A New Last-Come First-Served Preemptive Window Access Conflict Resolution Algorithm

Gillian M. Woodruff

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Computer Systems Research Institute
University of Toronto
Toronto, Canada
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Abstract

Random access protocols can be employed as distributed controls to ensure successful packet transmissions over a shared, multiple access communications channel. These schemes consist of a channel access algorithm (which governs when a station can first attempt to send a packet), and a conflict resolution algorithm (which governs when to try again if the attempt resulted in a collision). Some of these protocols are said to have the limited sensing property, which means that the actions necessary to transmit a given packet depend only on the state of the channel after the arrival of that packet. This is a significant practical advantage over continuous sensing algorithms, such as the first-come first-served (FCFS) window access algorithms, in which the actions necessary to transmit a given packet can depend on the channel history arbitrarily far into the past. However, current limited sensing protocols have other disadvantages: free access (or stack) algorithms are significantly less efficient than window access algorithms, and existing limited sensing window algorithms impose a delay penalty of several slots for new packet arrivals to acquire enough of the current state of the protocol to join in. Furthermore, their global scheduling is approximately last-come first-served (LCFS).

Here, a new LCFS preemptive window channel access method is proposed and analysed; it eliminates the state acquisition problem as long as each station operates with a common clock and window boundaries (assumed to be every two or three slots). Exact expressions for the packet delay distributions are derived assuming Poisson packet arrivals to the channel, and either a Standard Traversal or Modified Tree Algorithm (STA or MTA) for the conflict resolution. The mean and standard deviation of these delays are evaluated and compared to those for the FCFS sliding window channel access approach. The mean delays are shown to be comparable, but the standard deviation of the LCFS delay is significantly higher. The latter indicates the price of eliminating the continuous sensing problem, and may not be significant when the extra delay required for limited sensing is taken into account.

1 Introduction

In multiple access communications environments, such as local area networks (LANs), packet radio, and satellite networks, stations share a common channel and contend with each other for channel resources when they wish to send a packet. In many cases, there is only limited channel state information available to these stations, which leads to a non-zero probability of packet collisions. Several schemes to control the timing of packet transmissions in a distributed manner (in an attempt to efficiently utilise the channel) have been proposed and analysed. These schemes are based different degrees of channel state information available at each station ranging from ALOHA (no information) through Carrier Sense Multiple Access (CSMA — with carrier sensing) to CSMA with collision detection (CSMA/CD).

Each of these types of schemes employ both a *channel access algorithm*, which governs when a station can first attempt to send a packet, and a *conflict resolution algorithm*, which governs when to try again if the attempt resulted in a collision. In this project, attention is restricted to a class of conflict resolution algorithms called *binary preorder tree traversal algorithms* analysed extensively in [1]. These algorithms resolve conflicts between a group of contending stations (admitted into the contention by the channel access algorithm) during a Conflict Resolution Interval (CRI). In [1], the delay performance of this conflict resolution algorithm was studied assuming a first-come first-served (FCFS) fixed-size Simplified Window Algorithm (SWA) for channel access, which will be referred to here simply as the SWA method. With this method, the time axis can be visualised as being divided into fixed-size windows; at the end of the window, all packets arriving during that window are permitted to join the next CRI, when it occurs.

This method, studied in [1] in its simplest form, requires continuous sensing — all stations must know both where the current window, and the current CRI boundaries are. This is difficult to accomplish when new stations join an active channel, and has led to the development of limited sensing schemes, in which new stations monitor the channel to recognise certain idle/success/collision patterns that will indicate when the current CRI ends. Current limited sensing schemes have certain disadvantages: free access (or stack) algorithms are significantly less efficient than window access algorithms, and existing limited sensing schemes for window access impose a delay penalty of several slots for new packet arrivals to acquire enough of the current state of the protocol to join in. Furthermore, their global scheduling is approximately last-come first-served (LCFS). This problem has motivated the following objective of this project:

> *Objective:* To study the the delay performance of the same conflict resolution algorithm (binary preorder tree traversal algorithm) as in [1], but under a last-come first-served (LCFS) preemptive window access scheme. This scheme assumes a slotted channel, with window boundaries every 2 or 3 slots, and requires only that all stations have common knowledge of when these boundaries occur, e.g. using a common clock.

The conflict resolution algorithm and LCFS channel access scheme assumed here, and the delays encountered by a packet, are outlined in the next subsections.

1.1 Conflict Resolution Algorithm: Binary Preorder Tree Traversal

As in [1], it is assumed that the channel is slotted, with one packet allowed to be transmitted per slot, and that each station will know within a slot time (e.g. with acknowledgements) whether the transmission was successful. At the start of a CRI, each contending station permitted to join the

CRI attempts to transmit its packet (this is the root node of the 'tree'). If the attempt is successful, the CRI ends. However, if there is a collision, each station tosses a (possibly biased) two-sided coin to determine which 'branch' of the tree to take. If the left branch is taken, the station attempts to transmit again in the next slot; again, if the attempt is successful, this 'subepoch' ends with this leaf node and the right node is now visited, with the stations that took the right branch now transmitting. If the left node resulted in a collision, those colliding stations again toss a coin and create a new pair of left and right branches at the next level of the tree.

In this manner, each station traverses its own copy of the binary 'tree', with each successful left node transmission leading back to its right counterpart, and each successful right node transmission implying a success for its parent node. The CRI ends when all nodes in the tree have been traversed and all packets successfully sent. The 'epoch length' of the tree is the total number of nodes traversed for which a time slot was required. The Standard Traversal Algorithm (STA) results in one slot used per node created, and hence always results in an odd epoch length. The Modified Tree Algorithm (MTA) employs 'level skipping' by recognising that a collision node followed by an idle left node will always result in a collision right node; rather than wasting a slot on the right node, it is skipped, and its left successor node is visited in the next slot. This results in a shorter epoch length and CRI, and hence better delay performance and increased channel capacity. Figure 1 illustrates possible sample paths of the STA and MTA binary trees assuming that three packets are contending for transmission within the CRI.

Channel capacity is defined as the maximum successfully carried packet rate per slot for which the packet delay is finite. It is shown in [1] that for the fixed-size window SWA channel access method, the optimal fixed window sizes, i.e. those which maximise the capacity, are between 2 and 3. The capacities for a window size of 2 for the SWA/STA and SWA/MTA (unbiased) are .4196 and .4510 respectively, and are .4285 and .4614 respectively for a window size of 3.

1.2 Channel Access Algorithm: Last-Come First-Serve (LCFS)

It is assumed here that all stations have knowledge of slot boundaries and window boundaries, with windows being an integer number of slot times. Since the optimal window sizes for the STA and MTA are real numbers between 2 and 3, integer window sizes of 2 and 3 are considered here.

A new CRI is initiated at each window boundary with access permitted only to packets which have arrived during the previous window regardless of the number of steps remaining in any previous CRIs. The first slot in the window is always reserved for the root node of the new CRI. If there is a collision, the remainder of slots in the window are used to continue that same CRI, which is preempted by the arrival of a new window and a new CRI. If there is no collision at the root node of the new CRI, the remaining slots in the window are used to resume the execution of the most recent unresolved CRI. Remaining unresolved CRIs are served to completion in LCFS order during the 'extra' one (for window of 2) or two (for window of 3) slots available in windows which had a non-conflict root node in the first slot.

Figure 2 illustrates a possible LCFS CRI, labelled $W(i)$, within which all packets arriving within the i'th window are transmitted. In this example, a window size of 2 and a tree traversal epoch length of 5 are assumed; 5 is the number of slots which would be required for the CRI with a FCFS SWA channel access method. The random variables shown are defined further in Section 2.

Note that the LCFS non-idle CRIs will always end at a window boundary for the STA with window sizes of either 2 or 3, since the STA epoch length is odd (i.e. transmissions for a given CRI will be 2 contiguous slots initially and 1 per window thereafter, or 3 initially and 2 per window thereafter). However, for the MTA, this is true only for a window of size 2 since epoch lengths can be even. It will be seen that the delay analysis is tractable only when CRIs end on a window boundary; for this reason, the STA and MTA are considered in Section 2 for a window size of 2 but only the STA is considered in Section 3 for a window size of 3.

The advantage of the proposed LCFS preemptive scheme over existing limited sensing algorithms is that stations need only determine where the (constantly spaced) window boundaries are to know when to join the next CRI. The scheme also lends itself to easy delay analysis (compared to free access schemes, which allow a new station to immediately join a CRI in progress) for Poisson packet arrivals with an infinite population size because there is no complex interaction between new joining packets and old packets already being resolved within their CRI.

1.3 Packet Delay Components

The packet delay is defined as the time from the arrival of a packet at a station until its successful transmission over the channel (ending with the end of the slot during which the transmission was successful), and is expressed in units of slots.

In the LCFS scheme, the packet delay is the sum of two independent random delays [1]. [2] :

$T \triangleq$ delay from packet arrival until start of next window

$G \triangleq$ delay from start of the CRI at the next window boundary after packet arrival, until successful packet transmission

It will be assumed throughout that the arrival of transmissions to the common channel are Poisson, with an infinite population size and random addressing. With these assumptions, the initial random delay T is uniformly distributed across a window. If we denote the window size by w, then the Laplace transform of T is given by [1]:

$$T^*(w,s) \triangleq \frac{1-e^{-sw}}{sw}$$

with mean denoted by:

$$\overline{T}(w) \triangleq \frac{w}{2},$$

and standard deviation denoted by:

$$\sigma_T = \frac{w}{\sqrt{12}}.$$

The derivation of the distribution of delay within the CRI, G, is the subject of Sections 2 (for a window size of 2) and 3 (for a window size of 3).

[1] T corresponds to the first delay component, t_0, in [1]

[2] G corresponds to the third delay component, t_2, in [1], and should not be confused with the standard notation for arrival rate in a random access channel. The second delay component, t_1, in [1] is the time from the start of the next window boundary until the start of the CRI in which the given packet is transmitted; this delay is zero in the LCFS preemptive case — when t_1 is non-zero, sensing is required to detect when the pertinent CRI begins.

1.4 Notation

To a large extent, the notation adopted in [1] is used here. In particular, the following are defined:

$$
\begin{aligned}
w &\triangleq \text{window size} \\
\lambda &\triangleq \text{throughput or mean packet arrival rate per slot} \\
x &\triangleq \text{mean packet arrival rate to a new CRI} \\
&= w \times \lambda \\
\Phi(x) &\triangleq \text{probability of no packets in a new CRI (i.e. an idle root slot)} \\
&= \Pr\{i \mid x\} = e^{-x} \\
p(x) &\triangleq \text{probability of no packet collisions in a new CRI root slot} \\
&= \Pr\{\bar{c} \mid x\} = (1+x)e^{-x} \\
Q(x,z) &\triangleq \text{probability generating function of the STA or MTA} \\
&\ \text{epoch (i.e. SWA CRI) length} \\
Q^{(c)}(x,z) &\triangleq \text{probability generating function of the STA or MTA} \\
&\ \text{epoch length, conditioned on a collision in the root node} \\
&= \frac{Q(x,z) - p(x)z}{1 - p(x)}
\end{aligned}
$$

However, a 'looser' notation for random variables is used here, with the same basic symbol referring to a random variable (e.g. G), its probability generating function (e.g. $G(x,z)$) and the derivatives w.r.t. z (e.g. $G''(x,z)$), and its mean (e.g. $\overline{G}(x)$). With this notation, the symbol $\overline{Q}(x)$ is used to denote the mean epoch length of the STA or MTA, instead of the symbol $L(x)$ adopted in [1].

