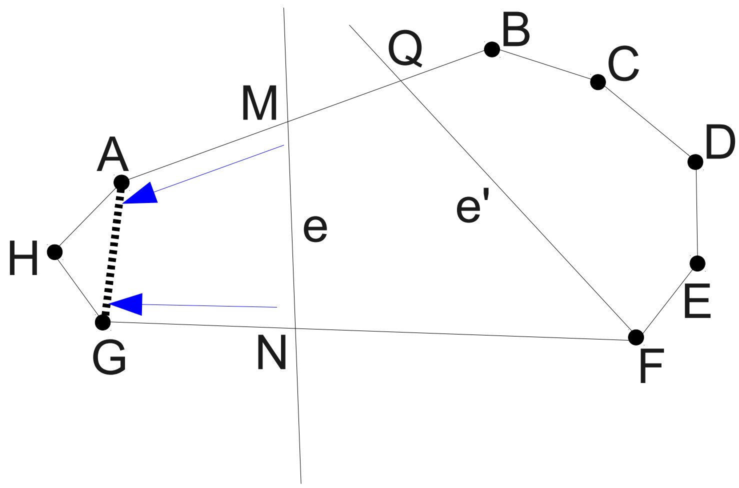

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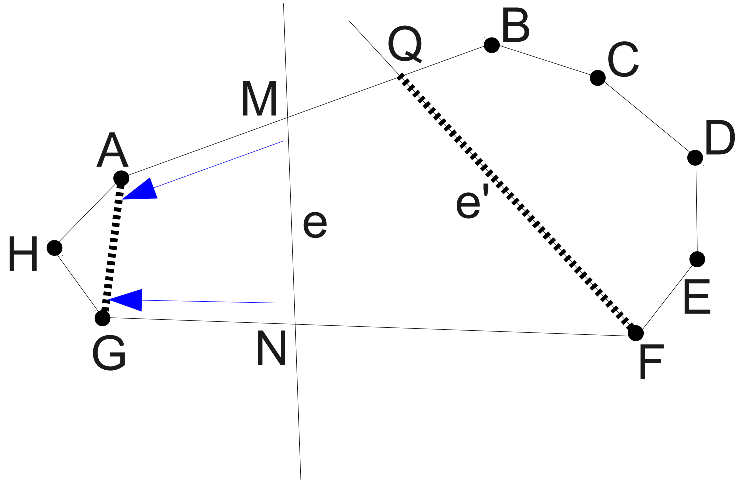

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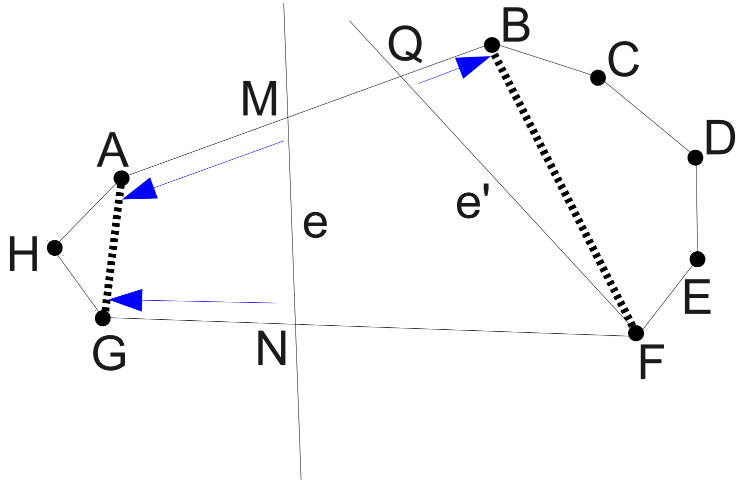

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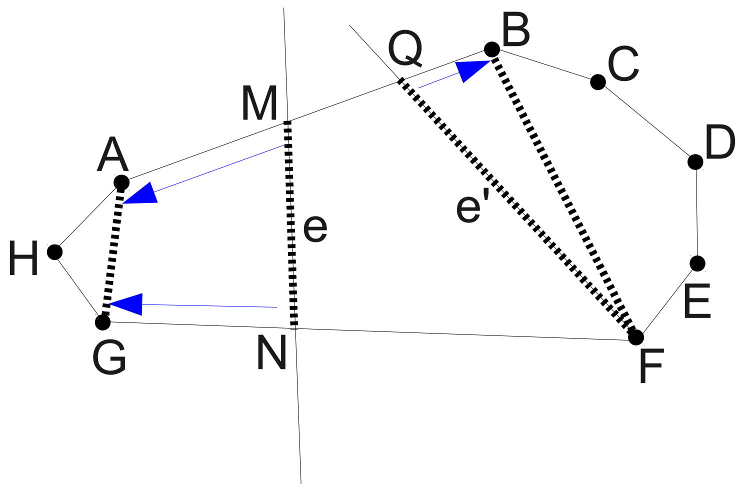

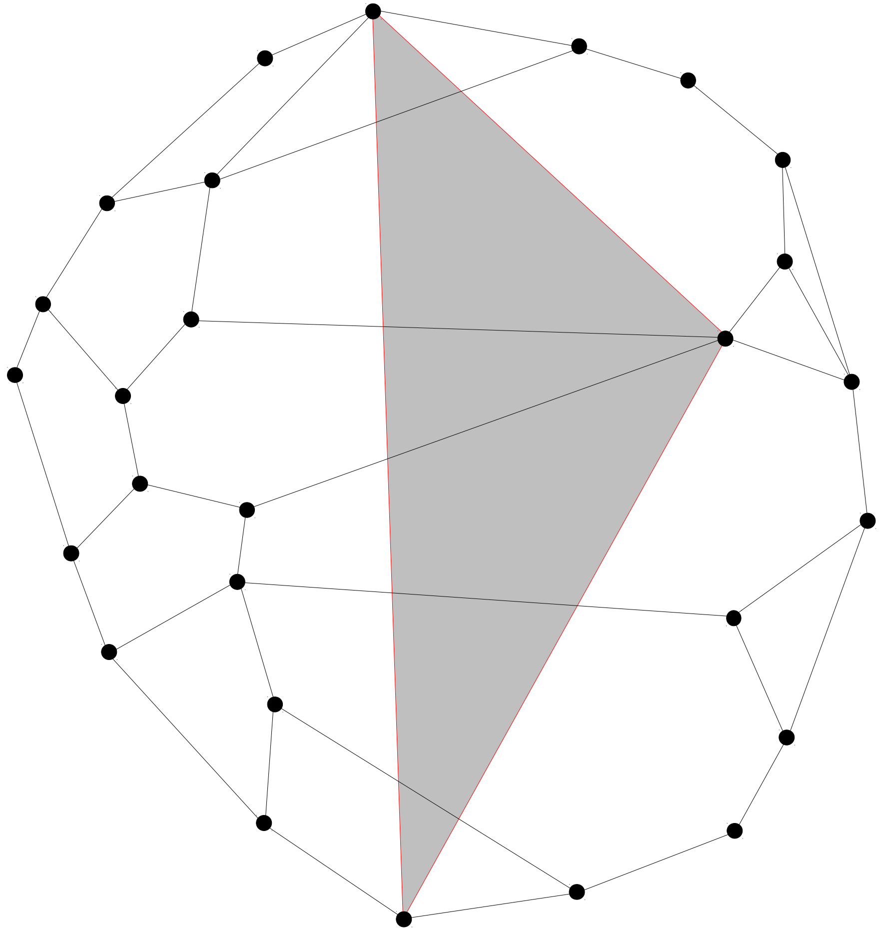

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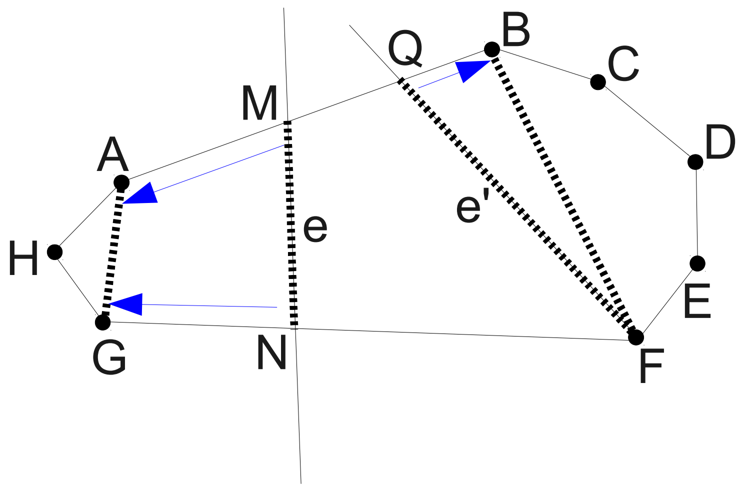

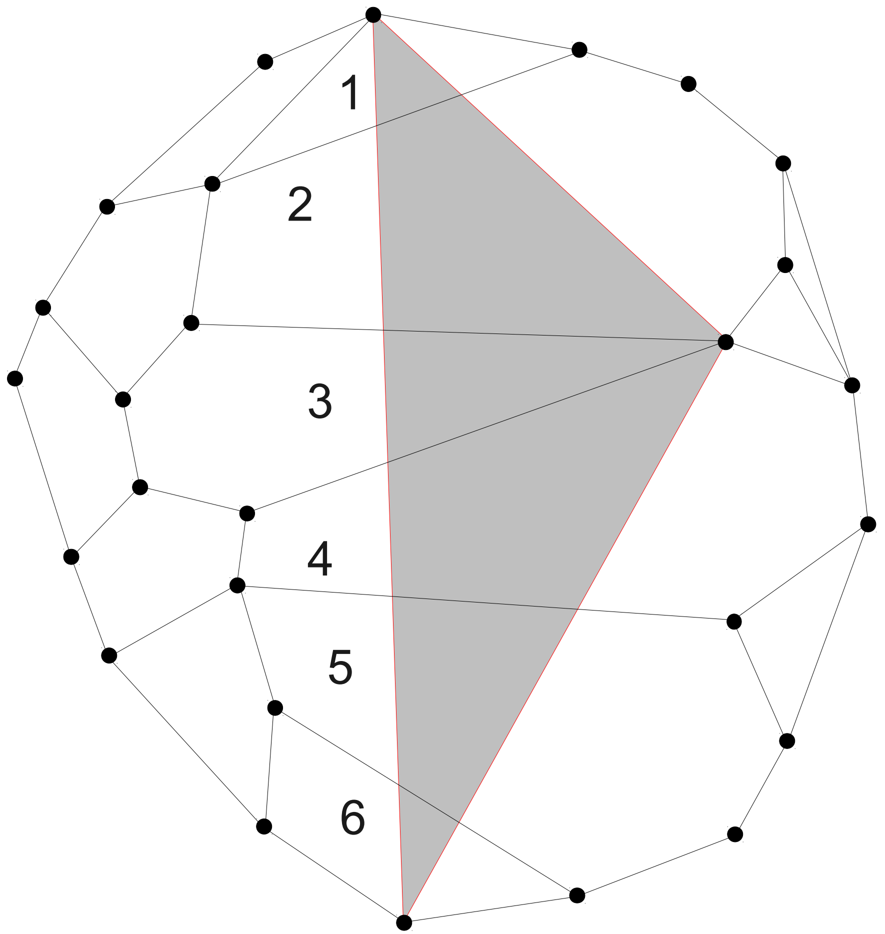

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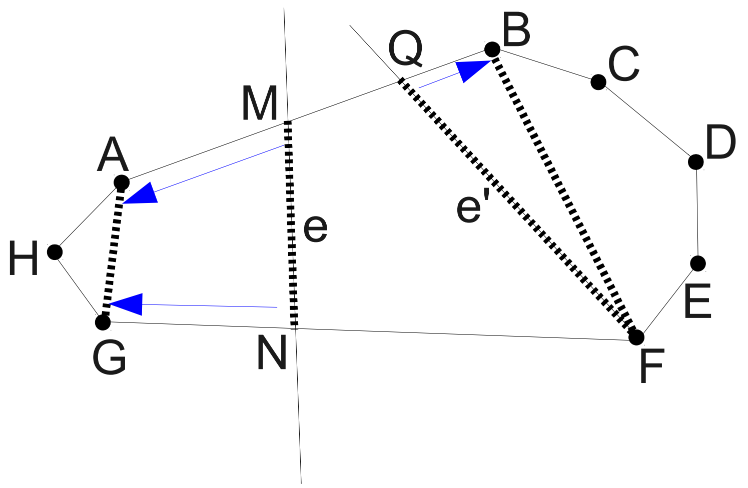

