5.1.B

We can show this in a circular way as in the Free-Trees Theorem 5.2.

$1 \Rightarrow 2$, $2 \Rightarrow 3$, $3 \Rightarrow 1$. ★

$1 \Rightarrow 2$: Construction.

I color the bipartite graph ($A \subseteq V$, $B \subseteq V$, $A \cup B = V$, $A \cap B = \phi$)

as follow all nodes of A : black

all nodes of B : white.

I will prove that this is a two-coloring of $G(V,E)$.

I.e. I need to prove that there are no adj. nodes with the same color.

First, recall that since A, B are my sets of the bipartite graph, by definition.

$$\forall (u,v) \in E \Rightarrow \begin{cases} u \in A \text{ and } v \in B \\ u \in B \text{ and } v \in A. \end{cases}$$

Both cases are ~~[illegible]~~ equivalent, so we can assume $u \in A$ and $v \in B$

Therefore, for every edge (u,v) the color $color(u) \neq color(v)$ are different

THUS: our scheme is a legitimate 2-coloring and the graph is 2-colorable.

★ Note

I could also do $1 \Rightarrow 2$, $2 \Rightarrow 1$, $1 \Rightarrow 3$, $3 \Rightarrow 1$

or $1 \Leftrightarrow 2$, $2 \Leftrightarrow 3$ etc.