## Bottom-up Parsing

*Basic Idea* :

- Scan the input string from left to right.
- Try to construct a parse tree starting at the bottom (i.e., the leaves) and working towards the root.

## Shift-reduce parsing :

*Basic Idea* : Apply a sequence of "*reductions*" to transform the input string to the start symbol of the grammar.

*reduction*: replace a substring matching the RHS of a production by the LHS.

*Example* : Consider the grammar

$$S \longrightarrow \mathbf{a}A B\mathbf{e}$$
$$A \longrightarrow A\mathbf{bc}$$
$$A \longrightarrow \mathbf{b}$$
$$B \longrightarrow \mathbf{d}$$

Input: $\mathbf{a}\underline{\mathbf{b}}\mathbf{bcde}$

$\leadsto \mathbf{a}\underline{A\mathbf{bc}}\mathbf{de}$

$\leadsto \mathbf{a}A\underline{\mathbf{d}}\mathbf{e}$

$\leadsto \underline{\mathbf{a}AB\mathbf{e}}$

$\leadsto S$

## Handles

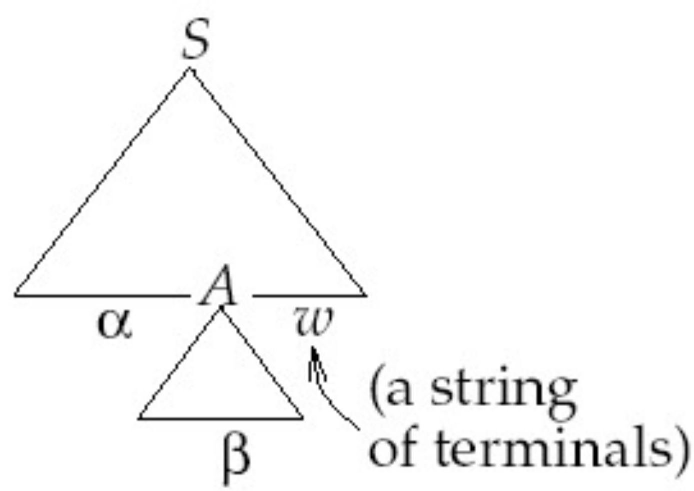
*Intuition* : A *handle* of a string $s$ is a substring $\alpha$ s.t.:

1. $\alpha$ matches the RHS of a production $A \longrightarrow \alpha$; and
2. replacing $\alpha$ by the LHS $A$ represents a step in the *reverse* of a *rightmost derivation* of $s$.

**Handles** : cont'd

**Definition** : A *handle* of a right-sentential form $\gamma$ is

1. a production $A \longrightarrow \beta$, and
2. a position in $\gamma$ where $\beta$ may be found and replaced by $A$ to produce the *previous* sentential form in a rightmost derivation of $\gamma$.




The handle $A \longrightarrow \beta$ in $\alpha\beta\omega$

# Shift Reduce Parsing

| Stack | Tokens | |
|---|---|---|
| $ | abc$ | |
| $abc | $ | Shifts |
| $S | $ | Reduce |

S → a b c

## Stack Implementation of Shift-Reduce Parsing:

**Data Structures** :

- *the stack*, its bottom marked by \$, initially empty.
- *the input string*, its right end marked by \$, initially $w$.

**Action** :

**repeat**

1. *Shift* zero or more input symbols onto the stack, until a handle $\beta$ is on the top of the stack.
2. *Reduce* $\beta$ to the LHS of the appropriate production.

**until** ready to accept.

**Acceptance** : When the stack contains the start symbol and the input is empty.

*Example* : Consider the grammar

$$S \longrightarrow \mathrm{a}AB\mathrm{e}$$
$$A \longrightarrow A\mathrm{bc}$$
$$A \longrightarrow \mathrm{b}$$
$$B \longrightarrow \mathrm{d}$$

Input: $\mathrm{a}\underline{\mathrm{b}}\mathrm{bcde}$

$\rightsquigarrow \mathrm{a}\underline{A\mathrm{bc}}\mathrm{de}$

$\rightsquigarrow \mathrm{a}A\underline{\mathrm{d}}\mathrm{e}$

$\rightsquigarrow \underline{\mathrm{a}AB\mathrm{e}}$

$\rightsquigarrow S$

| | | | | |
|---|---|---|---|---|
| | c | | e | |
| | b | d | B | |
| b | A | A | A | |
| a | a | a | a | S |
| $ | $ | $ | $ | $ |

## Conflicts during Shift-Reduce Parsing :

1. Can't decide whether to shift or to reduce ("*shift-reduce conflict*").

   Example : "dangling else":

   $$\begin{aligned} Stmt \longrightarrow\ & \textbf{if}\ Expr\ \textbf{then}\ Stmt \mid \\ & \textbf{if}\ Expr\ \textbf{then}\ Stmt\ \textbf{else}\ Stmt \mid \cdots \end{aligned}$$

2. Can't decide which of several possible reductions to make ("*reduce-reduce conflict*").

   Example :

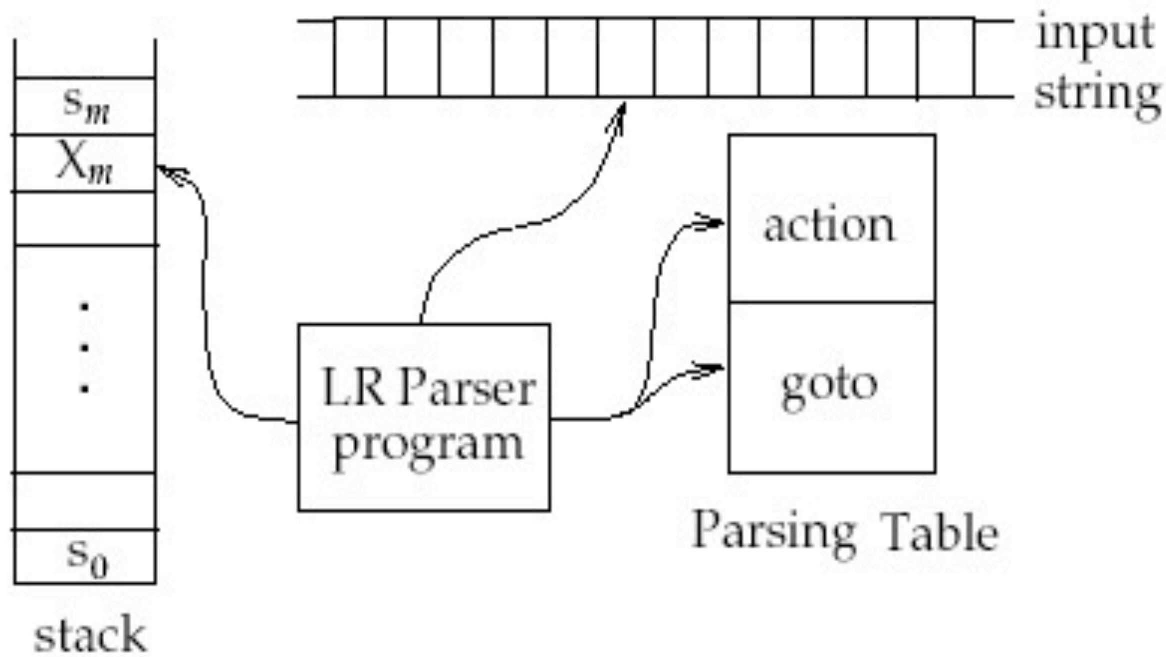
   $$\begin{aligned} Stmt &\longrightarrow \textbf{id}\ (\ params\ ) \mid Expr\ \textbf{:=}\ Expr \mid \cdots \\ Expr &\longrightarrow \textbf{id}\ (\ params\ ) \end{aligned}$$

   Given the input `A(I, J)` the parser doesn't know whether it's a procedure call or an array reference.

## LR Parsing

- Bottom-up.
- LR($k$) parser:
  - Scans the input **L**-to-R.
  - Produces a **R**ightmost derivation.
  - Uses $\underline{k}$-symbol lookahead.

## Schematic of an LR Parser :




- The driver program is the same for all LR parsers (SLR(1), LALR(1), LR(1), ... ) : only the parsing table changes.

- The stack holds strings of the form

$$s_0 X_1 s_1 X_2 s_2 \cdots X_m s_m$$

where $s_m$ is on top, the $s_i$ are "states", and $X_i$ are grammar symbols.

- The *configuration* of an LR parser is given by a pair $\langle$stack contents, unexpended input$\rangle$.

A configuration $\langle s_0 X_1 s_1 \cdots X_m s_m,\quad \mathrm{a}_i \mathrm{a}_{i+1} \cdots \mathrm{a}_n \rangle$ represents the right-sentential form

$$X_1 \cdots X_m \mathrm{a}_i \mathrm{a}_{i+1} \cdots \mathrm{a}_n$$

The sequence of symbols $X_1 \cdots X_m$ on the parser stack is called a *viable prefix* of the right sentential form.

## LR Parse Tables

- The parsing table consists of two parts: a parsing **action** function, and a **goto** function.