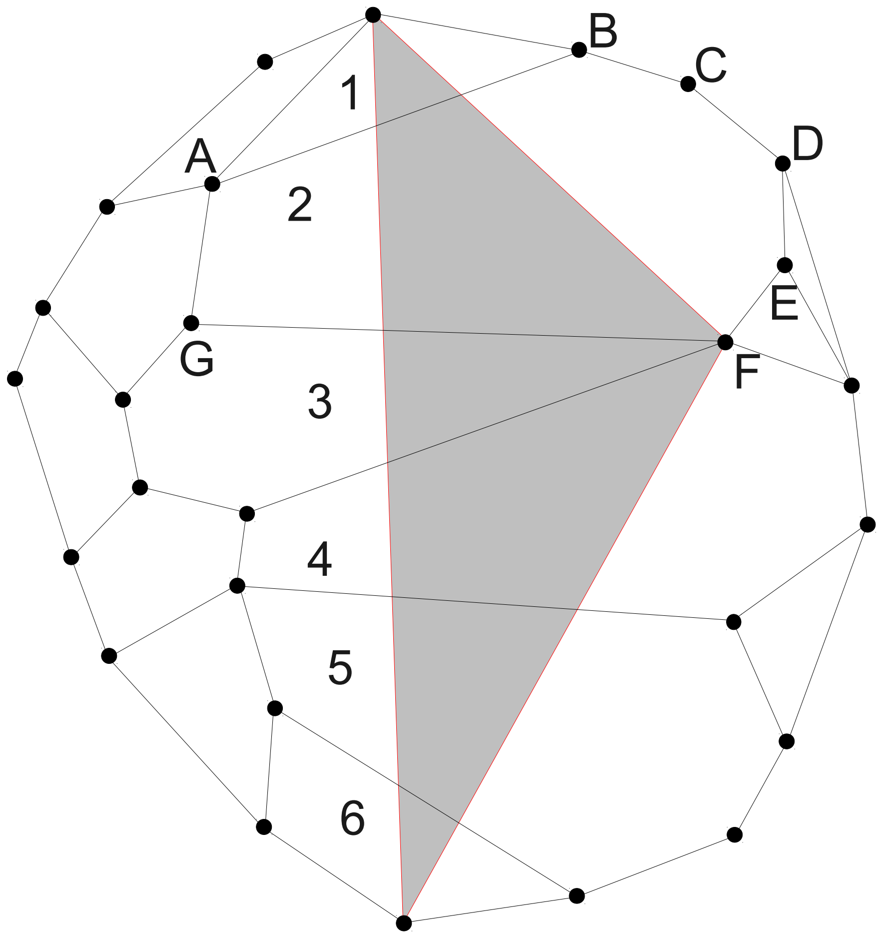

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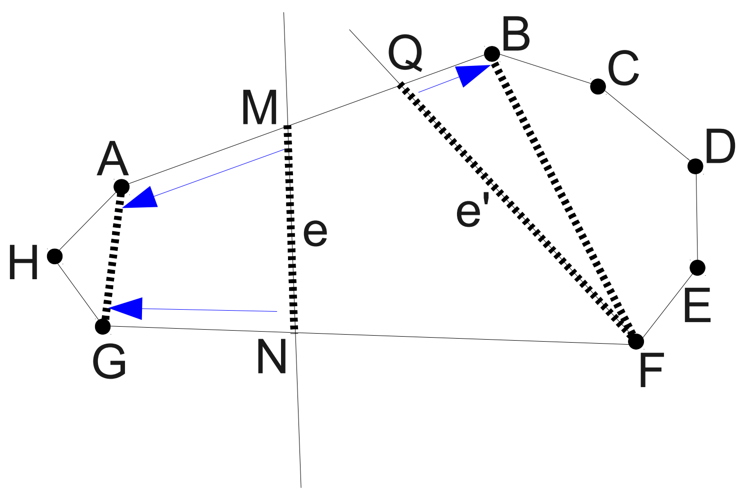

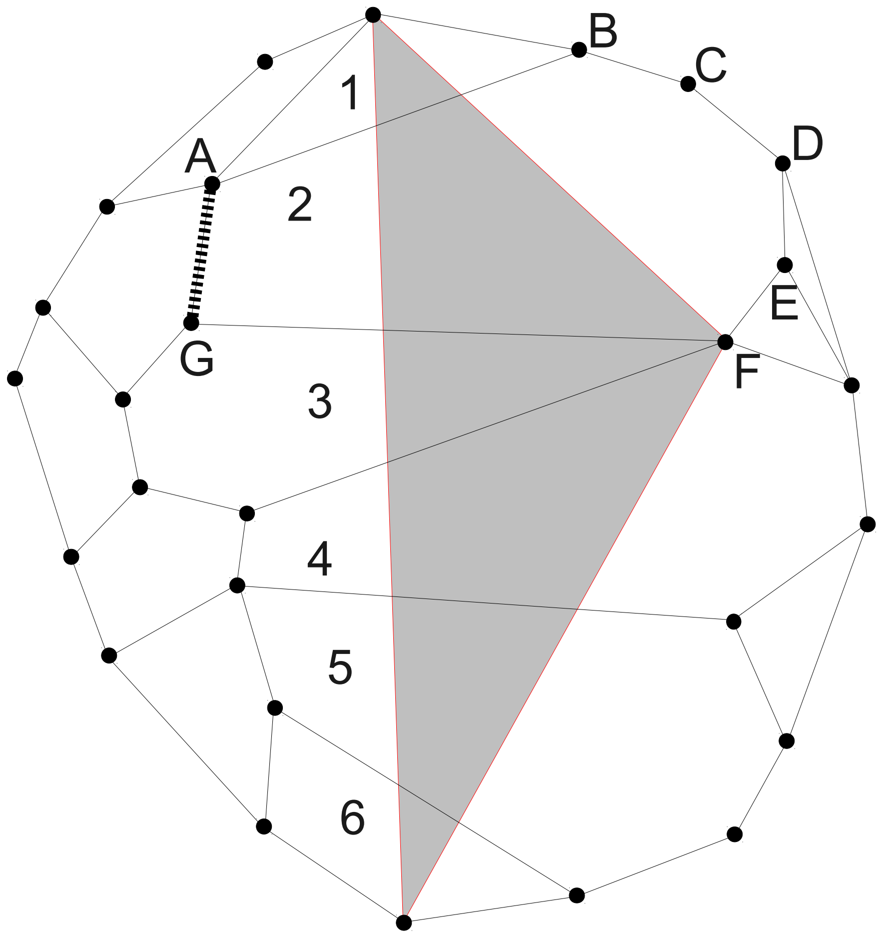

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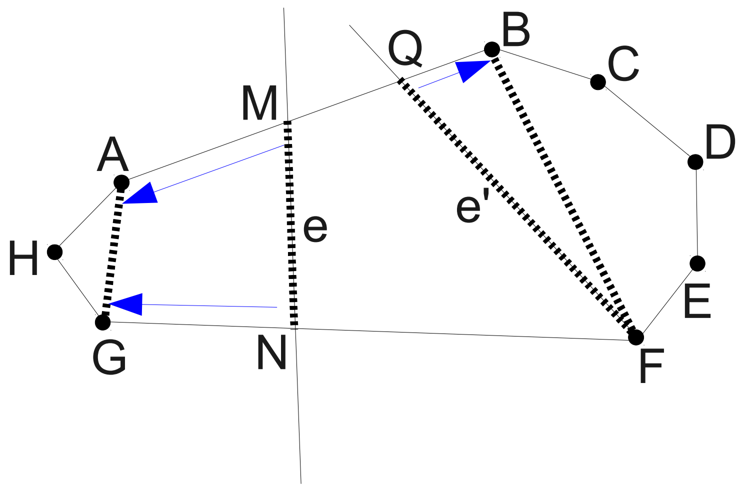

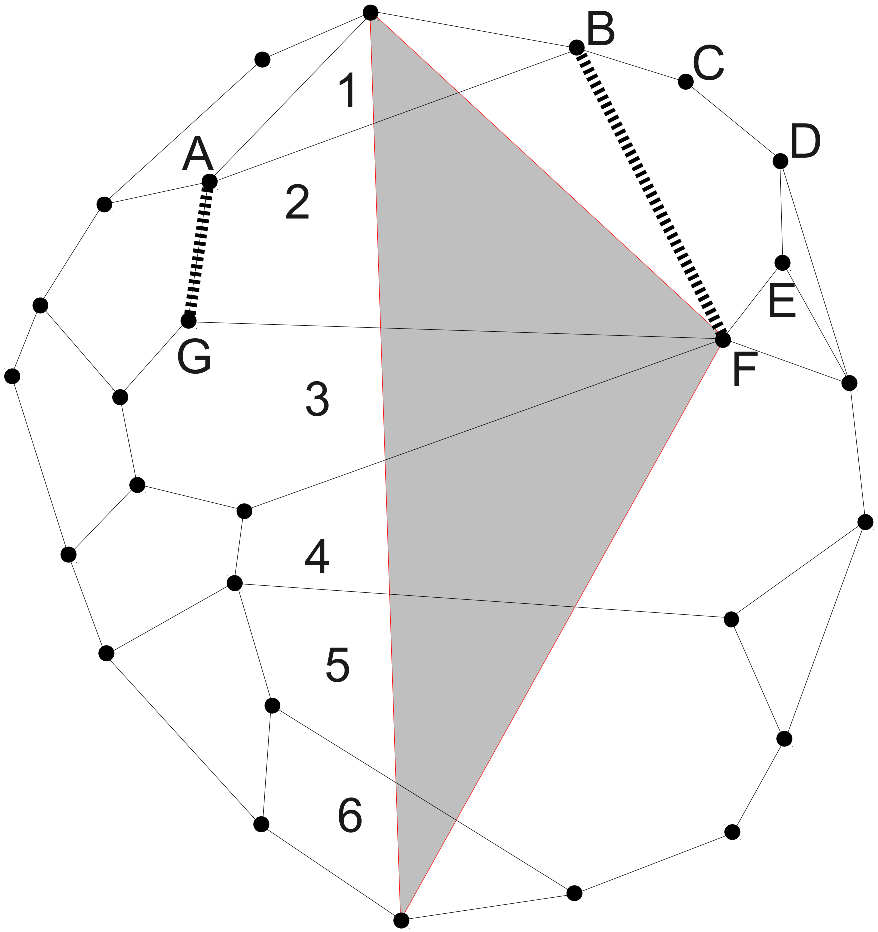

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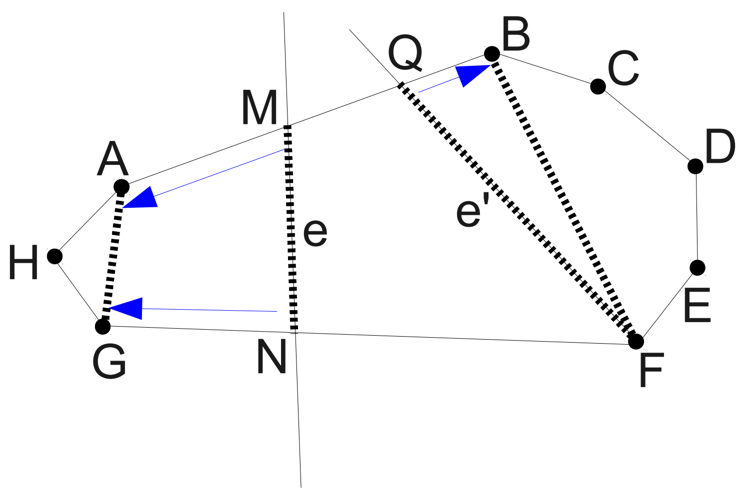