2 Delay Analysis for Window Size of 2

2.1 Distribution of CRI Length with the STA or MTA

Before deriving the distribution of the packet delay during the CRI, G, it is useful to obtain the distribution of the LCFS CRI length, which is denoted here by W:

$$
\begin{aligned}
W &\triangleq \text{time from the beginning of the } (i+1)\text{'th window,} \\
&\ \text{which is the start of the } i\text{'th window's CRI, until the} \\
&\ i\text{'th window's packets are all successfully transmitted.}
\end{aligned}
$$

The CRI length W will consist of a fixed window access STA or MTA epoch length Q[3], plus an interruption, of random length I, for every unfinished slot after the first two slots of the epoch (which is the maximum progress that can be accomplished given a window of size 2 before it is interrupted by the next CRI). In Figure 2, examples of the LCFS CRI lengths for slots i, $i+1$, $i+2$ etc. are shown as $W(i)$, $W(i+1)$, $W(i+2)$, etc. respectively. In this illustration, the i'th epoch length takes the value 5, and thus the i'th CRI is interrupted 3 times. Each interruption, I, ends when the most recent window's CRI has ended. In this example, the tagged packet of

[3] see Appendix A for the distributions of these random variables

interest is sent in the 4'th epoch slot, and thus the packet delay during the CRI, G, consists of the first two slots plus an interrupt I and single slot for each of the 2 additional epoch slots required after the first 2.

Denote the probability generating functions (PGFs) of the distributions of W and I by $W(x,z)$ and $I(x,z)$ respectively, and let $W^{(c)}$ denote the random variable W conditioned on a collision in the root node, with corresponding PGF $W^{(c)}(x,z)$. Then $W^{(c)}$ always ends on a window boundary, and the interruptions I are either equal to one slot (corresponding to no collision at the newly initiated CRI root node, with probability $p(x)$), or I is a geometrically distributed string of intervals $W^{(c)}$ ending with a no-collision CRI root node slot:

$$\begin{aligned} I(x,z) &= p(x)z + (1-p(x))\sum_{i=1}^{\infty}(1-p(x))^{i-1}p(x)\left[W^{(c)}(x,z)\right]^{i} z \\ &= \frac{p(x)z}{1-(1-p(x))W^{(c)}(x,z)}. \end{aligned}$$

Noting that

$$W(x,z) = p(x)z + (1-p(x))W^{(c)}(x,z),$$

the above equation yields an expression for $W(x,z)$ in terms of $I(x,z)$:

$$W(x,z) = 1 + p(x)z - \frac{p(x)z}{I(x,z)}. \tag{1}$$

To obtain another expression for $W(x,z)$ in terms of $I(x,z)$, note that

$$W(x,z) = p(x)z + (1-p(x))z^2 S(x,z)$$

where $S(x,z)$ is the PGF of the sum of a random number N of independent, identically distributed random variables $(I+1)$ (each with PGF $zI(x,z)$). The random number N is the number of unfinished STA or MTA epoch slots after the first two, given a collision in the first root node of the epoch:

$$N = Q^{(c)} - 2,$$

and $(I+1)$ corresponds to a random interrupt length plus the single epoch slot gained after this interrupt. The PGF of $S(x,z)$ can therefore be written [2] as the PGF of N evaluated at $zI(x,z)$:

$$S(x,z) = \left.\frac{Q^{(c)}(x,z)}{z^2}\right|_{z=zI(x,z)},$$

and the equation for $W(x,z)$ becomes:

$$W(x,z) = p(x)z + (1-p(x))z^2 \left.\left[\frac{Q(x,z)-p(x)z}{(1-p(x))z^2}\right]\right|_{z=zI(x,z)}.$$

Expanding the above, substituting the expression for $W(x,z)$ from equation (1), and simplifying, the following implicit equation for $I(x,z)$ is obtained:

$$I^2(x,z) = Q(x, zI(x,z)). \tag{2}$$

Moments of the LCFS CRI length W can be obtained by differentiating equations (1) and (2) w.r.t. z and setting $z=1$. Differentiating equation (2) yields:

$$2I(x,z)I'(x,z) = Q'(x, zI(x,z))\left[zI'(x,z) + I(x,z)\right]$$

and noting that $I(x,1) = 1$, $I'(x,1) = \overline{I}(x)$ and $Q'(x,1) = \overline{Q}(x)$, the mean interrupt length is given by:

$$\overline{I}(x) = \frac{\overline{Q}(x)}{2 - \overline{Q}(x)}. \tag{3}$$

Note that this is equivalent to:

$$\overline{I}(x) = \overline{Q}(x) + \overline{I}(x)\left[\overline{Q}(x) - 1\right],$$

which can be derived intuitively by noting that a mean interruption is equal to the mean epoch length plus a mean interruption for the mean of every unfinished slot of an augmented epoch after the first two. The augmented epoch is a normal epoch plus one slot to account for the extra interruption possible each time a CRI terminates on a window boundary.

Differentiating equation (1) yields:

$$I'(x,z)[1 + p(x) - W(x,z)] + I(x,z)[p(x) - W'(z)] = p(x)$$

and setting $z = 1$, the mean LCFS CRI length is given by:

$$\begin{aligned} \overline{W}(x) &= p(x)\overline{I}(x) \\ &= \frac{\overline{Q}(x)}{2 - \overline{Q}(x)}(1+x)e^{-x} \end{aligned} \tag{4}$$

It can be seen that the capacity of the LCFS scheme is the lowest upper bound on the throughput such that the mean CRI length is finite, i.e. the throughput for which $\overline{Q}(x) = 2$. This is the same capacity as for the SWA channel access method studied in [1], as would be expected; the SWA and LCFS schemes traverse the same sequence of conflict resolution trees — the only difference is a permutation of the order in which the nodes are visited. Figure 3 shows the mean CRI length $\overline{W}(2\lambda)$ and Figure 4 shows the mean interrupt length $\overline{I}(2\lambda)$ as a function of throughput for the LCFS/STA and LCFS/MTA (unbiased) schemes with a window size of 2, based on the equations for $\overline{Q}(x)$ given in Appendix A.

2.2 Distribution of Delay During the CRI with the STA

2.2.1 Method 1: Using Distribution of STA Epoch Length

To derive the packet delay during the CRI, G, the same basic approach of [1] is followed here i.e. the recursive structure of the STA and the LCFS interruptions are exploited to obtain a simple functional equation for G. In this subsection, it is assumed only that the distribution of the STA epoch length Q is given. In the following subsection, another method is outlined assuming that the distribution of delay during an SWA/STA epoch is also given.

In this section, a CRI conditioned on a tagged packet being present is assumed. Thus, the probability of a CRI (containing the tagged packet, with packet rate x) root node being idle is zero, of being a success is $\Phi(x)$, and of having a collision is $1 - \Phi(x)$.

The following random variables are defined:

$$
\begin{array}{rcl}
G^{(c)} & \triangleq & \text{delay during the CRI, } G\text{, given a root node collision} \\
G^{(L)} & \triangleq & \text{delay during the CRI, given the tagged packet takes the left branch} \\
G^{(R)} & \triangleq & \text{delay during the CRI, given the tagged packet takes the right branch} \\
G_2 & \triangleq & \text{delay during the CRI, given the CRI begins} \\
 & & \text{in the } \textit{second} \text{ slot of a window instead of the first} \\
W_2 & \triangleq & \text{LCFS CRI length, given the CRI begins} \\
 & & \text{in the } \textit{second} \text{ slot of a window instead of the first}
\end{array}
$$

Then using the notation from [1], e.g. $\Pr\{(s,i) \mid L\}$ to express the probability of a left success node and a right idle node given the tagged packet takes the left branch in the STA,

$$
\begin{aligned}
G^{(L)}(x,z) &= \Pr\{(s,i) \mid L\}z + \Pr\{(s,\bar{i}) \mid L\}z^2 + \Pr\{(c,\cdot) \mid L\}zG_2^{(c)}(x/2,z) \qquad (5) \\
G^{(R)}(x,z) &= \Pr\{(i,s) \mid R\}z + \Pr\{(\bar{i},s) \mid R\}zW_2^{(\bar{i})}(x/2,z)I(x,z)z \\
&\quad + \Pr\{(\cdot,c) \mid R\}zW_2(x/2,z)I(x,z)G_2^{(c)}(x/2,z). \qquad (6)
\end{aligned}
$$

The equation conditioned on the tagged packet taking the left branch states that the delay until transmission within the CRI will be either one slot (no initial collision), two slots (an initial collision but no collisions in the left node), or one slot for the initial root node collision plus another delay within a LCFS CRI *conditioned on starting with a collision in the second slot of the window* with packet rate $x/2$.

The equation conditioned on the tagged packet taking the right branch states that the delay will be either one slot (no initial collision), or will be the sum of the left branch LCFS CRI length starting in the second slot of the window instead of the first, plus a random interrupt I as defined in the previous section, plus the delay in the right branch CRI starting in the 2nd slot of a window. The interrupt I is added because the left branch CRI will always terminate at the end of a window, allowing a fresh CRI to be initiated before the given CRI can be resumed.

An expression for $G_2^{(c)}(x/2,z)$ can be found by observing that $G_2^{(c)} = G^{(c)} + I$ since after the initial collision in slot 2, the window boundary imposes an extra interruption I before the next node (normally served in slot 2 when considering $G^{(c)}$) can be visited:

$$
G_2^{(c)}(x/2,z) = G^{(c)}(x/2,z)I(x,z) = \left[\frac{G(x/2,z) - \Phi(x/2)z}{1 - \Phi(x/2)}\right] I(x,z) \qquad (7)
$$

An expression for $W_2(x/2,z)$ can be derived from first principles in a similar manner as for the expression for $W(x,z)$ in Section 2.1. This time, W_2 consists of a fixed window access STA epoch length Q, plus an interruption I for every unfinished slot after the first *single* slot of the epoch (which is the maximum which can be accomplished before the next window boundary):

$$
W_2(x/2,z) = p(x/2)z + (1 - p(x/2))z \left[\frac{Q(x/2,z) - p(x/2)z}{(1 - p(x/2))z}\right]\Bigg|_{z=zI(x,z)}.
$$

Simplifying the above,

$$
W_2(x/2,z) = \frac{Q(x/2,zI(x,z))}{I(x,z)}, \qquad (8)
$$

which allows an expression for $W_2^{(\bar{i})}(x/2, z)$ to be obtained:

$$W_2^{(\bar{i})}(x/2, z) = \frac{W_2(x/2, z) - \Phi(x/2)z}{1 - \Phi(x/2)}. \tag{9}$$

Substituting (7) and (9) into (5) and (6),

$$\begin{aligned} G^{(L)}(x, z) &= \Phi(x)z + \Phi(x/2)(1 - \Phi(x/2))z^2 + (1 - \Phi(x/2))z \left[\frac{G(x/2, z) - \Phi(x/2)z}{1 - \Phi(x/2)} \right] I(x, z) \\ G^{(R)}(x, z) &= \Phi(x)z + (1 - \Phi(x/2))\Phi(x/2)z^2 \left[\frac{W_2(x/2, z) - \Phi(x/2)z}{1 - \Phi(x/2)} \right] I(x, z) \\ &\quad + (1 - \Phi(x/2))zW_2(x/2, z) \left[\frac{G(x/2, z) - \Phi(x/2)z}{1 - \Phi(x/2)} \right] I^2(x, z). \end{aligned}$$

Substituting the above into

$$G(x, z) = \frac{1}{2}G^{(L)}(x, z) + \frac{1}{2}G^{(R)}(x, z),$$

and simplifying, an expression for $G(x, z)$ is:

$$\begin{aligned} G(x, z) &= z\frac{1}{2} \left[1 + W_2(x/2, z)I(x, z) \right] \left[G(x/2, z)I(x, z) + \Phi(x/2)z \left[1 - I(x, z) \right] \right] \\ &\quad + \Phi(x) \left[z - \frac{1}{2} \left[z^2 + I(x, z)z^3 \right] \right]. \end{aligned} \tag{10}$$

This can be interpreted as the sum of the initial root node slot, plus nothing (if the tagged packet takes the left branch) *or* the length of the left branch CRI starting in slot 2 plus an extra interrupt interval since the left branch CRI ends at a window boundary, plus the time until transmission within either the left or right branch CRI starting in slot 2. This latter time is equal to the delay starting in slot 1, plus an extra interrupt time I, with a correction term to account for the interrupt wrongly added when the root node is a success. The final term is a correction in the case of no collision in the initial root node of the CRI.