- For a given configuration of the parser, the next move is determined by the parse table entry

  action[$s_m$, $a_i$].

  where $s_m$ is the topmost state on the stack, and $a_i$ is the next input symbol.

- An action table entry can be of four types:

  1. **shift** $s$, where $s$ is a state.
  2. **reduce** by a grammar production $A \longrightarrow \beta$.
  3. **accept**
  4. **error**

## LR Parsing : cont'd

Suppose the parser configuration is

$$\langle s_0 X_1 s_1 \cdots X_m s_m, \quad \mathbf{a}_i \cdots \mathbf{a}_n \$ \rangle.$$

- if $\texttt{action}[s_m, \mathbf{a}_i] = \textit{shift } s$ then the parser executes a *shift* move. The new configuration is

$$\langle s_0 X_1 s_1 \cdots X_m s_m \underbrace{\mathbf{a}_i \; s}_{pushed}, \quad \mathbf{a}_{i+1} \cdots \mathbf{a}_n \$ \rangle.$$

- if $\texttt{action}[s_m, \mathrm{a}_i]$ = *reduce* $A \longrightarrow \beta$ then the parser does a <u>*reduce*</u> move. The new configuration is

$$\langle s_0 X_1 s_1 \cdots X_{m-r} s_{m-r} \underbrace{A\ s}_{new}, \quad \mathrm{a}_i, \cdots \mathrm{a}_n \$ \rangle.$$

  where

  - $r$ = length of $\beta$; and
  - $s = \texttt{goto}[s_{m-r}, A]$.

- if $\texttt{action}[s_m, \mathrm{a}_i]$ = *accept* then parsing is done.

- if $\texttt{action}[s_m, \mathrm{a}_i]$ = *error* the parser calls an error recovery routine.

## 5.2. Finite Automata to recognize Viable Prefixes

**Definition** : An *LR(0) item* of a grammar $G$ is is a production of $G$ with a dot '$\bullet$' added at some position in the RHS.

*Example* : The production $A \longrightarrow \mathrm{a}A\mathrm{b}$ gives the items

$$
\begin{aligned}
&A \longrightarrow \bullet\mathrm{a}A\mathrm{b}\\
&A \longrightarrow \mathrm{a}\bullet A\mathrm{b}\\
&A \longrightarrow \mathrm{a}A\bullet\mathrm{b}\\
&A \longrightarrow \mathrm{a}A\mathrm{b}\bullet
\end{aligned}
$$

**Intuition** : An item $A \longrightarrow \alpha\bullet\beta$ denotes:

- we have seen a string derivable from $\alpha$; and
- we hope to see a string derivable from $\beta$.

**Overall Goal** : Given a grammar with start symbol $S$,

- Construct an *augmented grammar* by adding a new start symbol $S'$ and production $S' \rightarrow S$;

- Starting with the item $S' \rightarrow \bullet S$, recognize the viable prefix $S' \rightarrow S\bullet$.

# Viable Prefix DFA

1. *closure* :

**Definition** : If $I$ is a set of items for a grammar $G$, then $closure(I)$ is the set of items constructed as follows:

**repeat**

1. add every item in $I$ to $closure(I)$;
2. if $A \longrightarrow \alpha_\bullet B\beta$ is in $closure(I)$ and $B \longrightarrow \gamma$ is a production of $G$, then add $B \longrightarrow {}_\bullet\gamma$ to $closure(I)$.

**until** no new item can be added to $closure(I)$.

**Intuition** : If $A \longrightarrow \alpha_\bullet B\beta$ is in $closure(I)$ then we hope to see a string derivable from $B$ in the input. So if $B \longrightarrow \gamma$ is a production of $G$, then we should hope to see a string derivable from $\gamma$ in the input. Hence, $B \longrightarrow {}_\bullet\gamma$ is in $closure(I)$.

## Viable Prefix DFA – cont'd:

**2. *goto* :**

**Definition** : If $I$ is a set of items for a grammar $G$ and $X$ a grammar symbol, then $goto(I, X)$ is the set of items

$$closure(\{A \longrightarrow \alpha X \bullet \beta \mid A \longrightarrow \alpha \bullet X\beta \in I\}).$$

**Intuition** :

- A set of items $I$ corresponds to a state.
- If $A \longrightarrow \alpha \bullet X\beta \in I$ then
  - we've seen a string derivable from $\alpha$; and
  - we hope to see a string derivable from $X\beta$;

- now suppose we see a string derivable from $X$ : the resulting state should be one in which:
  - we've seen a string derivable from $\alpha X$; and
  - we hope to see a string derivable from $\beta$;
- The item corresponding to this is $A \longrightarrow \alpha X \bullet \beta$.

## Constructing the Viable Prefix DFA for LR(0) Items

- Given a grammar $G$ with start symbol $S$, construct the *augmented grammar* by adding a special production

  $$S' \longrightarrow S$$

  where $S'$ does not appear in $G$.

- Algorithm for constructing the canonical collection of LR(0) items for an augmented grammar $G'$:

**begin**
    $C := \{closure(\{S' \longrightarrow .S\})\}$;
    **repeat**
        **for** each set of items $I \in C$ **do**
            **for** each grammar symbol $X$ **do**
                **if** $goto(I, X) \neq \emptyset$ **then**
                    add $goto(I, X)$ to $C$;
                **fi**
    **until** no new set of items can be added to $C$;
    **return** $C$;
**end**

# Example

**Original Grammar**

$E \rightarrow E + T \mid T$

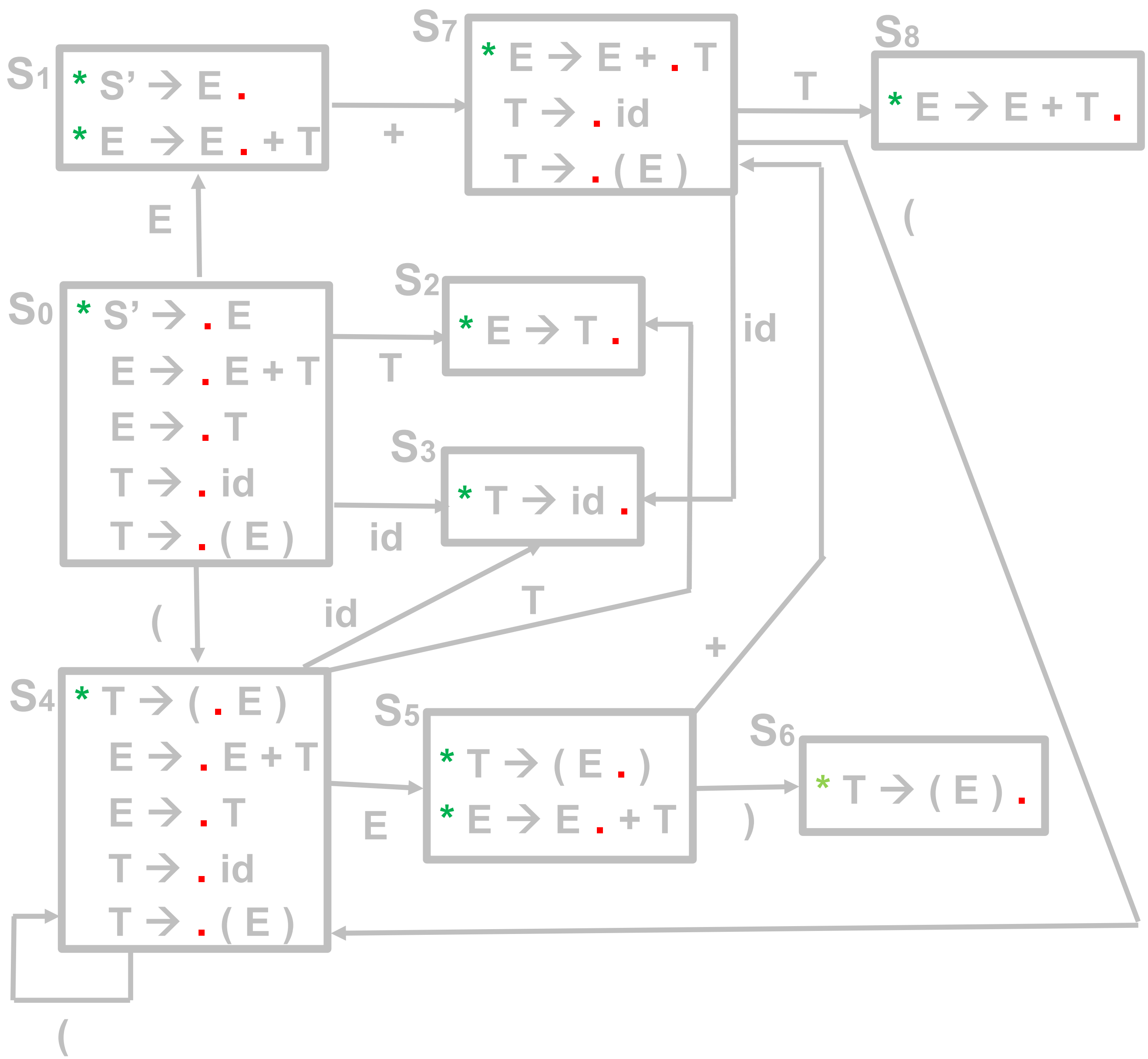
$T \rightarrow id \mid ( E )$

**Augmented Grammar**

$S' \rightarrow E$

$E \rightarrow E + T \mid T$

$T \rightarrow id \mid ( E )$




**Augmented Grammar**

**S' → E**
**E → E + T | T**
**T → id | ( E )**

**Kernel items are Marked with ***

**Rest of the items added by closure**

**. Tells where we are in the production**

## 5.3. Constructing an SLR(1) Parse Table

1. Given a grammar $G$, construct the augmented grammar $G'$ by adding the production $S' \longrightarrow S$.

2. Construct $C = \{I_0, \ldots, I_n\}$, the set of states of the viable prefix DFA for $G'$.

3. State $i$ is constructed from $I_i$, with parsing action determined as follows:

   (a) $A \longrightarrow \alpha \bullet \mathrm{a}\beta \in I_i$, **a** a terminal, $goto(I_i, \mathrm{a}) = I_j$: set $action[i, \mathrm{a}] = \underline{\text{shift } j}$.