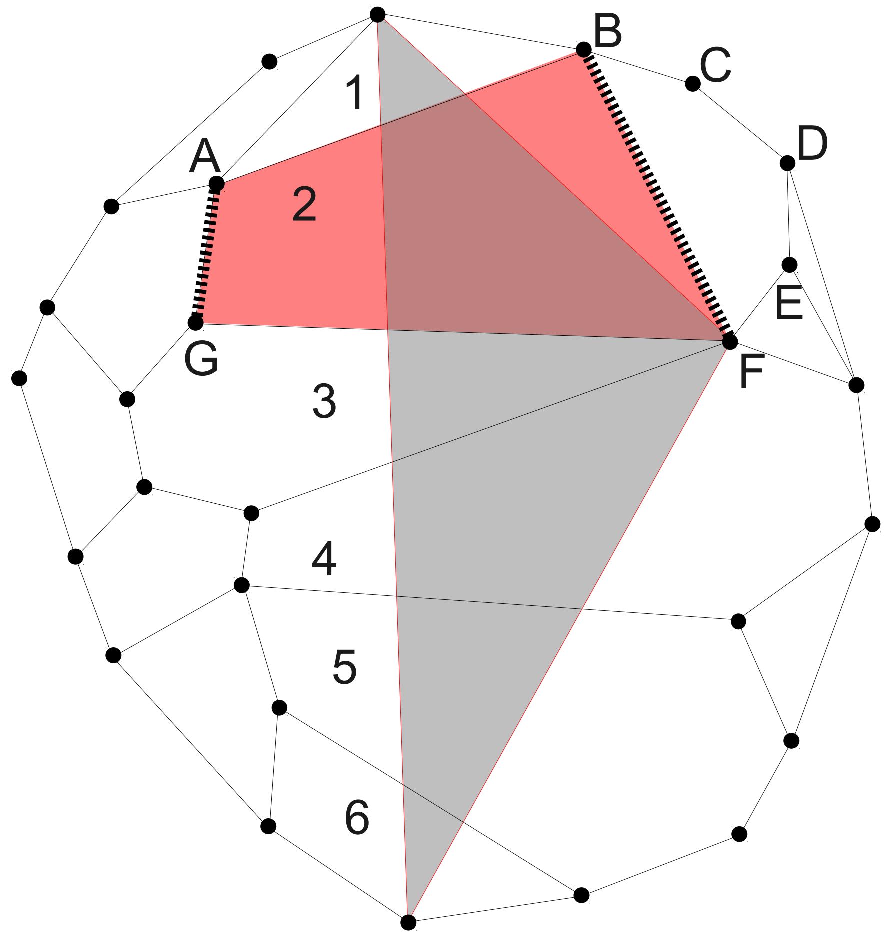

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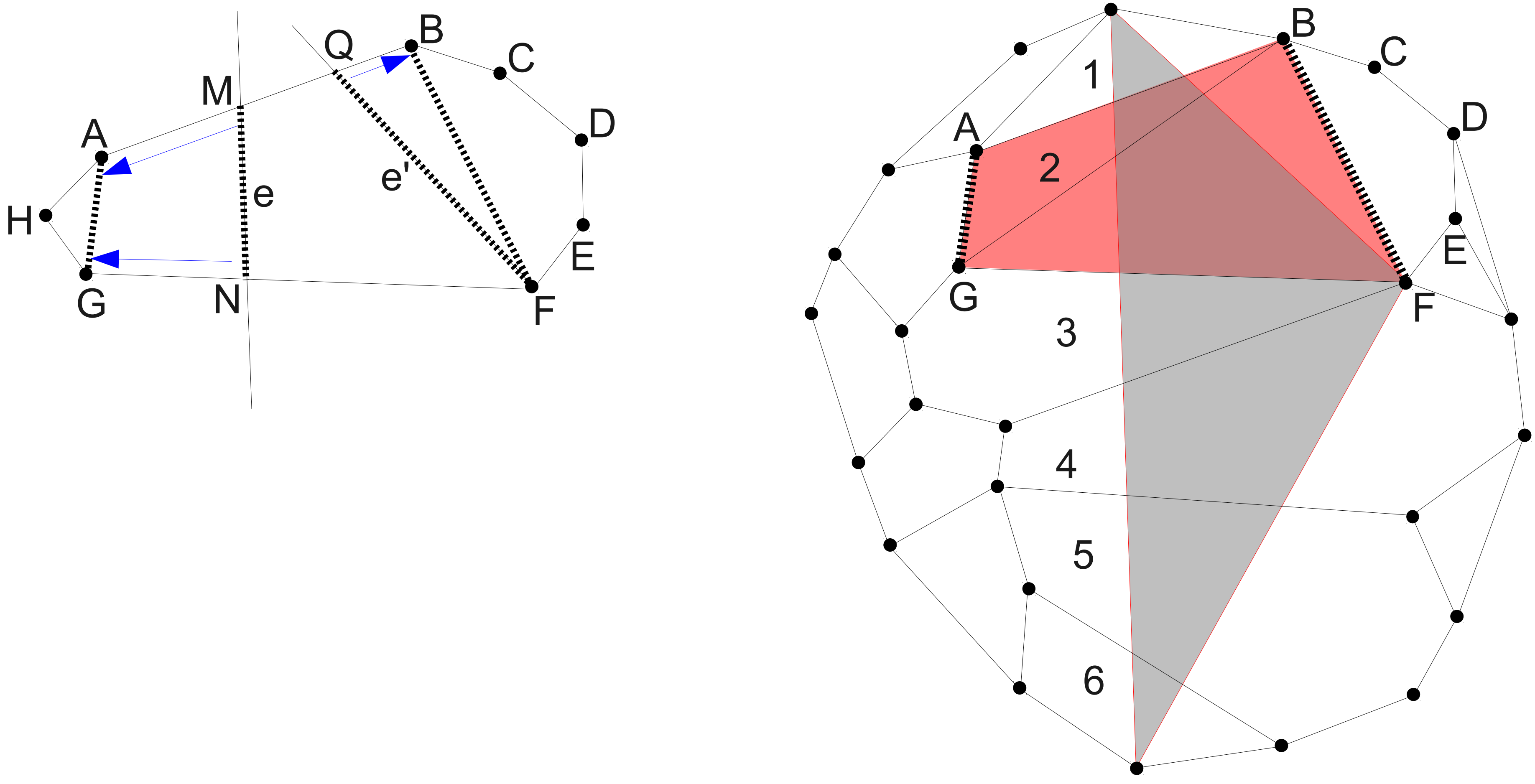

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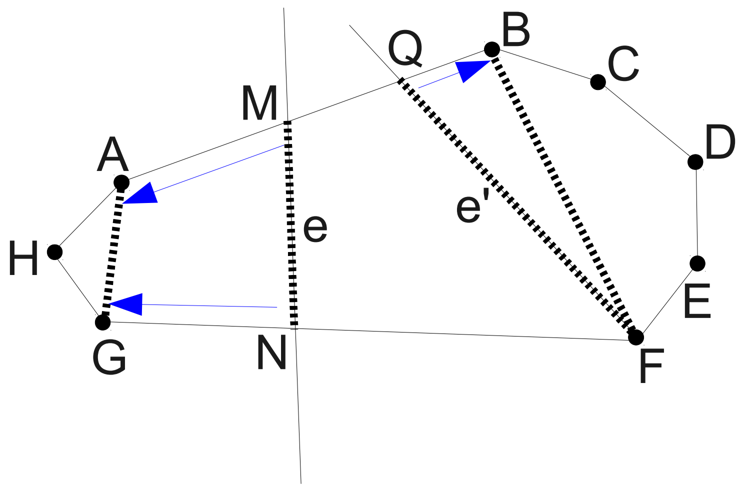

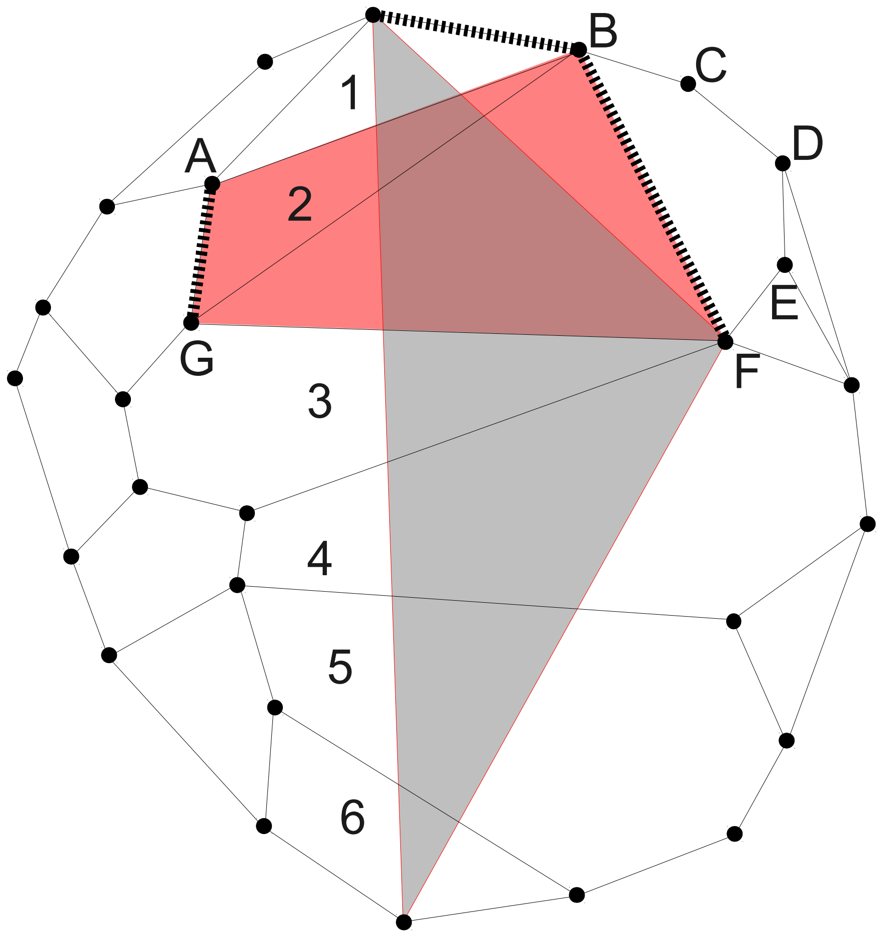

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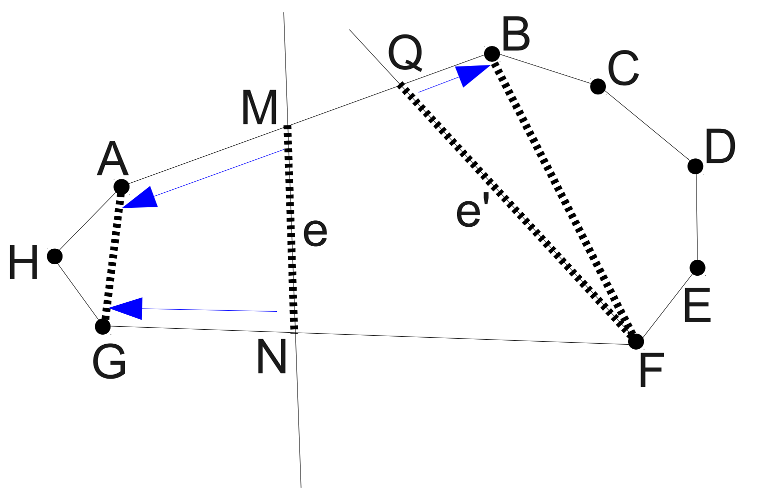