It is possible to obtain moments of the delay G within a CRI by differentiating equation (10). Differentiating and setting $z = 1$, the mean delay is obtained:

$$\overline{G}(x) = 1 + \overline{G}(x/2) + \overline{I}(x)(1 - e^{-x/2}) + \frac{1}{2} \left[\overline{W}_2(x/2) + \overline{I}(x) \right] - \frac{1}{2} \left[3 + \overline{I}(x) \right] e^{-x}.$$

An expression for $\overline{I}(x)$ is given in equation (3). Note that to find $\overline{W}_2(x/2)$, equation (8) can be differentiated to obtain:

$$\overline{W}_2(x/2) = \overline{Q}(x/2) + \overline{I}(x) \left[\overline{Q}(x/2) - 1 \right], \tag{11}$$

where the above simply says that the average CRI length starting in slot 2 is equal to the average epoch length plus an extra mean interrupt length for the mean number of unfinished epoch slots after the first. Substituting (11) into the equation for $\overline{G}(x)$ above,

$$\overline{G}(x) = 1 + \overline{G}(x/2) + \overline{I}(x)(1 - e^{-x/2}) + \frac{1}{2}\overline{Q}(x/2) \left[\overline{I}(x) + 1 \right] - \frac{1}{2} \left[\overline{I}(x) + 3 \right] e^{-x}. \tag{12}$$

Using the above, it is possible to obtain a series expansion of $\overline{G}(x)$ in powers of x, but the process is laborious due to the form of $\overline{I}(x)$ in equation (3); it has a power series $\overline{Q}(x)$ in the denominator, implying that several combinations of three power series must be multiplied and similar terms equated to obtain a power series expansion of $\overline{G}(x)$.

Instead, the expression was evaluated recursively by setting $\overline{G}(10^{-9}) = 1$ and $\overline{Q}(10^{-9}) = 1$ and using the recursive form of the equation for $\overline{Q}(x)$ given in Appendix A.1. Note that the value of $\overline{I}(x)$ is a *constant* during the recursion i.e. is evaluated once at the the parameter value x before the recursion since the interrupts must resolve a fresh window of packets. However, the arguments of $\overline{G}(x/2)$, $\overline{Q}(x/2)$, e^{-x} and $e^{-x/2}$ do vary and equal $x/2^n$, n integer, during the recursion. The total mean packet delay $[\overline{T}(2) + \overline{G}(2\lambda)]$, with $\overline{T}(2) = 1$, is shown in Figures 5 and 7 as a function of the packet throughput per slot and compared with the total delay for SWA channel access given in Appendix C. It can be seen that the LCFS method imposes a slightly higher mean packet delay, but the difference is negligible, especially at high load.

Differentiating equation (10) again w.r.t. z and setting $z = 1$, the following is obtained:

$$\begin{aligned} G''(x,1) &= G''(x/2,1) + I''(x,1)\left[\frac{3}{2} - \Phi(x/2)\right] + \frac{1}{2}W_2''(x/2,1) + 2\overline{I}(x)\left[\overline{G}(x/2) - \Phi(x/2)\right] \\ &\quad + \left[2 + \overline{W}_2(x/2) + \overline{I}(x)\right]\left[\overline{G}(x/2) + \overline{I}(x)(1 - \Phi(x/2))\right] + \overline{W}_2(x/2) \\ &\quad + \overline{I}(x)\left[1 + \overline{W}_2(x/2)\right] - \Phi(x)\left[4 + 3\overline{I}(x) + \frac{1}{2}I''(x,1)\right]. \end{aligned} \tag{13}$$

To obtain an expression for $I''(x,1)$, equation (2) is differentiated again to get:

$$I''(x,1) = \frac{Q''(x,1)\left[\overline{I}(x) + 1\right]^2 + 2\overline{I}(x)\left[\overline{Q}(x) - \overline{I}(x)\right]}{2 - \overline{Q}(x)}, \tag{14}$$

and to obtain an expression for $W_2''(x/2,1)$, equation (8) is differentiated again:

$$W_2''(x/2,1) = Q''(x/2,1)\left[\overline{I}(x) + 1\right]^2 + I''(x,1)\left[\overline{Q}(x/2) - 1\right] + 2\overline{I}^2(x)\left[1 - \overline{Q}(x/2)\right]. \tag{15}$$

All equations necessary to obtain the standard deviation of the total packet delay have now been derived. The variance of the total packet delay will be the sum of the variances of the random variables T and G since they are independent; the standard deviation of the total packet delay with $w = 2$ is therefore given by:

$$\sigma = \sqrt{\frac{w^2}{12} + G''(x,1) + \overline{G}(x) - \overline{G}^2(x)}. \tag{16}$$

Figures 6 and 8 compare the above LCFS/STA standard deviation to the standard deviation of the total delay with the SWA/STA scheme as derived in Appendix C. It can be seen that the LCFS method results in a significantly higher standard deviation, as would be expected since the SWA/STA scheme is essentially first-come first-serve.

2.2.2 Method 2: Using Distribution of Delay During an SWA/STA Epoch

Here, it is assumed that the distribution of delay during the epoch of transmission for the SWA/STA method is given. This delay will be referred to here as G_S, with the distribution as derived in [1] given in Appendix B.

The following probability is defined:

$$\begin{aligned} g_2(x) &\triangleq \text{probability that the tagged packet is sent in the 2nd slot} \\ &= \frac{1}{2}\left[e^{-x/2} - e^{-x}\right]. \end{aligned}$$

Then the PGF of the LCFS delay during the CRI can be expressed as:

$$G(x,z) = \Phi(x)z + g_2(x)z^2 + (1-\Phi(x)-g_2(x))z^2 \left[\frac{G_S(x,z) - \Phi(x)z - g_2(x)z^2}{(1-\Phi(x)-g_2(x))z^2}\right]\Bigg|_{z=zI(x,z)}, \quad (17)$$

since the LCFS delay is the SWA/STA delay within the epoch, plus an additional interruption I for every slot after the first two.

Differentiating equation (17) w.r.t. z and setting $z=1$, the mean delay during the CRI for the LCFS/STA method becomes:

$$\overline{G}(x) = \overline{G}_S(x) + \overline{I}(x)\left[\overline{G}_S(x) + e^{-x} - 2\right], \quad (18)$$

which when evaluated yielded identical results to those using equation (12) and shown in Figures 5 and 7. Differentiating equation (17) again w.r.t. z and setting $z=1$ results in:

$$\begin{aligned} G''(x,1) &= G_S''(x,1)\left[1+\overline{I}(x)\right]^2 + \overline{G}_S(x)\left[2\overline{I}(x) + I''(x,1)\right] \\ &\quad + \overline{I}(x)\left[2e^{-x} - 4\overline{G}(x)\right] + \overline{I}^2(x)\left[2e^{-x}-2\right] + I''(x,1)\left[e^{-x}-2\right]. \quad (19) \end{aligned}$$

Equation (19) was evaluated using (14) and the recursive equations given in Appendix B for $\overline{G}_S$ and G_S'', and was confirmed to give identical results to those given by equation (13) using Method 1.

2.3 Distribution of Delay During the CRI with the MTA

2.3.1 Method 1: Using the Distribution of MTA Epoch Length

For the MTA, we consider the general biased splitting case with a probability π of taking the left branch and $\overline{\pi} = 1-\pi$ of taking the right branch. The derivation method is similar to that for the STA, but using the MTA epoch length, Q, and accounting for level skipping. The following is obtained:

$$\begin{aligned} G^{(L)}(x,z) &= \Pr\{(s,i) \mid L\}z + \Pr\{(s,\bar{i}) \mid L\}z^2 + \Pr\{(c,\cdot) \mid L\}zG_2^{(c)}(\pi z, z) \\ G^{(R)}(x,z) &= \Pr\{(i,s) \mid R\}z + \Pr\{(\bar{i},s) \mid R\}zW_2^{(\bar{i})}(\pi x, z)I(x,z)z \\ &\quad + \Pr\{(i,c) \mid R\}z^2 I(x,z)\left[\frac{G^{(c)}(\overline{\pi}x,z)}{z}\right] \\ &\quad + \Pr\{(\bar{i},c) \mid R\}zW_2^{(\bar{i})}(\pi x,z)I(x,z)G_2^{(c)}(\overline{\pi}x,z). \end{aligned}$$

The last two lines above differ from the equation for the STA to account for the MTA level skipping. In particular, the second last line says that in the case of an idle left node when the tagged packet takes the right branch and has a collision, the delay will be two slots for the root and left node, plus an interruption due to the window boundary, plus the delay which would be obtained starting in a root slot 1 with a collision *but with the first node removed for level skipping.*

Substituting the appropriate probabilities, using

$$G(x,z) = \pi G^{(L)}(x,z) + \overline{\pi} G^{(R)}(x,z),$$

and simplifying, an expression for $G(x,z)$ is:

$$\begin{aligned} G(x,z) \quad = \quad & \pi z \left[G(\pi x, z) I(x,z) + \Phi(\pi x) z \left[1 - I(x,z) \right] \right] \\ & + \overline{\pi} z \left[W_2(\pi x, z) I(x,z) G(\overline{\pi} x, z) I(x,z) + \Phi(\overline{\pi} x) W_2(\pi x, z) z I(x,z) \left[1 - I(x,z) \right] \right] \\ & + \overline{\pi} \Phi(\pi x) \left[z^2 I(x,z) G(\overline{\pi} x, z) \left[\frac{1}{z} - I(x,z) \right] - \Phi(\overline{\pi} x) z^2 I(x,z) z \left[\frac{1}{z} - I(x,z) \right] \right] \\ & + \Phi(x) \left[z - \pi z^2 - \overline{\pi} z^3 I(x,z) \right]. \end{aligned} \tag{20}$$

The above equation can be derived intuitively. The first term is the left branch delay starting in slot 2, which is G plus an added interrupt term I and correction not to add I for the case of a success in the second slot (as for the STA). The second term accounts for the right branch; it is the sum of: the left branch CRI length, an added interrupt time due to the window boundary, and the right branch delay starting in slot 2 (which is equal to G plus an added interrupt time I), with the same type of correction term to account for when the right branch node is a success. The third term is a correction to account for level skipping when the left node is idle and the right is taken by the tagged packet — it says that rather than add an extra interrupt term to the delay G, just subtract 1 slot from G, except for the case where G is just one slot, i.e. no collision in the right node. The final term is an overall correction term to account for the case where the CRI ends in the first initial slot — only one slot is counted instead of the two for taking the left branch or the three plus interrupt for taking the right branch.