   (b) $A \longrightarrow \alpha \bullet \in I_i, A \neq S'$: for each a $\in$ FOLLOW($A$), set $action[i, \mathrm{a}] = \underline{\text{reduce } A \longrightarrow \alpha}$.

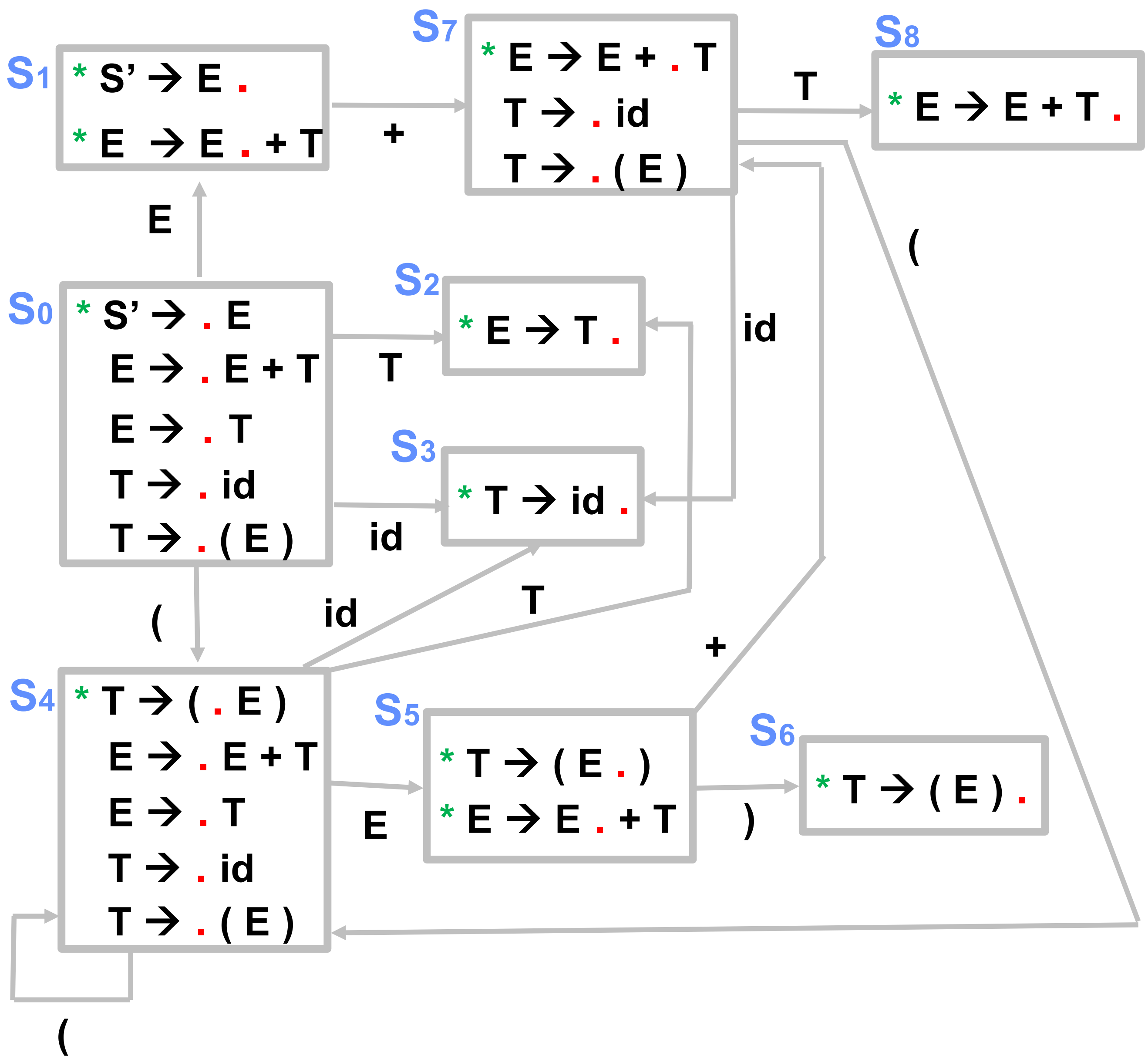
   (c) $S' \longrightarrow S \bullet \in I_i$ : set $action[i, \$] = \underline{\text{accept}}$.

4. goto transitions are constructed as follows: for each nonterminal $A$, if $goto(I_i, A) = I_j$ then $goto[i, A] = j$.

5. All entries not defined by the above steps are made *error*.

   If there are any multiply defined entries, then $G$ is not SLR.

6. Initial state of the parser: that constructed from $I_0 \sim S' \longrightarrow \bullet S$.




**Augmented Grammar**

S' → E
E → E + T | T
T → id | ( E )

**Kernel items are Marked with ***

**Rest of the items added by closure**

**. Tells where we are in the production**

| | ACTION | | | | | GOTO | | |
|---|---|---|---|---|---|---|---|---|
| | + | Id | ( | ) | $ | E | T | S |
| S0 | | S,S3 | S,S4 | | | S1 | S2 | |
| S1 | S,S7 | | | | accept | | | |
| S2 | R,#3 | | | R,#3 | R,#3 | | | |
| S3 | R,#4 | | | R,#4 | R,#4 | | | |
| S4 | | S,S3 | S,S4 | | | S5 | S2 | |
| S5 | S,S7 | | | S,S6 | | | | |
| S6 | R,#5 | | | R,#5 | R,#5 | | | |
| S7 | | S,S3 | S,S4 | | | | S8 | |
| S8 | R,#2 | | | R,#2 | R,#2 | | | |

#1 S' → E

#2 E → E + T

#3 E → T

#4 T → id

#5 T → ( E )

Follow(S') → { $ }

Follow(E) → { +, ), $ }

Follow(T) → { +, ), $ }

S - SHIFT    R - REDUCE

S# - Next State

#n - Production Rule Number

# The LR Parsing Algorithm

```
begin
    set ip to point to the first symbol of the input w$;

    while TRUE do
        let s be the state on top of the stack,
            a the symbol pointed at by ip;

        if action[s, a] = shift s' then
            push a then s' on top of the stack;
            advance ip to the next input symbol;

        else if action[s, a] = reduce A ⟶ β then
            pop 2* | β | symbols off the stack;
            let s' be the state now on top of the stack;
            push A then goto[A, s'] on top of the stack;

        else if action[s, a] = accept then return;

        else error();
        fi
    od
end
```

| Stack | Input | Action |
| --- | --- | --- |
| $ S0 | id + id $ | action[S0,id] = shift S3 |
| $ S0 id S3 | + id $ | action[S3,+] = reduce T→id |
| | | GOTO[S0,T] = S2 |
| $ S0 T S2 | + id $ | action[S2,+] = reduce E→T |
| | | GOTO[S0,E] = S1 |
| $ S0 E S1 | + id $ | action[S1,+] = shift S7 |
| $ S0 E S1 + S7 | id $ | action[S7,id] = shift S3 |
| $ S0 E S1 + S7 id S3 | $ | action[S3,$] = reduce T→id |
| | | GOTO[S7,T] = S8 |
| $ S0 E S1 + S7 T S8 | $ | action[S8,$] = reduce E→E+T<br>GOTO[S0,E] = S1 |
| $ S0 E S1 | $ | action[S1,$] = accept |

## Limitations of SLR Parsing

Cannot handle many "reasonable" grammars, e.g.:

$$S \longrightarrow R \mid L{=}R$$
$$L \longrightarrow * \; R \mid \text{id}$$
$$R \longrightarrow L$$

The SLR parse table contains a state

$$I = \{S \longrightarrow L_{\bullet}{=}R, R \longrightarrow L_{\bullet}\}$$

which causes a shift/reduce conflict on '=', since '=' is in FOLLOW($L$).

**Problem** : For an input

***id = id**

we want to remember enough "left context" after seeing * to make the right shift/reduce decision. SLR cannot do this adequately.