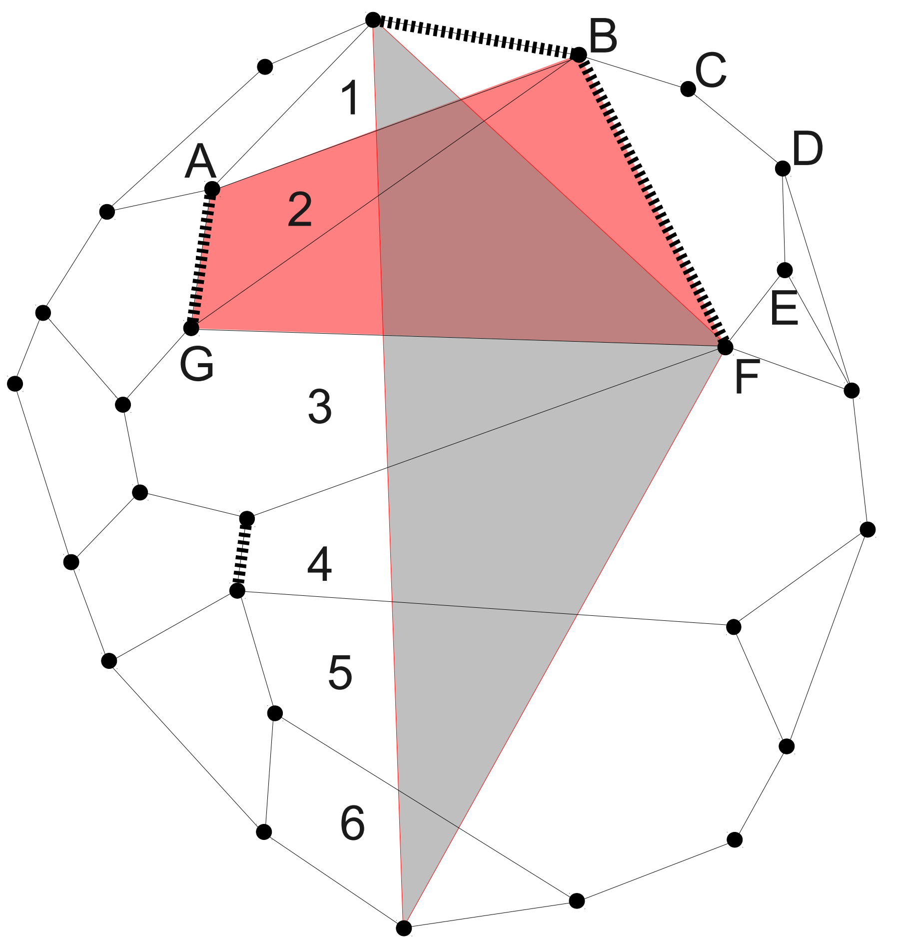

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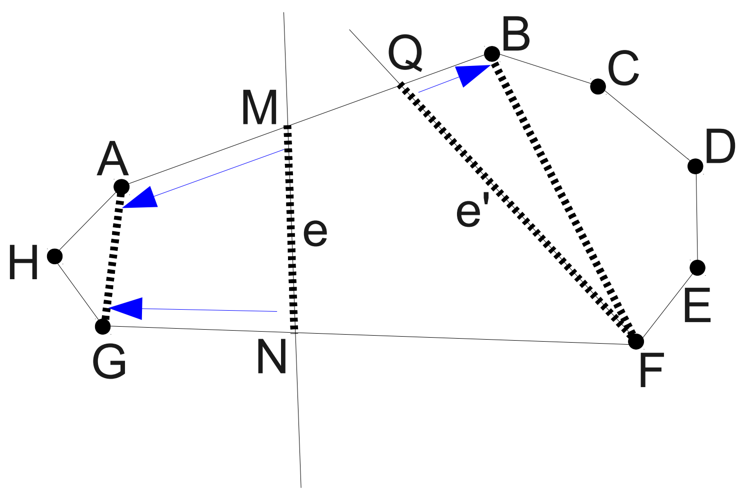

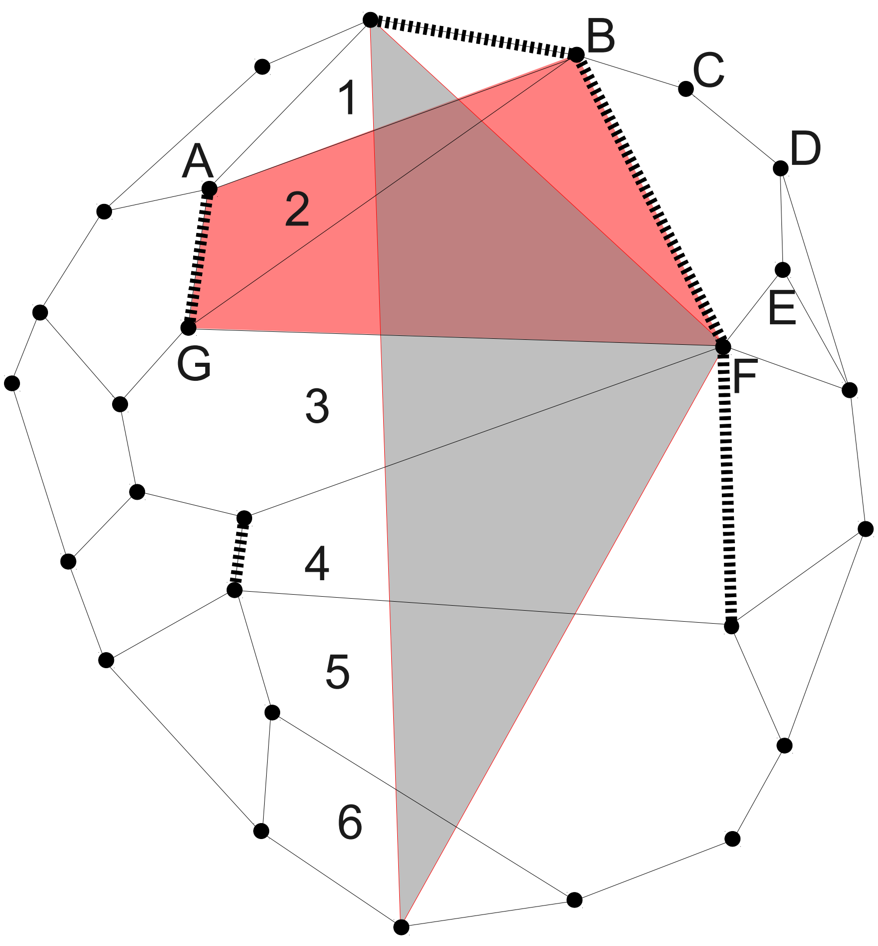

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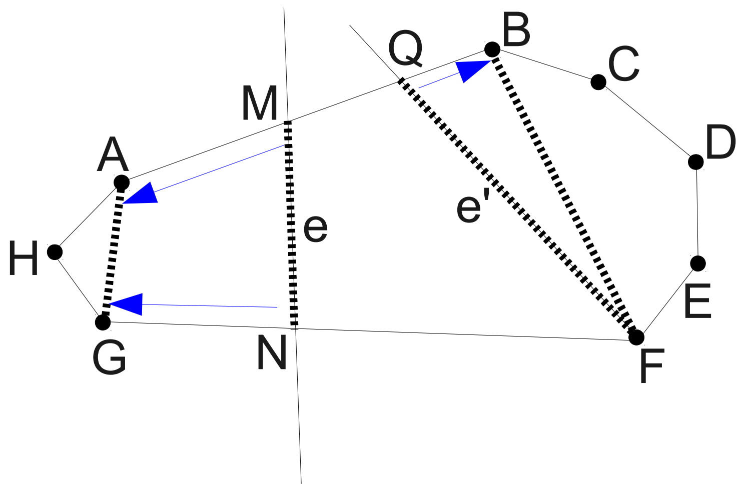

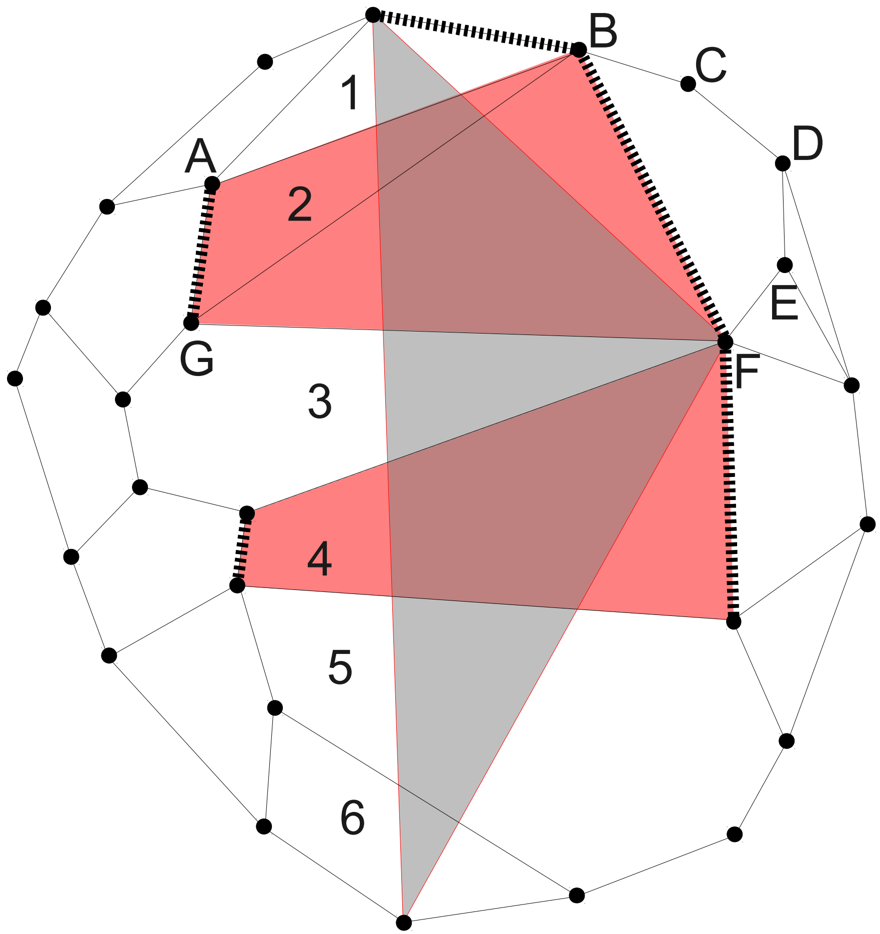

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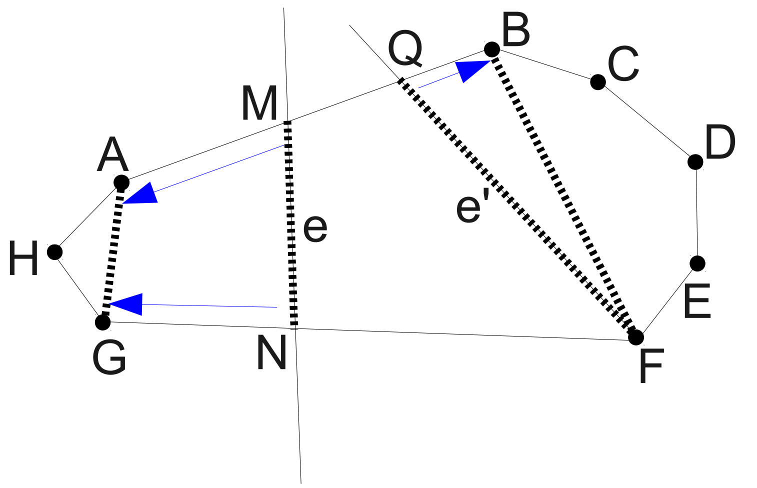