Differentiating the above equation and setting $z = 1$, the mean delay is:

$$\begin{aligned} \overline{G}(x) \quad = \quad & 1 + \pi \left[\overline{G}(\pi x) + \overline{I}(x)(1 - \Phi(\pi x)) \right] \\ & + \overline{\pi} \left[\overline{W}_2(\pi x) + \overline{I}(x) + \overline{G}(\overline{\pi} x) + \overline{I}(x) \left[1 - \Phi(\pi x) \right] \left[1 - \Phi(\overline{\pi} x) \right] - \Phi(\pi x) \left[1 - \Phi(\overline{\pi} x) \right] \right] \\ & - \Phi(x) \left[\pi + \overline{\pi} \left[2 + \overline{I}(x) \right] \right] \end{aligned}$$

which can also be derived intuitively; specifically, the second line (where the right branch is taken) corresponds to the left branch CRI length, plus an interrupt, plus the mean delay in the right branch starting at slot 2 (which is the mean delay $\overline{G}(\overline{\pi} x)$ starting in slot 1 with: an added interrupt if there was no level skipping and a collision in the right node; or with a slot removed if there was level skipping and a collision in the right node). This equation simplifies to:

$$\begin{aligned} \overline{G}(x) \quad = \quad & 1 - e^{-x} + \pi \left[\overline{G}(\pi x) + \overline{I}(x)(1 - e^{-\pi x}) \right] \\ & + \overline{\pi} \left[\overline{W}_2(\pi x) - e^{-\pi x} + \overline{I}(x)(1 - e^{-\pi x}) + \overline{G}(\overline{\pi} x) + \overline{I}(x)(1 - e^{-\overline{\pi} x}) \right]. \end{aligned} \tag{21}$$

This functional expression is hard to evaluate recursively because of the biased splitting. However, the unbiased case can be evaluated in a similar manner as for the STA. The case of $\pi = \frac{1}{2}$ yields a mean delay in the CRI given by:

$$\overline{G}(x) \quad = \quad 1 + \overline{G}(x/2) + \overline{I}(x)(1 - e^{-x/2}) + \frac{1}{2}\left[\overline{I}(x) + 1\right]\left[\overline{Q}(x/2) - e^{-x/2}\right] - e^{-x} \tag{22}$$

where equation (11) has been used for an expression for $\overline{W}_2(x/2)$.

The total mean packet delay $[\overline{T}(2) + \overline{G}(2\lambda)]$, with $\overline{T}(2) = 1$, is shown in Figure 5 for the unbiased MTA and is compared to the total mean packet delay for SWA channel access given in Appendix C. Just as for the STA, the LCFS channel access method imposes a slightly higher mean packet delay, but the difference is also negligible, especially at high load.

2.3.2 Method 2: Using the Distribution of Delay During an SWA/MTA Epoch

Following the same technique as in Section 2.2.2 for the STA, an identical equation is obtained by replacing G_S with G_M, the distribution of the SWA/MTA delay during the epoch of transmission given in Appendix B. The first and second moments of delay during the LCFS/MTA CRI (with unbiased splitting) are therefore:

$$\overline{G}(x) = \overline{G}_M(x) + \overline{I}(x)\left[\overline{G}_M(x) + e^{-x} - 2\right], \tag{23}$$

and

$$\begin{aligned} G''(x,1) \quad = \quad & G''_M(x,1)\left[1 + \overline{I}(x)\right]^2 + \overline{G}_M(x)\left[2\overline{I}(x) + I''(x,1)\right] \\ & + \overline{I}(x)\left[2e^{-x} - 4\overline{G}(x)\right] + \overline{I}^2(x)\left[2e^{-x} - 2\right] + I''(x,1)\left[e^{-x} - 2\right]. \end{aligned} \tag{24}$$

The mean using equation (23) was evaluated and found to be identical to the mean evaluated using equation (22), as would be expected. Using the recursive equation given in Appendix B for $G''_M(x,1)$, the second derivative in equation (24) was evaluated and used in the following equation for the standard deviation of the total packet delay for the LCFS/MTA (unbiased) scheme with $w = 2$:

$$\sigma = \sqrt{\frac{w^2}{12} + G''(x,1) + \overline{G}(x) - \overline{G}^2(x)}.$$

Figure 6 compares the above standard deviation with that of the total packet delay for the SWA/MTA (unbiased) scheme, using the equations given in Appendix C. As for the STA, the standard deviation with the LCFS method is substantially higher.

3 Delay Analysis for Window Size of 3

3.1 Distribution of CRI Length with the STA

Recall that with a window size of 3, if the first slot in the window results in no collision, the following 2 slots are used to resume the last unresolved CRI. When non-idle CRIs end on window boundaries, as is the case for the STA (since the CRI requires an odd number of slots), it is possible to analyse the delay with the same approach taken for the window size of 2 in Section 2.

When non-idle CRIs do not necessarily end on window boundaries, as is the case for the MTA, the analysis becomes exceedingly messy. For this reason, only the STA is considered here.

The CRI delay for a window size of 3 will consist of a fixed window access STA epoch length, Q, plus an interruption of random length, I, for each *pair* of unfinished slots after the first 3 slots of the epoch (which is the maximum that can be accomplished before the execution of the CRI is first preempted by the next window).

Using the same notation as in Section 2, since a non-idle CRI always ends on a window boundary, the interruptions I are again a geometrically distributed string of intervals $W^{(c)}$ ending with a no-collision CRI root node slot in the first slot of the window. This implies that equation (1) still holds for the relation between the PGFs of I and the CRI length W.

To derive a second relation between the PGFs of I and W, define an additional probability:

$$\begin{aligned} q_3(x) \;&\triangleq\; \text{probability of exactly 2 packets in the root CRI} \\ &\quad\;\; \text{and exactly 1 packet going to each of the left and right branches} \\ &=\; \frac{x^2}{4}e^{-x}, \end{aligned}$$

which corresponds to the probability that the CRI is exactly 3 slots long, implying no extra interrupt time is added to the STA epoch length in this case. Then $W(x,z)$ can be expressed as:

$$W(x,z) = p(x)z + q_3(x)z^3 + (1-p(x)-q_3(x))z^3 \left[\frac{Q(x,z) - p(x)z - q_3(x)z^3}{(1-p(x)-q_3(x))z^3}\right]\Bigg|_{z=\sqrt{z^2 I(x,z)}}.$$

The quantity in square brackets, when evaluated at $\sqrt{z}$, represents the PGF of the number of extra interruptions (one per pair of unfinished epoch slots) conditioned on a CRI of greater than 3 slots, and consists of only integer powers of z because the STA epoch length is always odd (i.e. $Q(x,z)$ contains only odd powers of z). This PGF is evaluated at $z^2 I(x,z)$ since this is the PGF of the interrupt length plus the two useful slots gained for that interrupt.

Substituting equation (1) for $W(x,z)$, the equation reduces to:

$$I^{3/2}(x,z) = Q(x, zI^{1/2}(x,z)). \tag{25}$$

Differentiating the above and setting $z = 1$, the mean interrupt length is obtained:

$$\overline{I}(x) = \frac{2\overline{Q}(x)}{3-\overline{Q}(x)}, \tag{26}$$

which can be rewritten as:

$$\overline{I}(x) = \overline{Q}(x) + \overline{I}(x)\left[\frac{\overline{Q}(x)-1}{2}\right],$$

with the same interpretation as for a window size of 2, i.e. the mean interrupt is equal to the mean epoch length plus a mean interrupt for the mean of each unfinished pair of an augmented epoch length minus the first three slots. The augmented epoch length here is a normal epoch length plus 2 slots to force the CRI to experience a fresh interrupt and end on a window boundary.

The mean LCFS CRI length becomes:

$$
\begin{aligned}
\overline{W}(x) &= p(x)\overline{I}(x) \\
&= \frac{2\overline{Q}(x)}{3-\overline{Q}(x)}(1+x)e^{-x}. \qquad (27)
\end{aligned}
$$

As can be seen, the capacity is the same as for the fixed-size window SWA channel access method, i.e. the throughput where $\overline{Q}(x) = 3$, since the mean time to resolve a CRI must be less than the time between new CRIs. The mean LCFS/STA CRI length, $\overline{W}(3\lambda)$, is shown in Figure 3, and the mean interrupt length, $\overline{I}(3\lambda)$, is shown in Figure 4 as a function of the packet throughput per slot.

3.2 Distribution of Delay During the CRI for the STA

3.2.1 Method 1: Using the Distribution of STA Epoch Length

The same approach is followed here as in Section 2.2.1 for a window size of 2, with the added random variable definition:

$$
G_3 \triangleq \text{delay during the CRI, given the CRI begins in the } \textit{third} \text{ slot of a window instead of the first.}
$$

Then the following equations are obtained:

$$
\begin{aligned}
G^{(L)}(x,z) &= \Pr\{(s,i) \mid L\}z + \Pr\{(s,\bar{i}) \mid L\}z^2 + \Pr\{(c,\cdot) \mid L\}zG_2^{(c)}(x/2,z) \\
G^{(R)}(x,z) &= \Pr\{(i,s) \mid R\}z + \Pr\{(\bar{i},s) \mid R\}zW_2^{(\bar{i})}(x/2,z)z \\
&\quad + \Pr\{(\cdot,c) \mid R\}zW_2(x/2,z)G_3^{(\bar{i})}(x/2,z).
\end{aligned}
$$

The equation conditioned on the tagged packet taking the left branch is the same as that for a window size of 2. When the right branch is taken, then either: with a left non-idle and right success, the total delay will be 1 for the root slot, plus $W_2^{(\bar{i})}$ for the left CRI time starting in slot 2, plus *only one extra slot since the left CRI will end at the beginning of slot 3 because the STA epoch is of odd length*; or with a right collision, the total delay will again be 1 for the root slot, plus W_2 for the left CRI time starting in slot 2, which always ends after slot 2, plus the right delay starting in slot 3, $G_3^{(c)}$. In the window = 3 case, no interrupt terms $I(x,z)$ appear in this equation because the left branch CRI ends after slot 2, allowing one more slot of the right branch to be gained before the next window boundary.

An expression for $G_3^{(c)}$ can be found by observing that $G_3^{(c)} = G^{(c)} + I$, since after the initial collision in slot 3, the window boundary imposes an extra interruption I before the next node is visited:

$$
G_3^{(c)}(x/2,z) = G^{(c)}(x/2,z)I(x,z) = \left[\frac{G(x/2,z) - \Phi(x/2)z}{1-\Phi(x/2)}\right] I(x,z)
$$

An expression for $W_2(x/2,z)$ is obtained in a similar manner as for the expression for $W(x,z)$ in Section 3.1. This time, W_2 consists of a fixed window access STA epoch length Q, plus an interruption, I, for every *pair* of unfinished slots after the first *two* slots of the STA epoch (which is the maximum that can be accomplished before the next window boundary). Since the STA epoch length is always odd, this leads to the following:

$$W_2(x/2,z) = p(x/2)z + (1-p(x/2))z^2 \left[\frac{Q(x/2,z) - p(x/2)z}{(1-p(x/2))z^3}\right]\Bigg|_{z=\sqrt{z^2 I(x,z)}} I(x,z)z.$$

The quantity in square brackets, when evaluated at $\sqrt{z}$, represents the PGF of the number of extra interruptions for each unfinished *pair* of epoch slots after the first two, with a last interruption and slot added for the final single slot, which ends after slot 2. This expression reduces to:

$$W_2(x/2,z) = \frac{Q(x/2, zI^{1/2}(x,z))}{I^{1/2}(x,z)}. \tag{28}$$

All that remains is to find an expression for $G_2^{(c)}(x/2,z)$, but this is easier said than done. The problem is that this delay is a highly non-linear function of the power series expansion of $G^{(c)}(x/2,z)$ in terms of z, with the coefficients of the even powers of z remaining the same, but with coefficients of the odd powers of z being multiplied by $I(x,z)$. Instead of finding an exact expression for $G_2^{(c)}(x/2,z)$, a pessimistic approximation is used here to obtain an upper bound for the delay:

$$G_2^{(c)}(x/2,z) \approx r(x/2)z^2 + \left[G^{(c)}(x/2,z) - r(x/2)z^2\right] I(x,z)$$

where

$$\begin{aligned} r(x/2) &= \text{probability the tagged packet is sent in the 2nd slot of} \\ &\quad \text{a CRI with packet rate } x/2\text{, given a collision in the root} \\ &= \left[\frac{g_2(x/2)}{1-\Phi(x/2)}\right] = \left[\frac{\frac{1}{2}(\Phi(x/4) - \Phi(x/2))}{1-\Phi(x/2)}\right]. \end{aligned}$$

This is pessimistic because all coefficients of powers of z greater than 2 in $G^{(c)}(x/2,z)$ are multiplied by $I(x,z)$, instead of just the coefficients of odd powers of z. However, since the probability of delay being 4 or more slots is expected to be small (actually, consider the probability of delay of 6 or more slots because the probability of delay of 4 slots is zero since the 4'th is a fresh CRI slot), this should be a reasonably tight upper bound.

