## CS/MATH111 ASSIGNMENT 3

**Problem 1:** Strings of length n are composed of the following strings: 1, 22, 23, 32, 33, 445 and 544. Let  $S_n$  be the number of strings of length n that can be formed in this way. For example, for n = 3, we can form the following strings:

111, 122, 123, 132, 133, 221, 231, 321, 331, 445, 544

and thus  $S_3 = 11$ . (Note that  $S_0 = 1$ , because the empty string satisfies the condition.)

(a) Derive a recurrence relation for the numbers  $S_n$ . Justify it.

(b) **Extra credit**. Let  $P_n$  be the number of strings of length n that can be formed from the given strings, considering that four 1's cannot be next to each other. (The substring 1111 is not allowed.) Derive a recurrence relation for the numbers  $P_n$ . Justify it.

**Problem 2:** Solve the following recurrence equation:

$$S_n = S_{n-1} + 4S_{n-2} + 2S_{n-3}$$
  
 $S_0 = 1$   
 $S_1 = 1$   
 $S_2 = 5$ 

Show your work (all steps: the characteristic polynomial and its roots, the general solution, using the initial conditions to compute the final solution.)

**Problem 3:** Solve the following recurrence equation:

$$D_{n} = 4D_{n-1} - 8D_{n-3} + 2n + 3$$
$$D_{0} = 0$$
$$D_{1} = 1$$
$$D_{2} = 1$$

Show your work (all steps: the associated homogeneous equation, the characteristic polynomial and its roots, the general solution of the homogeneous equation, computing a particular solution, the general solution of the non-homogeneous equation, using the initial conditions to compute the final solution.)

**Problem 4:** Solve the following recurrence equation:

$$A_n = A_{n-1} + 2A_{n-2} + 3^n$$
  

$$A_0 = 0$$
  

$$A_1 = 4$$

Show your work (all steps: the associated homogeneous equation, the characteristic polynomial and its roots, the general solution of the homogeneous equation, computing a particular solution, the general solution of the non-homogeneous equation, using the initial conditions to compute the final solution.)

Submission. To submit the homework, you need to upload the pdf file into gradescope and iLearn.