CS111 ASSIGNMENT 3

Problem 1: We want to tile an $n \times 1$ strip with 1×1 tiles that are green (G), blue (B), and red (R), 2×1 purple (P) and 2×1 orange (O) tiles. Green and blue tiles cannot be next to each other, and no three green or blue tiles in a row are allowed (no GB, BG, GGG, and BBB). Give a formula for the number of such tilings. Your solution must include a recurrence equation (with initial conditions!), and a full justification. You do not need to solve it.

Problem 2: Solve the following recurrence equations:

a)

$$\begin{aligned} f_n &= f_{n-1} + 4f_{n-2} + 2f_{n-3} \\ f_0 &= 0 \\ f_1 &= 1 \\ f_2 &= 4 \end{aligned}$$

Show your work (all steps: the characteristic polynomial and its roots, the general solution, using the initial conditions to compute the final solution.)

b)

$$\begin{aligned} f_n &= f_{n-1} + 4f_{n-2} + 2f_{n-3} + 2n \\ f_0 &= 0 \\ f_1 &= 1 \\ f_2 &= 4 \end{aligned}$$

Show your work (all steps: the associated homogeneous equation, the characteristic polynomial and its roots, the general solution of the homogeneous equation, computing a particular solution, the general solution of the non-homogeneous equation, using the initial conditions to compute the final solution.) You can use your work from (a).

c)

$$\begin{aligned} t_n &= 2t_{n-1} + t_{n-2} + 2^n \\ t_0 &= 0 \\ t_1 &= 2 \end{aligned}$$

Show your work (all steps: the associated homogeneous equation, the characteristic polynomial and its roots, the general solution of the homogeneous equation, computing a particular solution, the general solution of the non-homogeneous equation, using the initial conditions to compute the final solution.)

Problem 3: (a) Let T_n denote the number of moves needed to solve the Tower of Hanoi problem with n discs. (a) Set up (and justify) a recurrence relation for the sequence T_n and solve it. (b) Use mathematical induction to verify the formula obtained in (a).

Submission. To submit the homework, you need to upload the pdf file into Gradescope.