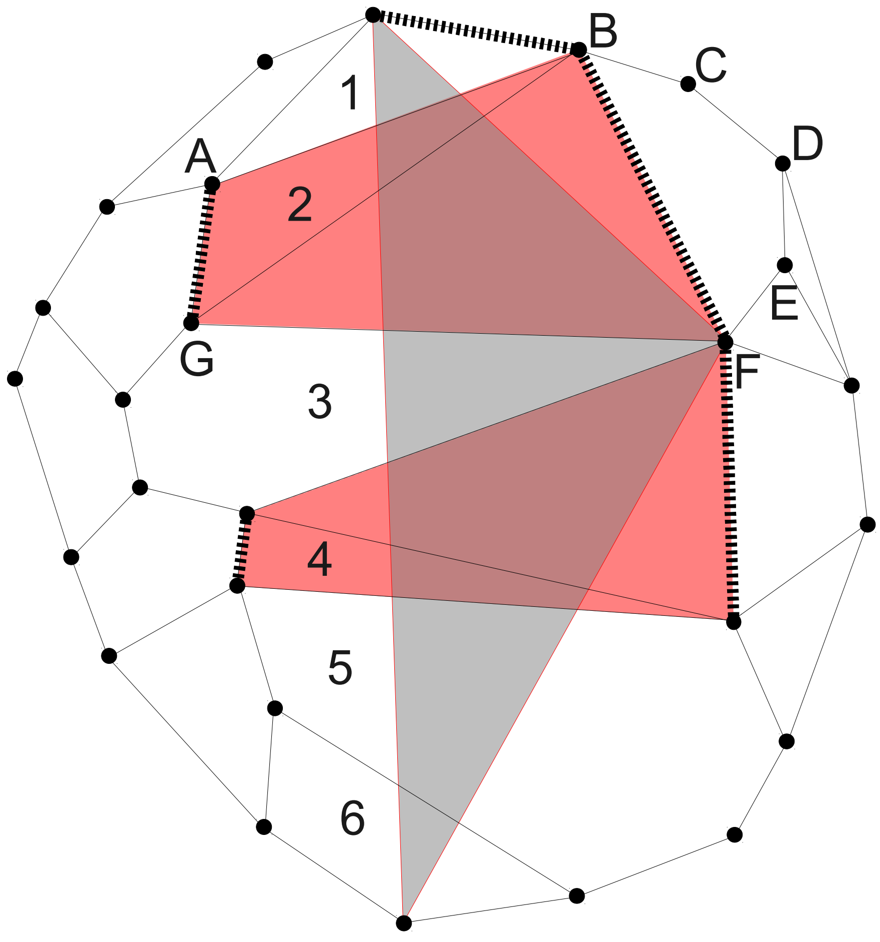

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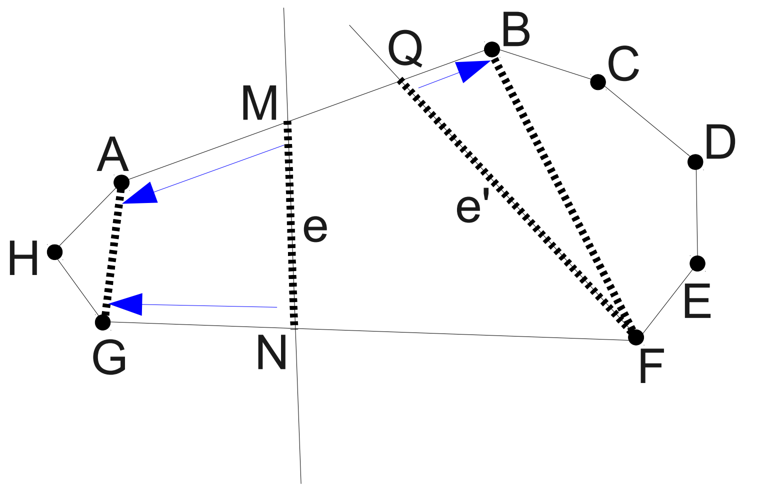

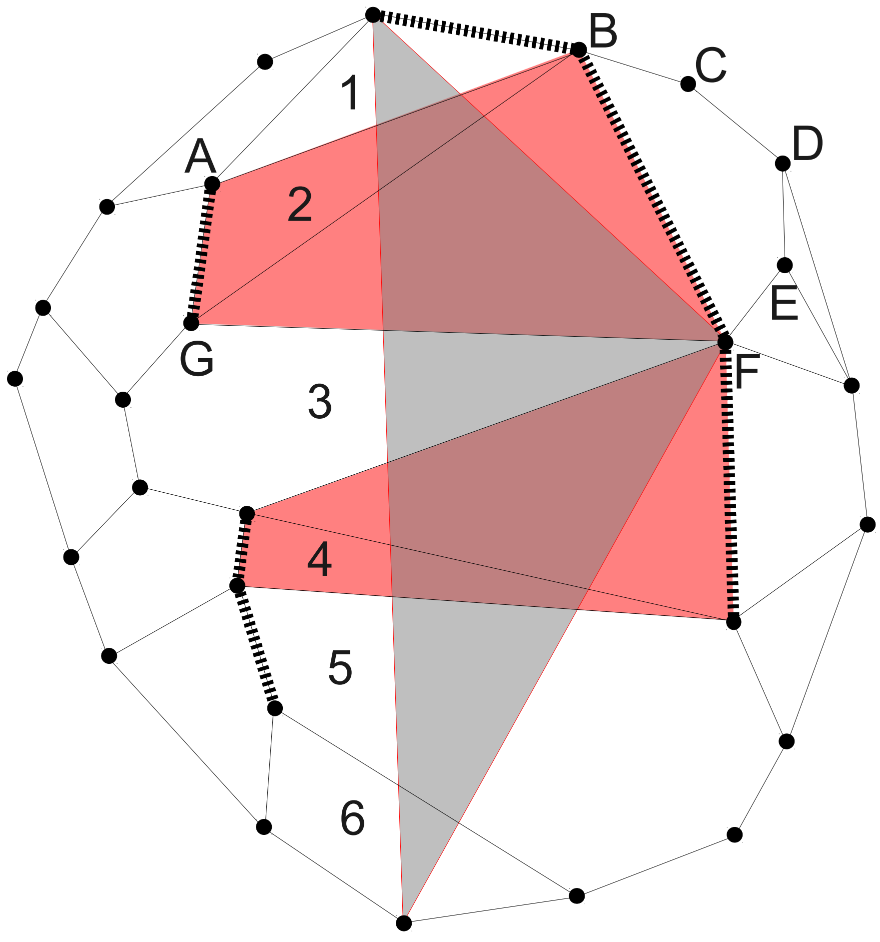

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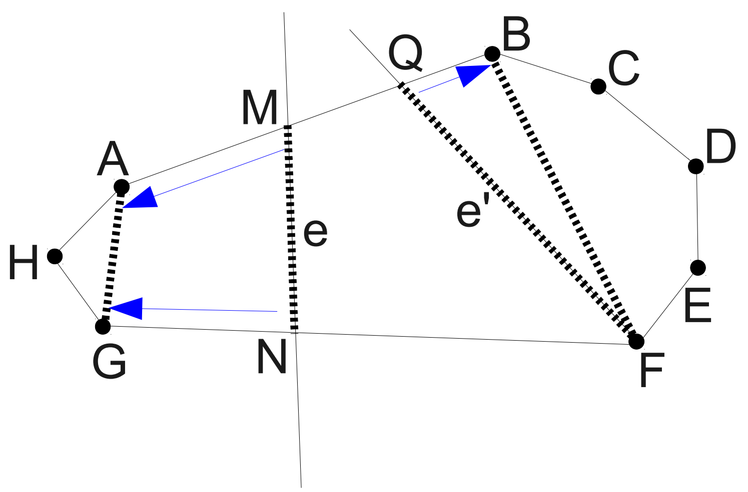
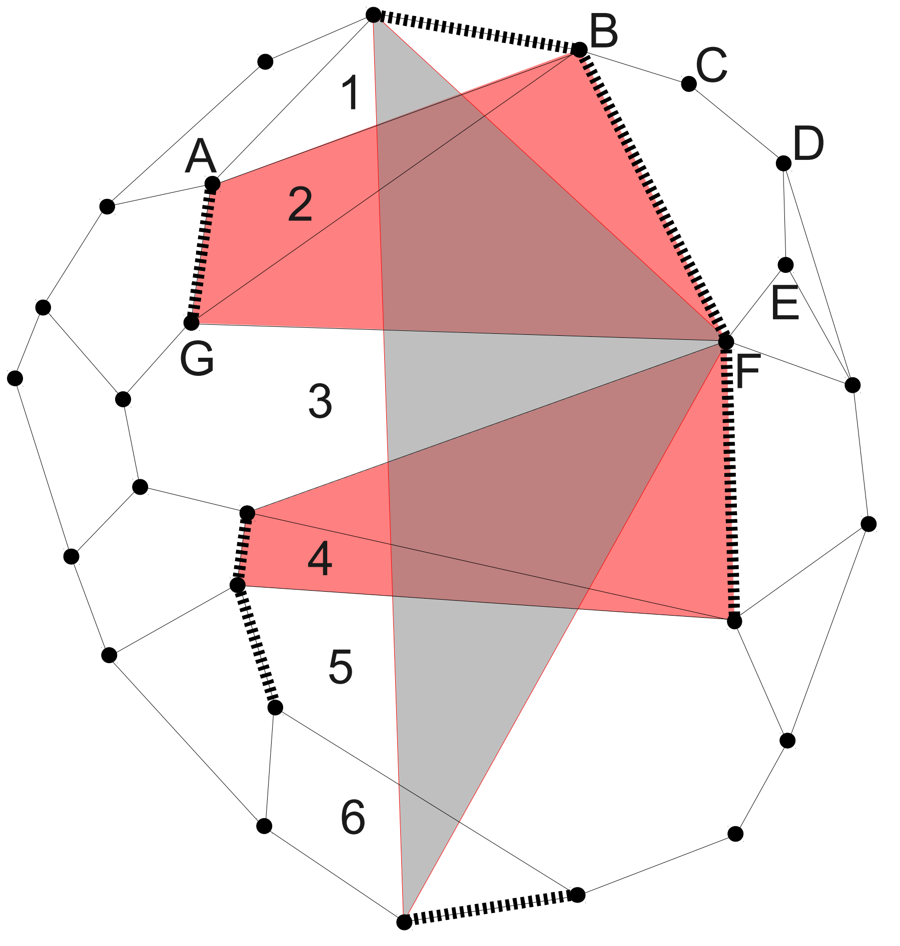
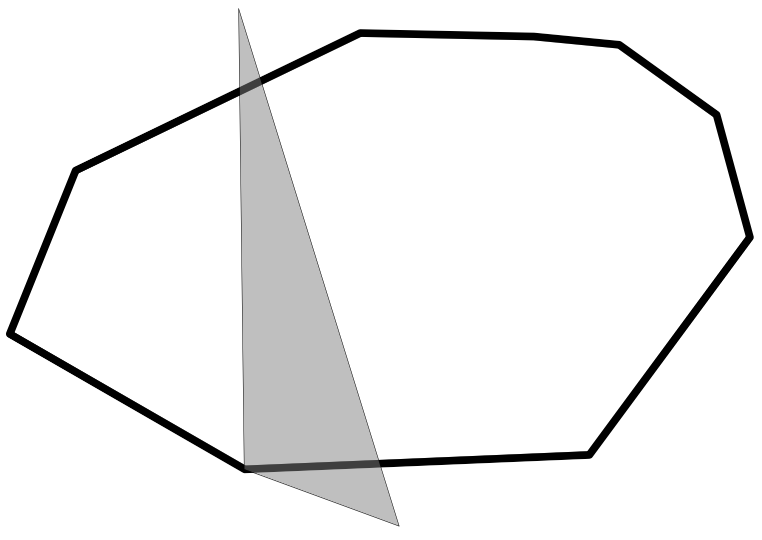




Feasibility



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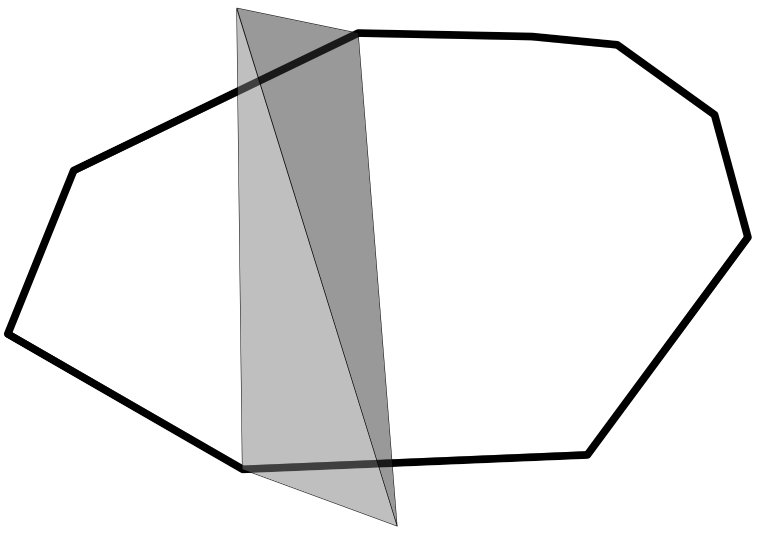