$S \rightarrow L = R$

$\rightarrow L = L$

$\rightarrow L = id$

$\rightarrow * R = id$

$\rightarrow * L = id$

$\rightarrow * id = id$

$S \rightarrow L = R$

$\rightarrow L = L$

$\rightarrow L = id$

$\rightarrow id = id$

**Once having reduced id to L and then seeing =,**

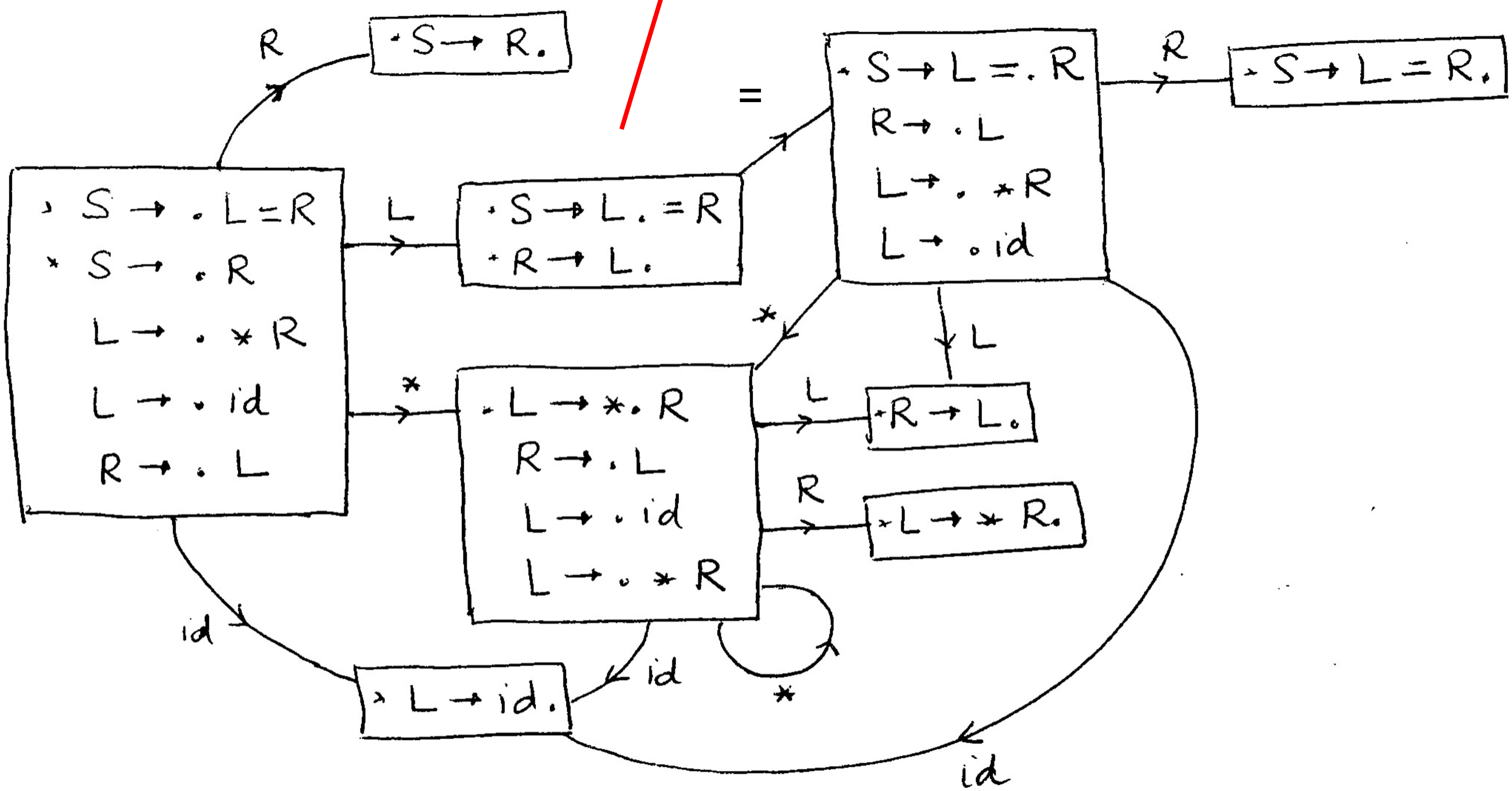
- **in the case on the left L is reduced to R**
- **in the case on the right = is shifted to the stack.**

**S → L = R | R**

**R → L**

**L → * R | id**

| | |
|---|---|
| **S → L . = R** | → **Shift on =** |
| **R → L .** | → **Since = is in Follow(R) so Reduce on =** |



## 5.4. LR(1) Parsing

**Idea** : Extend SLR parsing to incorporate lookahead.

**LR(1) Item** :

- Of the form $[A \longrightarrow \alpha \cdot \beta, \mathbf{a}]$, where **a** is a terminal or is the endmarker \$.
- The lookahead has no effect on items of the form $[A \longrightarrow \alpha \cdot \beta, \mathbf{a}]$, where $\beta \neq \varepsilon$.
- For items of the form $[A \longrightarrow \alpha \cdot, \mathbf{a}]$, reduce only if the next symbol is **a**.

**Note:** For an item of the form $[A \longrightarrow \alpha \cdot \beta, \mathbf{a}]$, $\mathbf{a} \in$ FOLLOW($A$). But there may be $\mathbf{b} \in$ FOLLOW($A$) for which there is no item $[A \longrightarrow \alpha \cdot \beta, \mathbf{b}]$.

# LR(1) Parsing: *closure* and *goto* Functions

1. *closure*$(I)$ :

**begin**
$S := I$;
  **repeat**
    **for(** each **item** $[A \longrightarrow \alpha \bullet B\beta, \mathrm{a}] \in I$,
      each production $B \longrightarrow \gamma$,
      each terminal $\mathrm{b} \in$ FIRST$(\beta\mathrm{a})$) **do**
    add $[B \longrightarrow \bullet\gamma, \mathrm{b}]$ to $S$;
  **until** no new item can be added to $S$;
  **return** $S$;
**end**

2. $goto(I, X)$ :

**begin**
  **let** $J = \{[A \longrightarrow \alpha X \bullet \beta, \mathrm{a}] \mid [A \longrightarrow \alpha \bullet X\beta, \mathrm{a}] \in I\}$;
  **return** $closure(J)$;
**end**

# Constructing the Viable Prefix DFA for LR(1) Items

- *Given* : An augmented grammar $G'$.

- *Algorithm* :

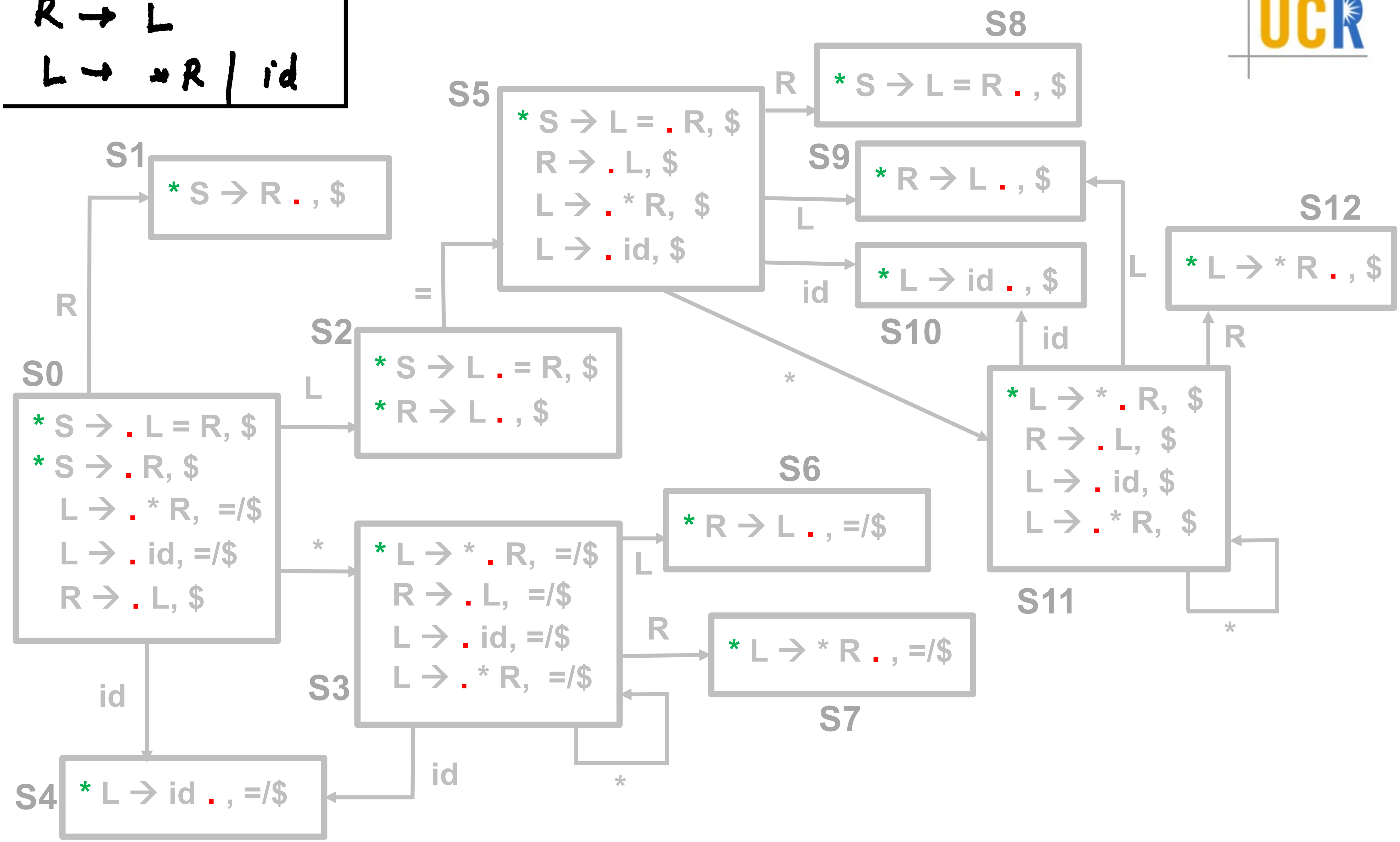
**begin**
  $C := \{closure(\{[S' \longrightarrow .S, \$]\})\}$;
  **repeat**
    **for** each set of items $I \in C$ **do**
      **for** each grammar symbol $X$ **do**
        **if** $goto(I, X) \neq \emptyset$ **then**
          add $goto(I, X)$ to $C$;
  **until** no new set of items can be added to $C$;
  **return** $C$;
**end**

