CS/MATH111 ASSIGNMENT 4

Problem 1: Give the asymptotic value (using the Θ -notation) for the number of letters that will be printed by the algorithms below. In each algorithm the argument n is a positive integer. Your solution needs to consist of an appropriate recurrence equation and its solution. You also need to give a brief justification for the recurrence.

Part (a)

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(i) Algorithm PRINTAS (n : integer)
           if n < 4
                  print("A")
            else
                  \operatorname{PRINTAs}(\lceil n/3 \rceil)
                  \operatorname{PRINTAs}(\lceil n/3 \rceil)
                  \operatorname{PRINTAs}([n/3])
                  \operatorname{PRINTAs}(\lceil n/3 \rceil)
                  \operatorname{PRINTAs}(\lceil n/3 \rceil)
                  for i \leftarrow 1 to 4n^2 do print("A")
(ii) Algorithm PRINTBS (n : integer)
             if n < 2
                   print("B")
             else
                   for j \leftarrow 1 to 8 do PRINTBS(|n/2|)
                   for i \leftarrow 1 to 10n^3 do print("B")
(iii) Algorithm PRINTCS (n : integer)
             if n < 3
                   print("C")
             else
                    \operatorname{PrintCs}(\lceil n/2 \rceil)
                    \operatorname{PrintCs}(\lceil n/2 \rceil)
                    \operatorname{PrintCs}(\lceil n/2 \rceil)
                    \operatorname{PrintCs}(\lceil n/2 \rceil)
                    for i \leftarrow 1 to 10 do print("C")
Part (b)
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(iv) Algorithm PRINTDS (n: integer)

if n < 2

print("D")

else

for j \leftarrow 1 to 4 do PRINTDS(\lfloor n/2 \rfloor)

for i \leftarrow 1 to 10n^k do print("D"),
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where k is a nonnegative integer.

Problem 2: Bill is buying his wife a bouquet of carnations, roses, tulips, daises, and lilies. The bouquet will have 30 flowers, with

- at most 2 carnations,
- between 3 and 7 roses,
- between 2 and 10 tulips,
- at least 3 daises, and
- at least 5 lilies.

How many different combinations of flowers satisfy these requirements? You need to use the counting method for integer partitions and show your work.

Problem 3: We have three sets P, Q, R with the following properties:

- (a) |P| = 3|Q| and |R| = |P|,
- (b) $|P \cap Q| = 11$, $|Q \cap R| = 14$, $|P \cap R| = 2|Q|$,
- (c) $7 \leq |P \cap Q \cap R| \leq 17$,
- (d) $|P \cup Q \cup R| = 100.$

Use the inclusion-exclusion principle to determine the number of elements in P. Show your work. (Hint: You may get an equation with two unknowns, but one of them has only a few possible values.)

Submission. To submit the homework, you need to upload the pdf file into Gradescope.