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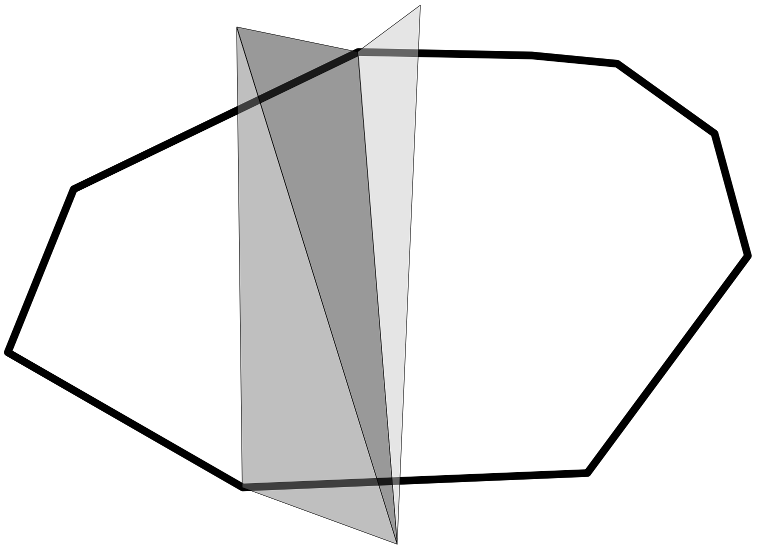

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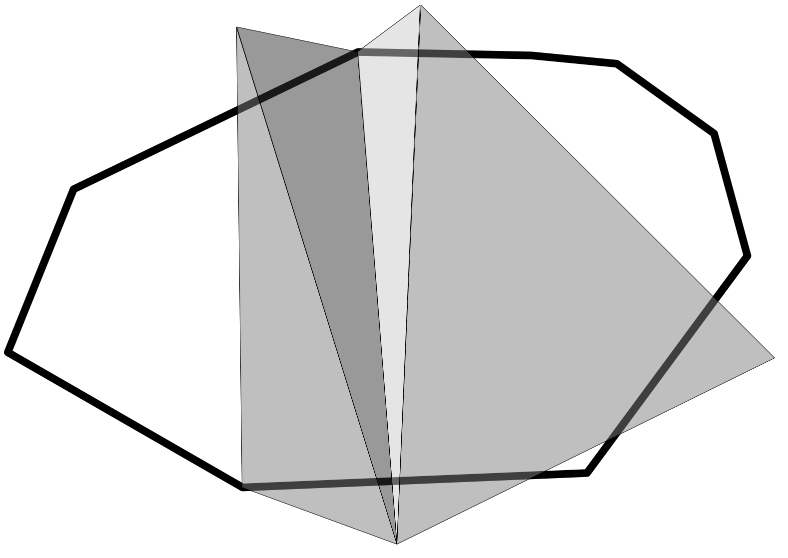

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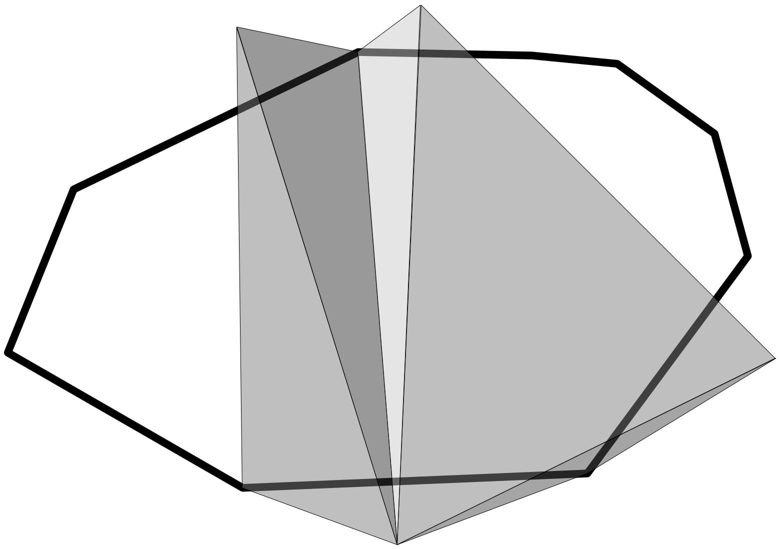

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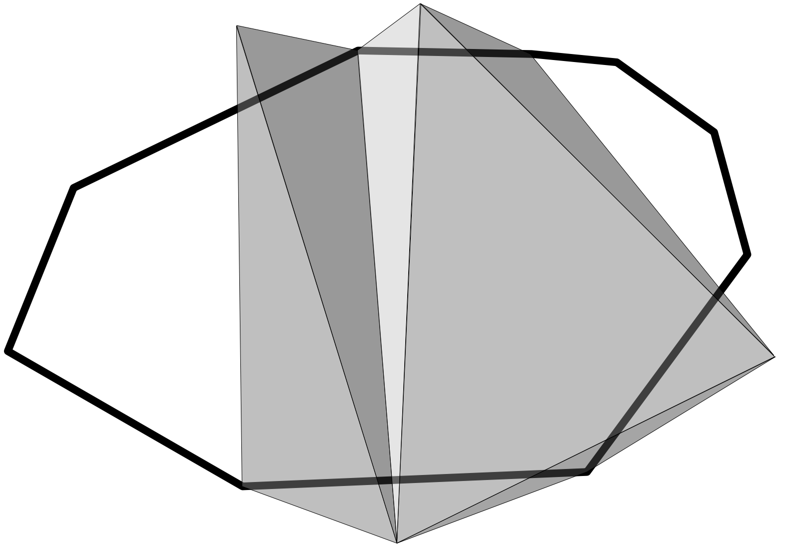

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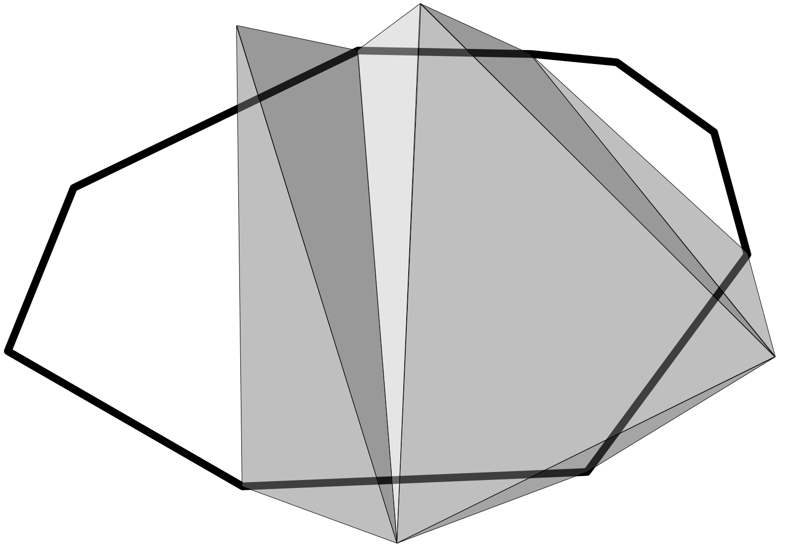

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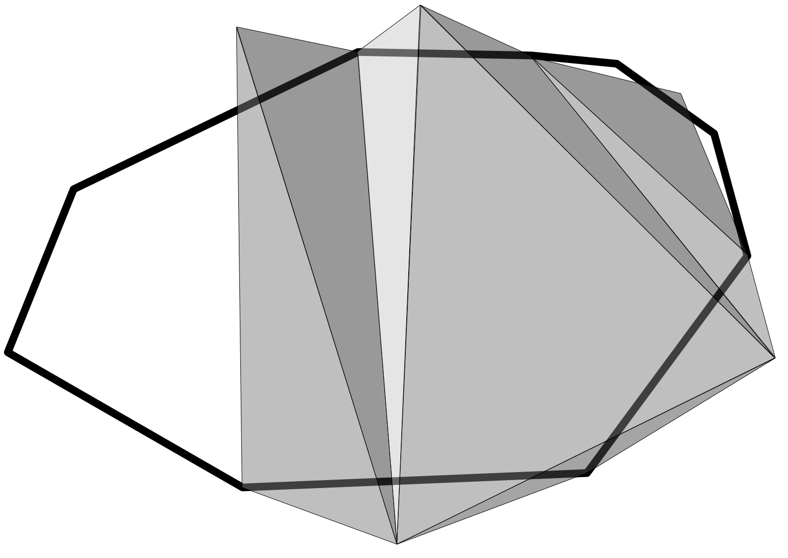

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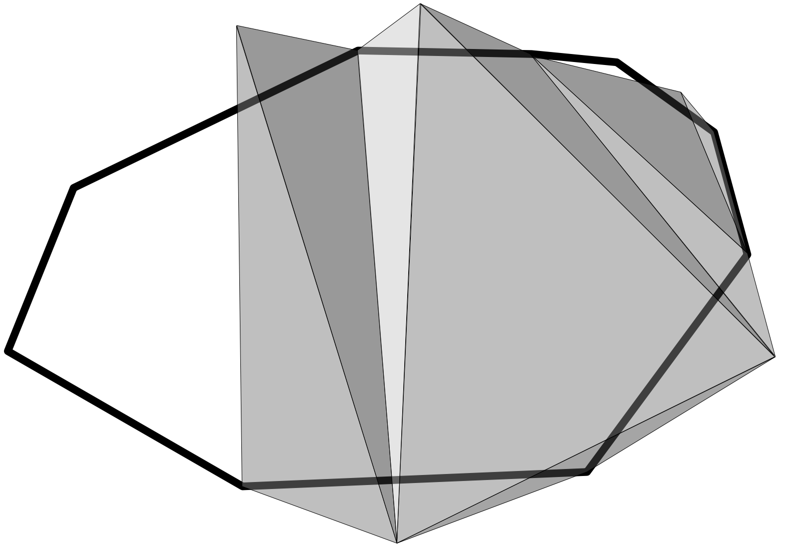

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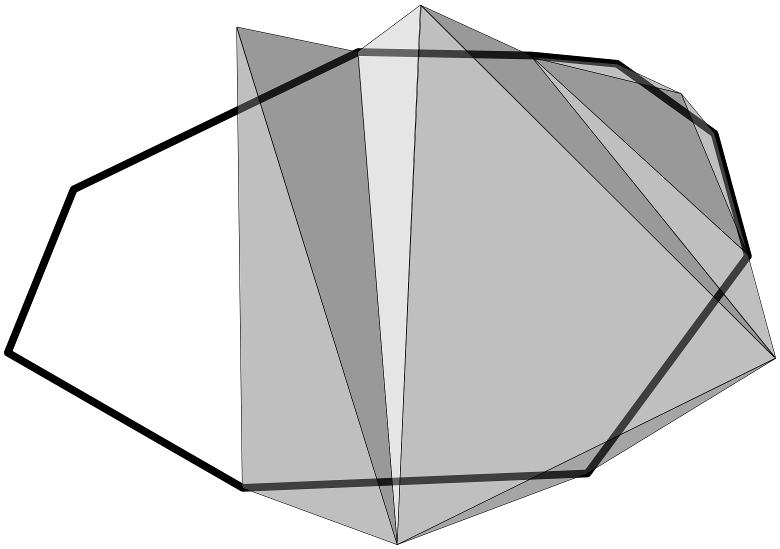