- **Note** : The set of items construction is essentially the same as for the SLR(1) case.

$S \rightarrow L = R \mid R$

$R \rightarrow L$

$L \rightarrow *R \mid id$



| | ACTION | | | | GOTO | | |
|---|---|---|---|---|---|---|---|
| | * | = | id | $ | S | R | L |
| S0 | S,S3 | | S,S4 | | | S1 | S2 |
| S1 | | | | accept | | | |
| S2 | | S,S5 | | R,R→L | | | |
| S3 | S,S3 | | S,S4 | | | S7 | S6 |
| S4 | | R,L→id | | R,L→id | | | |
| S5 | S,S11 | | S,S10 | | | S8 | S9 |
| S6 | | R,R→L | | R,R→L | | | |
| S7 | | R,L→*R | | R,L→*R | | | |
| S8 | | | | accept | | | |
| S9 | | | | R,R→L | | | |
| S10 | | | | R,L→id | | | |
| S11 | S,S11 | | S,S10 | | | S12 | S9 |
| S12 | | | | R,L→*R | | | |

S - SHIFT     R - REDUCE     S# - Next State     #n – Production Number

| | |
|---|---|
| $ S0 | * id = id $ |
| $ S0 * S3 | id = id $ |
| $ S0 * S3 id S4 | = id $ |
| $ S0 * S3 L S6 | = id $ |
| $ S0 * S3 R S7 | = id $ |
| $ S0 L S2 | = id $ |
| $ S0 L S2 = S5 | id $ |
| $ S0 L S2 = S5 id S10 | $ |
| $ S0 L S2 = S5 L S9 | $ |
| $ S0 L S2 = S5 R S8 | $ |

accept

# Constructing an LR(1) Parse Table

1. Given a grammar $G$, construct the augmented grammar $G''$ by adding the production $S'' \longrightarrow S$.

2. Construct $C = \{I_0, \ldots, I_n\}$, the viable prefix DFA for $G''$.

3. State $i$ is constructed from $I_i$, with parsing action determined as follows:

   (a) $[A \longrightarrow \alpha \bullet \mathrm{a}\beta, \mathrm{b}] \in I_i$, **a** a terminal, $goto(I_i, \mathrm{a}) = I_j$: set $action[i, \mathrm{a}]$ = <u>shift $j$</u>.

   (b) $[A \longrightarrow \alpha \bullet, \mathrm{a}] \in I_i, A \neq S''$: set $action[i, \mathrm{a}]$ = <u>reduce $A \longrightarrow \alpha$</u>.

   (c) $[S'' \longrightarrow S \bullet, \$] \in I_i$ : set $action[i, \$]$ = <u>accept</u>.

4. goto transitions are constructed as follows: for each nonterminal $A$, if $goto(I_i, A) = I_j$ then $goto[i, A] = j$.

5. All entries not defined by the above steps are made *error*.

   If there are any multiply defined entries, then $G$ is not LR(1).

6. Initial state of the parser: that constructed from $I_0 \sim [S' \longrightarrow \bullet S, \$]$.

## 5.4.3. LALR(1) Parsing

*Observation* : Every SLR grammar is an LR(1) grammar, but the LR(1) parser usually has many more states than the SLR parser.

Many of these states differ only on the lookahead token. But the lookahead token does not play any role except on reductions.

**Definition** : The *core* of a set of LR(1) items $I$ is

$$core(I) = \{J \mid [J, \mathbf{a}] \in I \text{ for some } \mathbf{a}\}$$

I.e., $core(I)$ is the set of first components of $I$.

*Example* : Suppose

$$\begin{aligned} I = \{&[A \longrightarrow \mathbf{c}_\bullet, \mathrm{a}], \\ &[A \longrightarrow \mathbf{c}_\bullet, \mathrm{b}], \\ &[B \longrightarrow \mathbf{c}_\bullet, \mathrm{c}]\} \end{aligned}$$

Then,

$$core(I) = \{A \longrightarrow \mathbf{c}_\bullet, B \longrightarrow \mathbf{c}_\bullet\}$$

## Merging sets of LR(1) Items

- If sets of items with the same core are merged, the parser behaves essentially as before.

  However, some redundant reductions might be done before an error is detected.

- $core(goto(I, X))$ depends only on $core(I)$, so *goto*'s of merged sets may themselves be merged.

- Suppose we take a set $C_0$ of sets of LR(1) items for a given grammar, and merge those sets of items that have the same core to get a set $C_1$ of sets of LR(1) items.

  LR(1) parse table construction using $C_1$ will not introduce any new shift/reduce conflicts compared to $C_0$.

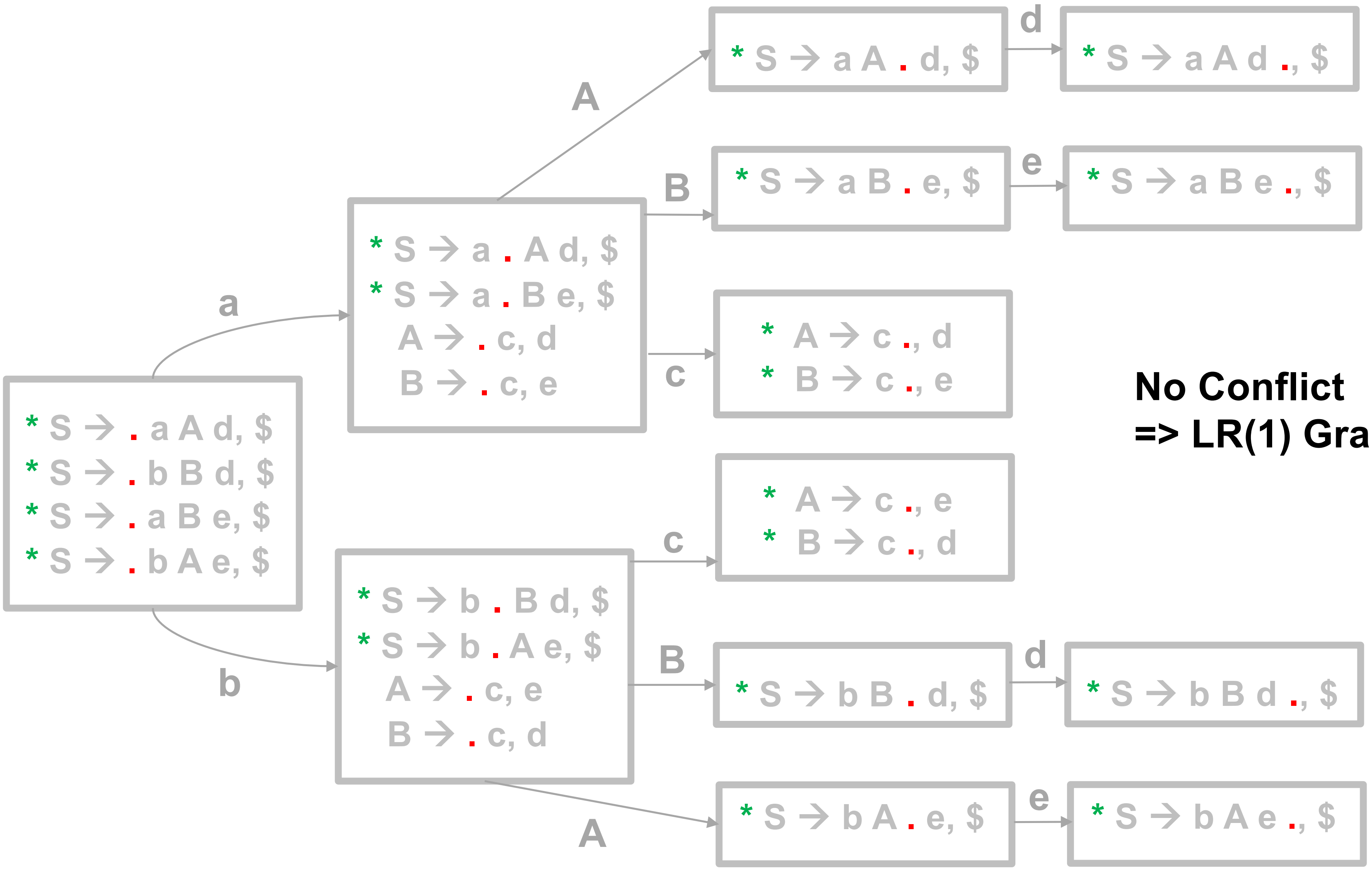
  However, this can introduce new reduce/reduce conflicts.

