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Sol B part-2.

3  $\Rightarrow$  1: By construction, I will show that there exists a partition  $A, B \subseteq V$ , and that forms a bipartite partition.

Given a Graph  $G(V, E)$ , select a node at random and run BFS ~~from~~ with it as a root. That will create a BFS tree, and every node will have a value  $d[u]$ .

It is easy to prove that there ~~are~~ <sup>is</sup> no edges between nodes  $u, v \in V$ , if  $d[u], d[v]$  are both odd or both even. (1)

Why?

Because then there would be a cycle of odd length. [You need to elaborate on this].

I define  $A = \{u : d[u] \text{ is odd}, u \in V\}$   
 $B = \{v : d[v] \text{ is even}, v \in V\}$ .

Given statement (1), the edges of  $E$  exist only between nodes of  $A$  and  $B$ :  
i.e.

$(u, v) \in E \Rightarrow \begin{cases} u \in A \text{ and } v \in B \\ u \in B \text{ and } v \in A \end{cases}$

Thus,  $A, B$  is a bipartite partition.  
and  $G(V, E)$  is bipartite.

Q.E.D.