Using this upper-bounding approximation,

$$\begin{aligned} G^{(L)}(x,z) &\approx \Phi(x)z + \Phi(x/2)(1-\Phi(x/2))z^2 + (1-\Phi(x/2))z \left[\frac{\frac{1}{2}(\Phi(x/4)-\Phi(x/2))z^2}{1-\Phi(x/2)}\right] \\ &\quad + (1-\Phi(x/2))z \left[\frac{G(x/2,z) - \Phi(x/2)z - \frac{1}{2}(\Phi(x/4)-\Phi(x/2))z^2}{1-\Phi(x/2)}\right] I(x,z) \\ G^{(R)}(x,z) &= \Phi(x)z + (1-\Phi(x/2))\Phi(x/2)z^2 \left[\frac{W_2(x/2,z) - \Phi(x/2)z}{1-\Phi(x/2)}\right] \\ &\quad + (1-\Phi(x/2))zW_2(x/2,z) \left[\frac{G(x/2,z)-\Phi(x/2)z}{1-\Phi(x/2)}\right] I^2(x,z). \end{aligned}$$

Substituting the above into

$$G(x,z) = \frac{1}{2}G^{(L)}(x,z) + \frac{1}{2}G^{(R)}(x,z),$$

and simplifying, a pessimistic approximation of $G(x,z)$ is:

$$\begin{aligned} G(x,z) \;\approx\;& z\frac{1}{2}\left[G(x/2,z)I(x,z)+\Phi(x/2,z)z\left[1-I(x,z)\right]+\frac{1}{2}(\Phi(x/4)-\Phi(x/2))z^3\left[1-I(x,z)\right]\right] \\ & + z\frac{1}{2}W_2(x/2,z)\left[G(x/2,z)I(x,z)+\Phi(x/2,z)z\left[1-I(x,z)\right]\right] \\ & + \Phi(x)\left[z-\frac{1}{2}(z^2+z^3)\right]. \end{aligned} \tag{29}$$

This expression is similar to that for a window size of 2, but with $G+I$ pessimistically approximating the delay in the left branch CRI starting in slot 2 (with no interrupt I added if there is no collision in the left branch node or if the tagged packet is sent in the 2nd slot of the left branch CRI), and with no extra interruption added to the left branch CRI length W_2 starting in slot 2 since it ends after a slot 2.

Differentiating the above and setting $z=1$, an upper bound on the mean delay in the CRI is:

$$\overline{G}(x) \le 1+\overline{G}(x/2)+\frac{1}{2}\overline{I}(x)\left[2-\frac{3}{2}e^{-x/2}-\frac{1}{2}e^{-x/4}\right]+\frac{1}{2}\overline{W}_2(x/2)-\frac{3}{2}e^{-x}. \tag{30}$$

An expression for $\overline{I}(x)$ for a window size of 3 is given in equation (26). To find $\overline{W}_2(x/2)$, equation (28) is differentiated to obtain:

$$\overline{W}_2(x/2)=\overline{Q}(x/2)+\overline{I}(x)\left[\frac{\overline{Q}(x/2)-1}{2}\right] \tag{31}$$

which says to add an extra mean interrupt for the mean of every pair of epoch slots unfinished after the first two (with only one subtracted to have an integer number of interrupts and finish after the 2nd slot).

The expression for the upper bound in (30) was evaluated recursively and the resulting total mean packet delay $[\overline{T}(3)+\overline{G}(3\lambda)]$, with $\overline{T}(3)=\frac{3}{2}$, is shown in Figure 7. Also shown is the total mean packet delay for the SWA/STA scheme. It can be seen that the delays of the two are very close, making this a very tight upper bound. In the next subsection an *exact* expression for the mean and standard deviation of delay is derived.

3.2.2 Method 2: Using the Distribution of Delay During an SWA/STA Epoch

When assuming that the PGF of the distribution of delay during the epoch of transmission for the SWA/STA method, $G_S(x,z)$, is given explicitly as a power series of z, it is possible to derive an exact expression for the LCFS delay distribution. Here it is assumed that $G_S(x,z)$ is of the form:

$$G_S(x,z)=\sum_{i=1}^{\infty} g_i(x)z^i$$

with the coefficients $g_i(x)$ that are derived in [1] given in Appendix B.

The PGF of the LCFS delay during the CRI is then equal to:

$$G(x,z)=g_1(x)z+g_2(x)z^2+g_3(x)z^3+\sum_{i=2}^{\infty}\left[g_{2i}(x)z^{2i}+g_{2i+1}(x)z^{2i+1}\right]I^{i-1}(x,z), \tag{32}$$

since there will be an added interrupt, I, for the next 2 slots after the first 3 slots, two added interrupts I for the two slots after these, and so on. The coefficients $g_i(x)$ can be calculated explicitly, and so it is possible to evaluate the moments of $G(x,z)$. Differentiating equation (32) and setting $z=1$,

$$\begin{aligned}\overline{G}(x) &= g_1(x)+2g_2(x)+3g_3(x)\\ &\quad+\sum_{i=2}^{\infty} g_{2i}(x)\left[2i+(i-1)\overline{I}(x)\right]+g_{2i+1}(x)\left[2i+1+(i-1)\overline{I}(x)\right]\\ &= \overline{G_S(x)}+\overline{I(x)}\sum_{i=2}^{\infty}(i-1)\left[g_{2i}(x)+g_{2i+1}(x)\right].\end{aligned}$$

Using the expression for $\overline{I}(x)$ in equation (26), the above exact expression for $\overline{G}(x)$ was evaluated and the resulting total mean delay $[\overline{T}(3)+\overline{G}(3\lambda)]$ is shown in Figure 7 as a dotted line. The Figure shows this delay is almost indistinguishable from the total mean delay for the SWA/STA method, and also that the upper bound in equation (30) is a good approximation.

Differentiating equation (32) again,

$$\begin{aligned}G''(x,1) &= 2g_2(x)+6g_3(x)+\sum_{i=2}^{\infty}\left[2i(2i-1)g_{2i}(x)+2i(2i+1)g_{2i+1}(x)\right]\\ &\quad+\sum_{i=2}^{\infty}2(i-1)\overline{I}(x)\left[(3i-2)g_{2i}(x)+(3i-1)g_{2i+1}(x)\right]+(i-1)I''(x,1)\left[g_{2i}(x)+g_{2i+1}(x)\right].\end{aligned}$$

An expression for $I''(x,1)$ is obtained by differentiating (25) again to get:

$$I''(x,1) = \frac{2}{3-\overline{Q}(x)}\left[Q''(x,1)\left[1+\frac{1}{2}\overline{I}(x)\right]^2+\overline{I}(x)\overline{Q}(x)-\frac{1}{4}\overline{I}^2(x)\left[3+\overline{Q}(x)\right]\right].$$

The above exact expression for $G''(x,1)$ was evaluated and used in the expression for the standard deviation of the LCFS total packet delay with $w=3$ and $x=3\lambda$:

$$\sigma=\sqrt{\frac{w^2}{12}+G''(x,1)+\overline{G}(x)-\overline{G}^2(x)}.$$

Figure 8 compares this standard deviation with that for the SWA/STA method; just as for the window size of 2, it can be seen that the standard deviation with the LCFS method is substantially higher.

4 Conclusions

A new LCFS window access algorithm has been described which eliminates the continuous sensing problem; stations are required only to have a common clock with common window boundaries every 2 or 3 slots. Exact expressions for the packet delay distribution for the LCFS preemptive channel access method have been derived using two different analytical approaches for the the STA and MTA conflict resolution algorithms with a fixed window size of 2, and for the STA with a window size of 3; the analysis becomes intractable for the MTA with a window size of 3. The second approach ('Method 2') is preferred over the first approach ('Method 1') as it makes use of formula developed in [1] for the components of the delay distribution for the STA and MTA under the Simplified Window Algorithm (SWA) for channel access; this simplifies the analysis and makes it possible to obtain exact expressions for moments of the LCFS delay distribution for a window size of 3 (which is not straightforward using Method 1).

The mean and standard deviation of these delays were evaluated with both analytical approaches for validation, and compared to those of the SWA access scheme (an unbiased MTA was assumed). The mean delay with the LCFS scheme is only marginally higher than the SWA scheme, but the standard deviation of the delay is significantly higher. If variation of delay is not a concern, the LCFS method may therefore be a good alternative to the SWA, with the price of eliminating the continuous sensing problem being increased delay variation. Further work should be carried out to compare the mean delay and delay variation of the LCFS algorithm with that of algorithms employing limited sensing. It is likely that the LCFS method will yield lower mean delays, and that its higher delay variation may not be significant (or may even be lower) when compared to these algorithms.

References

[1] G.C. Polyzos, 'A Queueing Theoretic Approach to the Delay Analysis for a Class of Conflict Resolution Algorithms', PhD Thesis, Computer Systems Research Institute, University of Toronto, 1988 (also Technical Report CSRI-224, January 1989).

[2] A. Leon-Garcia, 'Probability and Stochastic Processes for Electrical Engineering', Addison-Wesley Publishing Company, 1989.