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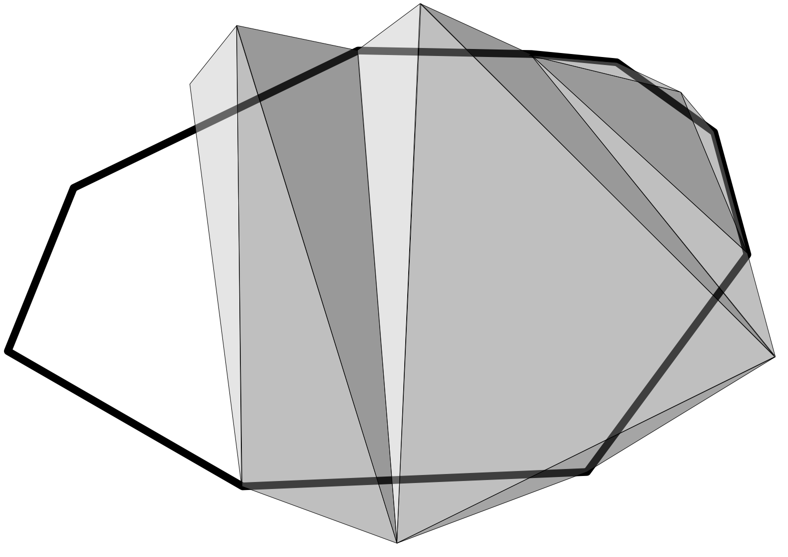

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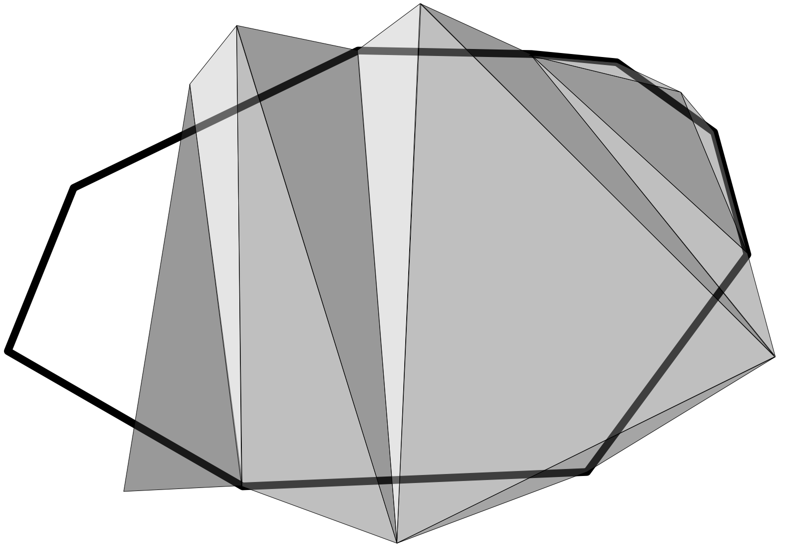

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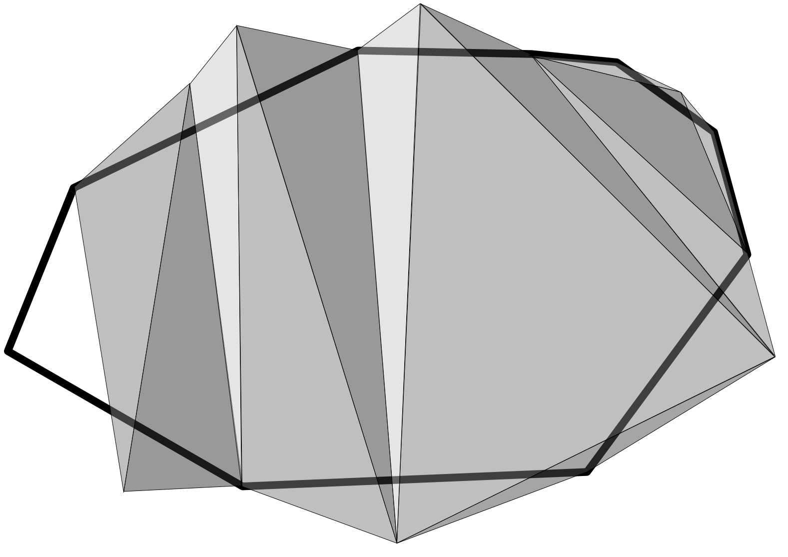

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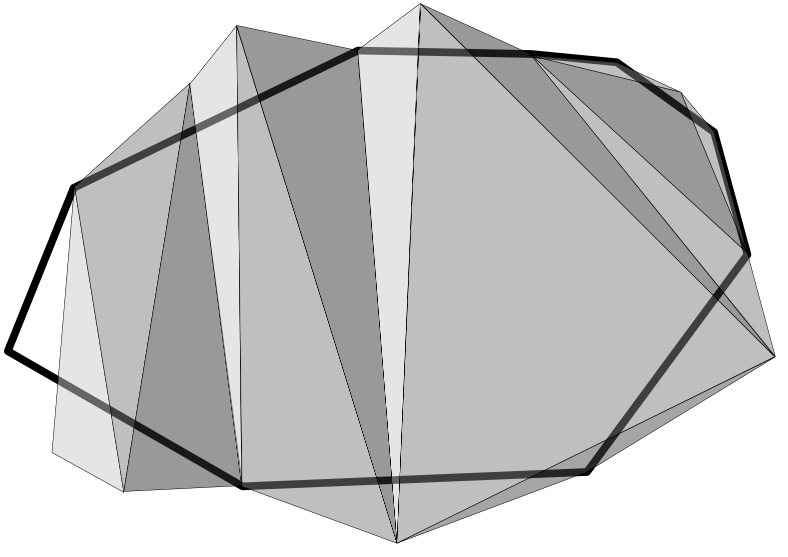

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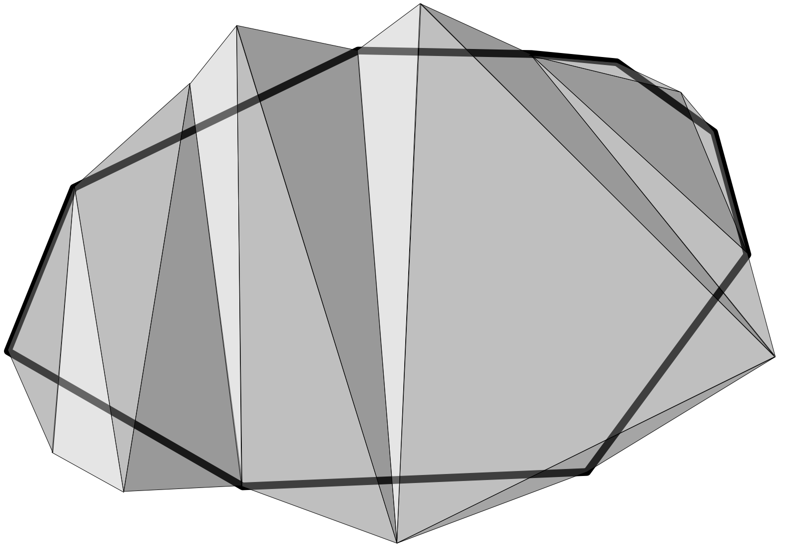

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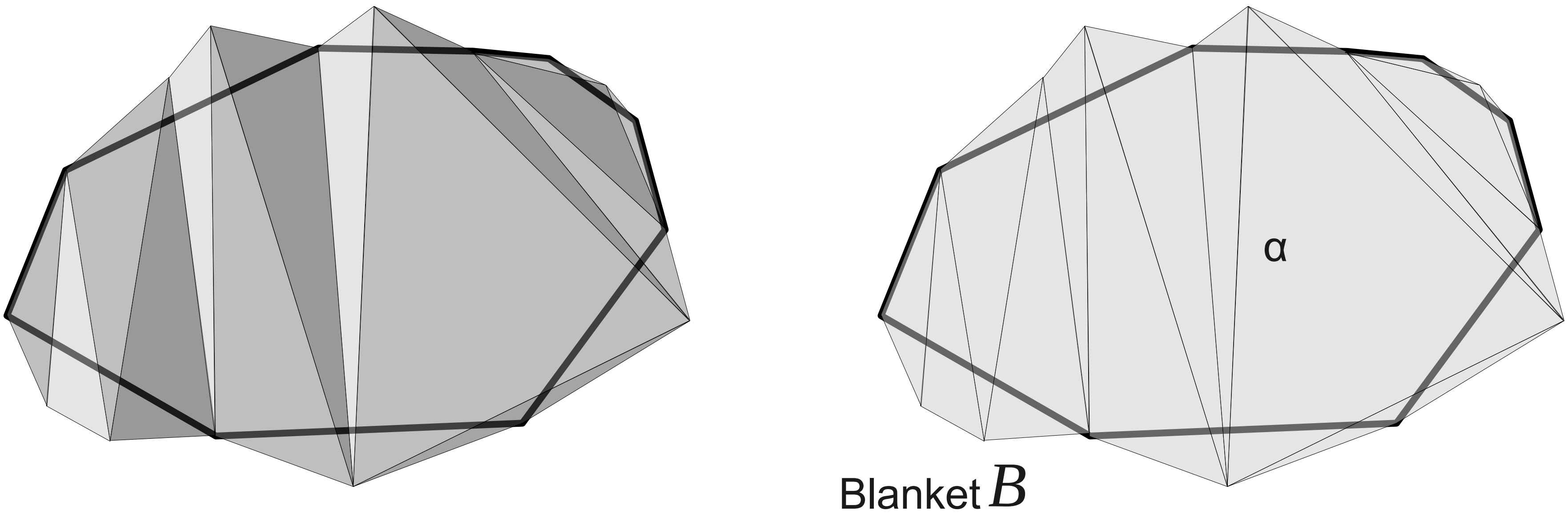

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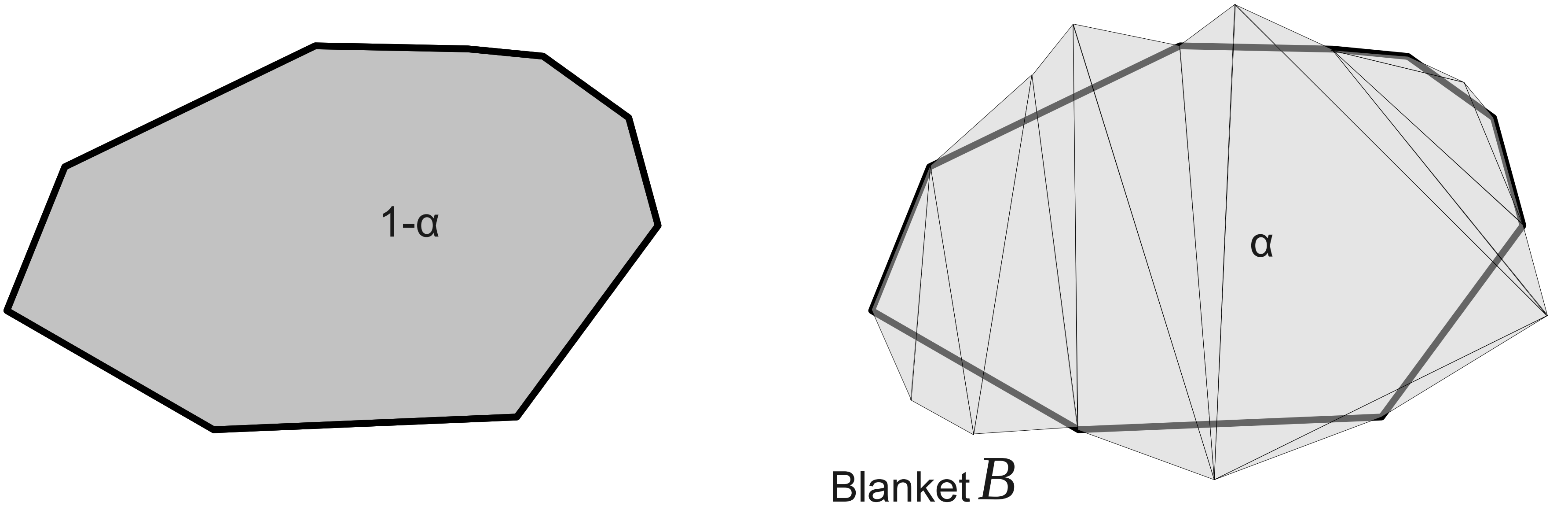

