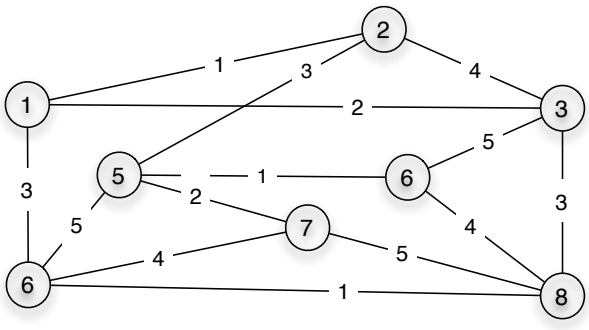
CS/MATH111 ASSIGNMENT 5

Problem 1: An *edge coloring* of a graph is an assignment of colors to edges such that any two edges that share an endpoint have different colors.

Here is an example of an edge coloring of a graph with 5 colors (colors represented by numbers):



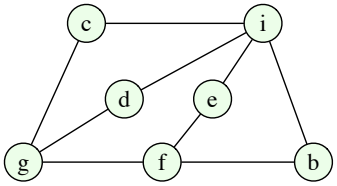
For the graph above, find (show on the graph) an edge coloring with at most 4 colors. The minimum required number of colors for the edges of a given graph is called the chromatic index of the graph. Is 4 the chromatic index of G?

(b) Let D denote the maximum vertex degree in a graph G. Prove that if $D \geq 1$ then G can be edge-colored with at most $2D - 1$ colors (the chromatic index of G is at most $2D - 1$). You need to give a direct proof, without using any results from the literature. Make your argument complete and rigorous.

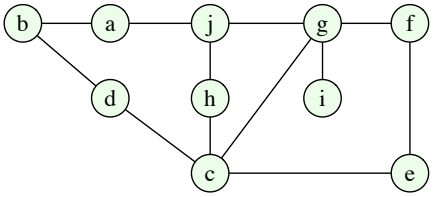
Hint: In class you proved that each graph of maximum vertex degree D has a vertex coloring with at most $D + 1$ colors. Follow the reasoning from that proof, modifying it slightly to work for coloring edges.

Problem 2: Let G and H be the graphs below. For each graph, determine whether it is bipartite. If the graph is bipartite, determine whether it has a perfect matching. Justify your answer.

Graph G:



Graph H:

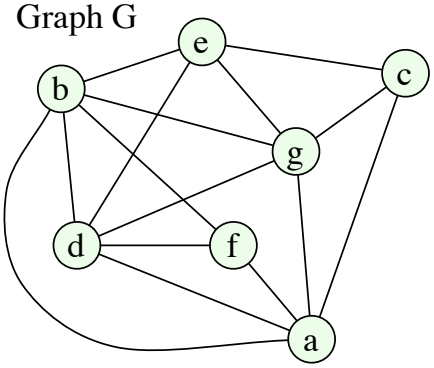


Problem 3: Let G be the Petersen graph. Answer questions (a-i) below. Justify and/or illustrate each answer.

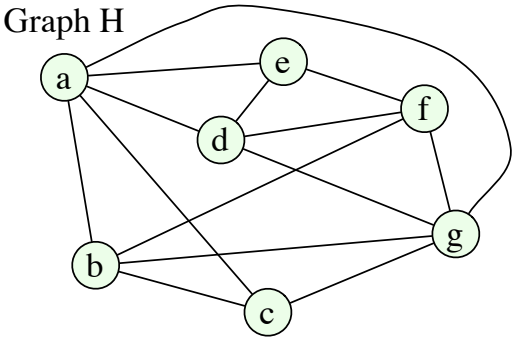
(a) Draw G.
(b) Does G have a Hamiltonian Path? Does G have a Hamiltonian Cycle?
(c) Does G have an Euler Tour?
(d) What is the chromatic number of G?
(e) What is the chromatic index of G?
(f) Is G bipartite? If it is, does it have a perfect matching?
(g) Show that the Petersen graph contains a minor isomorphic to the $K_{3,3}$ graph.
(h) Show that the Petersen graph contains a minor isomorphic to the K_5 graph.
(i) Is G a planar graph?

Problem 4: Determine which of the following two graphs is/are planar/nonplanar. Justify your answer. (You need to either show a planar embedding or use Kuratowski's theorem.)

Graph G



Graph H



Submission. To submit the homework, you need to upload the pdf file to Gradescope. Pictures should be imported into LaTeX in pdf or eps (see the source file to get an idea of how to do that). You can draw them in any drawing software and export in pdf, or draw by hand and scan.