(6)

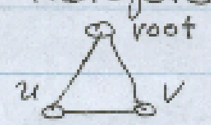
5.1.A.

Method 1: Construction. (Lengthier)

I will show that for every tree I can find a 2-coloring as follows:

- given a tree, pick a random node v as root.
- run BFS.
- colour nodes of odd depth : black.
  colour nodes of even depth : white.

Now, I need to prove that there does not exist two adj. nodes u, v $(u,v) \in E$ such that
$c[u] = c[v]$ where $c()$ is the color of a node.
I can prove this by contradiction:
Assume there is $u, v \in V$, $(u,v) \in E$
and $c(u) = c(v)$.
The main idea: this is impossible because then
a) Either $d[u] = d[v]$ in BFS and therefore there exists a cycle
(needs more elaborate proof)



b) or $d[u] = d[v] + 1$ (or vice versa)
and then one is even and one odd
so according to our coloring they should have different colours

Method 2: Induction on the number of nodes of a (Shorter) tree.
Hyp: a size n tree is two colorable, base for $n=2$.