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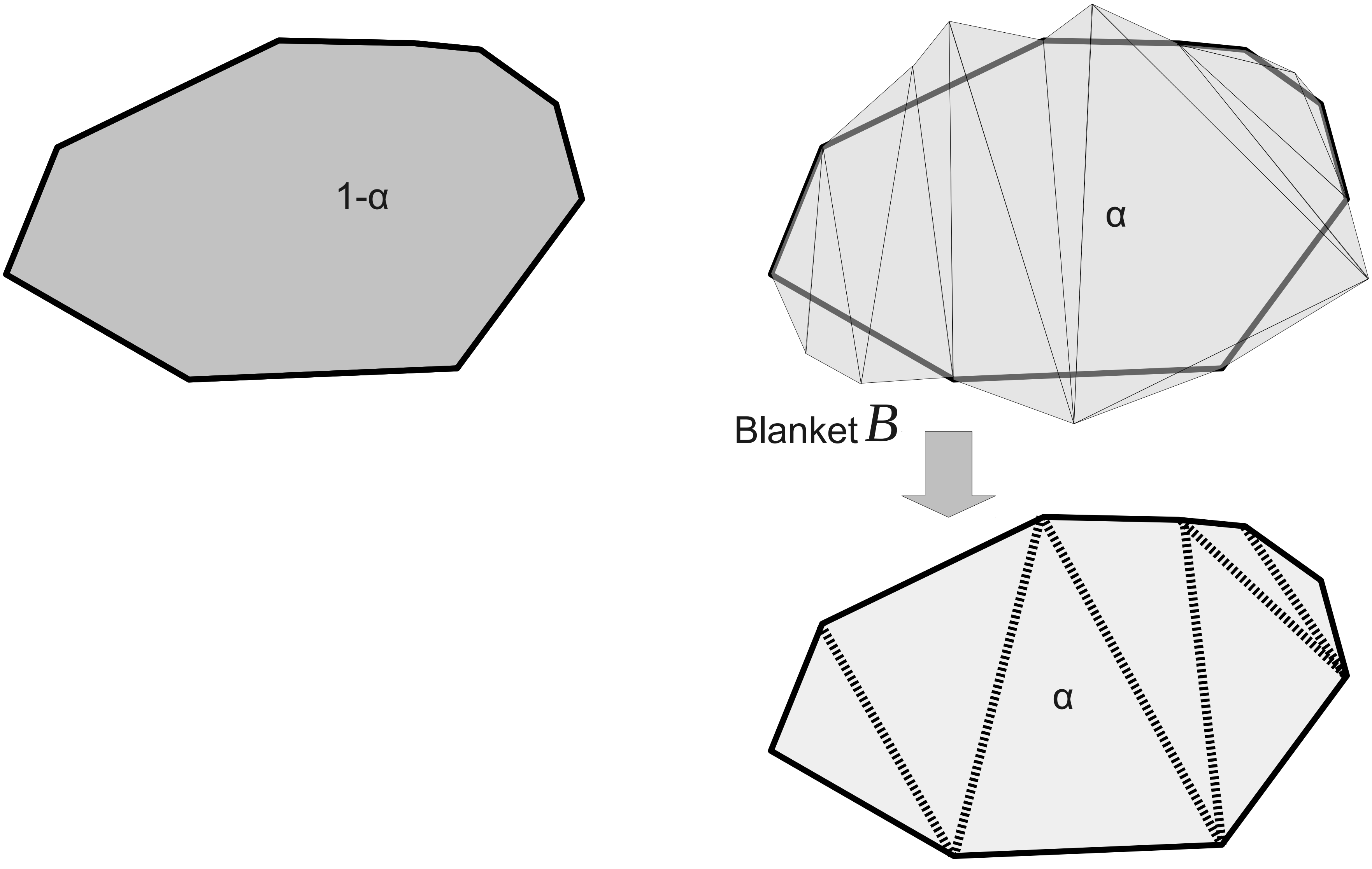

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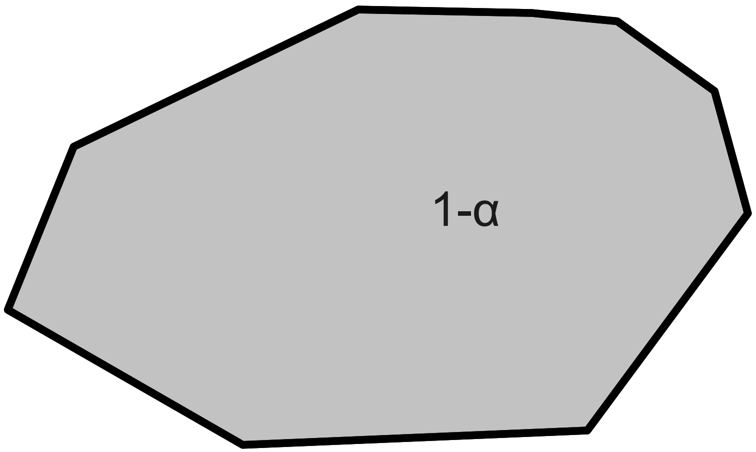
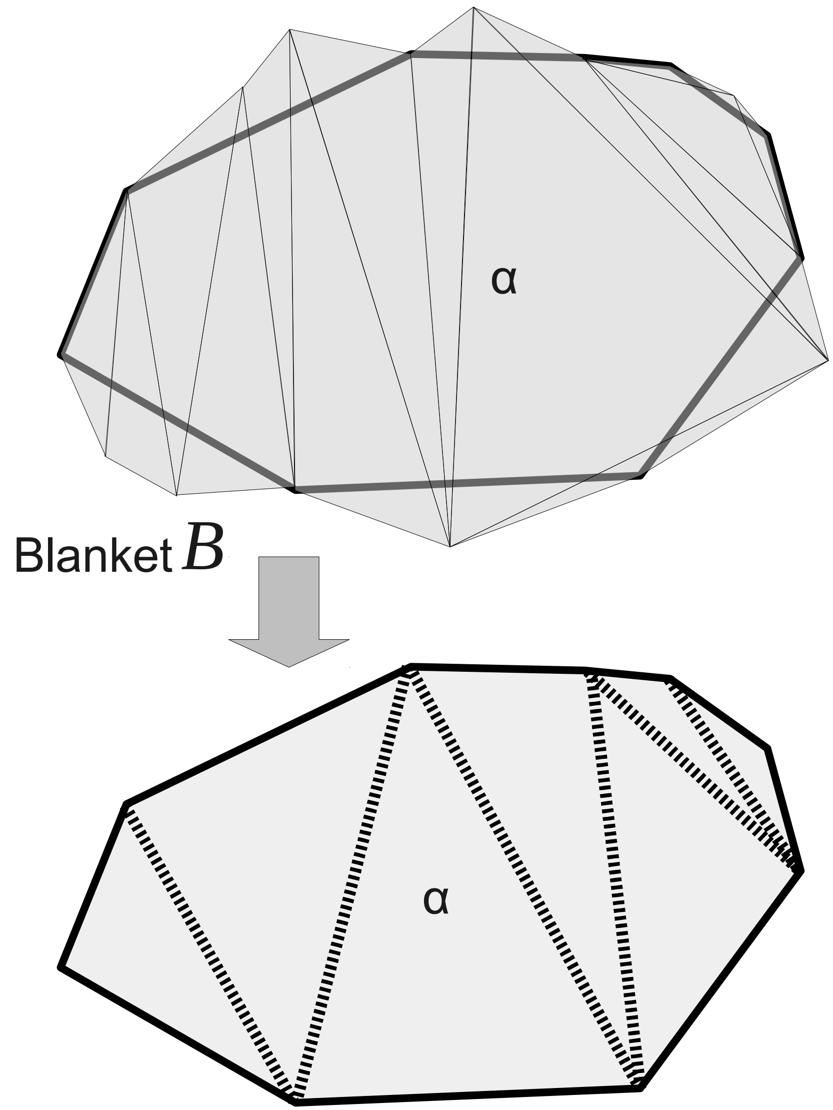




- Feasibility: Every point is covered with weight one.

$$\forall p \quad \sum_{t \ni p} X_t^f = 1$$

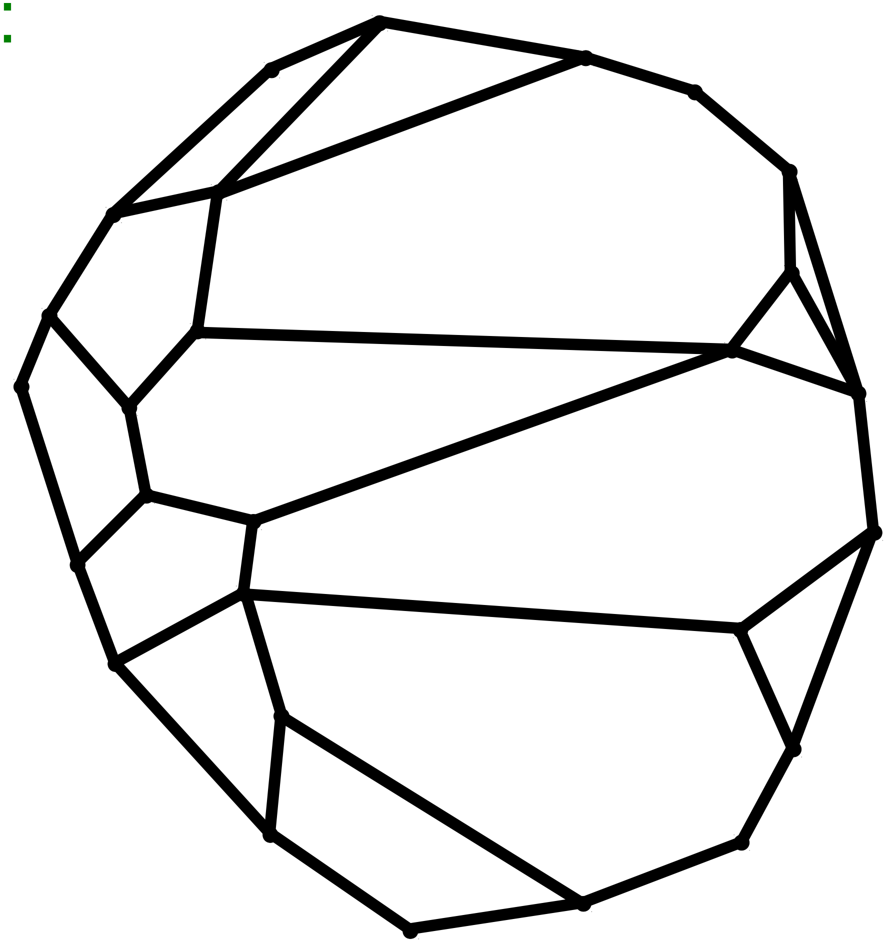
Cost Bound

[Levcopoulos and Krznaric '96]:

- There are constants λ and r and a convex partition (LK) such that:

  1) $|LK| \leqslant \lambda \cdot |MCP|$

  2) The edges of LK are all r-sensitive (r ≈ 4.45).



Cost Bound

- **Theorem:** If C is an arbitrary r-sensitive convex partition, then there is a triangulation T that costs at most $3|C| + 12r|OPT_F|$.

$$T \leqslant 3|LK| + 54|OPT_F|$$

$$T \leqslant 3\lambda|MCP| + 54|OPT_F| \quad \Rightarrow \quad T \leqslant (3\lambda + 54) \cdot |OPT_I|$$

- **Lemma:** $|MCP| \leqslant 18 \cdot |OPT_F|$

$$T \leqslant 3\lambda|MCP| + 54|OPT_F| \quad \Rightarrow \quad T \leqslant 54(\lambda + 1) \cdot |OPT_F|$$

Open Problems

- What is the integrality gap of the LP?

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- Does constant rounds of lift and project bring the integrality gap to 1+ε?

Thank you!