Review

- Logic
  - Propositional calculus: variables, boolean operations, truth tables, tautologies, De Morgan's Laws, distributive laws
  - Predicate calculus: predicates, quantifiers
- Sets
  - Notation (braces), how to specify/define sets (enumeration, using a predicate, ...)
  - Operations on sets: union, intersections, difference, complement, Cartesian product
  - Empty set
  - Infinite sets: countable vs non-countable
- Sequences
  - What's the difference between a sequence and a set?
  - Notation
- Relations
  - What is a relation?
  - Properties of relations (reflexive, symmetric, transitive, anti-symmetric)
  - Partial orders (posets), Hasse diagrams, topological sorting
  - Equivalence relations and equivalence classes
- Basic counting
  - Addition rule (union of disjoint sets), multiplication rule (for independent choices)
  - Subsets
  - Permutations
  - Functions
  - k-permutations
  - k-element subsets (combinations)
  - Basic probability
- Some important functions
  - Polynomials
  - Exponential functions
  - Logarithmic functions
- Some important numbers
  - Euler number e
  - π, circumference/diameter ratio for a circle
  - φ, golden ratio
- Important sequences and summation formulas
  - Finite arithmetic sequences
  - Finite geometric sequences
  - Infinite geometric sequences, Zeno's paradox
  - Harmonic numbers, $H_n = 1+1/2+1/3+ \ldots + 1/n$
  - Fibonacci numbers, $F_0 = F_1 = 1$, $F_n = F_{n-1} + F_{n-2}$
- Number theory basics
  - prime and composite numbers
  - prime factors, factorization
  - greatest common divisor (gcd)
  - least common multiple (lcm)
- Algebra
  - Solving quadratic equations
  - Solving polynomial equations with integer roots
  - Factoring polynomials
  - Solving systems of linear equations
  - Vectors, matrices, and operations on them
- What are proofs, and why do we care?
- Proofs for some summation formulas and bounds:
  - mathematical induction

        - sum of an arithmetic sequence, $1+2+ \ldots +n = n(n+1)/2$
        - sum of a finite geometric sequence, $1+2+\ldots+2^n = 2^{n+1}-1$
        - estimate for Fibonacci numbers: $1.5^n \leq F_n \leq 2^n$, for $n \geq 2$
    - sum of infinite geometric sequence: $1+x+x^2+ \ldots = 1/(1-x)$, for $0 < x < 1$
    - estimating Harmonic numbers: $(\log(n)-1)/2 \leq H_n \leq \log(n)+1$
- Other examples of proofs
    - If R is an equivalence relation on a set X, then the equivalence classes of R form a partition of X into disjoint subsets
    - Each finite poset has a topological sort (linear extension)
    - relations involving binomial coefficients, for example (n choose k) = (n-1 choose k) + (n-1 choose k-1)