*Example of reduce/reduce conflicts due to merging* :

Consider the grammar given by

$$S \longrightarrow \mathbf{a}A\mathbf{d} \mid \mathbf{b}B\mathbf{d} \mid \mathbf{a}B\mathbf{e} \mid \mathbf{b}A\mathbf{e}$$
$$A \longrightarrow \mathbf{c}$$
$$B \longrightarrow \mathbf{c}$$

$$S \longrightarrow \mathrm{a}A\mathrm{d} \mid \mathrm{b}B\mathrm{d} \mid \mathrm{a}B\mathrm{e} \mid \mathrm{b}A\mathrm{e}$$
$$A \longrightarrow \mathrm{c}$$
$$B \longrightarrow \mathrm{c}$$




**No Conflict**
**=> LR(1) Grammar**

$$S \longrightarrow \mathrm{a}A\mathrm{d} \mid \mathrm{b}B\mathrm{d} \mid \mathrm{a}B\mathrm{e} \mid \mathrm{b}A\mathrm{e}$$
$$A \longrightarrow \mathrm{c}$$
$$B \longrightarrow \mathrm{c}$$

A → c . , d
B → c . , e

A → c . , e
B → c . , d

Merge

A → c . , d/e

B → c . , d/e

Contains

reduce-reduce conflict

→ not LALR(1)

# Merging of LR(1) states cannot introduce a new shift-reduce conflict in LALR(1) parser

$A \rightarrow \alpha \, . \, a \, \beta$ , ...

$B \rightarrow \gamma \, .$ , ...

$A \rightarrow \alpha \, . \, a \, \beta$ , ...

$B \rightarrow \gamma \, .$ , $a$

**Conflict was already present in an LR(1) state!**

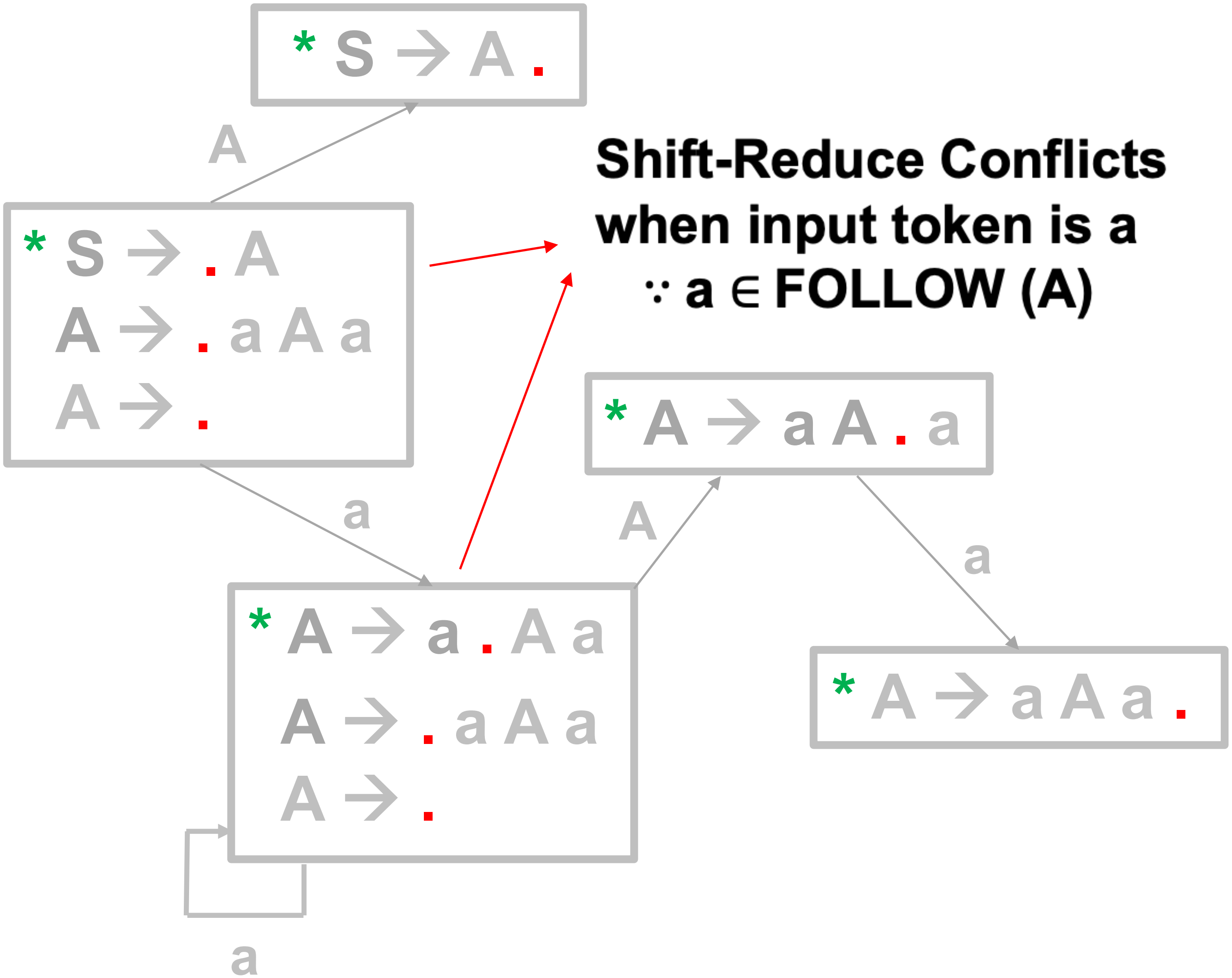
$A \rightarrow \alpha \, . \, a \, \beta$ , ...

$B \rightarrow \gamma \, .$ , $a$ / ...

