5-5.3

To prove that the graph is a tree, I need to show that:

a. it is connected.

b. it is acyclic.

Let us call $G_d(V_d, E_d)$, the directed and $G_u(V_u, E_u)$ the undirected

a. Connectivity:

We need to show that:

any two nodes u, w in V_u. have a path in G_u.

In the directed graph, There exists paths:

$v_0 \rightsquigarrow u : (x_1, \ldots, x_k) \quad x_1 = v_0,\ x_k = u$

$v_0 \rightsquigarrow w : (y_1, \ldots, y_m) \quad y_1 = v_0,\ y_m = w$

Because of the construction of the undirected graph we know that the above paths are also undirected paths in G_u. in other words the path

$(x_k, \ldots, x_1)$ is a legitimate undirected path in G_u.

Combining the two paths:

$(x_k, \ldots\ldots, x_1) \cup (\underset{v_0}{y_1}, \ldots\ldots, y_m)$ we get a path.

$(u, \ldots\ldots, x_2, v_0, y_2, \ldots, y_{m-1}, w)$

which is a path u and w.

b. Acyclicity: Do not extract any points for this part. only for (a)