Syllabus for CS111 Quiz 1

The entrance quiz will focus mostly on the topics from CS/MATH 11, but one question from algebra is possible too.

Topics:

- Logic. Prepositional and predicate calculi, conjunction, disjunction, negation, DeMorgan Laws, quantifiers.
  Examples:
  1. Negate the following sentence: "For each integer x, there is an integer y, such that for each integer z, $2xy - zx + 2 = 0$."
  2. Are the two statements below equivalent?
     - "It is not true that for each x, if x is omnilicious then x is bulganimic"
     - "There is an x that is omnilicious and not bulganimic"
- Sets. Notations for sets. Set operations (union, intersection, complement, difference), finite and infinite sets, countable sets and uncountable sets. Operations on sets: union, intersection, the power set, Cartesian products, other.
  Examples:
  1. List all elements of the Cartesian product of $X = \{a,c,x\}$ and $Y = \{b,z\}$.
  2. Let N denote the set of natural numbers. Give a 1-1 function between NxN and N.
- Functions. Functions onto and 1-1. The inverse and composition of functions.
- Relations. Properties of relations (reflexive, transitive, symmetric, anti-symmetric). Equivalence relations and equivalence classes. Partial orders.
  Examples:
  1. Let X be the set of integers 3,4,...,13. Define relation R on X as follows: xRy iff $x^2 = y^2 \pmod 3$. Prove that X is an equivalence relation and give its equivalence classes.
  2. Define relation R on the set of natural numbers as follows: xRy iff each each prime factor of x is a factor of y. Prove that X is a partial order.
- Basic algebra: solving quadratic equations, solving equations of degree 3 and higher (by guessing integral roots), solving systems of linear equations, matrices, matrix multiplication, determinants.
  Examples:
  1. Solve $x^2 + 3x + 4 = 0$
  2. Solve $x^3 -3x^2 + 4x -2 = 0$
  3. Find x,y such that $3x+2y = 7$ and $2x- y = -2$
  4. Find x,y such that $2x+y = 3$ and $x^2 + y + 1 = 0$
- Proof methods (induction, contradiction).
  Examples:
  1. Prove by induction that an n-element set has 2^n subsets.
  2. Prove that $1+2+...+n = n(n+1)/2$.
  3. Prove that $1^2+2^2+...+n^2 = n(n+1)(2n+1)/6$.

- Basic counting : permutations, combinations, subsets, functions, identities involving binomial expressions
  Examples:
  1. In how many ways we can order a set of 6 elements?
  2. There are 10 students in class. We choose 3 of them. In how many ways this can be done?
  3. There are 5 students in class. Each will be assigned a grade of A, B or C. In how many ways this can be done?
- Summation formulas, computing closed forms, arithmetic and geometric sums.
  Examples:
  1. What is the sum of 1+2+ ... + n?
  2. What is the sum of n + (n+1) + (n+2) + ... + 2n?
  3. What is the sum $1+ 3 + 3^2 + ... + 3^n$?
  4. What is the sum of 1 + 1/3 + 1/9 + 1/27 + ... ?
- Elementary number theory: prime numbers, factorization, relatively prime numbers, greatest common divisor, least common multiple.
  Examples:
  1. Compute the factorization of 5462.
  2. What is the greatest common divisor of 459 and 931?
