Ex. 5-4.4

Induction on the number of nodes. $n \geq 2$!

1. Basis. For $n=2$, at least one edge. $E=1$, $N=2$.

2. Hypothesis. I assume that $|E_k| \geq |V_k| - 1$
for all graphs with k nodes $|V_k| = k$

3. Prove for: Graph of $k+1$ nodes i.e. $|E_{k+1}| \geq |V_{k+1}| - 1$.
~~A graph of k+1 nodes can be constructed by a~~
Consider a graph of $k+1$ nodes.
Select a node u randomly.
If u does not have any edges and given that $n \geq 2$, there is at least one node in the graph, w, and u can't reach w.
Therefore, u has at least one edge.
Remove u and its edges. to create graph G' which is a graph of k nodes:
$|E'| \geq |V'| - 1$ (1)
We know by construction of G':
$|V'| = |V_{k+1}| - 1$ (2)
$|E_{k+1}| > |E'|$ (3) since we removed at least one edge.

(1), (2), (3) $\Rightarrow$ $|E_{k+1}| > |E'| \Rightarrow |E_{k+1}| \geq |E'| - 1 \geq |V'| - 1 - 1 =$
$= |V_{k+1}| - 1$