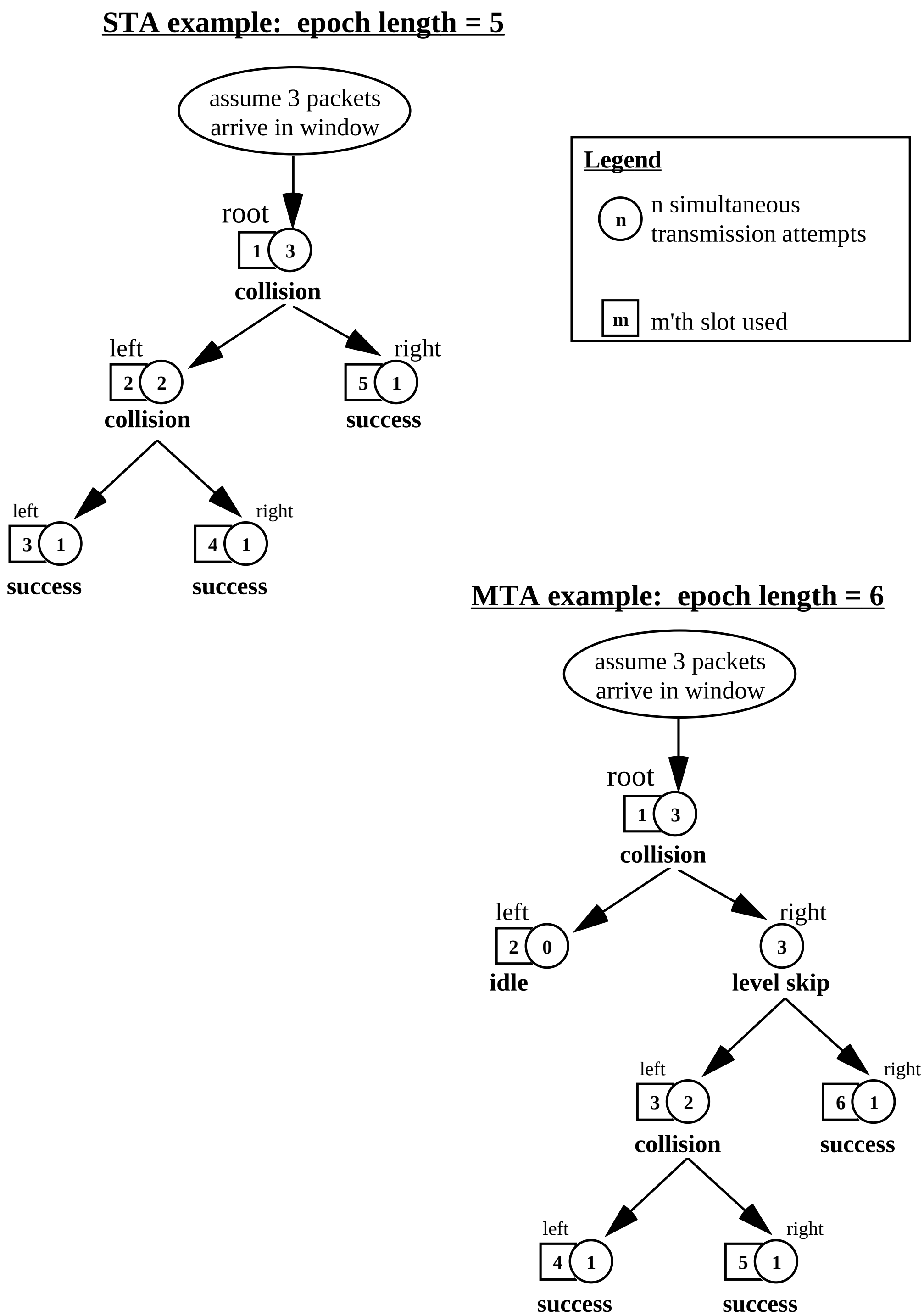

Figure 1: Sample paths of STA and MTA binary trees

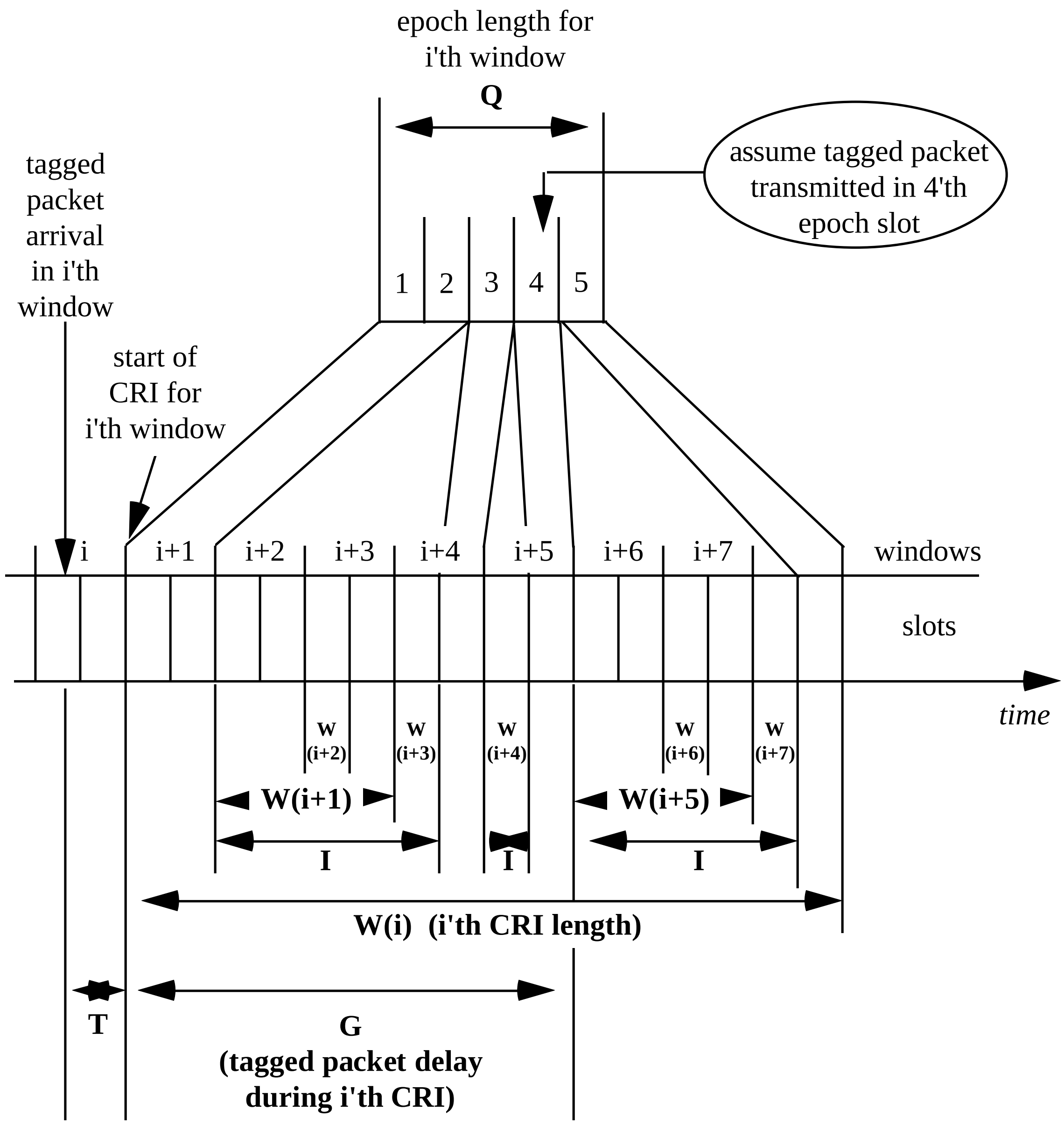

Figure 2: Example of LCFS CRI length (W) and delay within the CRI (G) for window size of 2

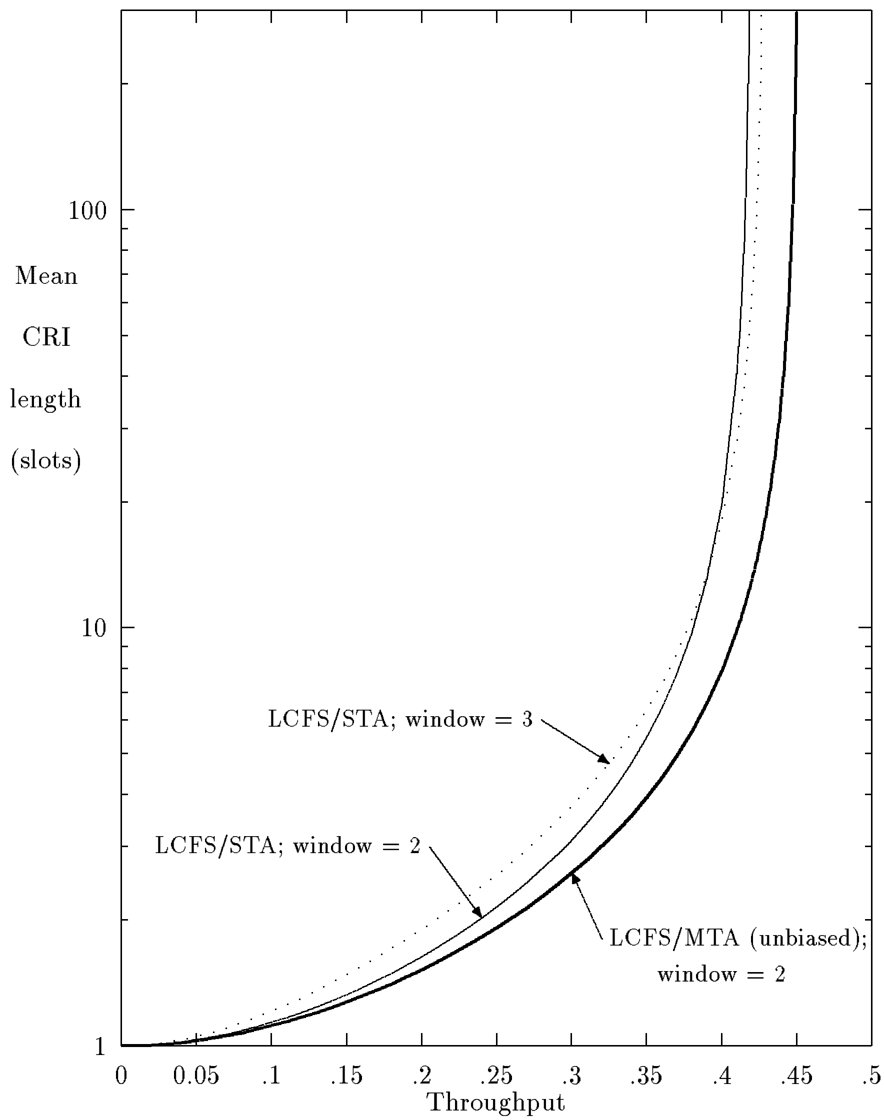

Figure 3: Mean CRI length for LCFS window access and window sizes of 2 and 3

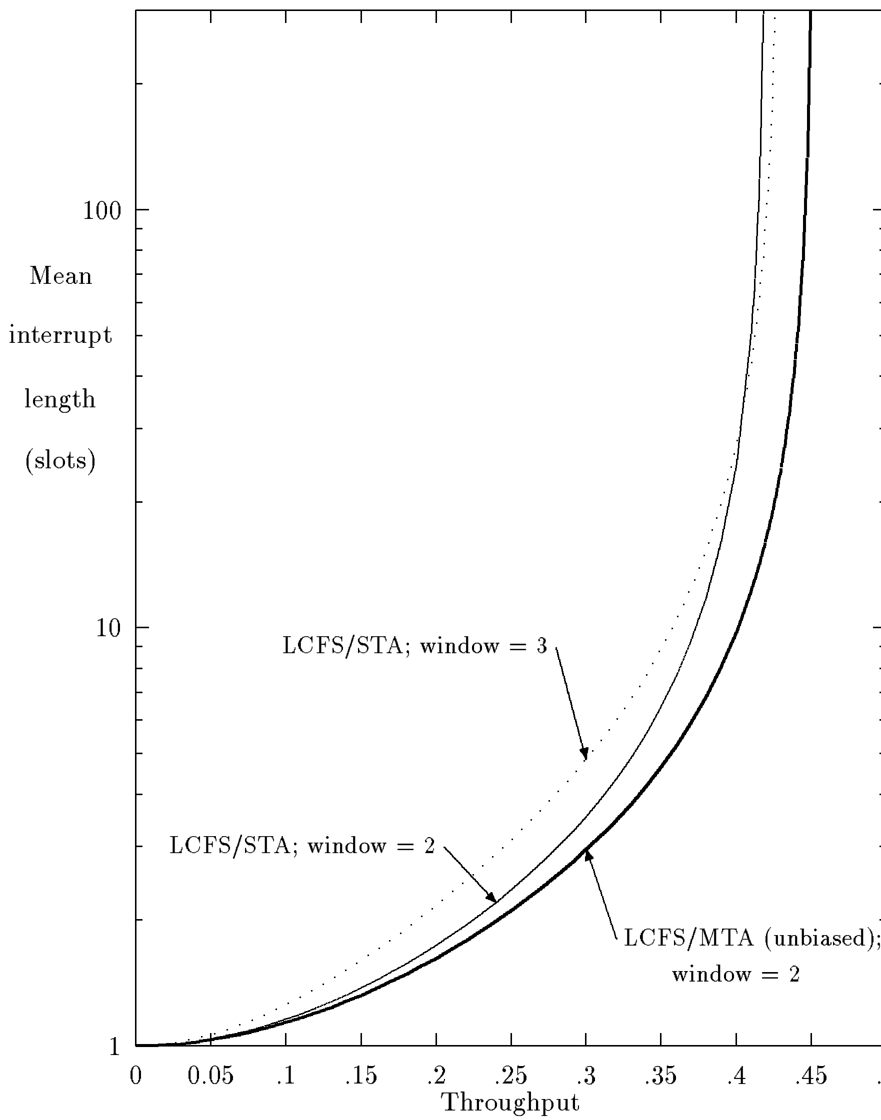

Figure 4: Mean interrupt length for LCFS window access and window sizes of 2 and 3