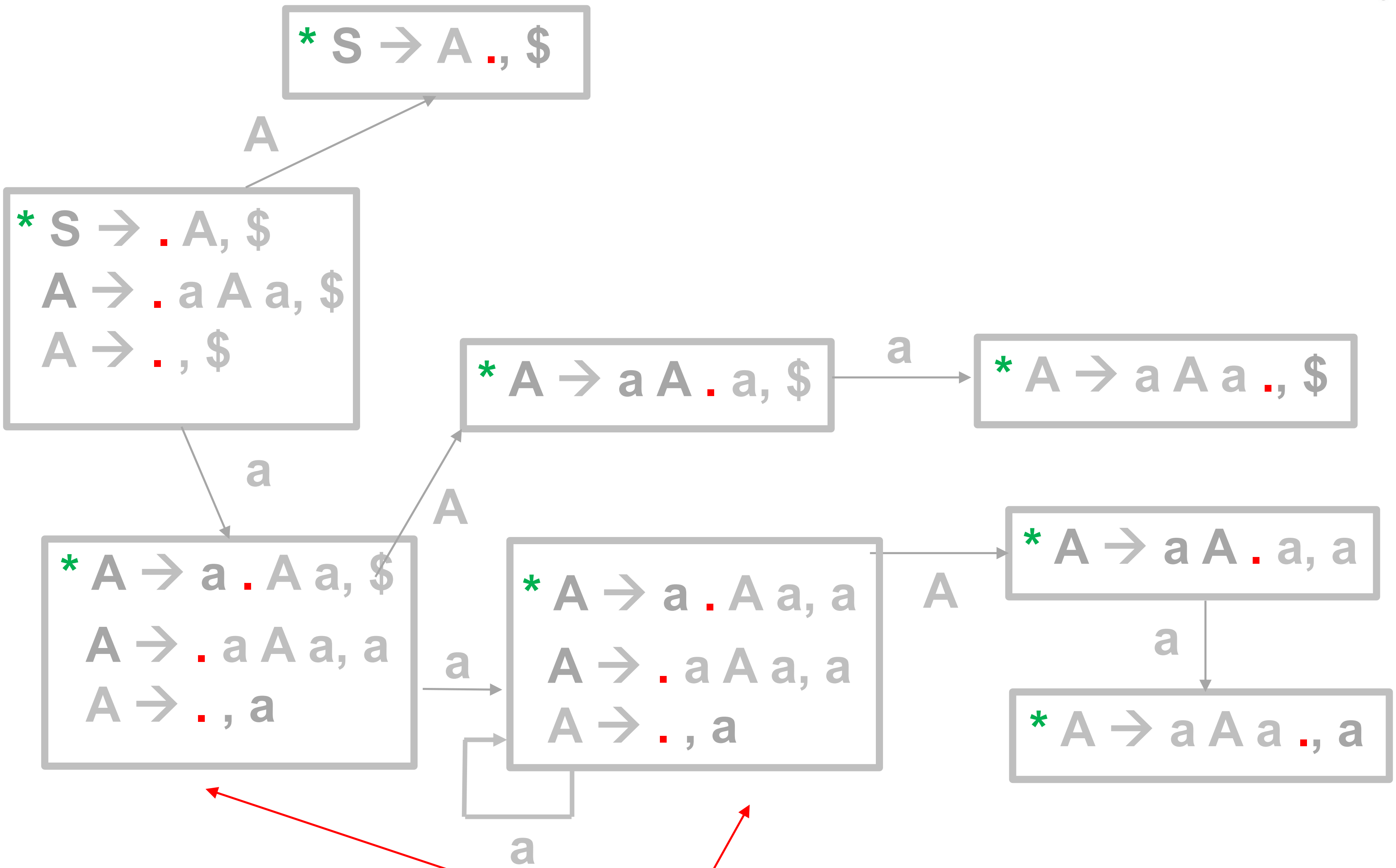
# SAMPLE PROBLEMS

$S \rightarrow A$

$A \rightarrow a\,A\,a \mid \epsilon$



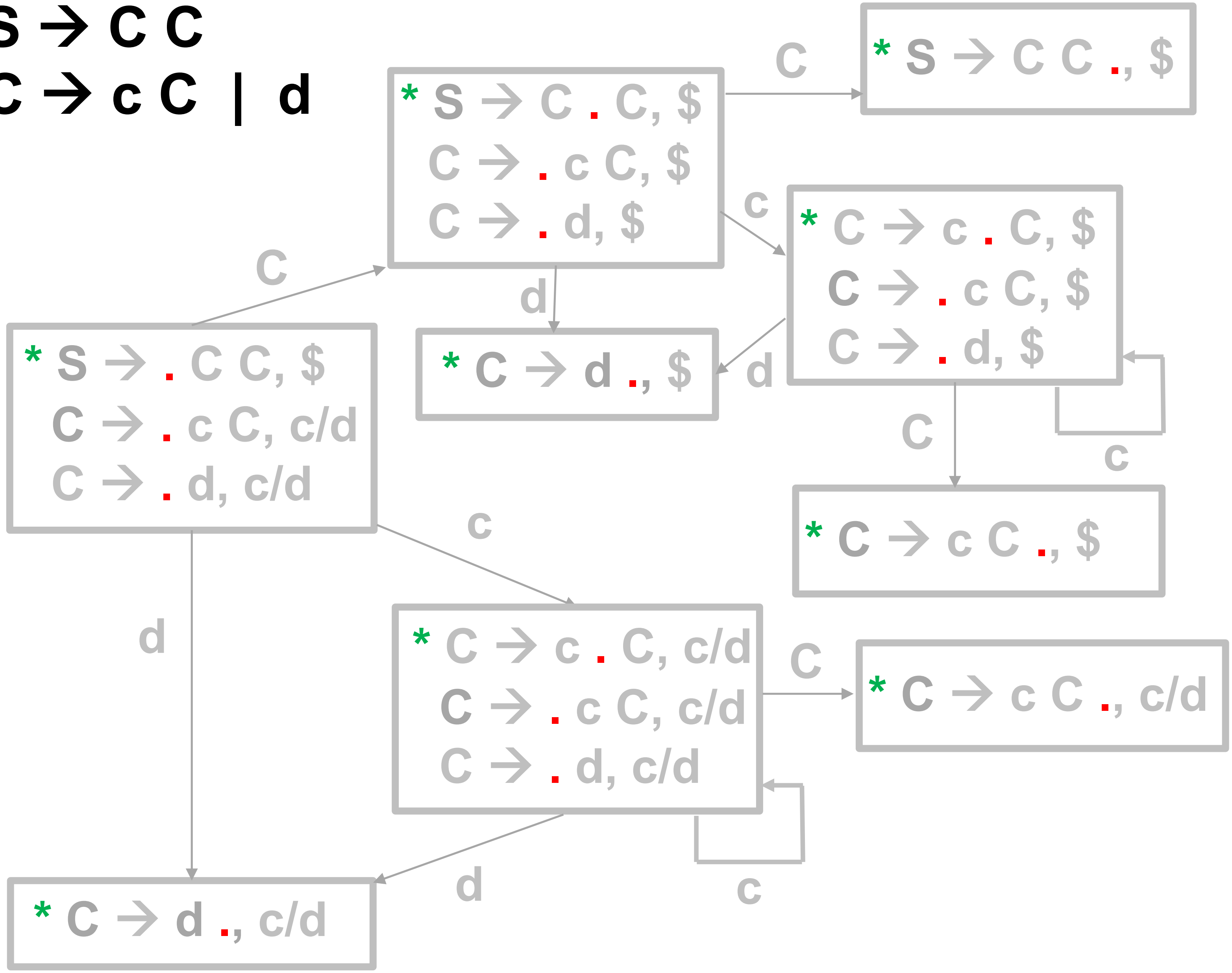
S → A
A → a A a | ϵ



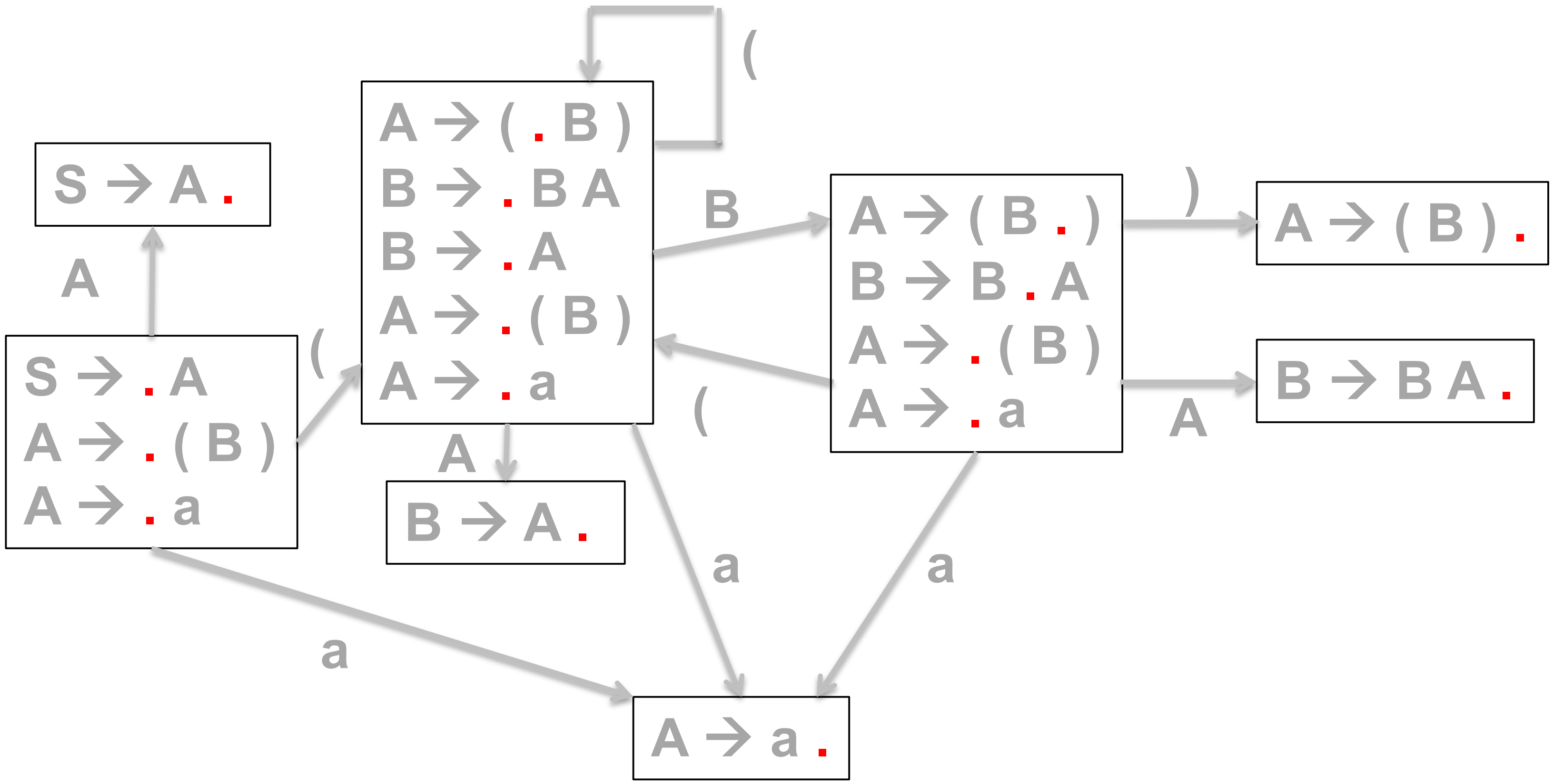

**Shift-Reduce Conflicts when input token is a**

S → C C

C → c C | d

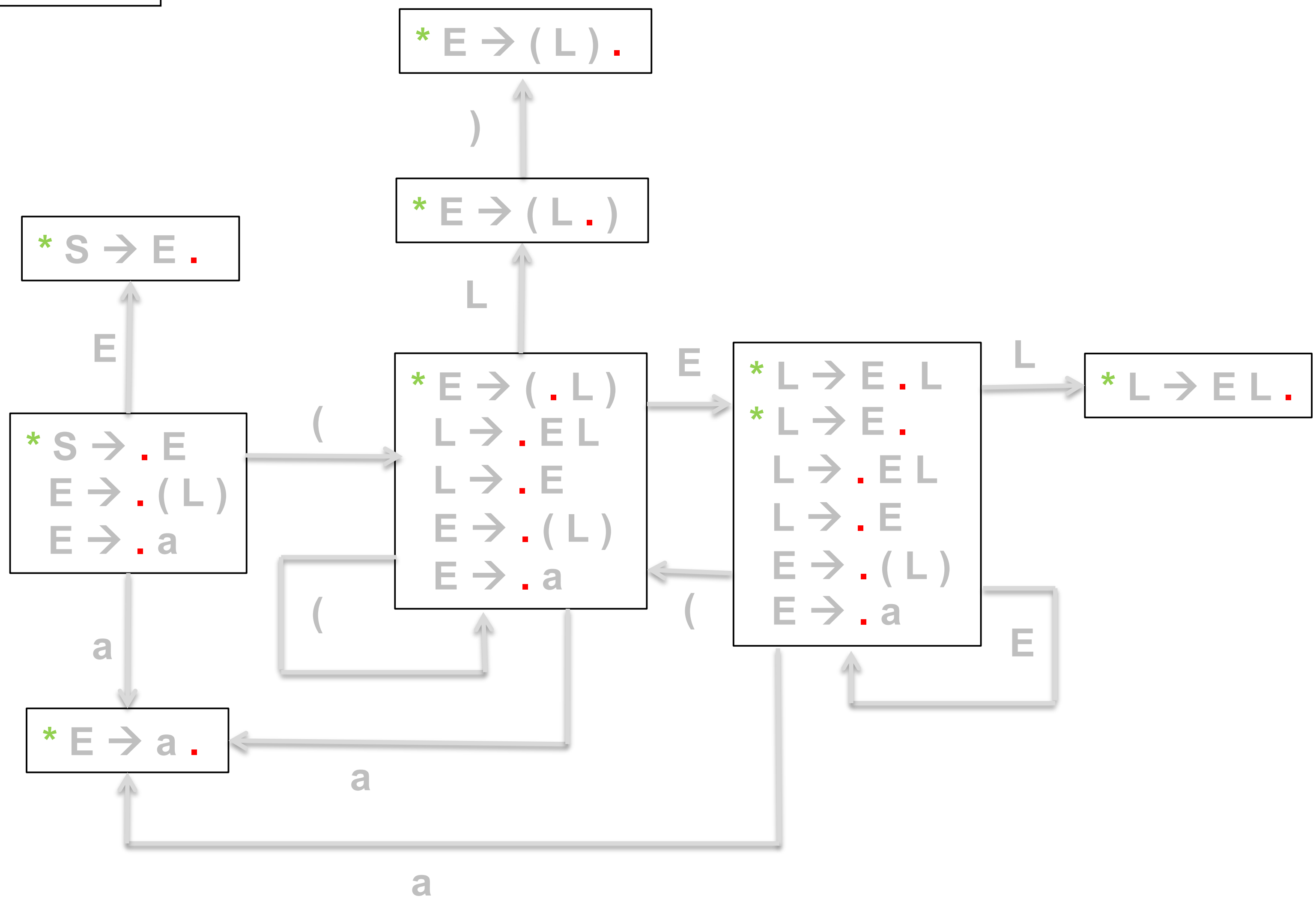

* S → C . C, $
C → . c C, $
C → . d, $
* S → C C ., $
* C → c . C, $
C → . c C, $
C → . d, $
* C → d ., $
* C → c C ., $
* S → . C C, $
C → . c C, c/d
C → . d, c/d
* C → c . C, c/d
C → . c C, c/d
C → . d, c/d
* C → c C ., c/d
* C → d ., c/d
C
c
d

S → A
A → ( B ) | a
B → B A | A



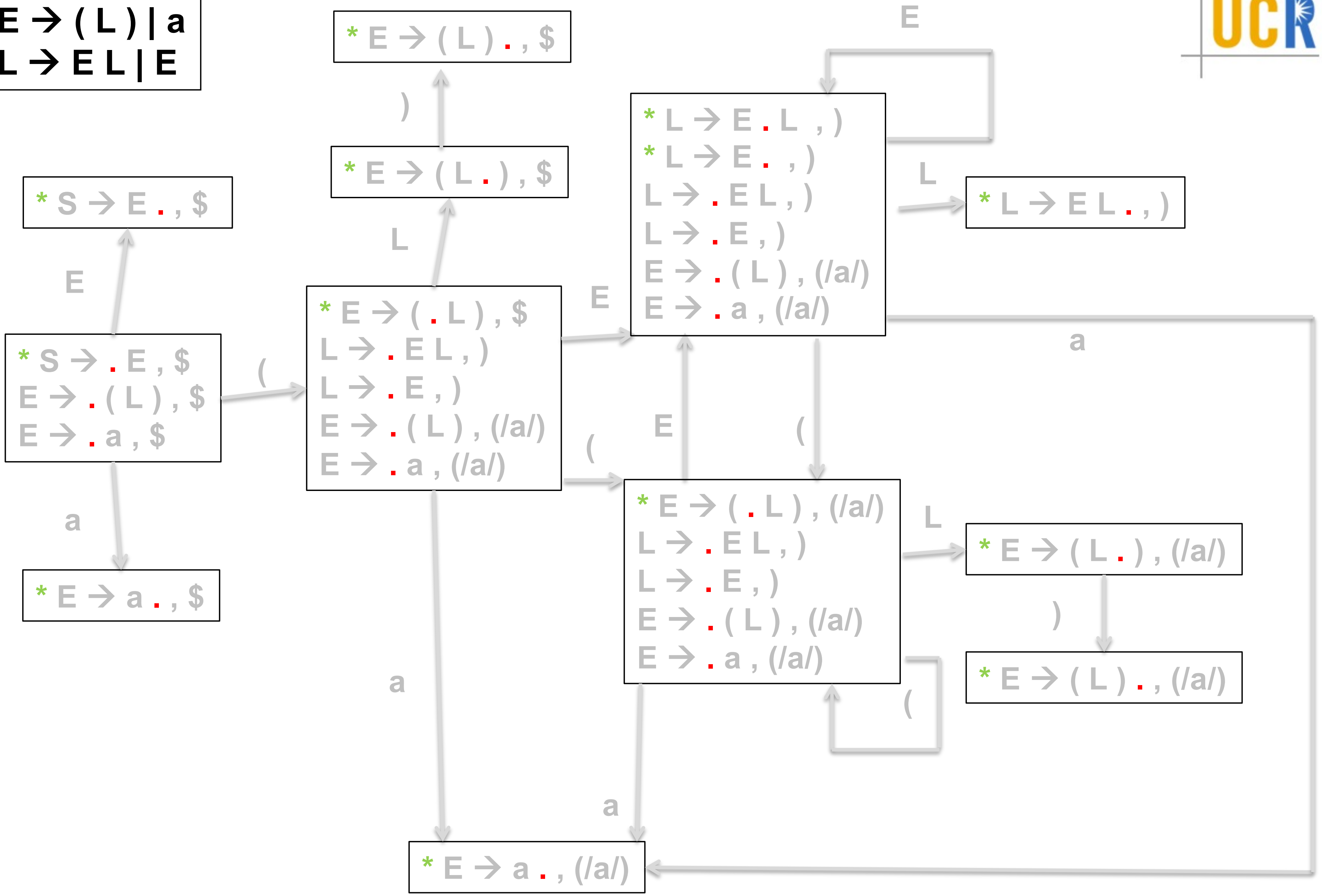

**No Conflicts!**

S → E
E → ( L ) | a
L → E L | E



* E → ( L ) .
)
* E → ( L . )
L
* S → E .
E
* S → . E
E → . ( L )
E → . a
(
* E → ( . L )
L → . E L
L → . E
E → . ( L )
E → . a
E
* L → E . L
* L → E .
L → . E L
L → . E
E → . ( L )
E → . a
L
* L → E L .
(
(
E
a
* E → a .
a
a

S → E
E → ( L ) | a
L → E L | E



* S → E . , $
* S → . E , $
E → . ( L ) , $
E → . a , $
* E → a . , $
* E → ( L ) . , $
* E → ( L . ) , $
* E → ( . L ) , $
L → . E L , )
L → . E , )
E → . ( L ) , (/a/)
E → . a , (/a/)
* L → E . L , )
* L → E . , )
L → . E L , )
L → . E , )
E → . ( L ) , (/a/)
E → . a , (/a/)
* L → E L . , )
* E → ( . L ) , (/a/)
L → . E L , )
L → . E , )
E → . ( L ) , (/a/)
E → . a , (/a/)
* E → ( L . ) , (/a/)
* E → ( L ) . , (/a/)
* E → a . , (/a/)
E
a
(
L
)