CS/MATH111 ASSIGNMENT 4

Problem 1: a) Give the asymptotic value (using the Θ-notation) for the number of letters that will be printed by the algorithms below. In each algorithm the argument n is a positive integer. Your solution needs to consist of an appropriate recurrence equation and its solution. You also need to give a brief justification for the recurrence (at most 10 words each).

(i) **Algorithm** PrintAs (n : **integer**)

```
if n < 4
    print("A")
else
    PrintAs(⌈n/3⌉)
    PrintAs(⌈n/3⌉)
    PrintAs(⌈n/3⌉)
    PrintAs(⌈n/3⌉)
    PrintAs(⌈n/3⌉)
    for i ← 1 to 4 do print("A")
```

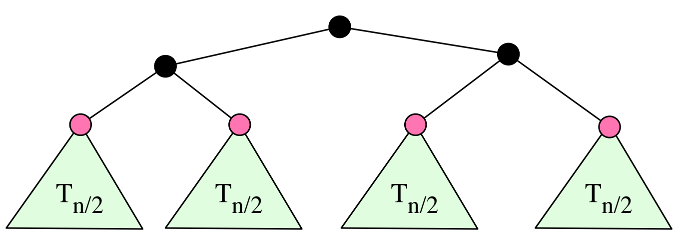
(ii) **Algorithm** PrintBs (n : **integer**)

```
if n < 2
    print("B")
else
    for j ← 1 to 6 do PrintBs(⌊n/2⌋)
    for i ← 1 to 10n^3 do print("B")
```

(iii) **Algorithm** PrintCs (n : **integer**)

```
if n < 3
    print("C")
else
    PrintCs(⌈n/2⌉)
    PrintCs(⌈n/2⌉)
    PrintCs(⌈n/2⌉)
    PrintCs(⌈n/2⌉)
    for i ← 1 to 20n^2 do print("C")
```

b) For each integer $n \geq 1$ we define a tree T_n, recursively, as follows. For $n = 1$, T_1 is a single node. For $n > 1$, T_n is obtained from four copies of $T_{\lceil n/2 \rceil}$ and three additional nodes, by connecting them as follows:



(In this figure, the subtrees are denoted $T_{n/2}$, without rounding, to reduce clutter.) Let $h(n)$ be the number of nodes in T_n. Give a recurrence equation for $h(n)$ and justify it. Then give the solution of this recurrence using the Θ() notation.

Problem 2: a) Bill is buying his wife a bouquet of daises, carnations, roses and tulips. The bouquet will have 25 flowers, with

- between 1 and 7 daises,
- between 2 and 11 carnations,
- at least 4 roses, and
- at most 6 tulips.

How many different combinations of flowers satisfy these requirements? You need to use the counting method for integer partitions and show your work.

b) We have three sets P, Q, R with the following properties:

(a) $|Q| = 2|P|$ and $|R| = 4|P|$,

(b) $|P \cap Q| = 11$, $|P \cap R| = 7$, $|Q \cap R| = 10$,

(c) $1 \leq |P \cap Q \cap R| \leq 11$,

(d) $|P \cup Q \cup R| = 121$.

Use the inclusion-exclusion principle to determine the number of elements in P. Show your work. (Hint: You may get an equation with two unknowns, but one of them has only a few possible values.)

Submission. To submit the homework, you need to upload the pdf file into Gradescope and iLearn .