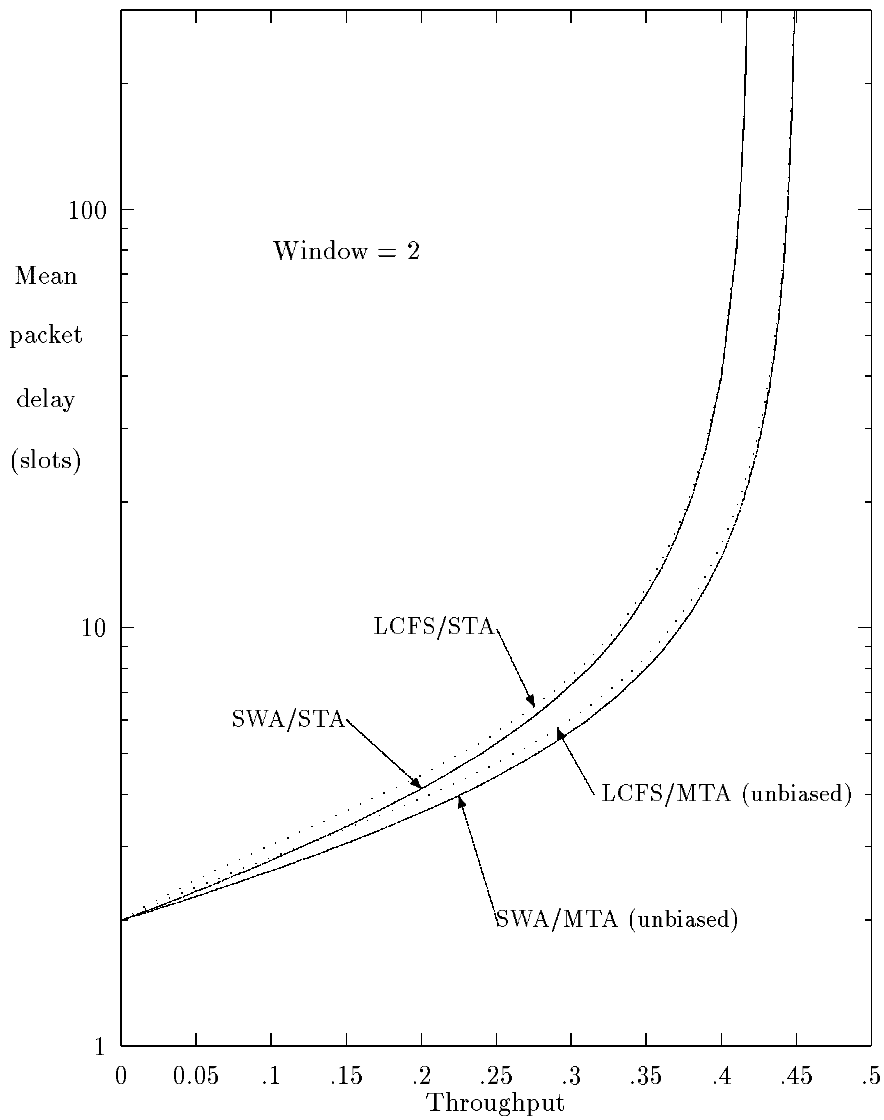

Figure 5: Comparison of STA and MTA mean packet delay for window size of 2

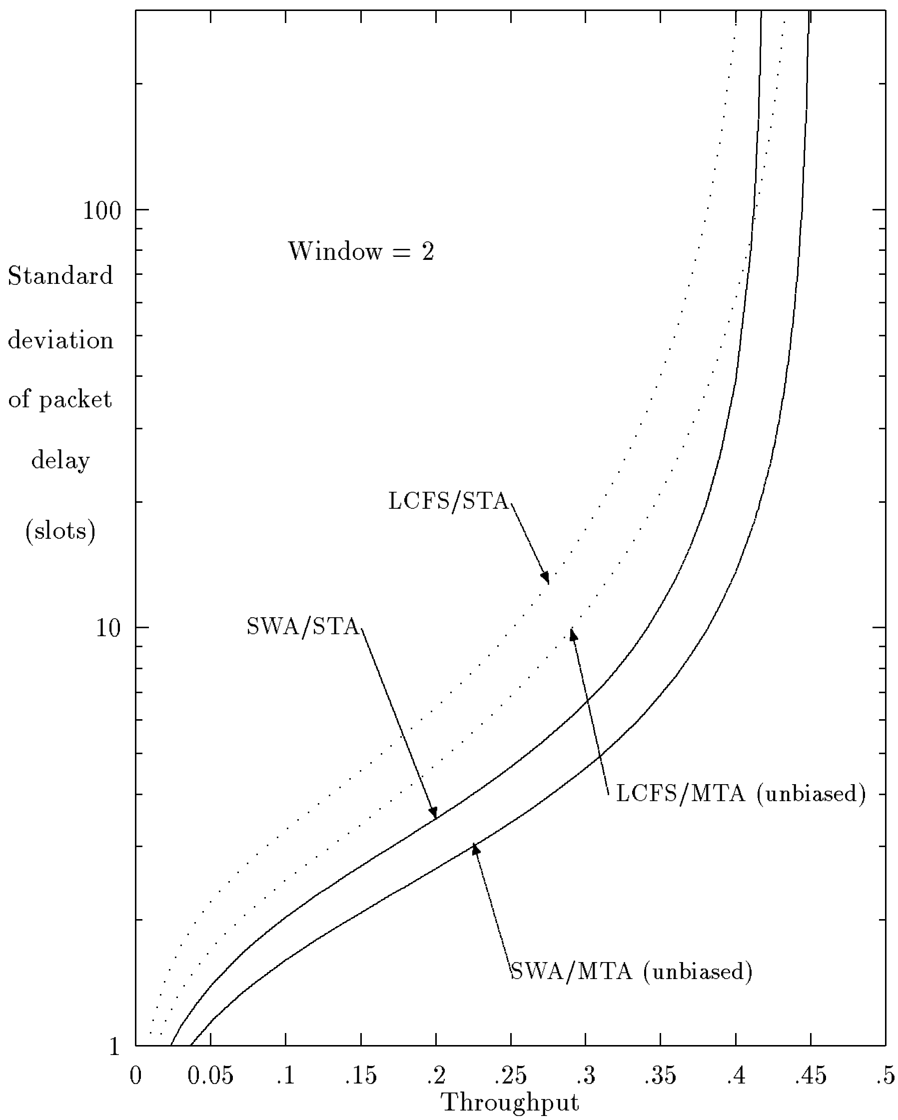

Figure 6: Comparison of standard deviation of STM and MTA packet delay for window size of 2

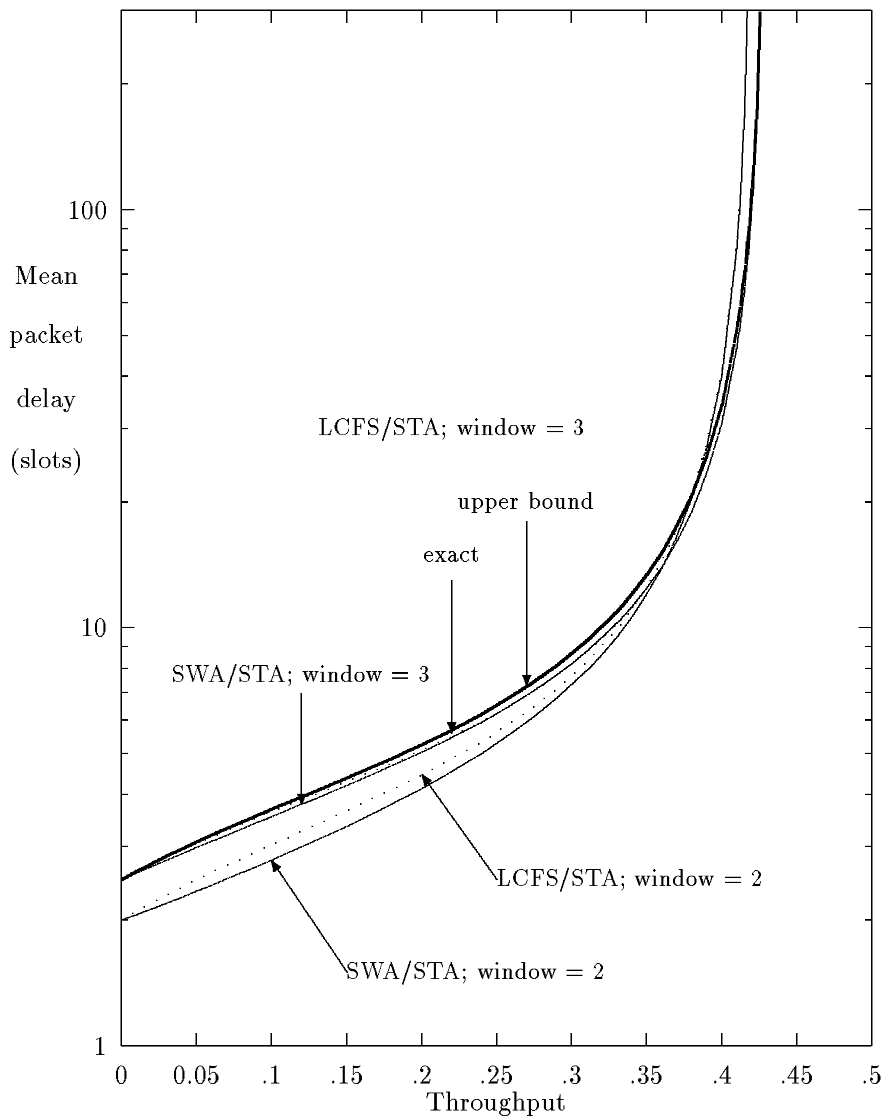

Figure 7: Comparison of STA mean packet delay for window sizes of 2 and 3

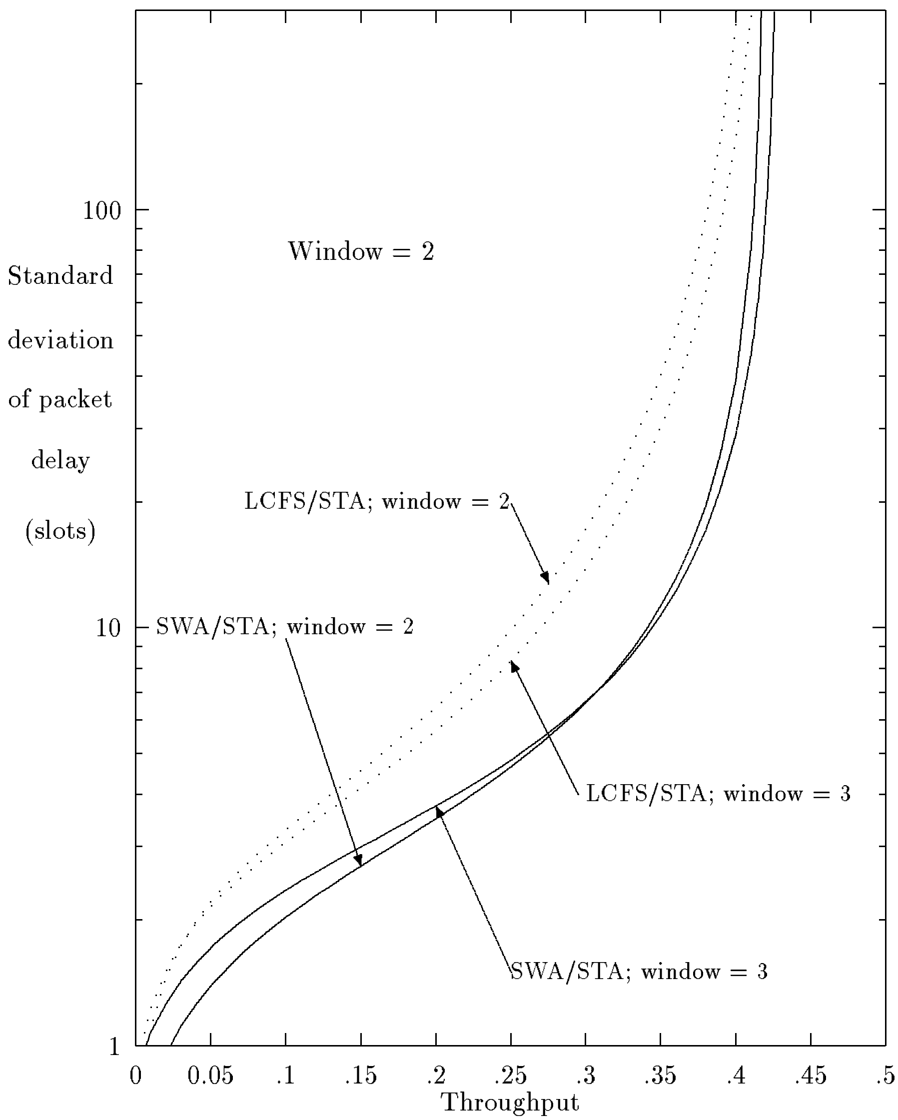

Figure 8: Comparison of standard deviation of STA packet delay for window sizes of 2 and 3

A Appendix: Epoch Length Distributions

A summary of the functional equations used for the STA and MTA epoch lengths as derived in [1] are included here. Further derivatives were obtained for use with the equations in Appendix B and C.

A.1 STA Epoch Length

The functional equation for the STA epoch length is:

$$Q_S(x,z) = zQ_S^2(x/2,z) + (z-z^3)(1+x)e^{-x}$$

and the corresponding equation for the mean is:

$$\overline{Q}_S(x) = 1 + 2\overline{Q}_S(x/2) - 2(1+x)e^{-x}.$$

The above equation was used for the numerical recursions. The power series expansion of the mean was used to validate the recursion method and to evaluate $\overline{I}(x)$ and is given by:

$$\overline{Q}_S(x) = \sum_{i=0}^{\infty} \alpha_i z^i$$

where:

$$\alpha_0 = 1, \qquad \alpha_1 = 0, \quad \text{and} \quad \alpha_i = (-1)^i \frac{2(i-1)}{i!(1-2^{1-i})} \quad i = 2,3,\cdots$$

Differentiating the functional equation further w.r.t. z,

$$Q_S''(x,1) = 2Q_S''(x/2,1) + 4\overline{Q}_S(x/2) + 2\overline{Q}_S^2(x/2) - 6(1+x)e^{-x},$$

and

$$Q_S'''(x,1) = 2Q_S'''(x/2,1) + 6Q_S''(x/2,1) + 6\overline{Q}_S(x/2)Q_S''(x/2,1) + 6\overline{Q}_S^2(x/2) - 6(1+x)e^{-x}.$$

A.2 MTA Epoch Length

The functional equation for the MTA epoch length is:

$$Q_M(x,z) = zQ_M(\pi x,z)Q_M(\overline{\pi}x,z) + (z-z^3)(1+x)e^{-x} + (z-z^2)\left[Q_M(\overline{\pi}x,z) - z(1+\overline{\pi}x)e^{-\overline{\pi}x}\right]e^{-\pi x}$$

and the corresponding equation for the mean is:

$$\overline{Q}_M(x) = 1 + \overline{Q}_M(\pi x) + \overline{Q}_M(\overline{\pi}x) - 2(1+x)e^{-x} + (1+\overline{\pi}x)e^{-x} - e^{-\pi x}.$$

For the unbiased MTA with $\pi = \frac{1}{2}$, the mean is:

$$\overline{Q}_M(x) = 1 + 2\overline{Q}_M(x/2) + e^{-x/2}((1+x/2)e^{-x/2} - 1) - 2(1+x/2)e^{-x/2},$$

and differentiating further,

$$\begin{aligned} Q_M''(x,1) &= 2Q_M''(x/2,1) + 4\overline{Q}_M(x/2) + 2\overline{Q}_M^2(x/2) - 6(1+x)e^{-x} \\ &\quad + e^{-x/2}\left[\, 3(1+x/2)e^{-x/2} - 2\overline{Q}_M(x/2) - 2\right], \end{aligned}$$

and

$$
\begin{aligned}
Q_M'''(x,1) \quad = \quad & 2Q_M'''(x/2,1) + 6Q_M''(x/2,1) + 6\overline{Q}_M(x/2)Q_M''(x/2,1) + 6\overline{Q}_M^2(x/2) \\
& - 6(1+x)e^{-x} - 3e^{-x/2}Q_M''(x/2,1) - 6e^{-x/2}\left[\overline{Q}_M(x/2) - (1+x/2)e^{-x/2}\right].
\end{aligned}
$$

B Appendix: Distribution of Delay in the SWA Epoch of Transmission

A summary of equations for the STA and MTA distributions of delay in the SWA epoch of transmission as derived in [1] is given here. The equations were further differentiated for use in the equations for the standard deviation in Appendix C.

B.1 Delay in the SWA/STA Epoch of Transmission

The functional equation is:

$$
G_S(x,z) = z\frac{1}{2}\left[Q_S(x/2,z) + 1\right]G_S(x/2,z) + \Phi(x)\left[z - \frac{1}{2}(z^2 + z^3)\right]
$$

. The power series expansion is:

$$
G_S(x,z) = \sum_{i=0}^{\infty} g_i z^i
$$

where:

$$
g_0(x) = 0, \quad g_1(x) = e^{-x}, \quad g_2(x) = \frac{1}{2}\left[e^{-x/2} - e^{-x}\right], \quad g_3(x) = \frac{1}{4}\left[e^{-x/4} - e^{-x/2} + xe^{-x}\right],
$$

$$
g_{i+1}(x) = \frac{1}{2}\left[g_i(x/2) + \sum_{m=0}^{i} q_{i-m}(x/2)g_m(x/2)\right], \qquad n \geq 3
$$

and where $q_i(x)$ are the coefficients of the STA epoch length PGF power series:

$$
q_{2i}(x) = 0, \quad q_1(x) = (1+x)e^{-x}, \quad q_3(x) = \frac{x^2}{4}e^{-x},
$$

$$
q_{2i+1}(x) = \sum_{m=0}^{2i} q_m(x/2)q_{2i-m}(x/2), \quad i = 2, 3, \cdots
$$

The mean delay is:

$$
\overline{G}_S(x) = 1 + \overline{G}_S(x/2) + \frac{1}{2}\overline{Q}_S(x/2) - \frac{3}{2}e^{-x},
$$

and differentiating again,

$$
G_S''(x,1) = G_S''(x/2,1) + \frac{1}{2}Q_S''(x/2,1) + \overline{G}_S(x/2)\left[\overline{Q}_S(x/2) - 1\right] + \overline{Q}_S(x/2) - 4e^{-x}
$$

B.2 Delay in the SWA/MTA Epoch of Transmission

For the unbiased MTA, the functional equation is:

$$G_M(x,z) = z\frac{1}{2}G_M(x/2,z) + z\frac{1}{2}G_M(x/2,z)\left[Q_M(x/2,z) + (1-z)e^{-x/2}\right] + \left[z - z^2\right]e^{-x},$$

with first moment:

$$\overline{G}_M(x) = 1 + \overline{G}_M(x/2) + \frac{1}{2}\left[\overline{Q}_M(x/2) - e^{-x/2}\right] - e^{-x},$$

and second derivative:

$$G''_M(x,1) = G''_M(x/2,1) + \frac{1}{2}Q''_M(x/2,1) + \overline{G}_M(x/2)\left[\overline{Q}_M(x/2) + 2 - e^{-x/2}\right] + \overline{Q}_M(x/2) - e^{-x/2} - 2e^{-x}.$$

C Appendix: Statistics of the SWA Packet Delay

C.1 The Mean

The mean SWA packet delay is the sum of the mean of its three component delays. For the STA this sum is:

$$\frac{w}{2} + \overline{L}(x) + \overline{G}_{TA}(x)$$

where L is the lag, i.e. the delay of the sliding windows before they are served by the next CRI, and where $TA = S$ or M denotes either the STA or MTA tree traversal algorithm respectively.

For a window size $w = 2$, the value $\overline{L}(x)$ is given by:

$$\overline{L}(x) = 1 - \frac{1}{2}\left[\frac{Q''_{TA}(x,1) - 2}{\overline{Q}_{TA}(x) - 2}\right].$$

For a window size $w = 3$,

$$\overline{L}(x) = \frac{3}{2} - \frac{1}{2}\left[\frac{Q''_{TA}(x,1) - 6}{\overline{Q}_{TA}(x) - 3}\right].$$

C.2 The Standard Deviation

The standard deviation of the SWA packet delay is:

$$\sigma_{TA} = \sqrt{\frac{w^2}{12} + L''(x,1) + \overline{L}(x) - \overline{L}^2(x) + G''_{TA}(x,1) + \overline{G}_{TA}(x,1) - \overline{G}^2_{TA}(x)}.$$

For a window size $w = 2$,

$$L''(x,1) = \frac{3\left[Q''_{TA}(x,1) - 2\right]^2 - 2\left[\overline{Q}_{TA}(x) - 2\right]\left[Q'''_{TA}(x,1) + 3Q''_{TA}(x,1) - 6\right]}{6\left[\overline{Q}_{TA}(x) - 2\right]^2}.$$

For a window size $w = 3$,

$$L''(x,1) = 1 - \frac{3}{2}\left[\frac{Q''_{TA}(x,1) - 6}{\overline{Q}_{TA}(x) - 3}\right] + \frac{1}{2}\left[\frac{Q''_{TA}(x,1) - 6}{\overline{Q}_{TA}(x) - 3}\right]^2 - \frac{1}{3}\left[\frac{Q'''_{TA}(x,1) - 6}{\overline{Q}_{TA}(x) - 3}\right].$$